Concurrent Trajectory and Vehicle Optimization for an Orbit Transfer

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Presentation Overview

- Motivation
- Single Objective Optimization
  - Problem Description
  - Mathematical Formulation
  - Design Variable Bounds
  - Verification
- Multi-Objective Optimization
  - Convergence
  - Sensitivity Analysis
- Vehicle Selection
  - Problem Formulation
  - Optimization
  - Sensitivity Analysis
- Future Work
Motivation

- Traditionally, orbit transfers are optimized with respect to a selected vehicle.

- Competing objectives of initial mass and time of flight are weighted by the preference of the customer.
  - High priority $\rightarrow$ minimize time of flight
  - Low priority $\rightarrow$ minimize initial mass

- Vehicle selection is as important as the trajectory.
  - The choice of vehicle drastically impacts both performance and cost.
  - Differing priorities would impact vehicle choice.

- Evaluate both the trajectory and vehicle selection for different preferences to show ‘optimal’ orbit transfer configurations.
Consider a co-planar orbit transfer from low Earth orbit (LEO) to geosynchronous Earth orbit (GEO) using a two-stage chemical rocket.

Minimize the initial mass of the system for a given payload mass.

Define three design variables:
- Transfer angle ($\nu$): Angle between first and second burn
- Specific impulse of first engine ($I_{sp_1}$)
- Specific impulse of second engine ($I_{sp_2}$)

Parameters:
- Payload mass ($m_p$) = 1000 kg
- Structural factor ($\alpha$) = 0.1 for both engines
- Initial radius ($r_0$) = 6628 km (250 km altitude)
- Final radius ($r_f$) = 42378 km (36000 km altitude)

Define two disciplinary models:
- Orbit transfer calculation: Assumes first burn is tangent to initial orbit
  - Input: the transfer angle, and the initial and final radii
  - Output: $\Delta V$ of each burn and time of flight
- Rocket equation: Assumes each burn is impulsive
  - Input: the $\Delta V$ of each burn, the specific impulse of each engine, the structural factor, and the payload mass
  - Output: the initial mass
Mathematical Problem Formulation

- Minimize $J(x) = m_i$

- Subject to the disciplinary model equations
  - Orbit transfer Equations
  - Rocket equation

- And subject to the variable bounds
  - $135 \text{ deg} \leq \nu \leq 180 \text{ deg}$
  - $300 \text{ sec} \leq I_{sp_1}, I_{sp_2} \leq 450 \text{ sec}$

Rocket Equation for Two stages

$$\frac{m_f}{m_i} = \Pi_{i=1}^2 \left(1 + \alpha_i\right) \exp \left(\frac{-\Delta V_i}{I_{sp_i} g_0}\right) - \alpha_i$$

Orbit Equations for One Tangent Burn

$$v_0 = \sqrt{\frac{\mu}{r_0}} \quad v_{TA} = \sqrt{\mu \left(\frac{2}{r_0} - \frac{1}{a_T}\right)}$$

$$v_f = \sqrt{\frac{\mu}{r_f}} \quad v_{TB} = \sqrt{\mu \left(\frac{2}{r_f} - \frac{1}{a_T}\right)}$$

$$e_T = \frac{r_0}{r_f - \frac{\mu}{v_f}} \quad \phi_T = \tan^{-1} \left(\frac{e_T \sin \nu}{1 + e_T \cos \nu}\right)$$

$$a_t = \frac{r_0}{1 - e_T}$$

$$\Delta V_A = \|v_{TA} - v_0\|$$

$$\Delta V_B = \sqrt{v_{TB}^2 + v_f^2 - v_{TB} v_f \cos \phi_T}$$

$$\Delta V_{Total} = \Delta V_A + \Delta V_B$$

$$\cos E = \frac{e_T + \cos \nu}{1 + e_T \cos \nu}$$

$$\sin E = \sqrt{1 - e_T^2} \frac{\sin \nu}{1 + e_T \cos \nu}$$

$$E = \tan^{-1} \left(\frac{\sin E}{\cos E}\right)$$

$$time = \sqrt{\left(\frac{\alpha f^2}{\mu}\right)} \left(2k\pi + E - e_T \sin E\right)$$
Transfer Angle Bounds

Initial Mass vs. Delta V for an Isp of 450 sec

Initial mass goes negative as $\Delta V$ becomes large

Time of flight vs. transfer angle

Time of flight goes to infinity as $\nu$ approaches 133 degrees
Model Verification

- Minimum initial mass solution is a Hohmann transfer
  - Transfer angle = 180 deg
  - Specific Impulse = 450 sec
  - Time of flight is half the transfer orbital period

- Using a SQP method, from any initial guess, model is verified

- Initial mass = 2721.2 kg

- Time of flight = 19086 sec (5.3 hours)
Multi-Objective Optimization

- The two objective to be minimized are initial mass and time of flight
- Scale each objective to be non-dimensional and approximately the same order of magnitude
  - Scale factor for mass is the payload mass (1000 kg)
  - Scale factor for time of flight is period of initial orbit (5370 sec)
  \[
  J_1 = \frac{m_i}{m_p}
  \]
  \[
  J_2 = \frac{\text{time}}{P_0}
  \]
- Use weighted sum approach
  \[
  J = \lambda J_1 + (1-\lambda)J_2
  \]

Initial mass vs. Time of flight

Minimum time solution*

Equal weights

Minimum initial mass solution

Utopia point

* Indicates that the solution is independent of engine selection

Minimum time of flight solution:
- \( \nu = 135 \) deg
- Time of flight = 8078.5 sec (2.2 hrs)
- Initial mass = 6598 kg
Comparison of SQP and SA Convergence

Convergence time for SQP = 0.32 sec
SQP parameters: objective function tolerance = 10^{-7}

Convergence time for SA = 33.48 sec
SA parameters: exponential cooling schedule,
\(dT = 0.75\), neq = 50, nfrozen = 40

\(\nu = 3.14\) deg
Isp\(_{1}\) = 450 sec
Isp\(_{2}\) = 450 sec
\(m_i = 2721\) kg

\(\nu = 3.14\) deg
Isp\(_{1}\) = 449.1 sec
Isp\(_{2}\) = 449.7 sec
\(m_i = 2725\) kg
Sensitivity Analysis

- **Sensitivity to design variables**
  - Minimum initial mass is most sensitive to specific impulse of first engine:
    
    \[
    \nabla J(x^+) = \begin{bmatrix}
    -3.3310^{-8} \text{kg/ rad} \\
    -1.4610^{-3} \text{kg/ sec} \\
    -8.3810^{-4} \text{kg/ sec}
    \end{bmatrix}
    \]

    \[
    \nabla J(x^+) = \begin{bmatrix}
    -3.810^{-11} \\
    -2.4110^{-4} \\
    -1.3810^{-4}
    \end{bmatrix}
    \]

  - Minimum time of flight is only sensitive to transfer angle
    
    \[
    \nabla J(x^+) = \begin{bmatrix}
    1.519 \text{kg/ rad} \\
    0 \text{kg/ sec} \\
    0 \text{kg/ sec}
    \end{bmatrix}
    \]

    \[
    \nabla J(x^+) = \begin{bmatrix}
    5.4210^{-4} \\
    0 \\
    0
    \end{bmatrix}
    \]

- **Sensitivity to Scale Factors**

As the scale factor for initial mass increases, the magnitude of $J_1$ decreases.

![Initial mass vs. Time of flight](image)

- **Minimum time of flight**
- **Equal weights**
- **Utopia point**
- **Minimum initial mass solution**
Vehicle Selection

- Define three design variables
  - Transfer angle ($\nu$)
  - Engine for first and second burns
- Select one of three different engines for each burn
  - Engine A: $\alpha = 0.08$, Isp = 300 sec
  - Engine B: $\alpha = 0.10$, Isp = 400 sec
  - Engine C: $\alpha = 0.12$, Isp = 450 sec
- Use SA, determine Pareto front
  - Engine C is chosen for both engines and for each set of weights, except $\lambda = 0$

* Indicates that the solution is independent of engine selection
Sensitivity to Engine Change

Pareto front

Minimum time solution
First stage uses Engine A
Second stage uses Engine B

First stage uses Engine B
Second stage uses Engine B

First stage uses Engine C
Second stage uses Engine C

Choice of Engines vs. Varying Structure Ratio for Minimum Initial Mass

Switch point from Engine C to Engine B

Initial mass (kg) vs. Time of flight (sec)

Engine C: $\alpha = 0.148$, $I_{sp} = 425$ sec
Conclusions and Future Work

- Global Optimality?
  - Single objective optimization
  - Multi objective optimization
  - Discrete engine selection
- Expand the model to include different types of engines
  - Allow for low thrust, high impulse engines
  - Modify model to include constant thrust trajectories
- Analyze more complicated problems
  - Allow for out of plane maneuvers
  - Examine different origin/destination pairs
- Examine the relationship between scaling factors and the Pareto front