Methodology for Combined Integration of Electric Vehicles and Distributed Resources into the Electric Grid

by

Samantha Joellyn Gunter

B.S., University of Illinois at Urbana-Champaign (2009)

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2011

Signature of Author ....
Department of Electrical Engineering and Computer Science
September 2, 2011

Certified by ....
David J. Perreault
Professor of Electrical Engineering and Computer Science
Thesis Supervisor

Certified by ....
Khurram K. Afridi
Visiting Associate Professor of Electrical Engineering and Computer Science
Thesis Supervisor

Accepted by ....
Leslie A. Kolodziejski
Professor of Electrical Engineering and Computer Science
Chair, Department Committee on Graduate Students

Methodology for Combined Integration of Electric Vehicles and Distributed Resources into the Electric Grid

by

Samantha Joellyn Gunter

Submitted to the Department of Electrical Engineering and Computer Science
on September 2, 2011, in partial fulfillment of the
requirements for the degree of
Master of Science

Abstract

Plug-in electric vehicles and distributed generation are expected to appear in growing numbers over the next few decades. Large scale unregulated penetration of plug-in electric vehicles and distributed generation can each have detrimental impact on the existing electric grid infrastructure. However, appropriate pairing of the two technologies along with some storage could mitigate their individual negative impacts.

This thesis develops a methodology and an optimization tool for the design of grid connected electric vehicle chargers that integrate distributed generation and storage into a single system. The optimization tool is based on a linear programming approach that identifies designs with the minimum system lifecycle cost. The thesis also develops the component and system cost models needed for this optimization. The tool can handle single and multiple charger systems with centralized or distributed generation and storage. To verify the tool's accuracy, a search-based optimization technique that works for single chargers with centralized generation and storage is also developed and used to validate the tool.

To demonstrate the usefulness of the optimization tool, it is used to design optimal architectures for a single-charger residential charging case and a multi-charger public charging case. It is shown that designs that draw the maximum available power from the grid have the lowest 20-year system lifecycle cost. When storage is needed because the grid cannot provide full charging power, optimal designs may or may not include solar PV based distributed generation depending on the location. For example, in locations with solar irradiation profiles like Los Angeles, CA, electric vehicle charger designs that include solar PV generation are optimal, while in locations like Eugene, OR, optimal designs do not include solar PV. It is also shown that with the available technology, wind turbines are not cost effective for use in residential chargers in locations with wind speeds similar to Los Angeles, CA and Boulder, CO. For the multi-charger public charging case, designs with centralized storage and generation are optimal.

Thesis Supervisor: David J. Perreault
Title: Professor of Electrical Engineering and Computer Science

Thesis Supervisor: Khurram K. Afridi
Title: Visiting Associate Professor of Electrical Engineering and Computer Science

Acknowledgments

It is almost difficult to express the gratitude I have for everyone who made this thesis possible. Not because it's lacking but rather because it's quite overwhelming. First and foremost, I would like to thank my advisors, David Perreault and Khurram Afridi, who have diligently guided me. You make a great team and I am glad I have you to help me every step of the way. I am extremely grateful for the research opportunity and the learning experience that you have provided for me. I can only hope that it has been as enjoyable for you as it was for me.

I also wish to acknowledge the guidance and support of Dr. Sindhu Suresh, Yaosuo Xue, Dr. Gerd Griepentrog and John Hurley of Siemens Corporate Technology; and Dr. Madhav Manjrekar and Paul Camuti of Siemens Smart Grid. I enjoyed our interactions and the insight you provided as I progressed through the development of this thesis. I eagerly look forward to future interactions.

I would like to thank Professor Salal Humair of Harvard University for valuable discussions regarding linear programming, and Ali Afridi of Lexington High School for help with processing the solar irradiation and wind speed data. Each element was vital in completing this thesis.

To my friends in LEES, you guys are the best. There is no other place I would rather be at MIT. Thank you, Andy, Minjie, and Richard for all the late nights, delicious lunches, study breaks and thoughtful discussions.

I'd like my friends in Boston to know that you put a smile on my face every day. It's not every lifetime that someone is blessed to know such an amazing, diverse group of people. I will never forget getting our fingers dirty at wings on Mondays, bowling for free on Tuesdays, trying to name that tune at trivia on Wednesdays and doing everything else possible the rest of the week. Thank you, Michael, Ian, Phil, Nick H., Dave, Seoyoung, Stephen, Aaron, Morgan, Nick L., Veronika, Chris, and Janet.

To my boyfriend, Justin Miller, thank you for being there literally every step of the way. I am not quite sure I would have ever finished without your love, support and knack for adventure. Thank you for helping me through my thesis and through my life.

I want to thank my parents and family. I love you all more than you could ever know. Thank you for always supporting me and believing in me. I am so very fortunate to have you in my life. How can I thank you enough.

Last, but certainly not least, I want to thank God.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Growing concern for climate change and energy security has renewed interest in Plug-in Hybrid Electric Vehicles (PHEVs) and Electric Vehicles (EVs), collectively referred to as Plug-in Electric Vehicles (PEVs). PEVs can offer benefits in terms of improved air quality and provide an opportunity for economic growth to regions that take the lead in its technological development. These vehicles connect to the electric grid in order to charge an internal battery which is later used, instead of or in addition to burning gasoline, to propel the vehicle. A shift towards PEVs could substantially reduce the US's dependence on foreign oil since 60% of oil is imported and 71% of the oil is consumed by the transportation sector, including 46% by cars and light trucks [1]. In the US, 27% of greenhouse gas (GHG) emissions due to human activities are caused by the transportation sector, and transportation is the fastest growing source of GHG emissions [1]. Depending on the grid's generation mix, using electricity instead of gasoline may result in substantial reductions in GHG emissions [2]. GHG emission reduction is expected in a number of East and West coast states whose generation mix has low emissions. On the other hand, GHG emissions reduction is not expected in states that depend heavily on coal power plants. When compared to existing hybrid electric vehicles, which do not connect to the grid but still employ an internal battery to make significant improvements in fuel efficiency, PEVs may have an *increase* in GHG emissions depending on the generation mix [3]. The incremental electricity required to charge a PEV should be generated from low emission sources such as nuclear or renewable generation to truly yield the potential emissions benefits often claimed from using PEVs instead of existing vehicles.

Much effort has already been made to integrate renewable energy into the generation mix. There is a strong legislative drive to mandate, or incentivize, large scale integration of renewable energy resources (such as wind and solar) into the electric grid. Twenty-nine US states and the District of Columbia have established renewable portfolio standards (RPS) that mandate, or set voluntary targets, to integrate substantial amount of renewable energy into the grid over the next decade [4]. Some of these RPS goals include carve-outs for distributed generation (DG). This type of generation specifically connects to the distribution system and consists of generation units rated from a few kW to a few MW. The prominent renewable DG that is seen today is rooftop solar photovoltaic (PV) systems although there has been some installation of small-scale wind turbines.

It is anticipated that over the next few decades, both PEVs and renewable DG will become more prominent. The increasing penetration level of each technology alone could present substantial challenges to the existing electric grid infrastructure if not integrated effectively. One way to mitigate the impact of PEV charging on the electric grid, while at the same time helping to reduce negative consequences of connecting DG to the grid, is to combine PEV charging, renewable DG and local storage into a single system. This solution would also help achieve RPS goals and yield the often-discussed emissions benefits of PEVs. This is an opportunity that has not been adequately exploited from a component size optimization standpoint, and is the subject of this thesis. The purpose of this thesis is to develop a methodology for sizing components to meet the charging needs of a PEV. To determine the optimal design out of all possible designs that meet the PEV requirements, a system lifecycle cost model complements the methodology. The design that yields the lowest system lifecycle cost is considered the optimal design in this thesis.

This chapter will first discuss the challenges of interconnecting PEVs and renewable DG to the electric grid which will drive the motivation to pair these two technologies together. Then it will review some of the past work done on PEV charging system optimization as well as existing PEV charging systems with integrated renewable generation. Moving forward, this chapter will present the scope and objectives of this thesis. Finally, it will introduce the content and organization of the chapters that follow.

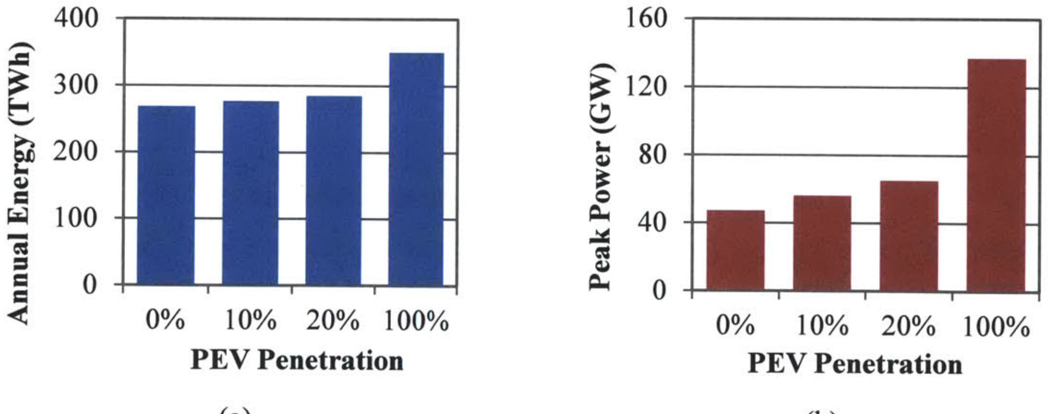
1.1 Motivation

The untraditional nature of PEVs and renewable DG can present many unique benefits as well as challenges to utility system operators. The impact of these technologies, both good and bad, will depend heavily on their penetration level. The next two sections will discuss the expected penetration levels and challenges of PEVs and renewable DG in order to motivate combined systems that may offer reduced system impact.

1.1.1 Plug-in Electric Vehicle Penetration and Impact

A number of automobile manufacturers have already released or plan to release PHEVs and EVs into the market [1], [5]. Although estimates vary, by 2020 roughly 2 million PEVs are expected to be on the road in the US, and this number is expected to climb to 13 million by 2030, representing about 4.5% of the light duty vehicle population [6], [7]. However, penetration is not expected to be uniform across the country. Some west coast utilities expect PEV penetration of around 10% in their service territories by as early as 2020 [8]. Such levels of PEV penetration will require large scale deployment of residential and public chargers. In 2010, the Society of Automotive Engineers (SAE) released an updated standard for PEV chargers that established recommended practices for Level 1 (up to 1.9 kW) and Level 2 (up to 19.2 kW) charging [9]. Although homegrown standards for fast (Level 3) charging are under development, chargers with power ratings of around 60 kW that meet Japanese rapid charging standards are already being piloted in the US [10]. Level 1 and Level 2 charging is expected to be used for residential charging while Level 2 and Level 3 charging is expected to be used in developing a public charging infrastructure [11].

Although the need for large scale deployment of PEV chargers is clear, their impact on the electric grid is less certain. The potential impact of a deployment of PEVs on the electric grid can be seen from the example of California shown in Fig. 1-1. As the penetration of PEVs increases, the additional annual energy demand due to these vehicles marginally increases the total annual electrical energy demand of the state (Fig. 1-1 (a)). Because generation asset capacity is sized to meet peak load demand, generation units are underutilized most of the time. This means that a generation unit might only operate for half of the year when there is enough demand and is available to operate more hours in the year if needed to serve the extra generation requirements of PEVs. This means that the increase in annual energy generated due to even a 100% penetration of PEVs in California is unlikely to require any additions to generation infrastructure due to existing idle capacity. However, the peak power demand of coincident peak PEV charging significantly increases the total peak power demand of the state (Fig. 1-1(b)). Most increases in peak power demand are undesirable as they usually result in costly investments in upgrading the capacity of existing transmission and generation infrastructure that would be used only a few hours each year. Even small, but geographically concentrated, PEV penetration could have substantial disruptive impact on local distribution systems, depending on the power rating of the chargers and the time of day when the vehicles are charged [15]-[17]. Note that a 7.2 kW Level 2 charger draws as much peak power as an entire house or two [18]. This sudden increase in power requirements at the distribution level could easily overload the existing infrastructure, primarily the components closest to the point of interconnection such as pole-mounted transformers.



(a) (b)

Figure 1-1: Impact on California's (a) annual electrical energy demand and (b) peak power demand for varying levels of PEV penetration out of the state's 30 million light duty vehicles. In (a), a 30 mile average daily commute at 4 miles/kWh vehicle efficiency is assumed. In (b), it is assumed that 50% of the vehicles are charging at the same time as the system peak at an average rate of 6 kW. Electricity and vehicle data for 2008 obtained from [12]-[14].

1.1.2 Renewable Distributed Generation Penetration and Impact

Sixteen states and the District of Columbia have an RPS with specific DG and/or solar provisions; compliance can be achieved by both large merchant DG and small residential DG facilities [3]. A few examples of these provisions are listed in Table 1-1. RPS goals and other financial incentives have the potential to greatly influence increasing numbers of DG connected to the distribution system. While an increased penetration of renewable DG may increase societal benefits, especially with respect to emissions, integrating large levels of renewable DG into the grid is challenging as the variable and unpredictable nature of wind and solar make grid operations more difficult [19].

Many of the challenges presented by DG relate to the fluctuation in feeder voltage. For example, this can result from a mismatch in voltage regulation coordination or from variable output power from the DG that occurs with a sudden cloud or a drop in wind speed [19], [20]. When this happens, the feeder voltage level may not comply with industry standards. These challenges can be mitigated through the use of storage at the DG location or along the feeder to buffer the fluctuation in voltage and properly regulate the voltage through appropriate power absorption or injection.

There are also complications associated with the disconnection protocol in use. This protocol was originally motivated by protection of worker safety during system outages [21]. DG is required to disconnect when there is a 12% drop in voltage and reconnection can only occur after five minutes of voltage levels returning to normal. The idea was that the DG will cease to supply power to what should be a deenergized line. This presents two problems: (1) any energy that could have been generated is lost and (2) the disconnection of several DG can occur even if there is no sustained fault on the local system. The first problem is detrimental to the investment made in DG technologies as well as decreases reliability benefits that could have been achieved. The second problem can result in a cascade of disconnecting DG leading to a severe sustained fault throughout a large region [22].

1.2 Past Work

On the commercial front, PEV chargers with renewable DG exist [23]-[26] but there is little indication that the power rating of the DG was optimized to charge a particular PEV. There are publications pertaining to sizing stand-alone renewable DG and/or storage systems to meet general load

Table 1-1: State RPS solar or DG generation requirements as a percentage of annual electricity sales [3].

Year	Arizona (DG)	Colorado (DG)	Illinois (Solar)	Nevada (Solar)
2011	.75%	1.00%	N/A	N/A
2015	1.5%	1.75%	0.27%	1.2%
2020	3.0%	3.00%	0.96%	1.5%
2025	4.5%	3.00%	1.41%	1.5%

requirements [27]-[30] but a lack of publications related to sizing grid-interfaced systems to meet PEV charging requirements. Likewise, there are publications describing the use of PEVs to mitigate impacts of variable renewable DG through the use of its internal battery as a storage unit [31], [32], but the renewable DG is not sized to specifically meet the needs of PEVs. The next two sections discuss the findings of past work and solutions and points out their limitations with respect to component interconnection and/or lack of component size optimization.

1.2.1 Existing PEV Charging Systems with Integrated Renewable Generation

Manufacturers and ecofriendly enthusiasts alike have begun installing charging stations with parking space shading provided by canopies mounted with solar PV panels. The vast majority of these systems are most likely meant to be statements about a particular company's efforts to generate low emission electricity. Often there is no storage associated with these charging stations, meaning that any renewable energy that is generated will only be supplied to a vehicle if it is plugged in. Also, the main purpose for the rooftop on which the solar PV panels are mounted is to provide shading for vehicles, so its dimensions may limit the use of an optimal number of solar PV panels needed to charge a PEV.

In the case of the National Renewable Energy Laboratory's prototype SolarTree^{TM} [23], the canopy provides shade for parked cars and a surface for solar PV panels to be mounted. The expected 1,520 kilowatt-hours (kWh) per year generated for the two charging stations could fully recharge one Chevrolet Volt's 16 kWh battery [33] from 20% state-of-charge (SOC) up to 118 times. If a Chevrolet Volt needs to be recharged every day, only one third of the total required energy each year will come from the solar PV panels if the vehicle is plugged in every minute that the solar PV panel is generating power. Keep in mind that these solar PV panels sit above two charging stations which makes solar energy utilization per vehicle even lower. It is unclear if the chosen number of solar panels is based on a predetermined canopy space or the type and frequency of PEVs that are expected to be charged.

The Solar EV Dock is another solution for charging PEVs using solar PV generated energy [24]. In this case, the integrated storage within the charger allows flexibility in time of charging. The PEV does not need to charge exactly coincident with solar PV power production. The two-vehicle charging station with associated canopy is claimed to produce between 4,000 kWh and 6,000 kWh a year. They further claim that this is enough to supply a 20-mile work commute each way for both cars each year. Using the Chevrolet Volt example again, 4,000 kWh will recharge it from 20% SOC up to 312.5 times. This is just under half the year if both of the chargers are used every day. 6,000 kWh will recharge a Chevrolet Volt 468 times – approximately 64% of the year if both chargers are used each day. This is close to the claim of supplying enough energy for two vehicles that have a work commute of 20 miles each way. The storage unit power rating and capacity are not listed.

PEV charging using renewable DG is not limited to parking lot canopies, as seen with the 40 kW solar PV installation on top of Nissan's headquarters in Japan [25]. It is claimed that this installation can charge 1,800 Nissan Leaf EVs per year with its three 50 kW chargers, four 3.3 kW chargers and 96 kWh lithium-ion storage. This comes out to charging just under five Nissan Leafs each day. For continued comparison, if it is assumed that 1,800 of the 24 kWh Nissan Leafs [34] are being charged from 20% SOC to 100% SOC each year, then this would translate to charging 2,700 of the 16 kWh Chevrolet Volt PHEVs from 20% SOC to 100% SOC each year. If there are seven chargers, each one could sufficiently recharge a Chevrolet Volt each day. In fact, each charger could recharge two Chevrolet Volts 21 days of the year if used every day.

Wind turbines, an alternative to solar PV panels, are also being explored. The Sanya Skypump will be a combination street light, GE Wattstation™ and 4 kW wind turbine that can either charge a PEV with renewable energy or send it to the grid when there is no vehicle charging [26]. There is no additional storage unit. At an average wind speed of 5 m/s, the wind turbine is estimated to produce 4,700 kWh which energy-wise could fully recharge a Chevrolet Volt from 20% SOC every day of the year assuming the vehicle is plugged in whenever the wind turbine is producing power.

Table 1-2 summarizes these commercially available products that pair renewable DG and PEV charging stations together. Here, the Chevrolet Volt is used as a comparison metric for each solution. If a PEV with a much larger battery was used instead, then the renewable DG might be insufficient to provide all the energy needs of the PEV. It is also unclear from each solution if any of the components were optimized on either an economic or energy basis.

1.2.2 Past Work on PEV Charging System Optimization

Each investment into a generation technology often takes careful calculation and analysis to sufficiently and adequately meet given load requirements. When the perfect balance between load and generation cannot be met in every instance of time, often storage is used to absorb or supply excess power. This storage becomes more crucial for generation sources, like wind and solar PV, where the

Table 1-2: Existing renewable energy based PEV Charging Solutions

Product/Charging Solution	Renewable DG Source	Storage?	Number of Times a Chevrolet Volt can be Recharged per Charger per Year[1]	Source
Solar Tree™	Solar PV	No	59	[23]
Solar EV Dock	Solar PV	Yes	156-234	[24]
Rooftop Solar (Nissan Headquarters)	Solar PV	Yes	386	[25]
Sanya Skypump	Wind Turbine	No	367	[26]

[1] Assuming that the Chevrolet Volt's 16 kWh battery will be recharged from 20% to 100% state-of-charge (SOC)

relatively uncontrollable output power is dependent on the wind and sun. At the same time, these systems must often make economic sense when compared to electricity that could be bought from the grid.

Standalone systems that are often found in remote locations require a DG unit that will supply all of the energy needs of the load. A storage unit is definitely necessary to buffer any mismatch between generation and load. Load control and curtailment are often necessary tools even when storage is present in the system. The optimal sizing of the DG and the storage unit indeed poses a complex problem [27]-[29]. IEEE has even established recommended practices for sizing lead-acid batteries for stand-alone solar PV systems [30]. However, because standalone systems are not grid-connected, these systems compose different architectures than the ones envisioned for this thesis.

Alternative publications on systems that do combine renewable DG, PEVs, and the grid often use PEVs instead of a separate storage unit to provide ancillary services in response to power fluctuations from the renewable DG. The use of PEVs in this manner often requires bi-directional power flow, in which case it is called Vehicle-to-Grid (V2G). Some systems do not interconnect the PEVs to the grid directly [31]. Systems that do interconnect PEVs to the grid are discussed with respect to power flow control but not component size optimization [32]. While power flow optimization is an important component in system sizing, previous work has assumed a fixed, predetermined amount of renewable DG. The main goal of these publications was to describe control schemes that adjust power flow between components to optimize utilization of each component rather than size components to optimize economic utilization.

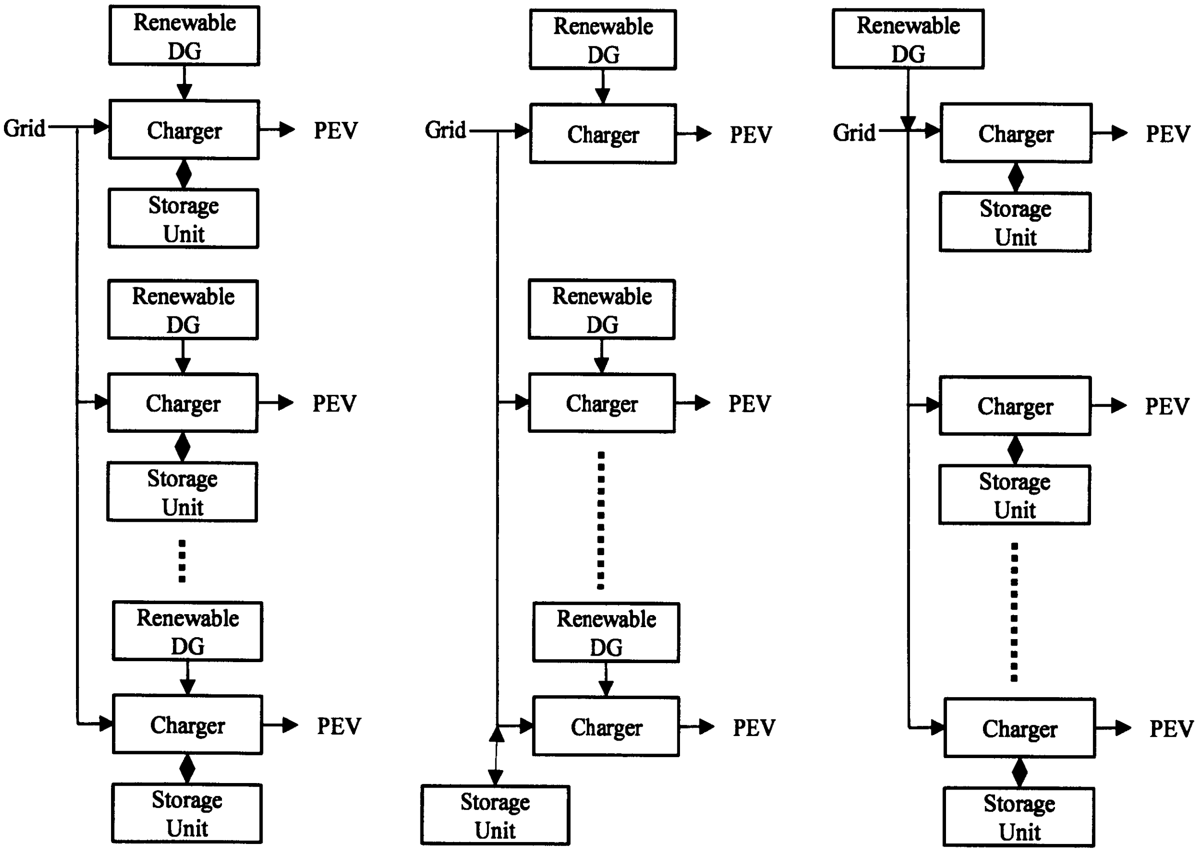
The next section will explain the scope of this thesis and how it differs from past work and the commercial solutions available today primarily from a component size optimization standpoint but also with respect to flexibility in interconnecting components (e.g., multiple chargers can have centralized or distributed renewable DG) and power flow control options (e.g., the grid can charge the storage unit).

1.3 Thesis Scope and Objectives

There are a number of different ways to configure and control a system consisting of PEV chargers, renewable DG, energy storage and electric grid interface. The system might not include storage and/or renewable DG. Alternatively, it might be a self-sufficient, stand-alone system with substantial renewable DG and storage, but no grid interface. If renewable DG, storage and grid interface are all present, their relative ratings can be quite different depending on how power flow is managed. For example, the system may or may not prefer to draw power from storage and/or renewable DG before drawing power from the grid; and it may impose different limits on the power that can be drawn from or delivered to the grid. If there are multiple chargers in the same geographical vicinity, alternative configurations can also be conceived depending on whether the renewable DG and/or storage is centralized or distributed. Some possible configurations based on centralized versus distributed renewable generation sources and/or storage are shown in Fig. 1-2. Different designs can also be developed using alternate technologies for renewable DG (e.g., solar PV versus wind turbine) or storage (e.g., electrochemical battery versus flywheel).

Without a quantitative comparison of these possible designs in terms of system attributes of interest, it is not clear which design is the most appropriate. The system attributes that are of importance for a PEV charging system are cost, efficiency and reliability. This thesis will compare alternative designs in terms of system lifecycle cost, which includes initial capital costs and operating costs (consisting of energy and maintenance costs). By including energy and maintenance costs in system lifecycle cost, the impact of system efficiency and reliability is incorporated in the results of the comparative analysis.

The main objective of this thesis is to develop a methodology paired with a system lifecycle model to find the optimal cost design for a given architecture and configuration. In its development, the methodology will be flexible enough to incorporate different inputs including PEV charging



(a) (b) (c)

Figure 1-2: Sample alternative configurations for integrating PEV chargers, renewable sources of energy and storage into the electric grid: (a) fully distributed renewable sources and storage, (b) distributed renewable sources but centralized storage, and (c) distributed storage but centralized renewable sources.

requirements, renewable DG output power profiles, component cost models, and design constraints. While this methodology is helpful in understanding the sensitivity of the various inputs, its usefulness would be disregarded if the optimal designs did not make practical sense. Therefore, the second objective of this thesis is to develop realistic, data-based models for the inputs including detailed incremental cost analysis of the various components and historical output power profiles for the renewable DG. The third and final objective is to analyze the results from applying the methodology and models to both residential and public charging cases.

This thesis represents a major step in assisting the industry to develop cost optimized PEV charging solutions that directly use low emission energy generated from renewable sources. These solutions can be optimized for various PEV types, applications (residential and public), and geographic locations.

1.4 Thesis Organization

There are six chapters in this thesis. Following this introductory chapter, Chapter 2 develops a system lifecycle cost model that is used to compare alternative designs. It also investigates various component costs necessary for the system lifecycle cost model. Vendor data is gathered and modeled for solar PV and wind turbine generation units; lead-acid battery, lithium-ion battery and flywheel storage units; and grid energy and connection charges.

Chapter 3 presents a methodology for determining an optimal design for a grid-interfaced PEV charging system with integrated generation and storage. Two approaches are formulated for implementing this methodology. The first approach is based on linear programming, while the second is based off a limited search approach.

The two approaches are used to determine and explore optimal designs for a residential charger case in Chapter 4. In developing this case, solar irradiation and wind speed data for two US locations is obtained and used to create realistic models for solar PV and wind turbine output power profiles. Chapter 4 also presents a sensitivity analysis for lithium-ion batteries.

The linear programming formulation developed in Chapter 3 is expanded and applied to a multi-charger case in Chapter 5. The multi-charger formulation includes two PEV chargers, a single grid connection, and a combination of centralized or distributed generation and storage. In total, four configurations involving two PEV chargers are analyzed in this chapter using solar irradiation data from one US location.

Chapter 6 presents a summary, conclusions and directions for future work.

Included in this thesis are two Appendices. A PLECS simulation model developed to validate the resulting design is presented in Appendix A. The MATLAB code for the linear programming and search-based optimization approaches developed in Chapter 3 and Chapter 5 are included in Appendix B.

Chapter 2

System Lifecycle Cost Model

As various PEV charger designs are generated, it is important to be able to understand how one design compares to another. In this thesis, system lifecycle cost is chosen as the comparison metric. When comparing a design with a large solar PV system to a design with a small solar PV system or to a design that only uses the electric grid, cost is a predominant factor in that decision. Most renewable DG systems have high initial capital costs but low operating costs. On the other hand, purchasing energy from the grid has low initial connection cost but the cumulative energy costs over a given time period can be substantial. This is why total cost over the system life is used.

2.1 Development of System Lifecycle Cost Model

The system lifecycle cost, C, used in this thesis includes both the initial capital costs and the operating costs of the system:

$$C = C_{DG} + C_S + C_G + C_M \ , \tag{2.1}$$

where C_{DG} and C_S are the initial capital costs for the renewable DG and the storage unit, respectively, including any costs associated with their power electronic interface; C_G is the cost associated with getting energy from the grid, and C_M is the maintenance cost of the system. Collectively C_G and C_M represent the operating costs of the system over its lifetime.

The initial capital costs of the renewable DG and storage unit are modeled as:

$$C_{DG} = C_{DG,0} + C'_{DG} P_{DG,r} \ , \tag{2.2}$$

$$C_S = C_{S,0} + C'_S P_{S,r} + C''_S E_{S,r} \ , \tag{2.3}$$

where $P_{DG,r}$, and $P_{S,r}$ are the power ratings of the renewable DG and storage unit, respectively, and $E_{S,r}$ is the energy storage capacity of the storage unit. The cost parameters are defined in Table 2-1.

The cost associated with getting energy from the grid is modeled as:

$$C_G = C_{G,0} \frac{T_{life,sys}}{T_{bp}} + C''_G \frac{T_{life,sys}}{T} E_{G,r} + C'_G \frac{T_{life,sys}}{T_{bp}} P_{G,r} \ , \tag{2.4}$$

where T is the time period over which the battery state-of-charge (SOC) has to be maintained; $E_{G,r}$ is the energy drawn from the grid over time period T; and $P_{G,r}$ is the peak power drawn from the grid. The remaining cost and time parameters are defined in Table 2-1. The first two terms of (2.4) capture the consumption charge (consisting of distribution and energy charges) and the third term represents the demand charge.

The maintenance cost is modeled as a cost associated primarily with the cost of replacing the renewable DG and the storage unit at the end of their respective lives. It is given by:

$$C_M = C_{DG} \cdot \mathrm{int}\left(\frac{T_{life,sys}}{T_{life,DG}}\right) + C_S \cdot \mathrm{int}\left(\frac{T_{life,sys}}{T_{life,S}}\right) , \tag{2.5}$$

where int() is the floor function which rounds its argument down to the nearest integer. The time parameters used in (2.5) are defined in Table 2-1.

Table 2-1: Cost and time duration parameters used in the system lifecycle cost model.

Parameter	Description	Value	Unit
$C_{DG,0}$	Fixed capital cost of the renewable DG	200 (solar) 3,750 (wind)	\$
C'_{DG}	Variable capital cost of the renewable DG	4,400 (solar) 5,500 (wind)	\$/kW
$C_{S,0}$	Fixed capital cost of the storage unit	110 (lead-acid) 80 (Li-ion)	\$
C'_S	Variable capital cost of the storage unit depending on the power rating	200	\$/kW
C''_S	Variable capital cost of the storage unit depending on the energy capacity rating	220 (lead-acid) 800 (Li-ion)	\$/kWh
$C_{G,0}$	Fixed distribution charge from the grid per billing period, T_{bp}	9 (residential) 15 (commercial/no demand) [1] 60 (commercial/demand) [1]	\$
C'_G	Demand charge from the grid per billing period, T_{bp}	0 (no demand) [1] 10 (demand)[1]	\$/kW
C''_G	Variable distribution and energy charge from the grid	0.14	\$/kWh
$T_{life,sys}$	Expected life of the system	20	years
$T_{life,DG}$	Expected life of the renewable DG	25	years
$T_{life,S}$	Expected life of the storage unit	6 (lead-acid) 12 (Li-ion)	years
T_{bp}	Length of billing period	1/12	years
T	Time period over which the storage unit SOC is to be maintained	1/365	years

Note: The values of these cost and time duration parameters are discussed in Section 2.2 and Section 2.3, respectively.

[1] Depending on the peak power drawn during the billing period, a commercial customer may incur a demand charge. The values of $C_{G,0}$ and C'_G will depend on the presence of a demand charge.

By combining (2.1)-(2.5), the system lifecycle cost in terms of the system ratings is given by:

$$\begin{aligned} C = {} & C_{DG,0} \cdot \mathrm{int}\left(1 + \frac{T_{life,sys}}{T_{life,DG}}\right) + C_{S,0} \cdot \mathrm{int}\left(1 + \frac{T_{life,sys}}{T_{life,S}}\right) + C_{G,0}\frac{T_{life,sys}}{T_{bp}} \\ & + C'_{DG} \cdot \mathrm{int}\left(1 + \frac{T_{life,sys}}{T_{life,DG}}\right) P_{DG,r} + C'_{S} \cdot \mathrm{int}\left(1 + \frac{T_{life,sys}}{T_{life,S}}\right) P_{S,r} + C'_{G}\frac{T_{life,sys}}{T_{bp}} P_{G,r} \qquad (2.6) \\ & + \mathrm{C}''_{\mathrm{S}} \cdot \mathrm{int}\left(1 + \frac{T_{life,sys}}{T_{life,S}}\right) E_{S,r} + \mathrm{C}''_{\mathrm{G}}\frac{T_{life,sys}}{T} E_{G,r} \end{aligned}$$

The cost and time duration parameters used in (2.2)-(2.6) are defined in Table 2-1, which also lists the values for these parameters as used in this thesis. These cost and time duration parameter values are developed in the next two sections.

2.2 Development of the Cost Parameters

The cost data used in the development of the cost parameters is in 2011 US dollars (USD) unless otherwise stated. When cost data is not available in 2011 USD, it is adjusted for inflation using a GDP deflator [35]. Because the GDP deflator only has data up through 2010, any values adjusted for inflation will be reflected in 2010 USD. It is not expected that the difference between 2010 USD and 2011 USD will be significant.

2.2.1 Renewable Generation

Renewable sources of generation connecting to the grid have become more prevalent over the past decade through various government incentive programs. As a result, the cost of such systems has fallen as the market for said technology grows making it difficult to determine a value for the cost parameters. This analysis will present data to support the values listed in Table 2-1 for renewable DG. It will present data that reflects realistic, current component and system costs. Subsidies and financial incentives are not taken into account even though the cost reductions provided by these programs could amount to significant savings.

Solar PV

This section will present a number of sources that aggregate past and current cost data on solar PV systems [36]-[39]. The difficulty of an aggregation of past data is that it usually entails total system installation costs but it does not necessarily reflect present costs, as the average installed cost of residential and small commercial solar PV systems has dropped from about $11/W in 1998 to about $8/W in 2007 [36]. On the other hand, aggregation of current cost data usually does not include installation or balance of system (BOS) costs, as they use mainly module and inverter cost data available from retail manufacturers. In this sense, both types of data are relevant and useful.

To understand the installed cost of solar PV systems, it is insightful to understand the breakdown of these costs based on system components. In 2006, the US Department of Energy (DOE) published cost information for residential and commercial solar PV installations [37]. The residential installation cost data was based on 5200 residential PV systems installed between 2000 and 2005 while the commercial installation cost data was based on 330 installations. The installed system cost is broken down into 5 categories: (1) module, (2) inverter, (3) BOS, (4) installation, and (5) other/indirect. BOS includes all hardware other than modules and inverters including mounting frames, fuses, disconnects, cables and combiner boxes. Installation cost includes the cost of labor and equipment needed to perform on-site installation. The other/indirect category contains additional costs such as design, engineering, site preparations, permitting and profits. The breakdown of these costs for both residential and commercial units is given in Table 2-2. Also according to the DOE report, the annual operation and maintenance cost of these systems in 2005 was 0.5% and 0.45% of the installed cost for residential and commercial systems, respectively. However, the DOE report projected that these operation and maintenance costs would drop to 0.3% for the year 2011 which is relatively small. For this reason, the maintenance cost term of the system lifecycle cost model ignores it and only considers the replacement cost at the end of the solar PV system's life.

A slightly more recent report by the US DOE utilizes a much larger dataset for installed solar PV system costs [38]. This dataset includes 52,000 installations between 1998 and 2008 representing 71% of all grid-connected solar PV capacity through 2008. Using information in the report that was provided by installers, Table 2-3 shows the breakdown of system element costs by percent for residential, small commercial and large commercial systems.

As can be seen in both reports, the cost of the module composes around 50% of the total installed system cost. This percentage is also agreed upon by a Rocky Mountain institute study [40]. Due to the

Table 2-2: Installed costs for residential and commercial solar PV systems installed between 2000 and 2005 based on data from [37].

System Element	Unit	Residential System	Commercial System
Module	$/W	4.44	3.89
Inverter	$/W	1.00	0.67
BOS	$/W	0.68	0.60
Installation	$/W	1.84	0.61
Other/Indirect	$/W	1.44	1.22
Total	$/W	9.40	6.99

Table 2-3: Breakdown of average installed costs for residential, small commercial and large commercial solar PV systems based on percentage data from [40].

System Element	Residential System %	Small Commercial System %	Large Commercial System %
Module	48	52	52
Inverter	7	7	6
Other Materials	7	12	11
Installation Labor	9	9	10
Other/Indirect	29	20	21
Total	100	100	100

age of the installations, the actual values of the cost data are already outdated. Instead, current cost data for modules will be used to calculate present installed solar PV system costs.

Aggregate industry data suggests that as of August 2011, module costs are \$2.84/W [39]. The lowest prices seen are around \$1.50/W but only 42.4% of module prices are under \$3/W depending on the retailer. The aggregate industry data is not broken down into fixed and variable costs. Therefore, one solar manufacturer was chosen for further investigation in this thesis to pinpoint fixed and variable costs of modules. Due to the range of module sizes available and the accessibility of web-based price, BP Solar modules were chosen [41]. Many other solar module manufacturers exist but the information needed to conduct this analysis was more readily available for BP Solar. Price data was collected from a number of retail sites [42]-[47] and each module price was plotted against the module output power rating. Once this was done, linear trendlines for each retailer were fitted against the data to calculate the fixed and variable cost components. Using this method, the fixed cost is the intercept of the linear trendline and the variable cost is the slope of the linear trendline. The resulting plot is shown in Fig. 2-1 and the costs are listed in Table 2-4. The trendlines produce fixed costs between \$55 and \$165 and most of variable costs are between \$2/W and \$2.25/W. The variable costs from the retailers are slightly lower than the aggregate industry dataset but there is also a nonzero fixed cost. Considering this, the data presented in Table 2-4 corresponds well to the aggregate industry data if the fixed costs were absorbed into the variable costs.

The same analysis can be done for inverters. The source of aggregate industry data presented earlier prices inverter variable costs at \$0.714/W [39]. Repeating the same procedure done for calculating module fixed and variable costs, one inverter manufacturer, Enphase, was selected for a detailed investigation [48]. The same retailers were used to find cost data [42], [44]-[47]. Enphase grid-tied solar microinverters are sized to convert power from a single PV module rather than from a group of modules. Due to the close proximity of rated intput power, most trendlines do not have a positive slope resulting in a fixed cost that is higher than the actual price of the microinverter. The datapoints that did have a

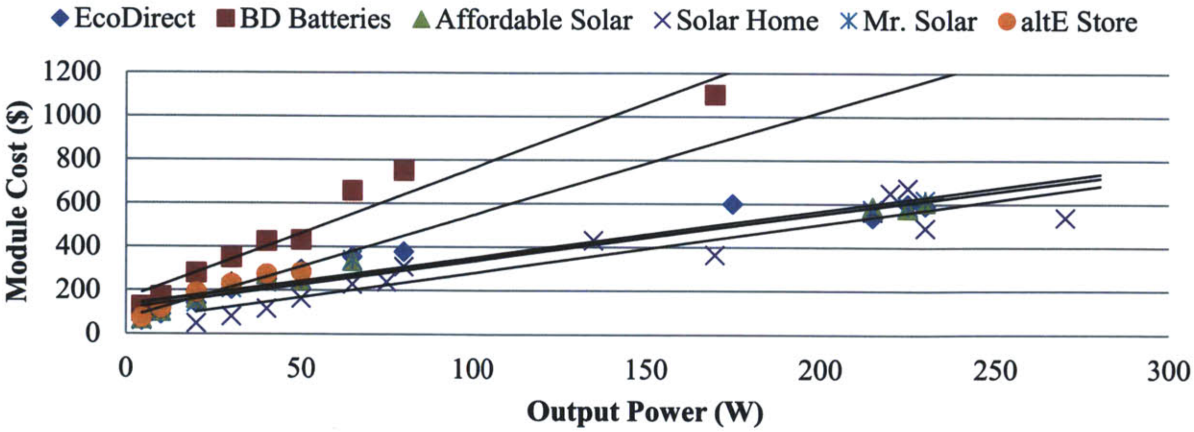


Figure 2-1: BP Solar module cost and corresponding fitted linear trendlines as a function of module rated output power based on data from six retailers [42]-[47].

Table 2-4: Fixed and variable module costs based on the linear trendlines in Fig. 2-1.

Retailer	Fixed Cost ($)	Variable Cost ($/W)	R^2 Value
EcoDirect	135.65	2.1580	0.9077
BD Batteries	164.08	5.9836	0.9509
Affordable Solar	132.31	2.0993	0.9505
Solar Home	55.194	2.2533	0.8841
Mr. Solar	117.59	2.1822	0.9607
altE Store	71.707	4.7506	0.9452

positive slope between them resulted in a negative fixed cost. Therefore, the best determination of fixed and variable costs is to simply assume a zero fixed cost and attribute all of the cost into the variable cost. To calculate this, the price was divided by the input power rating for each microinverter sold by each retailer. All of the calculated costs were then averaged. The average cost was $0.76/W which matches relatively well to the aggregate manufacturer data.

To conduct a similar analysis for the remaining components of a solar PV system would require data that is not readily available. However, since it is generally accepted that the module cost is roughly 50% of the total installed cost of a solar PV system, a reasonable estimate can be made using the module cost [37], [38], [40]. As can be seen from Table 2-4, the fixed cost of the module is roughly $100. Doubling this yields a fixed installed cost for solar PV systems (in the power range of interest in this thesis) of $200. Also, the variable cost of the module is approximately $2.2/W. Hence, the variable cost of the installed solar PV system would be roughly $4.4/W which includes the inverter, BOS, installation and other costs. Therefore, in this thesis, the variable cost of an installed solar PV system is chosen to be $4.4/W, or equivalently $4,400/kW as presented in Table 2-1.

Wind Turbine

Wind turbines that can be used at the residential or small commercial level are less common than small solar PV systems. Many MW level wind turbines are available, but these are not appropriate for individual residential or small commercial applications. Also, cost data for small wind turbines is not readily available. According to the American Wind Energy Association, the cost of distributed wind turbines is between \$3/W and \$6/W for the period 2007 to 2009 [49]-[51]. However, it is not clear whether this is just the cost of the wind turbine or the total installed wind turbine system cost.

A reasonable way to determine fixed and variable costs is through the compilation of manufacturer data as was done for solar modules. This task is made complex by the variety of products offered by retailers. Some retailers sell the wind turbine itself while others offer a package that includes an inverter, a tower, or both. Some wind turbines even have the power conditioning modules already integrated into the turbine hub. The process to determine fixed and variable installed costs is made even more complicated by the fact that the cost of towers is dependent on both the power rating of the turbine and the desired tower height. The following data is presented as coherently as possible given the broad range of data and possible system combinations.

Installed system costs are generally difficult to obtain. However, SoleVento, LLC has publically listed costs for installing a number of wind turbines from different manufacturers [52]. The installed system costs are plotted against turbine power rating in Fig. 2-2. The installed fixed and variable costs according to the fitted trendline are \$15,984 and \$5.67/W, respectively. While the variable cost is within the expected range, the fixed cost is too high to be reasonable. It happens to be higher than some of the installed costs within the set itself. It is possible that the dataset would be better fit with two linear trendlines; one trendline for lower output power and one trendline for higher output power. That analysis

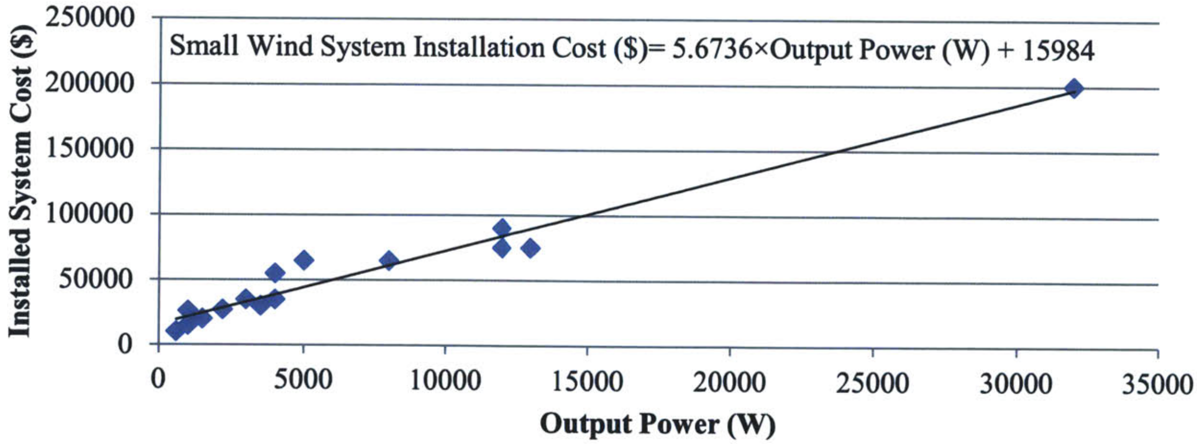


Figure 2-2: Small wind system installation costs and corresponding fitted linear trendline as a function of turbine rated output power based on data from one installer [52].

will not be done here in order to remain consistent with the cost model and the analysis done with solar PV systems. Instead, a closer look at the individual cost components is in order.

As mentioned previously, some retailers [42], [53]-[58] list only wind turbines without any towers or power conditioning units (i.e., inverters). Those prices are shown in Fig. 2-3. As before, the wind turbine cost is plotted against its rated output power and a linear trendline is fitted. The fixed and variable costs come out to $1,092 and $2.82/W, respectively. Both these values seem reasonable. These units will also require an inverter for grid-connected systems. The SMA Windy Boy inverter series is mentioned often and is the basis for the inverter cost analysis [59]. Due to a slight difference in price between vendors [45], [56], [57], [60], the R-squared value of the linear trendline in Fig. 2-4 is low however, the fixed and variable costs of $803 and $0.38/W, respectively, do not appear unreasonable.

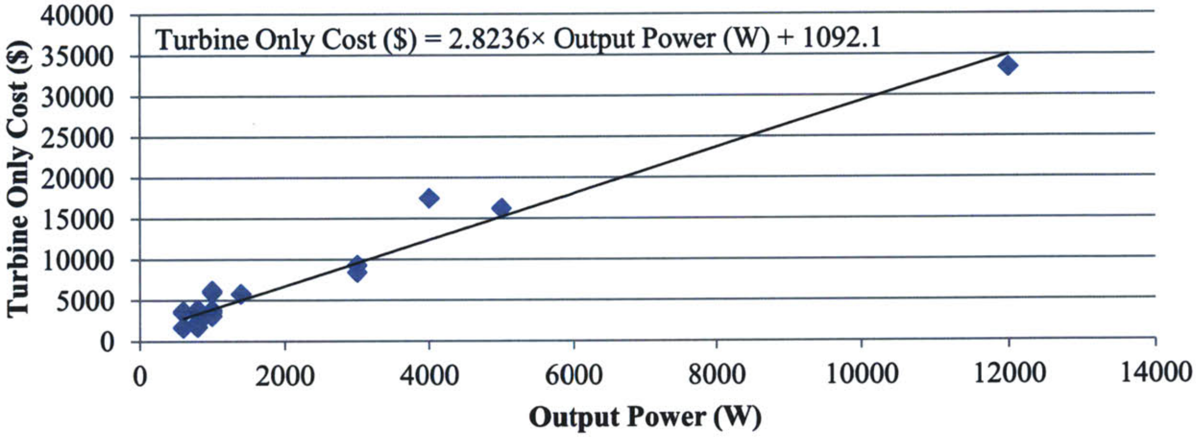


Figure 2-3: Small wind turbine costs and corresponding fitted linear trendline as a function of turbine rated output power based on data from seven retailers [42], [53]-[58].

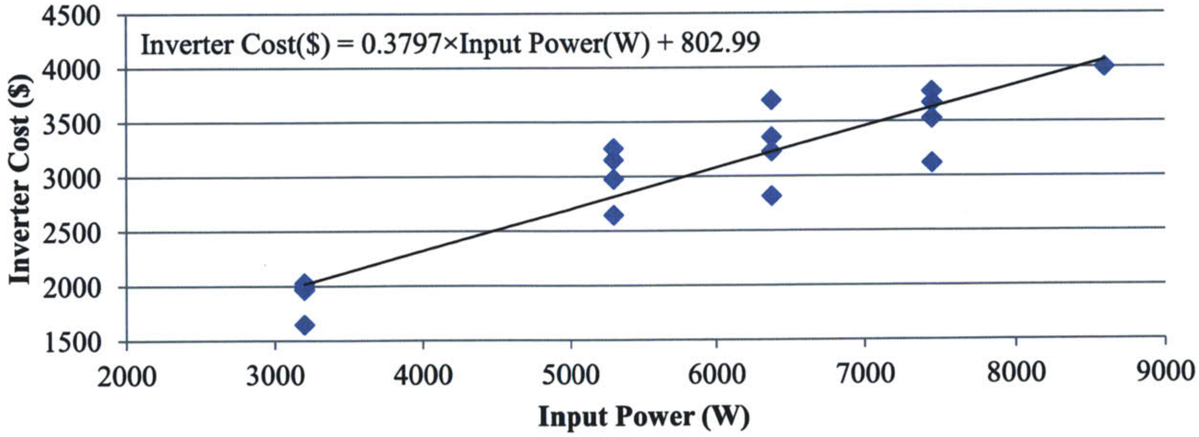


Figure 2-4: SMA Windy Boy inverter costs and corresponding fitted linear trendline as a function of inverter rated input power based on data from four retailers [45], [56], [57], [60].

When individual linear trendlines are fitted for data from each retailer separately, the fixed and variable costs have very similar values. After summing the turbine and inverter costs, the values for fixed and variable costs are $1,895 and $3.20/W, respectively.

Some wind turbine manufacturers [61]-[65] offer the power conditioning units packaged with the turbine itself thereby eliminating the need to purchase a separate inverter. The cost data from retailers [42], [47], [53], [54], [58], [60], [66], [67] for these power conditioned wind turbines is used to compare against the sum of the wind turbine and inverter costs that was just presented. The fitted linear trendline results in a positive variable cost of $4.18/W but a negative fixed cost of -$870.93. Instead, separate linear trendlines for each retailer is presented in Fig. 2-5 and the resulting fixed and variable costs are listed in Table 2-5. The average values of these costs are $2,592 and $3.45/W which are not the same as the values calculated for separate turbine and inverter components but they are relatively close.

Taking the middle ground between the two cost points for wind turbine and inverter costs, it is reasonable to assign fixed and variable costs of $2,250 and $3.30/W, respectively. The remaining installation costs would be attributed to the towers, installation labor, fees, etc. Due to lack of such information, it would be very difficult to do a similar analysis with any accuracy. If wind turbine systems

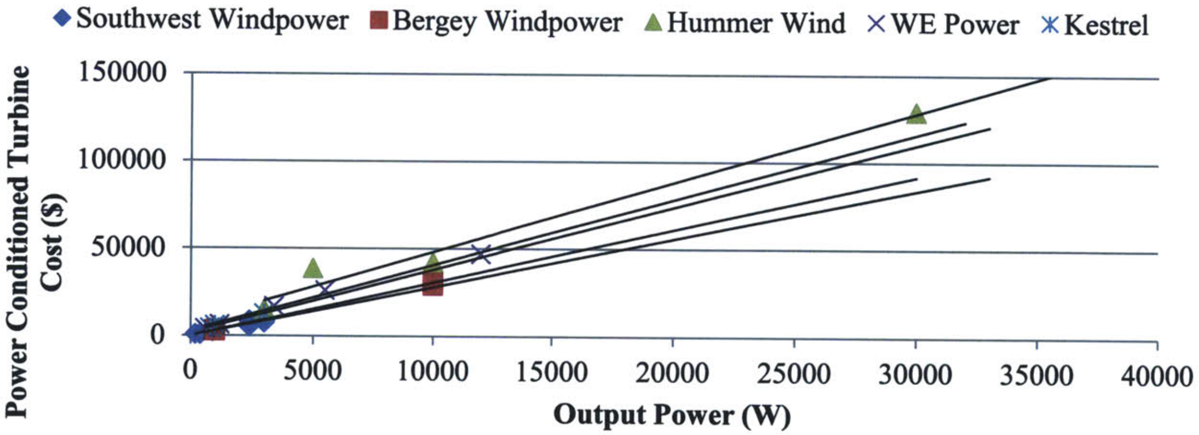


Figure 2-5: Power conditioned wind turbine costs and corresponding fitted linear trendlines as a function of turbine rated output power based on data from eight retailers [42], [47], [53], [54], [58], [60], [66], [67].

Table 2-5: Fixed and variable power conditioned turbine costs based on various data presented in Fig. 2-5.

Manufacturer	Fixed Cost ($)	Variable Cost ($/W)	R^2 Value
Southwest Windpower	90.9	2.7880	0.9427
Bergey Windpower	- 78.4	3.0588	0.9957
Hummer Wind	8,011.6	4.0131	0.9772
WE Power	3,000.9	3.7663	0.9915
Kestrel	1,932.5	3.6020	0.9943

followed the same cost trends as solar PV systems, then the wind turbine and inverter would compose about 60% of the total installation cost based on percentage breakdown in Table 2-3. Using this percentage, the overall system installation fixed and variable costs are approximately \$3,750 and \$5.5/W, respectively. This assigns a variable cost that is consistent with the values stated by the American Wind Energy Association and the variable cost calculated from the trendline in Fig. 2-2. While the fixed cost is much lower than calculated earlier, it is more reasonable. Therefore, \$3,750 and \$5.5/W are used in this thesis for wind DG systems.

2.2.2 Storage Unit

There are three different types of storage units that will be investigated in this thesis. The first type is lead-acid batteries, the standard storage unit often seen in renewable energy projects. The second type is lithium-ion (Li-ion) batteries, an emerging alternative to lead-acid batteries. These have gained increased use in consumer electronics and now EVs. While still a relatively new technology for renewable energy projects, it is useful to understand how it compares to lead-acid batteries both in terms of cost and in life. The last storage technology explored is flywheel energy storage. This kinetic energy based technology is not new, but has recently been promoted as a grid-scale energy storage technology for frequency regulation [68]. Therefore, it is worthwhile to do a brief cost analysis for this alternative.

Lead-Acid Battery

The lead-acid battery is the most common deep-cycle battery technology available for renewable energy projects. Although it is not always operated in such a way, a deep-cycle battery is capable of discharging down to 20% SOC without significant damage to the battery. This is a beneficial feature for a renewable energy application. For example, on a rainy day, the solar PV system may not supply sufficient energy and the storage unit would have to be safely drained below normal. As of August 2011, a survey of battery pricing indicates that batteries cost around \$213/kWh while the most common batteries used average between \$239/kWh and \$260/kWh [39]. One important note on these calculations pertains to the kWh basis. Battery manufacturers list a voltage for the battery and a few amp-hour (Ah) ratings. Each Ah rating is based on how quickly the battery is discharged. In order to calculate the energy a particular battery can deliver, one would follow:

$$E_B = \frac{V_B \times Ah_B}{1000} \tag{2.7}$$

where E_B is the energy delivered by the battery in kWh, V_B is the voltage supplied by the battery in volts (V), and Ah_B is the manufacturer provided Ah value in Ah. Ah ratings are typically given for 5-hour, 20-hour, and 100-hour discharging rates. Batteries are more efficient when they are charged and discharged slowly. Therefore Ah ratings for a 100-hour discharge will be higher than Ah ratings for a 20-hour or 5-

hour discharge. The kWh values provided by [39] are based on a 20-hour discharge rate. While the data from [39] is useful, it would be beneficial to know the fixed and variable costs of lead-acid batteries.

Cost data for lead-acid batteries is available from most retailers that sell solar modules and/or wind turbines [42], [44], [46], [47], [69]. Among these retailers, there are batteries from five common battery manufacturers [70]-[74] and these batteries will be the basis for this cost analysis. The values used in Fig. 2-6 are based on a 100-hour discharge rate which is typically the highest capacity that is stated on manufacturer specification sheets. This means the anticipated variable cost should be lower than the \$239/kWh and \$260/kWh values presented earlier. As seen in Fig. 2-6, the calculated fixed cost is \$24 and the variable cost is \$169/kWh. When the same analysis is done at the 20-hour rated energy capacity, the fixed and variable costs are \$36 and \$242/kWh. This matches well with the values from [39]. For this thesis, an averaged fixed cost of \$30 and variable cost of \$220/kWh for lead-acid batteries is used based on data from all three datasets analyzed because the practical capacity of the battery can vary between the values given in the specification sheets.

Lithium-Ion (Li-ion) Battery

The market for Li-ion batteries has increased due to use in personal electronics and now in electric vehicles. However, it is still a new storage technology and fairly expensive. It is expected that the price of Li-ion batteries will continue to drop as their market increases. Today, there are few retailers supplying Li-ion batteries for large energy storage systems. The analysis done for lead-acid batteries cannot be duplicated for Li-ion batteries due to a lack of data. Fortunately, the Electric Power Research Institute recently released a report analyzing the costs for various storage technologies that can be used for grid-scale storage [75]. In general, they conclude that Li-ion battery systems are a relatively new

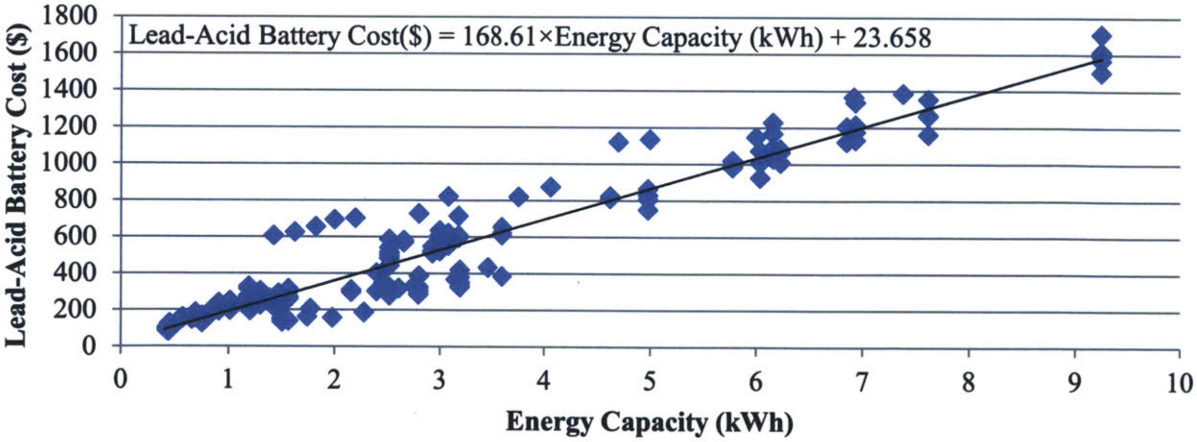


Figure 2-6: Lead-acid battery costs and corresponding fitted linear trendline as a function of lead-acid battery 100-hour rated energy capacity based on data from five retailers [42], [44], [46], [47], [69].

option that is only in the demonstration stage. They are only somewhat confident about their cost estimates which are based on vendor quotes and system installation estimates. Table 2-6 summarizes their cost findings for each application. Disregarding the very large energy storage systems presented in Table 2-6, it appears that the average cost of Li-ion energy storage systems are between $800/kWh and $3,600/kWh. The very high energy and power applications exhibit much higher variable costs. Because the variation in cost is so great, it is very difficult to estimate a single variable cost for Li-ion batteries.

While using Li-ion batteries for a slightly different application, a report by Deutsche Bank estimates that Li-ion batteries used in EVs cost $650/kWh in 2010 and this will drop to $315/kWh in 2015 and $250/kWh in 2020 [75]. These are much lower than the costs listed in Table 2-6. This is because the Electric Power Research Institute cost estimates include installation, interconnection and grid integration costs which include power electronic interfaces, transformers, communication and controls. They note that significantly lower costs may be possible if combined with renewable DG systems. Therefore, the battery-only costs are lower than cost data presented in Table 2-6. Also, installation and power conditioning components were already incorporated into the fixed and variable costs for the renewable DG. Therefore, this thesis will use battery-only cost data. It will be chosen as $800/kWh for Li-ion batteries which is slightly higher than the EV battery cost but on the low end of the grid storage system costs. Fixed cost, due to lack of data, is assumed to be zero.

Table 2-6: Li-ion energy storage averaged system cost estimates by application [75].

Application	Capacity (kWh)	Power (kW)	Duration (hours)	Cost ($/kWh)
Frequency Regulation Power Quality Defer Capital Cost	250-25,000	1,000-100,000	0.25-1	4,340-6,200
Utility T&D Substation Grid Support Peak Shaving, Reliability Frequency Regulation	4,000-24,000	1,000-10,000	2-4	900-1,700
Commercial and Industrial Energy Management Power Quality, Reliability	100-800	50-200	2-4	950-1,900
Distributed Energy Storage Distribution Deferral, Peak Shaving Reliability Frequency Regulation	25-50	25-50	1-4	950-3,600
Residential Energy Storage Back-up Power, Reliability Home PV Time Shifting	7-40	1-10	1-7	800-2,250

Note: A large part of these costs may be due to power electronics, BOS and installation.

Flywheel Storage

Flywheel energy storage technology is attractive because it has very fast response (less than 4 ms), high power density (5 to 10 times that of batteries), high round-trip energy efficiency (~ 93%), and long lifetime (20 years) [75]. It has not had widespread use in grid energy storage systems because of its low energy density. For example, although exact density is unpublished, a 1 MW flywheel system will take several hundred square feet to store 250 kWh.

The market for flywheels is even smaller than the market for Li-ion batteries. Fortunately, the same Electric Power Research Institute report also analyzes flywheel energy storage technologies. Unfortunately, there is only one current application for flywheel energy storage systems - frequency regulation. (The time scale for energy buffering in frequency regulation is relatively short – on the order of minutes and below, but not hours.) From the capacity and power data given, it can be assumed that the cost data is based on a single flywheel frequency regulation project. The cost presented is \$7,800/kWh to \$8,800/kWh which will include any costs due to power electronics, BOS and installation [75]. This is an order of magnitude higher than the cost of battery technologies. It is not likely that this is a viable technology option for EV charging energy storage systems. Therefore, flywheel technology is not explored any further in this thesis.

Charge Controller

The last component necessary for storage units, particularly of the battery variety, is a charge controller which prevents damage to the storage unit as it charges or discharges. Aggregate industry data indicates that charge controllers cost \$5.93/A [39]. However, the cost model developed does not have output current from the charge controller as a variable. Instead, the power rating of the storage unit is used. Therefore, the output current must be translated into power. Most batteries offered are 12V although 2V and 6V batteries are occasionally used. Batteries can be placed in series for larger voltage drops (such as 24V or 48V) however balancing circuits would be required to ensure the voltage across each battery in the series is safe. If 12V is assumed to be the standard battery voltage and subsequently the required output voltage of the charge controller, then dividing \$5.93/A by 12V will yield a cost of \$0.494/W or equivalently, \$494/kW.

The cost of charge controllers provided by retailers is not simply based on output current alone. As can be seen graphically in Fig. 2-7, charge controllers with the same output current can have a spread of prices. As with other components, it is reasonable to believe that the variation of cost in Fig. 2-7 is due to the difference between manufacturers [77]-[84] and retailers [42], [44], [46], [47], [69]. But even when cost data is plotted for one manufacturer [82] as sold by one retailer [42], there are multiple cost points for a given output current (see Fig. 2-8). A single manufacturer can produce several charge controllers that have the same output current. The differences between charge controllers are the input voltage, the output voltage, number of input phases and other characteristics such as meters or type of control methodology used. A single charge controller unit is even capable of several output voltages ranging from 12V to 72V. This is why there is a cost spread for a given output current. For each fitted linear trendline in Fig. 2-7 and Fig. 2-8, the fixed cost of $18 and $12 is quite low and the variable cost of $6.83/A and $5.11/A are about a dollar off from the aggregate industry cost.

If instead, the cost data was plotted against maximum output power, the spread of data points does not see significant improvement (see Fig. 2-9). In this case, the maximum output power is the output current times the maximum output voltage for each unit, which varies from 12V to 72V. The fixed and variable costs in this case are roughly correlated to $110 and $123/kW, respectively. This is lower than the $494/kW calculated from the aggregate industry data almost exactly by a factor of four indicating that the average designed output voltage that should be used is 48V, not 12V. If only charge controllers with a maximum output of 12V are used in the cost analysis, the fixed cost comes out to $50 and the variable cost is $284/kW. The 12V charge controllers have less than a 1.2 kW maximum output power. As seen in Fig. 2-9, it is very plausible that the cost points for output powers less than 1.2 kW have a steeper linear variation than the cost points for output powers greater than 1.2 kW.

Because the output voltage of the charge controller will depend on the system setup and specific battery bank that is used, the 12V only charge controller data cannot be used exclusively. On the other hand, because most batteries are 12V, it is reasonable to believe that most systems may only require a 12V output charge controller. To remedy this, the fixed cost that is used is $80 which is halfway between $50 and $110 and the variable cost that is used is $200/kW which is halfway between $123/kW and $284/kW.

Summary of Storage Unit Costs

In summary, only lead-acid and Li-ion batteries will be used for the storage units. Flywheel energy storage systems are not cost competitive for this storage application. Lead-acid battery costs are $30 fixed and $220/kWh. Li-ion battery cost is $800/kWh. Each system with a battery will require a charge controller to ensure that the battery is not damaged when charged. The charge controller costs are $80 fixed and $200/kW. The lead-acid storage unit will have a fixed capital cost ($C_{S,0}$) of $110, which is the summation of $30 and $80. The Li-ion storage unit will have a fixed capital cost ($C_{S,0}$) of $80. The variable cost of the entire storage unit that depends on the rated power (C'_S) will be $200/kW for either battery technology used. The variable cost that depends on the energy capacity (C''_S) will be $220/kWh for lead-acid battery systems and $800/kWh for Li-ion battery systems.

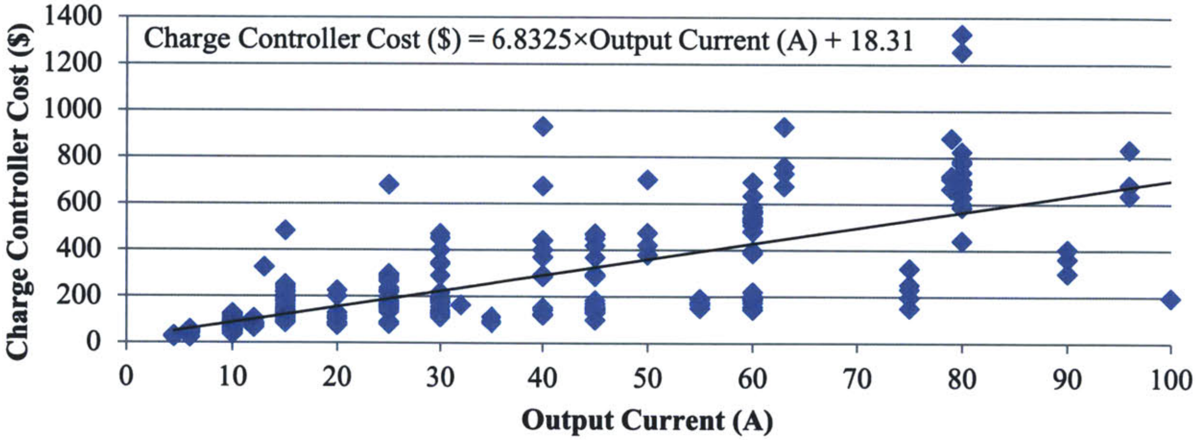


Figure 2-7: Charge controller cost and corresponding fitted linear trendline as a function of output current based on data from five retailers [42][44][46][47][69].

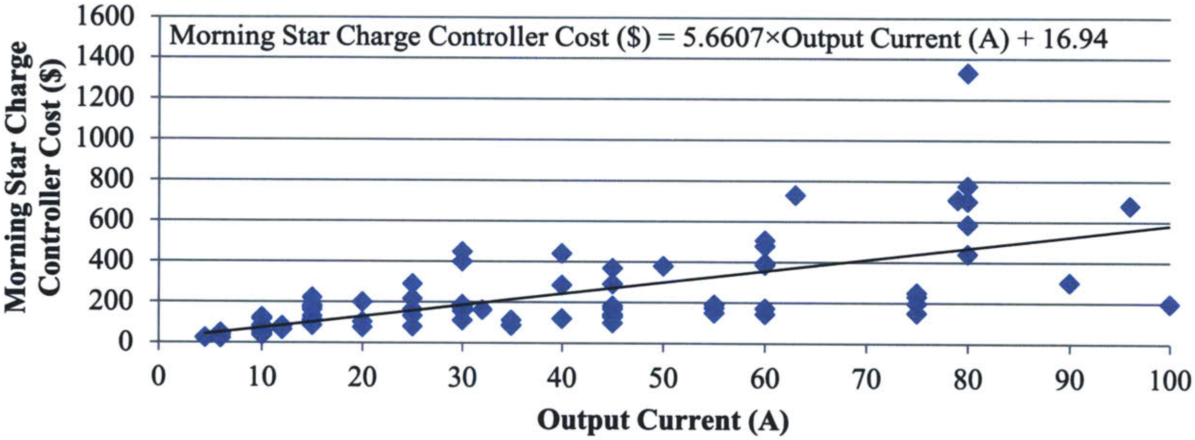


Figure 2-8: Morning Star charge controller cost and corresponding fitted linear trendline as a function of output current based on data from one retailer [42].

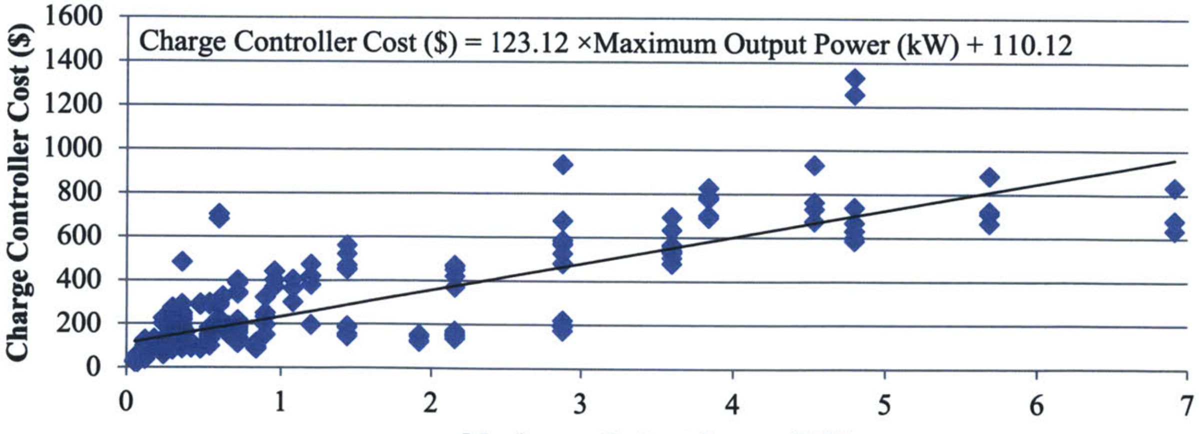


Figure 2-9: Charge controller cost and corresponding fitted linear trendline as a function of maximum output power based on data from five retailers [42][44][46][47][69].

2.2.3 Electric Grid

When purchasing electricity from the grid, there are three typical charges that appear on utility bills. The most familiar charge is the retail price of electricity that customers pay for each kWh they consume. Lesser known are the fixed distribution charge that can appear on each bill for connecting to the grid and the demand charge based on the peak power consumed within the billing cycle.

The fixed distribution charge allows distribution utilities to recover fixed costs associated with the delivery of electricity. These costs do not depend on how many kWh are delivered in a particular billing period. For example, depending on the utility, an employee may have to go to each individual consumer and read their meter to report back how many kWh were delivered that month to that customer. This meter reader visits the meter whether the consumer used only a few kWh or many kWh and his salary is considered a fixed cost. The fixed distribution charge varies by utility. Some utilities may not have a fixed monthly charge on top of their energy charge for their residential customers, although they may still have a minimum charge if no energy was sold that month to that customer. Other utilities may have charges in excess of $15. It would be a very daunting task to seek out all of the various monthly charges for the thousands of utilities in the US. Instead, the fixed distribution charges for the top five utilities (by consumers) are listed in Table 2-7 along with the average for each consumer type [85]-[89]. Using the average values, $C_{G,0}$ is taken to be $9 for residential consumers, $15 for commercial customers who do not incur a peak monthly demand charge and $60 for commercial customers who do incur a peak monthly demand charge.

Also shown in Table 2-7 are the peak demand charges. These charges are based on the consumer's peak demand over a 15 minute interval for the entire month. This charge is in place to

Table 2-7: Monthly fixed distribution charges and demand charges for the top five distribution utilities by number of consumers [85-89].

Utility	Residential Monthly Charge ($)	Commercial (no Demand) Monthly Charge ($)	Commercial (with Demand) Monthly Charge ($)	Peak Demand Charge ($/kW)
Pacific Gas & Electric Company	0	3.94	3.94	18.05
Southern California Edison	0.88	22.30	134.17	12.15
Florida Power & Light Company	5.90	6.89	16.44	6.5
Commonwealth Edison Company	16.50	10.95	19.37	5.14
Consolidated Edison Company of New York	15.76	24.06	117.14	17.22
Average	8.71	13.63	58.21	10.01

Note: Commercial (no Demand) is typically retail customers whose peak monthly demand is less than 10kW or 20kW depending on the utility. Commercial (with Demand) is typically customers whose peak monthly demand is greater than 10 kW or 20kW depending on the utility.

incentivize lowering the overall system peak for better capacity utilization. Since demand charge is typically applicable only to industrial and large commercial customers, C'_G is assumed to be zero for the residential charging case. The public charging case will depend on the charging system setup. Some commercial customers will not incur a demand charge if their peak monthly demand is less than 10 kW or 20 kW depending on the utility. If a single public charging station is only rated at 7.2 kW, then it is likely that this charging station will not incur a demand charge. If there are several charging stations together or if the charging rate of a single charger is above 20 kW, then this charging scenario will incur a demand charge. Therefore, C'_G is assumed to be zero for charging stations which are not expected to incur a demand charge and $10/kW for charging stations that are expected to incur a demand charge.

The last cost parameter for using the grid is the variable distribution and energy charge which is based on the kWh consumed by the customer. This cost is derived from the electricity that the distribution utility purchases from the generation units and also the cost of the infrastructure of wires to deliver this electricity. In 2010, the average residential and commercial retail price of electricity was $0.112/kWh and $0.0997/kWh, respectively [90]. Table 2-8 breaks up the average price for residential and commercial customers by state (and region). The New England, Middle Atlantic, and Pacific Noncontiguous regions have average retail prices that are much higher than the nationwide average, while the East South Central and the West North Central regions have retail prices that are much lower than the nationwide average. The areas that are more prone to see large penetration of electric vehicles and renewable DG are going to be those with strong incentives and history for alternative vehicles, charging infrastructure, and/or renewable DG. California is a prime example of a state that is expected to have a high penetration of both technologies based on existing number of hybrid electric vehicles, EV charging stations, and incentives or laws promoting EVs [91], [92]. Likewise, using renewable DG instead of the grid is more cost competitive in areas where retail electricity prices are high. Therefore, an average retail price of $0.14/kWh is chosen which is higher than the nationwide average but lower than the average retail price in the state of California and the regions of New England, Middle Atlantic, and Pacific Noncontiguous. The difference between residential and commercial sectors is small and $0.14/kWh will be used for C''_G independent of residential or public charging scenarios.

2.3 Time Duration Parameters

This section describes how the values to the time duration parameters listed in Table 2-1 were determined. The system lifecycle cost will be calculated over the system life, $T_{life,sys}$. The system life is chosen to be close to but less than the life of the renewable DG so that the DG does not have to be replaced. For this thesis, a value of 20 years is chosen. Also of great importance within this thesis is the period, T, over which the storage unit will return to its initial SOC. For this thesis, it is assumed that the

Table 2-8: Average retail price of electricity to ultimate customers for 2010 [90].

Census Division and State	Residential (₵/kWh)	Commercial (₵/kWh)	Census Division and State	Residential (₵/kWh)	Commercial (₵/kWh)
New England	**16.62**	**15.03**	**West North Central**	**8.79**	**7.29**
Connecticut	19.42	16.61	Iowa	9.65	7.32
Maine	15.62	12.49	Kansas	9.43	7.87
Massachusetts	15.44	15.23	Minnesota	9.92	7.91
New Hampshire	15.97	13.98	Missouri	8.02	6.67
Rhode Island	8.71	13.63	Nebraska	8.01	7.27
Vermont	15.34	13.32	North Dakota	7.35	6.68
			South Dakota	8.28	7.22
Middle Atlantic	**15.24**	**13.33**	**West South Central**	**10.54**	**8.9**
New Jersey	15.93	13.42	Arkansas	8.74	7.51
New York	18.12	15.31	Louisiana	8.72	8.61
Pennsylvania	12.45	10.12	Oklahoma	8.56	6.87
			Texas	11.59	9.41
South Atlantic	**10.71**	**9.19**	**Mountain**	**10.01**	**8.48**
Delaware	13.37	11.37	Arizona	10.29	8.94
District of Columbia	13.25	13.8	Colorado	10.78	8.78
Florida	11.09	9.57	Idaho	7.79	6.7
Georgia	9.64	9.01	Montana	8.74	8.23
Maryland	14.39	11.61	Nevada	12.71	10.35
North Carolina	10.05	8.08	New Mexico	10	8.36
South Carolina	10.22	8.62	Utah	8.35	6.9
Virginia	10.33	7.7	Wyoming	8.37	7.36
West Virginia	8.47	7.51			
East North Central	**10.92**	**9.29**	**Pacific Contiguous**	**12.08**	**11.04**
Illinois	10.9	8.67	California	15.09	12.54
Indiana	9.22	8.22	Oregon	8.67	7.64
Michigan	12.01	9.98	Washington	7.76	7.24
Ohio	10.8	9.74			
Wisconsin	12.29	9.74			
East South Central	**9.15**	**9.05**	**Pacific Noncontiguous**	**22.6**	**19.96**
Alabama	10.47	10.17	Alaska	16.26	13.83
Kentucky	8.12	7.48	Hawaii	27.46	25.43
Mississippi	9.61	9.32			
Tennessee	8.64	9.13			
Average US	11.2	9.97			

EV charger will be used daily. Therefore, T is chosen to be one day in order to minimize the battery size.

2.3.1 Renewable Generation

To determine the expected life of renewable DG, the warranty is generally a good indicator although the life of the unit can easily go past the warranty timeframe. For solar PV modules, many manufacturers have a product and parts warranty of 5 years [41], [93]-[100]. What is more useful is the performance warranty which typically guarantees at least a 90% of rated power output for 10 years and 80% of rated power output for 25 years. This indicates a reasonable life for solar PV modules that is greater than or equal to the system life. The Enphase microinverter is also backed by a 25-year limited warranty [48].

Wind turbines also typically come with a 5-year product warranty [61]-[64]. They do not come standard with the same performance warranty as solar PV modules. Most of the manufacturers used in the cost analysis above claim a design life of 20 years but it is reasonable for the life to extend beyond 20 years [61], [62], [65]. The SMA Windy Boy inverters come with a 10-year warranty and it can be extended for another 10 years [59].

In this thesis, 25 years is used for $T_{life,DG}$, although the analysis could use anything above 20 years and system lifecycle cost would remain the same.

2.3.2 Storage Unit

The life of the storage unit depends on the number of times it has been charged and discharged. One charge and discharge is called a cycle. Battery manufacturers sometimes provide graphs that present the number of cycles a battery can sustain based upon the depth of discharge (DOD) of the battery. A graph for SunXtender lead-acid batteries is used in the cost analysis above and is shown in Fig. 2-10 [71]. The lower the DOD, the more cycles the battery can provide. If a battery is drained all the way and recharged, it can only survive few cycles. If the battery is drained halfway and recharged, it can sustain more cycles. For each case just presented, the amount of energy provided from each cycle is different.

The following equation is used to determine the life of the battery:

$$T_{life,S} = \frac{N_{cycles}}{365} \tag{2.8}$$

where N_{cycles} is the number of cycles expected out of a given battery and 365 indicates that the battery will go through 365 cycles a year or one cycle per day.

The energy that will be required out of the storage unit is fixed. The rated energy capacity of the storage unit will have to be adjusted to ensure that the required amount of energy is delivered when the DOD is varied:

$$E_{S,r} \propto \frac{E_{delivered}}{DOD} \tag{2.9}$$

where $E_{delivered}$ is the fixed energy that must be delivered and DOD is the depth of discharge that the battery will incur. The system lifecycle cost from (2.6) is proportionate to the number of cycles and the depth of discharge for each battery:

$$C \propto \frac{\text{int}\left(1 + \frac{T_{life,sys}}{N_{cycles}} \times 365\right)}{DOD} = C_{E,factor} \tag{2.10}$$

where $C_{E,factor}$ is defined as a comparative factor between various pairs of DOD and N_{cycles}. This is used to generate the values seen in Table 2-9 from the data presented in Fig. 2-10. These values do not represent the actual system lifecycle cost but rather the relative cost of the energy portion of the storage system for comparison. The higher the value, the higher the overall system lifecycle cost will be. It can be seen that the lowest cost factor occurs when the DOD is limited to 20%. The battery life that this corresponds to is just under 8 years. However, there will be days when the clouds may cover the sun or the wind may not

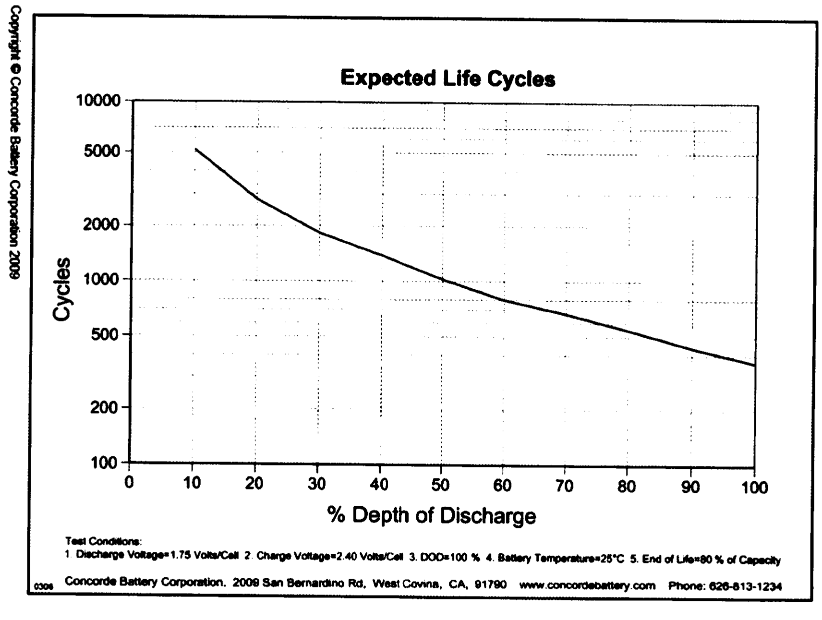


Figure 2-10: SunXtender lead-acid battery cycle life versus depth of discharge [71].

Table 2-9: Relative system lifecycle energy cost factors for SunXtender lead-acid batteries [71].

% DOD	Approximate Cycles	$C_{E,factor}$
100	380	20
80	520	18.75
50	1000	16
30	1800	16.67
20	2900	15

blow. In these cases, the battery may be drained past 20% DOD. Likewise, over time, it is expected that the battery capacity may decrease. Toward the end of the battery life, $E_{delivered}$ will comprise a much higher percentage of the current battery capacity. The battery would be regularly drained past 20% DOD. For these reasons, the expected life, $T_{life,S}$, of the lead-acid battery is chosen to be 6 years rather than 8 years because the DOD will likely go beyond 20%.

It is difficult to duplicate the analysis for Li-ion batteries because there is a lack of cycles versus depth of discharge graphs. Instead, EV battery lifetimes will be used. EV manufacturers desire batteries that can last many years for warranty purposes but at the same time do not want a very oversized battery for space and weight purposes. Discharging to 80% DOD has become a standard among auto manufacturers [101]. Both Nissan and GM expect their Li-ion batteries will experience a capacity degradation of about 30% over their life and they will warranty the battery for 8 years although the degradation will not be covered [33], [34]. For the same reliability and capacity degradation reasons stated for lead-acid batteries, it would be preferred if the Li-ion batteries also only had a 20% DOD. It is assumed that Li-ion batteries will have similar DOD and cycle life characteristics as lead-acid batteries. If this is true, it would be reasonable for Li-ion batteries to have at least a 10 year life for 20% DOD. For the 20 year system life used in this thesis, any Li-ion battery lifespan above 10 years and below 20 years would not affect the system lifecycle cost because the battery would still need to be replaced once. Assuming a 20-year Li-ion battery life would be too far of a jump based on the current data. In this thesis, the expected life, $T_{life,S}$, of the Li-ion battery is chosen to be 12 years.

For both battery technologies, a designed maximum DOD is chosen to be 20% but it is assumed that the battery will go through instances of increased discharge in actual system use.

2.3.3 Electric Grid

The last time parameter to consider is T_{bp}, the length of the billing period associated with connecting to the electric grid. Most consumers receive a utility bill once a month and the fixed and demand charge cost data that was gathered was based on monthly intervals. Therefore, in this thesis, T_{bp} is taken to be one month.

Chapter 3

Optimization Methodology for a Single Charger

In this thesis, an optimal architecture is one with the lowest system lifecycle cost, as modeled by (2.1). Hence, a method is needed to size the individual elements of the system so as to yield designs with minimum lifecycle cost that meet all system requirements. The system elements that need to be sized are the distributed renewable generation source and the storage unit. The renewable DG's power rating, $P_{DG,r}$, needs to be determined and the storage unit's power and energy capacity ratings, $P_{S,r}$ and $E_{S,r}$, respectively, also need to be determined. In order to calculate system lifecycle cost, the peak power, $P_{G,r}$, and the energy drawn from the grid over period T, $E_{G,r}$, are needed. These ratings depend upon $P_C(t)$, the power drawn by the PEV charger as a function of time t, as well as the design constraints and the power flow control methodology.

3.1 Development of Optimization Methodology

Design constraints arise from the maximum power that can be drawn from and delivered to the grid, $P_{G,max(p)}$ and $P_{G,max(n)}$, respectively; limits on the maximum rating of the renewable DG, $P_{DG,max}$; the maximum power and energy rating of the storage unit, $P_{S,max}$ and $E_{S,max}$, respectively; and the maximum and minimum state-of-charge allowed for the storage unit, SOC_{max} and SOC_{min}, respectively. These constraints can be expressed as follows:

$$-P_{G,max(n)} \leq P_G(t) \leq P_{G,r} \leq P_{G,max(p)} \quad , \tag{3.1}$$

$$0 \leq P_{DG}(t) \leq P_{DG,r} \leq P_{DG,max} \quad , \tag{3.2}$$

$$-P_{S,max} \leq -P_{S,r} \leq P_S(t) \leq P_{S,r} \leq P_{S,max} \quad , \tag{3.3}$$

$$E_{S,r} \cdot SOC_{min} \leq E_S(t) \leq E_{S,r} \cdot SOC_{max} \leq E_{S,max} \cdot SOC_{max} \quad , \tag{3.4}$$

where $P_G(t)$, $P_{DG}(t)$, and $P_S(t)$ are the instantaneous powers delivered by the grid, the renewable DG, and the storage unit, respectively; and $E_S(t)$ is the instantaneous energy stored in the storage unit. The maximum power available from the grid could be limited due to the limited rating of a distribution transformer or a feeder line. The limits on the DG and storage could be due to space constraints. These design constraints along with their assumed values for the purpose of analyzing example cases in this thesis are summarized in Table 3-.

Additional constraints are imposed by physical laws and the connections between the components; including the following from energy conservation:

Table 3-1: Design constraints.

Design Constraint	Description	Unit	Value
$P_{G,max(p)}$	Maximum power allowed from the grid	kW	5
$P_{G,max(n)}$	Maximum power allowed into the grid	kW	0
$P_{DG,max}$	Maximum power rating of the renewable distributed generation source	kW	10
$P_{S,max}$	Maximum power rating of the storage unit	kW	10
$E_{S,max}$	Maximum energy capacity of the storage unit	kWh	100
SOC_{max}	Maximum fractional SOC of the storage unit (between 0 and 1)	-	1
SOC_{min}	Minimum fractional SOC of the storage unit (between 0 and 1)	-	0.80

$$P_G(t) + P_{DG}(t) + P_S(t) = P_C(t). \tag{3.5}$$

An additional constraint is on the instantaneous power from the renewable DG:

$$P_{DG}(t) = P_{DG,r} f_{DG}(t) \ , \tag{3.6}$$

where $f_{DG}(t)$ is the normalized output power profile of the renewable distributed generation source, and varies between 0 and 1 due to variation in solar irradiation or wind speed.

Also, the energy in the storage unit is related to the power drawn from it by:

$$E_S(t) - E_S(t - \Delta t) = -\int_{t-\Delta t}^{t} \tilde{P}_S(t')dt' \ , \tag{3.7}$$

where $\tilde{P}_S(t)$ is given by:

$$\tilde{P}_S(t) = \begin{cases} \dfrac{P_S(t)}{\sqrt{\eta}}, & P_S(t) > 0 \\ \sqrt{\eta} P_S(t), & P_S(t) \le 0 \end{cases} \ , \tag{3.8}$$

and incorporates the effect of the round-trip efficiency of the storage unit, η. It is assumed that the round-trip energy loss is divided equally between discharging and charging of the storage unit. When the storage unit is discharging, it will supply less energy than what was stored. To account for this, when $P_s(t)$ is positive, $\tilde{P}_S(t)$ becomes greater than $P_s(t)$ since $\sqrt{\eta}$ is less than one. There will be more energy drawn internally from the storage unit, but due to the losses, the output power $P_s(t)$ is smaller than $\tilde{P}_S(t)$ and the output energy is less than what is drawn internally. When the storage unit is charging, it will store less energy than is supplied to it. This is why when $P_s(t)$ is negative, $\tilde{P}_S(t)$ becomes less than $P_s(t)$ since $\sqrt{\eta}$ is less than one. The lower $\tilde{P}_S(t)$ effectively accounts for the energy losses and therefore the actually energy that is stored is less than the energy that was supplied.

Additionally, the system must return the storage unit to its original SOC over some time period, T, i.e.,

$$E_S(T) - E_S(0) = -\int_0^T \tilde{P}_S(t')dt' = 0 . \tag{3.9}$$

As mentioned in chapter 2, *T* is taken to be one day, unless otherwise specified.

Finally, the energy and instantaneous power drawn from the grid are related as follows:

$$E_{G,r} = \int_0^T P_G(t')dt' . \tag{3.10}$$

3.2 Linear Programming Formulation

In order to find the minimum lifecycle cost architecture the optimization problem described in the previous section is framed as a linear programming one. First it is framed for the case of one PEV charger, one renewable DG, one storage unit, and one connection with the grid. This setup is shown in Fig. 3-2.

A linear programming problem can be expressed in conical form as:

$$\min_{\boldsymbol{x}} (\boldsymbol{c}^T\boldsymbol{x}) \quad \text{subject to} \begin{cases} \boldsymbol{A}\cdot\boldsymbol{x} \le \boldsymbol{b} \\ \boldsymbol{D}\cdot\boldsymbol{x} = \boldsymbol{g} \\ \boldsymbol{l}_{min} \le \boldsymbol{x} \le \boldsymbol{l}_{max} \end{cases} , \tag{3.11}$$

where $\boldsymbol{c}^T\boldsymbol{x}$ is the cost function to be minimized, $\boldsymbol{c}$ is the cost vector and $\boldsymbol{x}$ is the feasible vector consisting of the decision variables [102]. In this case the cost function $\boldsymbol{c}^T\boldsymbol{x}$ is equal to the variable part of the system lifecycle cost expression developed in Chapter 2. The vectors $\boldsymbol{b}$, $\boldsymbol{g}$, $\boldsymbol{l}_{min}$ and $\boldsymbol{l}_{max}$ and the matrices $\boldsymbol{A}$ and $\boldsymbol{D}$ model the constraints of the system, in our case given by (3.1)-(3.10). Based on the system lifecycle cost model of (2.1), the decision variables in $\boldsymbol{x}$ could simply be $P_{G,r}$, $P_{DG,r}$, $P_{S,r}$, $E_{S,r}$, and $E_{G,r}$. However, since some of the constraints are on the instantaneous values of $P_G(t)$, $P_{DG}(t)$, $P_S(t)$ and $E_S(t)$, these must also be represented in $\boldsymbol{x}$. Hence, $\boldsymbol{x}$ can be expressed as:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}_G \\ P_{G,r} \\ \boldsymbol{p}_{DG} \\ P_{DG,r} \\ \boldsymbol{p}_S \\ P_{S,r} \\ \boldsymbol{\Delta e}_S \\ E_{S,r} \\ E_{G,r} \end{bmatrix} , \tag{3.12}$$

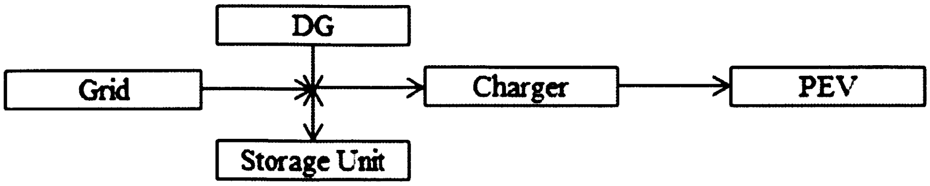


Figure 3-1: Residential PEV charging configuration.

where $\boldsymbol{p}_G$, $\boldsymbol{p}_{DG}$, and $\boldsymbol{p}_S$ are vectors representing the discrete time output power profiles of the grid, renewable DG and storage unit, respectively; and $\boldsymbol{\Delta e}_S$ represents the delta in $E_s(t)$, the energy stored in the storage unit, i.e.:

$$\boldsymbol{p}_G = \begin{bmatrix} p_{G1} \\ p_{G2} \\ p_{G3} \\ \vdots \\ p_{GN} \end{bmatrix}, \tag{3.13}$$

$$\boldsymbol{p}_{DG} = \begin{bmatrix} p_{DG1} \\ p_{DG2} \\ p_{DG3} \\ \vdots \\ p_{DGN} \end{bmatrix}, \tag{3.14}$$

$$\boldsymbol{p}_S = \begin{bmatrix} p_{S1} \\ p_{S2} \\ p_{S3} \\ \vdots \\ p_{SN} \end{bmatrix}, \tag{3.15}$$

$$\boldsymbol{\Delta e}_S = \begin{bmatrix} \Delta e_{S1} \\ \Delta e_{S2} \\ \Delta e_{S3} \\ \vdots \\ \Delta e_{SN} \end{bmatrix}. \tag{3.16}$$

These four vectors are of length N, where N equals the time period, T, divided by the desired time step, Δt. Hence, $\boldsymbol{x}$ is of length $4N+5$. To match the variable part of (2.1) to $\boldsymbol{c}^T\boldsymbol{x}$, $\boldsymbol{c}$ is expressed as:

$$\boldsymbol{c} = \begin{bmatrix} \boldsymbol{0} \\ C_G' \cdot \dfrac{T_{life,sys}}{T_{bp}} \\ \boldsymbol{0} \\ C'_{DG} \cdot \mathrm{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,DG}}\right) \\ \boldsymbol{0} \\ C'_S \cdot \mathrm{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,S}}\right) \\ \boldsymbol{0} \\ C_S'' \cdot \mathrm{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,S}}\right) \\ C_G'' \cdot \dfrac{T_{life,sys}}{T} \end{bmatrix}, \tag{3.17}$$

where each zero vector is of length N.

3.2.1 Development of the Constraint $\boldsymbol{A} \cdot \boldsymbol{x} \leq \boldsymbol{b}$

This inequality will be used to model the constraints given by (3.1)-(3.4). These constraints can be expressed in such a way that $\boldsymbol{b}$ is a zero vector. Its length is the same as the number of rows in $\boldsymbol{A}$. In our formulation $\boldsymbol{A}$ is a matrix with $4N+5$ columns and $4N$ rows. It takes the form:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_3 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_4 & \boldsymbol{0} \end{bmatrix}, \tag{3.18}$$

where each submatrix, $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3, \boldsymbol{A}_4$ has N rows and $N+1$ columns.

$\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ model the constraint on the output power of the grid and the renewable DG, respectively. At each time instant the output power from the grid must be less than or equal to $P_{G,r}$, i.e.:

$$p_{Gn} - P_{G,r} \leq 0 \text{ for all } 1 \leq n \leq N\,. \tag{3.19}$$

Similarly, the output power from the renewable DG must be less than or equal to $P_{DG,r}$ at each time instant, i.e.:

$$p_{DGn} - P_{DG,r} \leq 0 \text{ for all } 1 \leq n \leq N\,. \tag{3.20}$$

Due to its location, $\boldsymbol{A}_1$ is multiplied against the portion of $\boldsymbol{x}$ that contains $\boldsymbol{p}_G$, and $P_{G,r}$. $\boldsymbol{A}_2$ is multiplied against the portion of $\boldsymbol{x}$ that contains $\boldsymbol{p}_{DG}$, and $P_{DG,r}$. Hence, $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ can be expressed as:

$$\boldsymbol{A}_1 = \boldsymbol{A}_2 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & -1 \\ 0 & 1 & 0 & \cdots & 0 & 0 & -1 \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix}. \tag{3.21}$$

When formulated this way, the first row of $\boldsymbol{A}_1$ multiplied against $\boldsymbol{p}_G$, and $P_{G,r}$ will yield:

$$p_{G1} + 0 + 0 + \cdots + 0 + 0 - P_{G,r} \leq 0\ , \tag{3.22}$$

where the zero on the right hand side of the inequality comes from $\boldsymbol{b}$, a zero vector. Repeating this process for every row will ensure that (3.19) is upheld. The same process is true for $\boldsymbol{A}_2$ multiplied against $\boldsymbol{p}_{DG}$, and $P_{DG,r}$ to uphold (3.20).

Development of $\boldsymbol{A}_3$ is a little more involved. The storage unit can either supply power or draw power. $P_{S,r}$ is determined by the magnitude of the maximum power supplied or magnitude of the maximum power drawn. In any given situation, it is not clear which magnitude is greater. This means that $P_{S,r}$ is equal to the absolute value of the largest magnitude element of $\boldsymbol{p}_S$. Hence, the structure of $\boldsymbol{A}_3$ depends on the sign (positive or negative) of each element of $\boldsymbol{p}_S$. If the storage unit is charging, then $\boldsymbol{p}_{Sn}$ is negative or if the storage unit is discharging, then $\boldsymbol{p}_{Sn}$ is negative. The power drawn by the charger $\boldsymbol{p}_C$, where:

$$\boldsymbol{p}_C = \begin{bmatrix} p_{C1} \\ p_{C2} \\ p_{C3} \\ \vdots \\ p_{CN} \end{bmatrix}, \tag{3.23}$$

is used to determine if the storage unit is charging or discharging. It is assumed that there will not be excess generation to supply both the charger and the storage. If the PEV is charging, then the storage unit will not also be charging, it can only be discharging. When the vehicle is charging, p_{Cn} and p_{Sn} are non-negative. The power rating, $P_{S,r}$, has to be greater than the value of p_{Sn}:

$$p_{Sn} - P_{S,r} \leq 0 \quad \text{if}\, p_{Cn} > 0 \ . \tag{3.24}$$

If there is no PEV charging, the storage unit should not be discharging. This relates to the power flow control. The optimal storage unit size is to supply the PEV with sufficient energy when the grid or the renewable DG cannot. Therefore, if p_{Cn} is zero or negative, then the storage unit should only be capable of charging and p_{Sn} will be negative. The power rating, $P_{S,r}$, has to be greater than the negative of the value of p_{Sn}:

$$-p_{Sn} - P_{S,r} \leq 0 \quad \text{if}\, p_{Cn} \leq 0 \ . \tag{3.25}$$

In order to ensure (3.24) and (3.25) are satisfied, A_2 has to incorporate p_{Cn} in its development:

$$A_3 = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 & 0 & -1 \\ 0 & a_{22} & 0 & \cdots & 0 & 0 & -1 \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & a_{NN} & -1 \end{bmatrix} \text{where}\, a_{nn} = \begin{cases} -1, & p_{Cn} \leq 0 \\ 1, & p_{Cn} > 0 \end{cases} , \tag{3.26}$$

where n is the row of A_3. Due to its location, A_3 is multiplied against the portion of x that contains p_S, and $P_{S,r}$.

Lastly, A_4 relates to the energy capacity of the storage. There are constraints on the minimum and maximum SOC for the storage unit. These constraints were established in chapter 2 in order to extend the life of the storage unit and to increase reliability for contingencies. Under designed operation, the storage unit should not go below the minimum SOC or above the maximum SOC. The minimum SOC is determined by multiplying $E_{S,r}$ by SOC_{min} and the maximum SOC is determined by multiplying $E_{S,r}$ by SOC_{max}. The storage unit will charge and discharge such that the stored energy will remain between the minimum SOC and the maximum SOC. When the storage unit is charging, Δe_S will increase. When the storage unit is discharging, Δe_S will decrease. The energy capacity of the storage unit, $E_{S,r}$, will be determined by the maximum of Δe_S, the minimum SOC and maximum SOC. The maximum of Δe_S is:

$$\max(\Delta e_S) = (SOC_{max} - SOC_{min})E_{S,r} \ . \tag{3.27}$$

This is used to determine $E_{S,r}$ as follows:

$$\Delta e_{Sn} - (SOC_{max} - SOC_{min})E_{S,r} \leq 0 \ . \tag{3.28}$$

In this way, $E_{S,r}$ is compared to every value in Δe_S because it is unknown which element of Δe_S is the maximum. To implement this, A_4 is:

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & -(SOC_{max} - SOC_{min}) \\ 0 & 1 & 0 & \cdots & 0 & 0 & -(SOC_{max} - SOC_{min}) \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & -(SOC_{max} - SOC_{min}) \end{bmatrix} . \tag{3.29}$$

A_4 is multiplied against the portion of $\boldsymbol{x}$ that contains Δe_S and $E_{S,r}$.

3.2.2 Development of the Constraint $\boldsymbol{D} \cdot \boldsymbol{x} = \boldsymbol{g}$

This equality ensures that the constraints given by (3.5)-(3.10) are met. $\boldsymbol{D}$ is a matrix with $4N$+5 columns and $3N$+1 rows; $\boldsymbol{g}$ will be a vector whose length is $3N$+1, based on the number of rows in $\boldsymbol{D}$. $\boldsymbol{D}$ takes the following form:

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{D}_1 & \boldsymbol{D}_1 & \boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_3 & \boldsymbol{D}_4 & \boldsymbol{0} \\ \boldsymbol{d}_5 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -1 \end{bmatrix} , \tag{3.30}$$

where each submatrix $\boldsymbol{D}_1$, $\boldsymbol{D}_2$, $\boldsymbol{D}_3$ and $\boldsymbol{D}_4$ has N rows and N+1 columns and the row vector $\boldsymbol{d}_5$ has N+1 columns.

$\boldsymbol{D}_1$ is used to model the conservation of energy established by (3.5), i.e.:

$$p_{DGn} + p_{Sn} + p_{Gn} = p_{Cn} \ , \tag{3.31}$$

where n is the row of $\boldsymbol{D}_1$. The vectors $\boldsymbol{p}_{DG}$, $\boldsymbol{p}_S$ and $\boldsymbol{p}_G$ are contained in the vector $\boldsymbol{x}$ and will by multiplied by $\boldsymbol{D}_1$. Therefore, $\boldsymbol{D}_1$ has the following form to satisfy (3.31):

$$\boldsymbol{D}_1 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix} . \tag{3.32}$$

Since $\boldsymbol{p}_C$ is not within $\boldsymbol{x}$, it has to be represented in $\boldsymbol{g}$:

$$\boldsymbol{g} = \begin{bmatrix} \boldsymbol{p}_C \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} . \tag{3.33}$$

Here, the remainder of the vector $\boldsymbol{g}$ is zero as the remaining equality constraints can be satisfied with $\boldsymbol{D}_2$, $\boldsymbol{D}_3$, $\boldsymbol{D}_4$ and $\boldsymbol{d}_5$ similarly to what was done in the previous section with $\boldsymbol{b}$ as a zero vector.

$\boldsymbol{D}_2$ is used to model (3.6) in order to scale the output power of the renewable DG to $f_{DG}(t)$, i.e.:

$$p_{DGn} - P_{DG,r} \, f_{DGn} = 0 \ , \tag{3.34}$$

where f_{DGn} is the n^{th} element of $\boldsymbol{f}_{DG}$, the discrete time version of $f_{DG}(t)$:

$$\boldsymbol{f}_{DG} = \begin{bmatrix} f_{DG1} \\ f_{DG2} \\ f_{DG3} \\ \vdots \\ f_{DGN} \end{bmatrix} . \tag{3.35}$$

$\boldsymbol{D}_2$ will have the following form which incorporates the $\boldsymbol{f}_{DG}$ factor:

$$\boldsymbol{D}_2 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & -f_{DG1} \\ 0 & 1 & 0 & \cdots & 0 & 0 & -f_{DG2} \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & -f_{DGN} \end{bmatrix} . \tag{3.36}$$

Next, $\boldsymbol{e}_S$ is calculated using $\boldsymbol{D}_3$ and $\boldsymbol{D}_4$. To calculate the n^{th} element of $\Delta\boldsymbol{e}_S$, both the previous element of $\Delta\boldsymbol{e}_S$ and n^{th} element of $\boldsymbol{p}_S$ are required. Inefficiency is incorporated depending on whether the storage unit is charging or discharging as seen in equation (3.8). Using the same assumption as before, the storage unit should only be discharging when the vehicle is charging because it is assumed that the storage unit and the vehicle will not be charging simultaneously. When the vehicle is charging, the relationship between the delta of the stored energy and the power drawn from the storage unit is:

$$\Delta e_{Sn} - \Delta e_{Sn-1} + \frac{p_{Sn}}{\sqrt{\eta}} \cdot \Delta t = 0 \text{ when } p_{Cn} > 0 \ , \tag{3.37}$$

where Δt is the time step. When the vehicle is not charging, the delta of the stored energy and power drawn are related as:

$$\Delta e_{Sn} - \Delta e_{S\,n-1} + p_{Sn} \cdot \sqrt{\eta} \cdot \Delta t = 0 \text{ for } p_{Cn} \leq 0 \ . \tag{3.38}$$

$\boldsymbol{D}_3$ is multiplied against the portion of $\boldsymbol{x}$ which contains $\boldsymbol{p}_S$. By doing so, the elements of $\boldsymbol{p}_S$ will be scaled based off of $\boldsymbol{p}_C$ to account for the third term in (3.37) and (3.38):

$$\boldsymbol{D}_3 = \begin{bmatrix} d_{11} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & d_{22} & 0 & \cdots & 0 & 0 & 0 \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & d_{NN} & 0 \end{bmatrix} \text{ where } d_{nn} = \begin{cases} \sqrt{\eta}\Delta t, & p_{Cn} \leq 0 \\ \dfrac{1}{\sqrt{\eta}}\Delta t, & p_{Cn} > 0 \end{cases} . \tag{3.39}$$

$\boldsymbol{D}_4$ is multiplied with the portion of $\boldsymbol{x}$ that contains $\boldsymbol{e}_S$. In doing so, the necessary elements of $\boldsymbol{e}_S$ will be summed together according to the first two terms in (3.37) and (3.38):

$$\boldsymbol{D}_4 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & -1 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 & 0 \end{bmatrix} . \tag{3.40}$$

In $\boldsymbol{D}_4$, the first row shows that the last element in $\boldsymbol{e}_S$ is subtracted from the first element in $\boldsymbol{e}_S$. In doing so, (3.9) is satisfied because it ensures that the difference between the final SOC and the initial SOC is attributed to the storage power and not from a mismatch in energy balance.

Lastly, $E_{G,r}$ is simply a scaled summation of all of the elements in $\boldsymbol{p}_G$ based on (3.10):

$$(p_{G1} + p_{G2} + \cdots + p_{GN}) \cdot \Delta t - E_{G,r} = 0 \, . \tag{3.41}$$

This is implemented by the row vector $\boldsymbol{d_5}$. Due to its location, $\boldsymbol{d_5}$ is multiplied with the portion of $\boldsymbol{x}$ that contains $\boldsymbol{p_G}$. $\boldsymbol{d_5}$ is simply given by the scalar coefficient for all but the last term in (3.41)

$$\boldsymbol{d}_5 = [\Delta t \cdots \Delta t \ 0]\,. \tag{3.42}$$

The $E_{G,r}$ term in (3.41) is accounted for directly in $\boldsymbol{D}$ where the last element of the matrix is a negative one.

3.2.3 Development of the Constraint $\boldsymbol{l}_{min} \leq \boldsymbol{x} \leq \boldsymbol{l}_{max}$

The last part of developing the linear programming framework is the maximum and minimum constraints on the decision variables as defined by (3.1)-(3.4). The $\boldsymbol{l}_{min}$ vector represents the minimum constraints and the $\boldsymbol{l}_{max}$ vector represents the minimum constraints. These constraints apply to the vectors, $\boldsymbol{p}_{DG}$, $\boldsymbol{p}_S$, $\boldsymbol{p}_G$ and $\boldsymbol{e}_S$, as well as the scalar ratings, $P_{DG,r}$, $P_{S,r}$, $P_{G,r}$, $E_{S,r}$, and $E_{S,r}$. The two vectors are quite simply:

$$\boldsymbol{l}_{min} = \begin{bmatrix} -\boldsymbol{p}_{G,max(n)} \\ \boldsymbol{p}_{DG,min} \\ \boldsymbol{p}_{S,min} \\ SOC_{min}\boldsymbol{e}_{S,min} \\ E_{G,min} \end{bmatrix}, \tag{3.43}$$

$$\boldsymbol{l}_{max} = \begin{bmatrix} \boldsymbol{p}_{G,max(p)} \\ \boldsymbol{p}_{DG,max} \\ \boldsymbol{p}_{S,max} \\ SOC_{max}\boldsymbol{e}_{S,max} \\ E_{G,max} \end{bmatrix}, \tag{3.44}$$

where each vector, $\boldsymbol{p}_{G,max(n)}$, $\boldsymbol{p}_{DG,min}$, $\boldsymbol{p}_{S,min}$, $\boldsymbol{e}_{s,min}$, $\boldsymbol{p}_{G,max(p)}$, $\boldsymbol{p}_{DG,max}$, $\boldsymbol{p}_{S,max}$, and $\boldsymbol{e}_{s,max}$, is of length N+1. The value of each element within each vector is equal to corresponding minimum or maximum constraint listed in Table III. For example:

$$\boldsymbol{p}_{G,max(n)} = \begin{bmatrix} p_{G,\max(n)} \\ p_{G,\max(n)} \\ p_{G,\max(n)} \\ \vdots \\ p_{G,\max(n)} \end{bmatrix}. \tag{3.45}$$

Minimum constraints that are not listed in Table III are taken to be zero. Maximum constraints not listed in Table III are taken to be large enough in order to not limit the design space with respect to that constraint.

This linear programming formulation is implemented in MATLAB.

3.3 Search based Formulation

To validate the results of the linear programming based optimization, and to investigate its design decisions, a search based optimization technique is also developed and implemented in MATLAB. In this approach the renewable DG and storage unit are sized for different design options. That is to say, one design may be generated with a very large renewable DG system and one design may be generated with a very small renewable DG system. The alternate designs are compared in terms of their system lifecycle cost. To keep the search manageable the search is performed across only a limited set of feasible designs. Hence, the success of this approach requires ensuring that the feasible designs considered include the optimal design. For the cases consider in chapter 4, a feasible design must include storage, however, designs with and without renewable DG are feasible.

The sizing algorithm depends upon whether or not the renewable DG is present. In architectures where a renewable distributed generator is present, the algorithm takes an iterative approach. It starts with $P_{DG,r}$ equal to zero, computes the change in the SOC of the storage unit over the time period T using (3.9), and keeps increasing the size of the renewable DG until the change in the state-of-charge is zero. For this SOC calculation, $P_S(t)$ must be known and is computed using (3.5), where $P_{DG}(t)$ is given by (3.6) and $P_G(t)$ is calculated from:

$$P_G(t) = \min\left(P_{G,r}, \max\left(-P_{G,max(n)}, P_C(t) - P_{DG}(t)\right)\right) \ . \tag{3.46}$$

In developing (3.46) it is assumed that the power flow in the system is controlled in such a way that the PEV charger first takes power from the renewable DG. When the renewable DG cannot meet the full power needs of the charger, the remaining power is first drawn from the grid. Only when the grid power hits its limit is power drawn from the storage unit. This power flow control assumption limits the architectures that are searched. However, since this control methodology minimizes the amount of storage needed, this approach will search through the space of potential least cost architectures that contain renewable DG. This sizing algorithm is parameterized in terms of $P_{G,r}$, and used to generate multiple designs by sweeping across different values of $P_{G,r}$ in the range of 0 through $P_{G,max(p)}$.

In architectures where no renewable DG is present, a slightly different approach is followed. This is necessary because now instead of the renewable DG, the grid will have to charge the storage unit. It will do so when the electric vehicle is not charging. To incorporate this change in the control methodology, the energy that has to be supplied to the charger from the storage unit when the electric vehicle is charging must be calculated by:

$$E_{S2C} = \int_0^{\mathrm{T}} P_{S2C}(t)dt \ , \tag{3.47}$$

where $P_{S2C}(t)$ is the power supplied to the charger from the storage unit according to:

$$P_{S2C}(t) = \begin{cases} 0, & P_C(t) \leq 0 \\ P_C(t) - P_{G,r}, & P_C(t) > 0 \end{cases} \ . \tag{3.48}$$

The energy taken from the storage unit must be returned to it from the grid when the electric vehicle is not charging. Hence, the output power flowing from the storage unit at all times can be expressed as:

$$P_S(t) = \begin{cases} -\frac{1}{\eta} \cdot \frac{E_{S2C}}{T_{C,off}}, & P_C(t) \leq 0 \\ P_C(t) - P_{G,r}, & P_C(t) > 0 \end{cases} . \tag{3.49}$$

Here it is assumed that the system controller recharges the battery evenly across the time period when the PEV is not charging, $T_{C,off}$. Now that the output power profile of the storage unit is known, the instantaneous power drawn from the grid is:

$$P_G(t) = \min\left(P_{G,r}, \max\left(-P_{G,max(n)}, P_C(t) - P_S(t)\right)\right) . \tag{3.50}$$

For either case (with or without DG), once $P_G(t)$, $P_{DG}(t)$ and $P_S(t)$ that satisfy the state-of-charge requirement have been determined, the sizing algorithm proceeds to calculate the remaining three system ratings needed for system lifecycle cost calculation. Of these $E_{G,r}$ is calculated using (3.10); and the other two are calculated using:

$$P_{S,r} = \max_{\mathrm{t}}(|P_S(t)|) , \tag{3.51}$$

$$E_{S,r} = \frac{\max_{\mathrm{t}}(\Delta \mathrm{E}_s(t)) - \min_{\mathrm{t}}(\Delta \mathrm{E}_s(t))}{SOC_{max} - SOC_{min}} , \tag{3.52}$$

where $\max_t()$ and $\min_t()$ return the maximum and minimum value of their argument over $0 \leq t \leq T$, and $\Delta E_s(t)$ is given by:

$$\Delta E_S(t) \equiv E_S(t) - E_S(0) = -\int_0^t \tilde{P}_S(t')dt' . \tag{3.53}$$

Once the system has been designed the algorithm also checks to make sure that the following design constraints are satisfied:

$$P_{DG,r} \leq P_{DG,\max} , \tag{3.54}$$

$$P_{S,r} \leq P_{S,\max} , \tag{3.55}$$

$$E_{S,r} \leq E_{S,\max} . \tag{3.56}$$

If these are not satisfied the design is discarded. This search based formulation is represented graphically in Fig. 3-2.

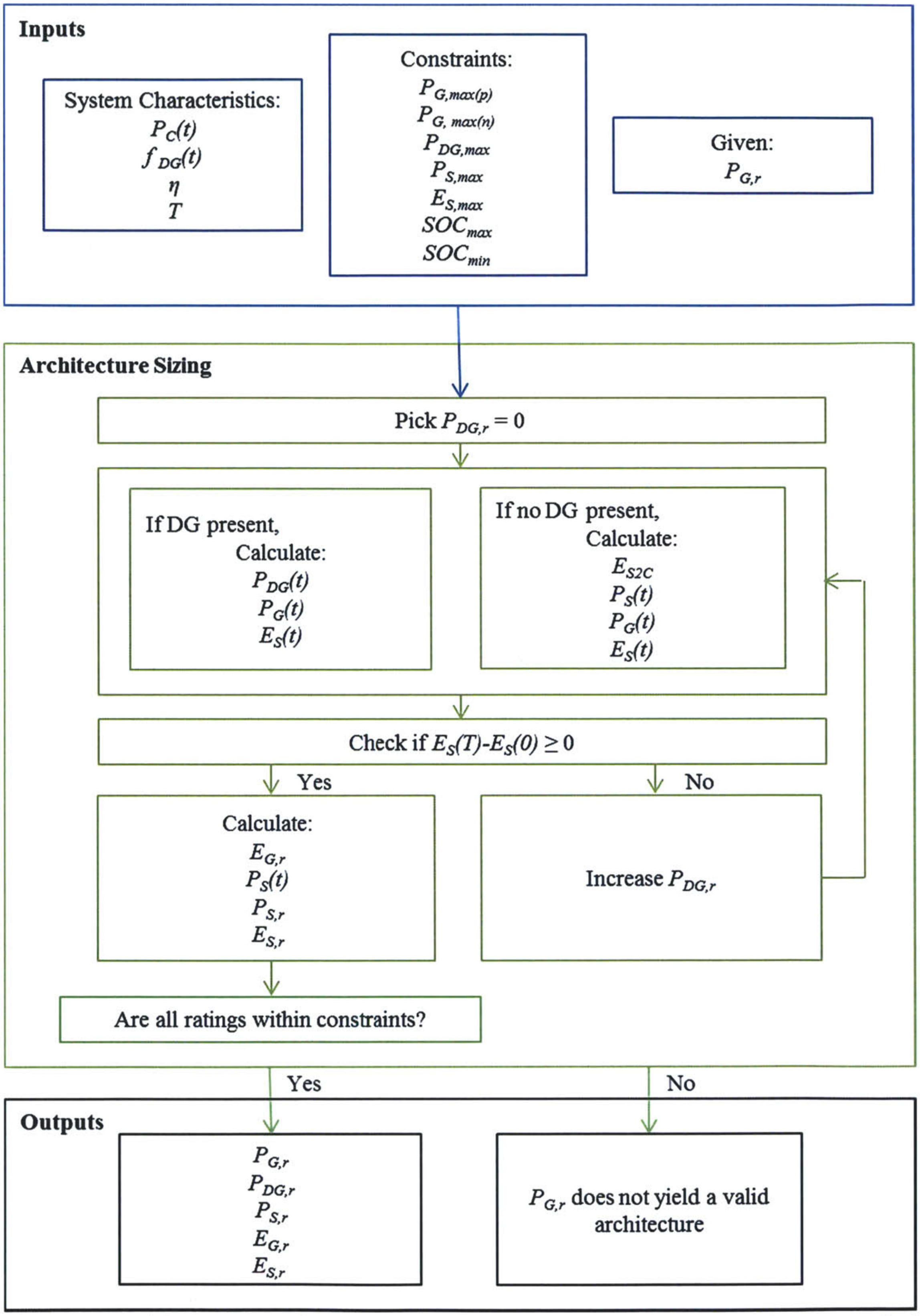


Figure 3-2: Search based process for sizing a single design given EV charging requirements and other system constraints.

Chapter 4

Optimal Design Results and Discussion for a Single Charger

The optimization methodology developed in the previous chapter is applied to the design of a residential PEV charging case. The two formulations of the optimization methodology developed in Chapter 3 are used and their results compared. The analysis uses the cost parameters developed in Chapter 2. It also uses solar irradiation and wind speed profile data for various locations in the US.

4.1 Residential Charging Case Study Overview

The first case analyzed using the optimization methodology is that of a residential charging system. It is assumed that an electric vehicle with a 30 kWh battery has to be fully recharged every evening from 20% state-of-charge. It is also assumed that the owner desires to charge the vehicle within 4 hours using a 6 kW Level 2 charger starting at 7 pm each day. However, the neighborhood in which the house is located is serviced by a 25 kVA transformer and the typical load between 7 pm and 11 pm of all the houses serviced by this transformer is 20 kW, excluding the charging needs of the EV. Hence, the maximum power that can be drawn from the grid, $P_{G,max(p)}$, without overloading the transformer is 5 kW.

Clearly the system requirements cannot be met with a grid-only charging solution, as the charging rate (6 kW) exceeds the maximum allowable power draw from the grid. Hence, a storage unit and/or renewable DG will be needed. The storage unit will be necessary with solar PV systems as the solar panel cannot produce any power in the late evening hours when the vehicle is to be charged. In this thesis, designs with a solar PV distributed generator and a lead-acid electrochemical storage unit (battery system) are considered for two locations in the US: Eugene, OR and Los Angeles, CA. Then, designs with a solar PV distributed generator and a Li-ion electrochemical storage unit are compared with the designs using a lead-acid electrochemical storage unit. Finally, designs with a small wind turbine system and a lead-acid electrochemical storage unit are considered for two locations as well: Los Angeles, CA and Boulder, CO.

For each location, an optimal design is determined based on a 20-year system lifecycle cost using the linear programming formulation of Chapter 3. The parameters and design constraints given in Table 2-1 and Table 3-1 are used and the round-trip efficiency of the storage unit, η, is assumed to be 85%. The results are validated using the search-based optimization approach. The linear programming approach is an order of magnitude faster than the search-based one (3.88 s versus 41.14 s for the residential case). However, it requires large memory resources to store matrices $\boldsymbol{A}$ and $\boldsymbol{D}$. For the residential charging case, it takes four times more memory to run the linear programming tool than it does to run the search-based one (~800 MB versus ~200 MB).

4.2 Dependence on Solar Irradiation Profile

An important parameter in the optimization methodology is the normalized output power profile of the renewable DG, f_{DG}. Figure 4-2 shows the variation in f_{DG} over a day for high (95th percentile), median (50th percentile) and low (5th percentile) output power profile cases using 5-minute interval irradiation data for Eugene, OR from 1995-2011 [103]. The 95th, 50th, and 5th percentile f_{DG} represent profiles that are above 95%, 50% and 5% of the data at a particular time of the day, respectively. The 95th percentile f_{DG} has a flat top because typical solar module name-plate output power ratings are based on 1000 W/m^2 solar irradiation, while the 95th percentile irradiation for Eugene goes above 1000 W/m^2 around 12 noon.

When the 95th percentile f_{DG} is used in the design of the charging system, the lowest lifecycle cost solution is one that includes a solar PV DG. The ratings of the individual components for this optimal design are given in Table 4-1. The power flow and stored energy as a function of time for this optimal design are shown in Fig. 4-2 (a). As expected the stored energy returns to its initial value after 24 hours. Note that the optimal design uses the maximum available power from the grid when the vehicle is charging. This is expected since without a high demand charge, the grid is the lowest cost alternative for peak power. However, notice that the battery is charged using the DG instead of the grid. This is because the cost of solar PV is lower than the cost of energy from the grid over the 20-year system life.

When the median f_{DG} is used in the design of the system, the lowest lifecycle cost solution is one that does not include a solar PV generator. Instead, the system relies on the grid to charge the battery for use when the vehicle is charging. This is because the lower normalized energy under the median f_{DG} profile necessitates the need for a DG with roughly twice the rating compared to the DG needed for the 95th percentile case. As a result the cost of the DG is more than the cost of energy from the grid over the system life. The power flow and stored energy for this optimal design are shown in Fig. 4-2(b).

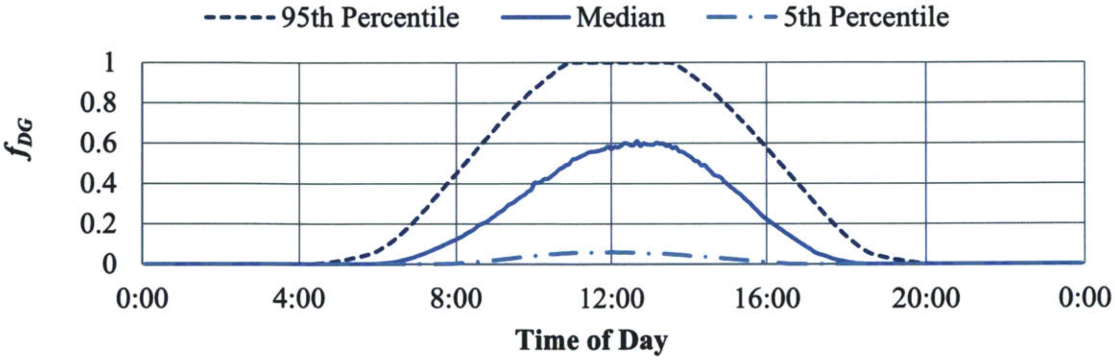


Figure 4-1: Normalized output power profile for solar PV generation at different levels of solar irradiation. Irradiation data is for Eugene, OR from 1995 to 2011 [103].

Table 4-1: Optimal design ratings and lifecycle costs for designs with solar PV systems.

f_{DG} (percentile)	$P_{DG,r}$ (kW)	$P_{S,r}$ (kW)	$P_{G,r}$ (kW)	$E_{S,r}$ (kWh)	$E_{G,r}$ (kWh)	C ($)
95th	0.578	1	5	21.64	20	45,631
50th	0	1	5	21.69	24.1	47,739

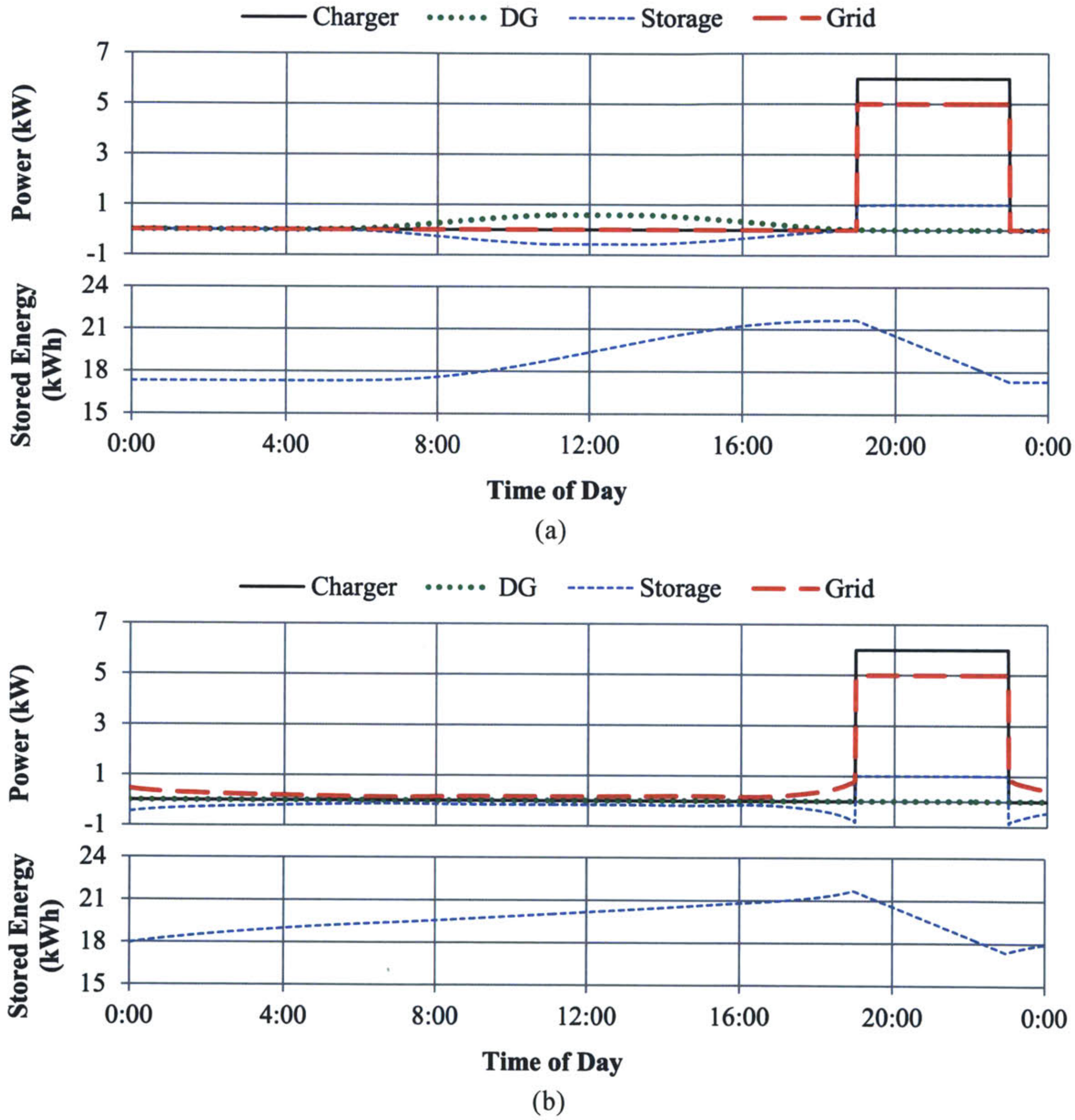


Figure 4-2: Power and energy profile for the optimal design of a residential EV charger with integrated solar PV system and lead-acid battery, assuming (a) 95th percentile and (b) median (50th percentile) normalized output power profiles of Fig. 4-1.

Clearly, whether or not the DG based design is optimal depends on the solar irradiance profile. Fig. 4-3 presents the system lifecycle cost of the optimal design for different f_{DG} profiles, ranging from 5th percentile to 95th percentile using Eugene, OR data. Up to the 50th, percentile the optimal design does not utilize a solar PV. However, once the solar irradiance is at or above the 53rd percentile level, corresponding to a noon-time irradiance of about 720 W/m^2, the design with solar PV is optimal. The median irradiance of Eugene, OR is just below this threshold. Fig. 4-3 also displays that the linear programming and search based approaches both identify the same optimal design.

4.3 Optimization Across Non-identical Days

The analysis so far has implicitly assumed that all days have identical solar irradiation profiles. In order to investigate optimal designs for systems operating over dissimilar irradiance days, an f_{DG} profile was created that models the variation in irradiance over a full year using 6 day-length irradiance profiles, where each day's irradiance represents the median values at each time point for the days of two consecutive months. This 6-day f_{DG} profile is shown for Eugene, OR and Los Angeles, CA in Fig. 4-4. The Los Angeles f_{DG} profile is based on 1-minute interval irradiation data from 2010-2011 [104]. Notice that Eugene has lower average irradiation than Los Angeles.

When the f_{DG} profiles of Fig. 4-4 are used in the design of the charging system with T set to 6 days, the optimal design for Eugene does not have a solar PV generation source, while the optimal design

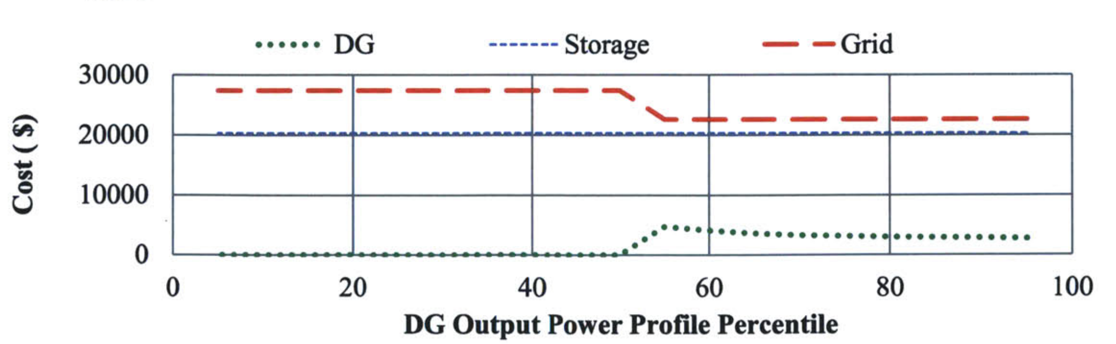
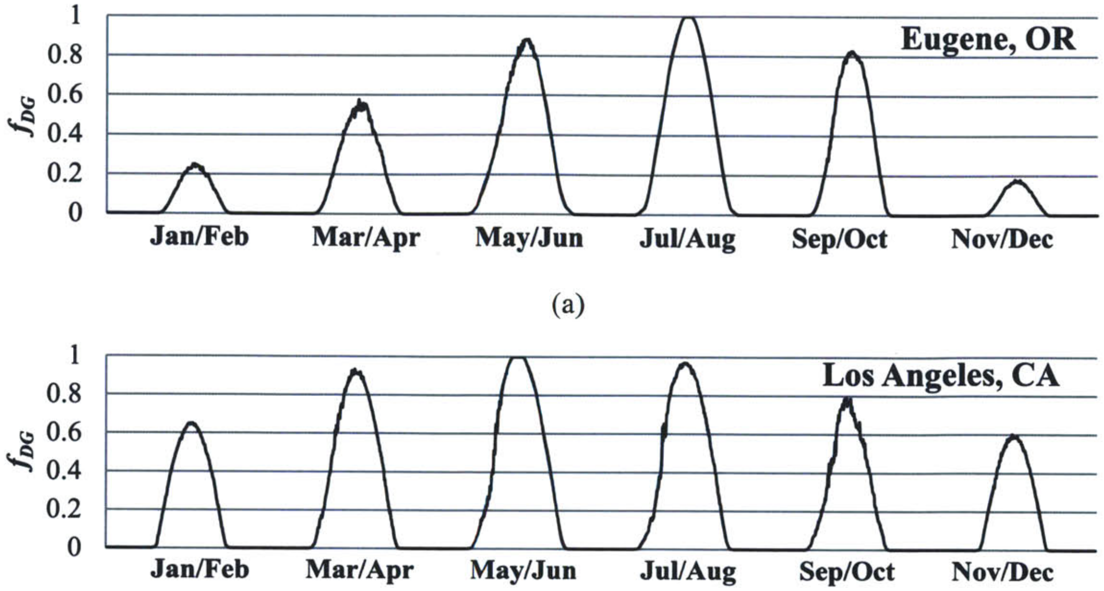


Figure 4-3: (a) System lifecycle cost and (b) components costs as the solar PV system's normalized output power profile is varied from the 5th percentile to the 95th percentile using Eugene, OR data.



(a)

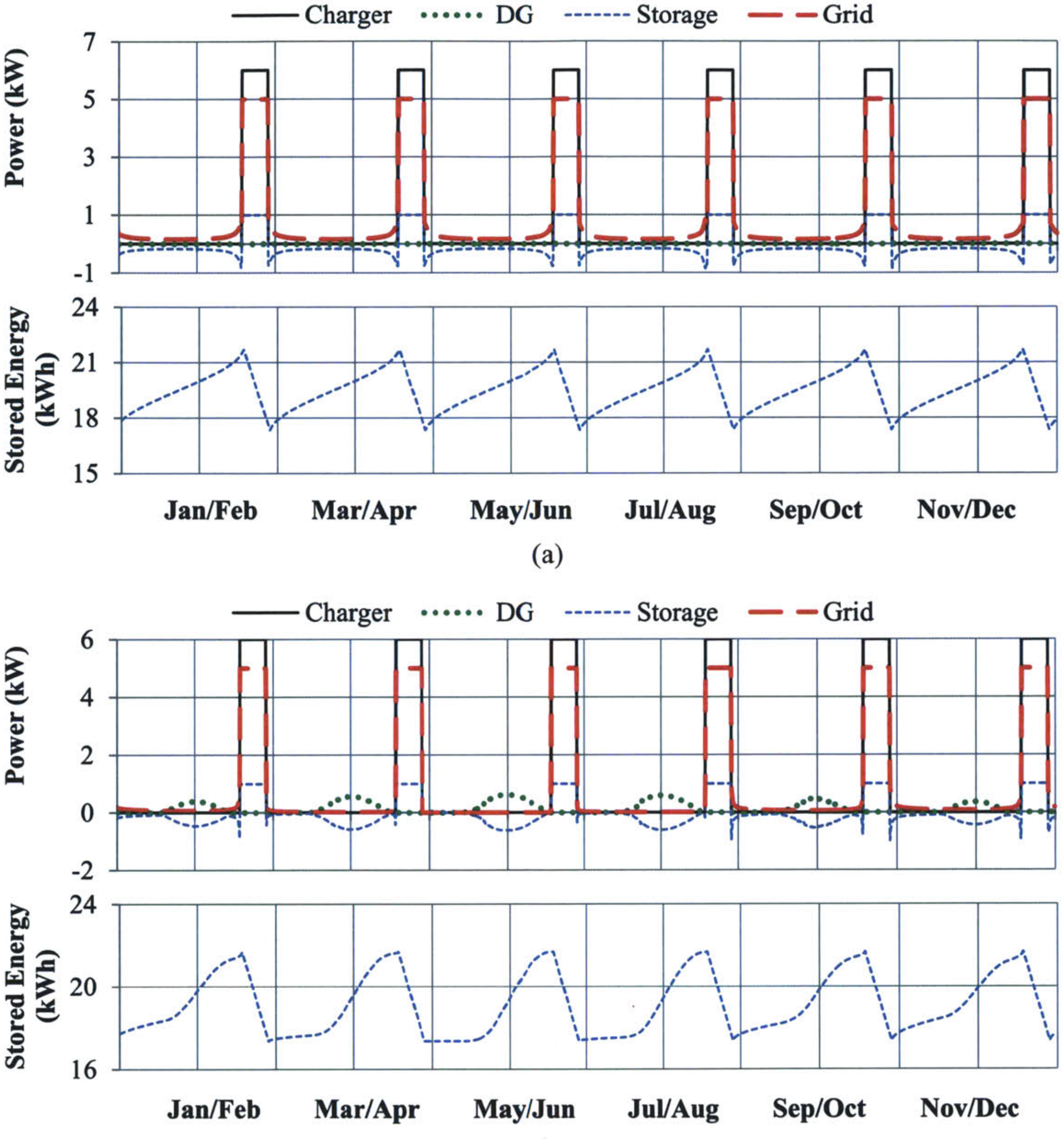
(b)

Figure 4-4: Two-month median normalized output power profiles across a year for (a) Eugene, OR and (b) Los Angeles, CA [103,104].

for Los Angeles does. The power flow and stored energy waveforms for Eugene and Los Angeles optimal designs are shown in Fig. 4-5 a) and (b), respectively. As before, the lowest cost design uses the maximum available power from the grid when the vehicle is charging. Notice also that the optimal design returns the stored energy to its initial level after each day, even though the imposed constraint is over 6 days. This is because the battery cost is a very large fraction of the total lifecycle cost, and by bringing the battery SOC to its original position after each 24-hour period, the size of the battery can be minimized.

4.4 Exploration of Li-ion Electrochemical Batteries

As can be seen from Fig. 4-3, the cost of storage is nearly 50% of the system lifecycle cost. One reason this cost is so large is that the lead-acid batteries have a life of only six years. Hence in the 20-year life of the system, four lead-acid batteries have to be purchased. Because lead-acid batteries are a very mature technology, major advancements in increasing their life or reducing their cost seems unlikely. On the other hand, Li-ion batteries are a relatively new technology with longer life than lead-acid units and with the potential for significant cost reductions over the next decade. Assuming 12-year life Li-ion batteries, the price, C''_S, at which Li-ion battery technology becomes cost competitive with lead-acid batteries is investigated. It is further examined whether or not this price for Li-ion is reasonable to expect over the next decade.



(a)

(b)

Figure 4-5: Power and energy profiles for the optimal design of a residential PEV charger with integrated solar PV generation and lead-acid storage, assuming the normalized output power profile of (a) Eugene, OR and (b) Los Angeles, CA as depicted in Fig.4-4.

When the median solar irradiation profile for Los Angeles, CA is used to find the optimal design for a PEV charging system with a solar PV generation source and a lead-acid battery, the resulting system lifecycle cost is \$46,793 of which \$20,330 is the storage cost. The lifecycle cost of systems with Li-ion storage technology depends on the specific cost, C''_S (in \$/kWh), of Li-ion batteries. The system lifecycle costs for designs with Li-ion storage technology are plotted as a function of Li-ion specific cost in Fig. 4-6. Also plotted for reference in Fig. 4-6 is the lifecycle cost of the system with a lead-acid

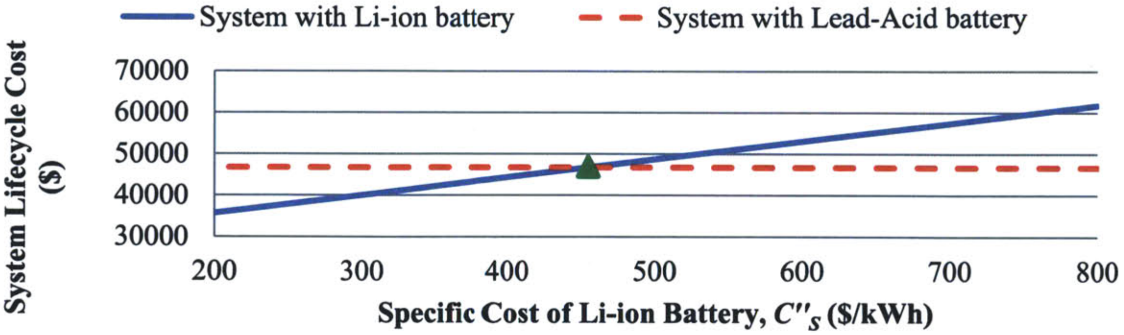


Figure 4-6: System lifecycle cost as a function of specific cost of Li-ion batteries. The triangle indicates the specific cost below which Li-ion storage technology is cost competitive with lead-acid storage technology, when it is assumed that the Li-Ion battery achieves a 12-year life while the lead-acid battery has a 6-year life.

battery. The breakeven point between Li-ion and lead-acid is marked by a triangle. When the specific cost of Li-ion storage drops below $455/kWh, Li-ion storage becomes more attractive than lead-acid storage. The current cost of Li-ion batteries as used in this thesis is $800/kWh. However, some industry analysts expect this cost to reduce by 60% over the next decade [76]. Hence, these are battery only prices and do not include other costs associated with a complete storage unit. Given these costs, it is quite feasible for Li-ion storage technology to become cost competitive with lead-acid storage technology for PEV charging applications over the next decade. However, with the current cost of Li-ion technology, designs with Li-ion batteries result in system lifecycle costs that are almost 33% higher than designs with lead-acid batteries. Therefore, all the remaining designs explored in this thesis use lead-acid battery storage.

4.5 Dependence on Wind Speed Profile

Another option for distributed renewable generation is a wind turbine. This option may offer a more cost effective solution for charging systems at locations that have reasonable wind, since the wind blows even at night when the PEV is to be charged and could reduce the required battery size. The output power of a wind turbine is a highly nonlinear function of wind speed. A normalized output power versus wind speed model for wind turbines is developed using the averaged output power data of five wind turbines from three manufacturers [35], [62], [63]. This normalized output power versus wind speed is shown in Fig. 4-7 and given by the following polynomial expression:

$$\begin{aligned} f_{DG,wind}(v_w) = &-0.000000136v_w^6 + 0.000015661v_w^5 - 0.000513088v_w^4 \\ &+ 0.005882602\,v_w^3 - 0.013104647\,v_w^2 - 0.015410784v_w \\ &+ 0.058081524 \end{aligned} \tag{4.57}$$

$$\text{for } 2.5\ m/s \le v_w \le 20\ m/s$$

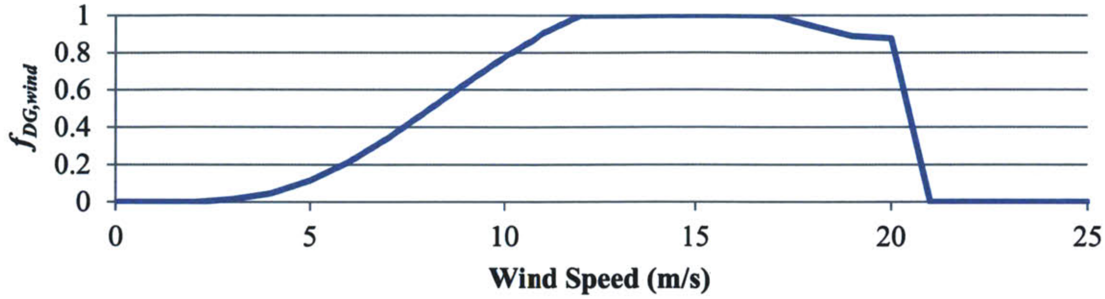
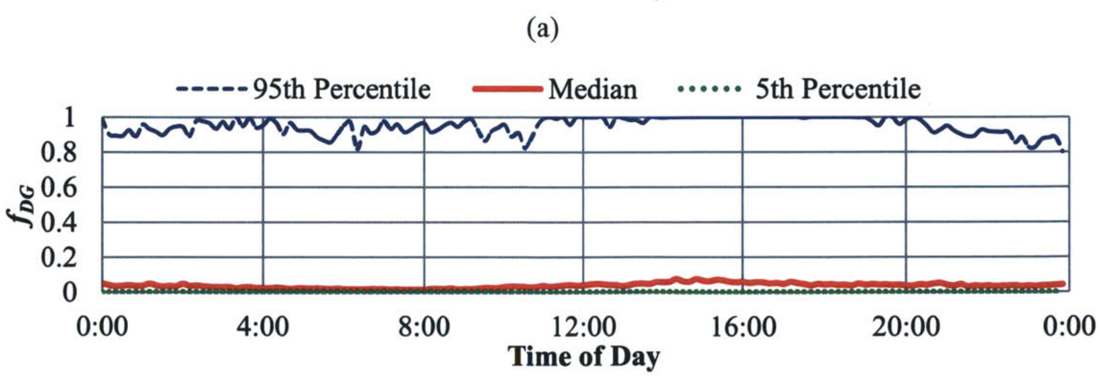


Figure 4-7: Normalized output power versus to wind speed of wind turbines used in this thesis.

As in the case of solar PV, $f_{DG,wind}$ was clipped at its maximum value of 1, even if (4.1) gave a higher value. Wind turbines have a cut-in wind speed below which they produce no power. Wind turbines also have a cut-out wind speed beyond which they produce no power in order to protect themselves from damage. In this model, the cut-in wind speed is 2.5 meters per second (m/s) and the cut-out wind speed is 20 m/s. Using this model, the output power from wind turbines installed in Los Angeles, CA and in Boulder, CO is calculated and plotted in Fig. 4-8 using actual wind speed data for 2010-2011 [104]. Figure 4-8 shows the high (95th percentile), median (50th percentile), and low (5th percentile) output power profiles for these two locations.

As can be seen from Fig. 4-8(a), in Los Angeles, even the 95th percentile wind profile utilizes less



(b)

Figure 4-8: Normalized output power profile for wind turbine generation at different percentiles of wind speed for (a) Los Angeles, CA and (b) Boulder, CO [104].

than 15% of the rated output power capability of the wind turbine. This is extremely poor utilization of the wind turbine and leads to optimal designs that do not use a wind turbine. Hence solar PV is a better option than small wind turbines for Los Angeles, CA.

In the case of Boulder, CO, the median output power profile is also quite poor and not favorable for a wind turbine in an optimal PEV charger design. On the other hand, the 95th percentile profile favors a wind turbine. These results are summarized in Table 4-2. The power flow for the 95th percentile case is shown in Fig. 4-9. As seen in Fig. 4-9, the output of the wind turbine is fairly flat across the day. This means that the storage unit is slowly charged during the day when the PEV is not charging, and when the PEV is charging, the wind turbine directly supplies it part of its required power. However, it is not reasonable to design the PEV charging system based on the 95th percentile output. Also, the wind data used in this analysis is for 50 meters height which may be on the high side for a residential application.

Therefore, while wind can be an attractive resource as it can be available at night (as well as when solar is not an option) and therefore reduce the need for storage, the available wind technology is

Table 4-2: Ratings and lifecycle costs for optimal PEV charger designs with wind turbine as the renewable generation option.

f_{DG} (percentile)	$P_{DG,r}$ (kW)	$P_{S,r}$ (kW)	$P_{G,r}$ (kW)	$E_{S,r}$ (kWh)	$E_{G,r}$ (kWh)	C ($)
95th	0.200	0.840	5	17.62	20	44,065
50th	0	1	5	21.69	24.1	47,739

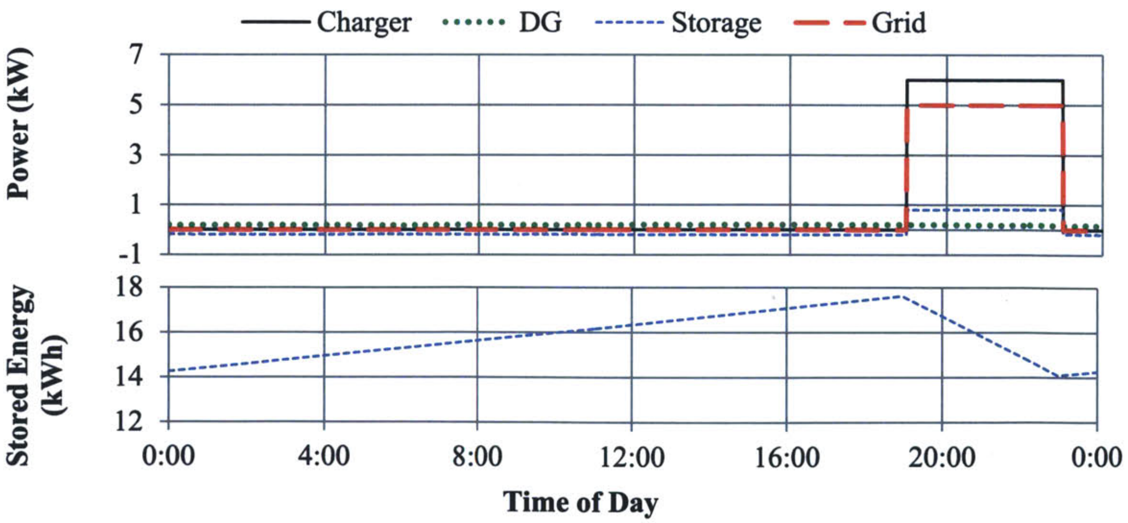


Figure 4-9: Power and energy profile for the optimal design of a residential PEV charger with integrated wind turbine and lead-acid battery, assuming the 95th percentile normalized output power profile for Boulder, CO as shown in Fig. 4-8(b).

not suitable for capturing energy from low wind speeds. It is possible that if the cut-in wind speed was lower and if the wind turbine could reach its rated turbine output power at lower wind speeds, then the utilization factor of wind turbines would be much greater. However, with the available technology, wind turbines are not cost effective for use in PEV chargers at least in locations with wind speeds similar to Los Angeles, CA and Boulder, CO.

Chapter 5

Multiple Charger Optimization Methodology and Analysis

This chapter will present modifications to the linear programming formulation in order to model the multiple charger case. Here only a two PEV charger case is formulated. However, the steps taken to expand the linear programming formulation to model systems with two PEV chargers can be repeated to expand the formulation to model more than two PEV chargers. Three configurations involving a combination of centralized or distributed generation and storage will be implemented. The implemented configurations are applied to a two PEV charger public charging case using solar irradiation data for Los Angeles, CA.

5.1 Modifications for the Multiple Charger Case

Table 5-1 defines each of the vectors that will be used in the two charger case. Because two PEV chargers are used, there will be two charging profiles for the PEVs: $P_{C1}(t)$ and $P_{C2}(t)$. The grid connection

Table 5-1: Vectors used in the formulation of the two charger case.

Vector	Description
$\boldsymbol{p}_{C1}$	Discrete time power profile of charger 1
$\boldsymbol{p}_{C2}$	Discrete time power profile of charger 2
$\boldsymbol{p}_{G1}$	Discrete time power drawn from the grid by charger 1
$\boldsymbol{p}_{G2}$	Discrete time power drawn from the grid by charger 2
$\boldsymbol{p}_{G}$	Discrete time total power drawn from the grid
$\boldsymbol{p}_{DG1}$	Discrete time output power profile of DG 1 or the power drawn from the centralized DG by charger 1 depending on the configuration
$\boldsymbol{p}_{DG2}$	Discrete time output power profile of DG 2 or the power drawn from the centralized DG by charger 2 depending on the configuration
$\boldsymbol{p}_{DG}$	Discrete time output power profile from the centralized DG
$\boldsymbol{p}_{S1}$	Discrete time output power profile of storage unit 1 or the power drawn from the centralized storage unit by charger 1 depending on the configuration
$\boldsymbol{p}_{S2}$	Discrete time output power profile of storage unit 2 or the power drawn from the centralized storage unit by charger 2 depending on the configuration
$\boldsymbol{p}_{S}$	Discrete time output power profile from the centralized storage unit
$\boldsymbol{\Delta e}_{S1}$	Discrete time change in energy of storage unit 1
$\boldsymbol{\Delta e}_{S2}$	Discrete time change in energy of storage unit 2
$\boldsymbol{\Delta e}_{S}$	Discrete time change in energy of the centralized storage unit

point is shared between the two chargers. There is only one grid constraint based on the sum of power that is drawn from the grid by both chargers but there is no constraint on how the power from the grid is divided between the two chargers. Here, the total grid power, $P_G(t)$, will also be represented by the power from the grid that goes into each charger, $P_{G1}(t)$ and $P_{G2}(t)$ where:

$$P_{G1}(t) + P_{G2}(t) = P_G(t)\,. \tag{5.1}$$

Each charger will also be powered by a DG and a storage unit and the associated power balance for each one is:

$$P_{G1}(t) + P_{DG1}(t) + P_{S1}(t) = P_{C1}(t)\ . \tag{5.2}$$

$$P_{G2}(t) + P_{DG2}(t) + P_{S2}(t) = P_{C2}(t)\ . \tag{5.3}$$

where the discrete time vector representations of each power profile listed is defined in Table 5-1. As implemented here, a distributed DG can only supply power to the charger to which it is connected or to a centralized storage unit. A distributed storage unit can only supply power to the charger to which it is connected.

b will still be a zero vector in each configuration and its length is still determined by the number of rows in ***A***. ***g*** is the same between each configuration:

$$\boldsymbol{g} = \begin{bmatrix} \boldsymbol{p}_{C1} \\ \boldsymbol{p}_{C2} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{5.4}$$

where the length is still dependent on the number of rows in ***D***. The remaining changes in ***x***, ***c***, ***A***, and ***D*** will depend on the configuration.

5.1.1 Configuration A: Distributed DG and Distributed Storage

In configuration A shown in Fig. 5-1, each PEV charger has its own DG and own storage unit. The power from the renewable DG that is connected to one charger does not supply power to the other

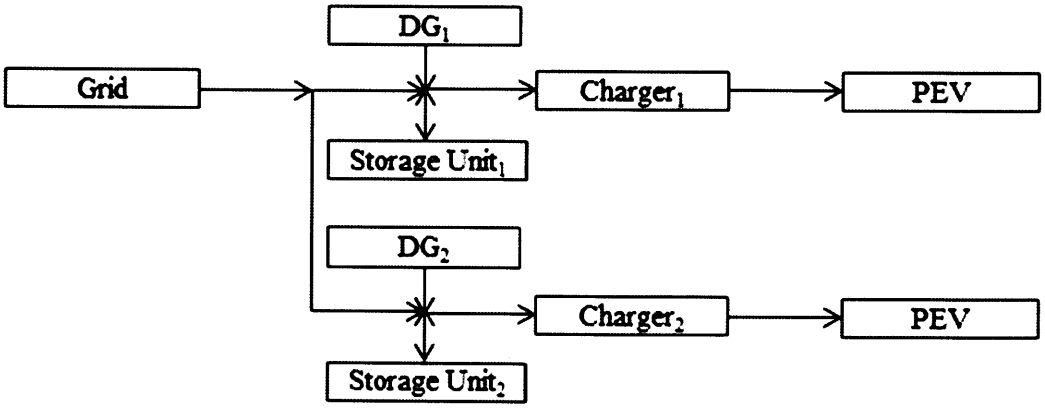


Figure 5-1: Configuration A for a two charger system consists of distributed renewable DG and distributed storage units.

charger. The same is true for the storage unit. For this system, $\boldsymbol{x_a}$ is:

$$\boldsymbol{x_a} = \begin{bmatrix} \boldsymbol{p}_{G1} \\ P_{G1,r} \\ \boldsymbol{p}_{G2} \\ P_{G2,r} \\ \boldsymbol{p}_{G} \\ P_{G,r} \\ \boldsymbol{p}_{2DG} \\ \boldsymbol{p}_{2S} \\ \boldsymbol{\Delta e}_{2S} \\ E_{G,r} \end{bmatrix}, \tag{5.5}$$

where the subscript of $\boldsymbol{x}$ indicates the configuration and $\boldsymbol{p}_{2DG}$, $\boldsymbol{p}_{2S}$ and $\boldsymbol{\Delta e}_{2S}$ are represented by:

$$\boldsymbol{p}_{2DG} = \begin{bmatrix} \boldsymbol{p}_{DG1} \\ P_{DG1,r} \\ \boldsymbol{p}_{DG2} \\ P_{DG2,r} \end{bmatrix}, \tag{5.6}$$

$$\boldsymbol{p}_{2S} = \begin{bmatrix} \boldsymbol{p}_{S1} \\ P_{S1,r} \\ \boldsymbol{p}_{S2} \\ P_{S2,r} \end{bmatrix}, \tag{5.7}$$

$$\boldsymbol{\Delta e}_{2S} = \begin{bmatrix} \boldsymbol{\Delta e}_{S1} \\ E_{S1,r} \\ \boldsymbol{\Delta e}_{S2} \\ E_{S2,r} \end{bmatrix}. \tag{5.8}$$

The corresponding $\boldsymbol{c_a}$ is:

$$\boldsymbol{c_a} = \begin{bmatrix} \mathbf{0} \\ 0 \\ \mathbf{0} \\ 0 \\ \mathbf{0} \\ C'_G \cdot \dfrac{T_{life,sys}}{T_{bp}} \\ \boldsymbol{c}_{2DG} \\ \boldsymbol{c}_{2S} \\ C''_G \cdot \dfrac{T_{life,sys}}{T} \end{bmatrix}, \tag{5.9}$$

where each zero vector is of length N and $\boldsymbol{c}_{2DG}$ and $\boldsymbol{c}_{2S}$ are the cost for two distributed renewable DG and two distributed storage units, respectively:

$$\boldsymbol{c}_{2DG} = \begin{bmatrix} \mathbf{0} \\ C'_{DG} \cdot \operatorname{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,DG}}\right) \\ \mathbf{0} \\ C'_{DG} \cdot \operatorname{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,DG}}\right) \end{bmatrix}, \tag{5.10}$$

$$\boldsymbol{c}_{2S}=\begin{bmatrix}\mathbf{0}\\ C'_S\cdot\mathrm{int}\left(1+\dfrac{T_{life,sys}}{T_{life,S}}\right)\\ \mathbf{0}\\ C'_S\cdot\mathrm{int}\left(1+\dfrac{T_{life,sys}}{T_{life,S}}\right)\\ \mathbf{0}\\ \mathrm{C}''_{\mathrm{S}}\cdot\mathrm{int}\left(1+\dfrac{T_{life,sys}}{T_{life,S}}\right)\\ \mathbf{0}\\ \mathrm{C}''_{\mathrm{S}}\cdot\mathrm{int}\left(1+\dfrac{T_{life,sys}}{T_{life,S}}\right)\end{bmatrix}, \tag{5.11}$$

where each zero vector is of length N.

Due to the new $\boldsymbol{x}_a$, $\boldsymbol{A}_a$ is in the following form:

$$\boldsymbol{A}_a=\begin{bmatrix}\boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{3,1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{3,2} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_4 & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_4 & \mathbf{0}\end{bmatrix}. \tag{5.12}$$

The ratings are determined using the same setup as before where $\boldsymbol{A}_1$ corresponds to the power rating of the grid, $\boldsymbol{A}_2$ corresponds to the power rating of the renewable DG, $\boldsymbol{A}_3$ corresponds to the power rating of the storage unit, and $\boldsymbol{A}_4$ corresponds to the energy capacity rating of the storage unit. The only difference can be seen in $\boldsymbol{A}_{3,1}$ and $\boldsymbol{A}_{3,2}$ where the second number in the subscript indicates which discrete time vector of the charging profile, $\boldsymbol{p}_{C1}$ or $\boldsymbol{p}_{C2}$, is used to develop the matrix.

$\boldsymbol{D}_a$ is also modified to incorporate (5.1)-(5.3) in the new configuration and the additional components:

$$\boldsymbol{D}_a=\begin{bmatrix}\boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{D}_1 & \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \boldsymbol{D}_1 & \boldsymbol{D}_1 & -\boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_{2,1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_{2,2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_{3,1} & \mathbf{0} & \boldsymbol{D}_4 & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_{3,2} & \mathbf{0} & \boldsymbol{D}_4 & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{d}_5 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1}\end{bmatrix}. \tag{5.13}$$

The first two rows correspond to (5.2) and (5.3). The third row matches (5.1). $\boldsymbol{D}_{2,1}$ and $\boldsymbol{D}_{2,2}$ are determined by $\boldsymbol{f}_{DG,1}$ and $\boldsymbol{f}_{DG,2}$, the discrete time vectors for the output power profiles of DG$_1$ and DG$_2$. $\boldsymbol{D}_{3,1}$ and $\boldsymbol{D}_{3,2}$ scale the output power of storage unit 1 and storage unit 2 based on $\boldsymbol{p}_{C1}$ and $\boldsymbol{p}_{C2}$. $\boldsymbol{D}_{3,1}$ and $\boldsymbol{D}_{3,2}$ are each paired with $\boldsymbol{D_4}$ to complete calculating the change in energy stored in each storage unit.

Finally, the constraints are established by:

$$l_{min,a} = \begin{bmatrix} -\boldsymbol{p}_{G,max(n)} \\ -\boldsymbol{p}_{G,max(n)} \\ -\boldsymbol{p}_{G,max(n)} \\ \boldsymbol{p}_{DG,min} \\ \boldsymbol{p}_{DG,min} \\ \boldsymbol{p}_{S,min} \\ \boldsymbol{p}_{S,min} \\ SOC_{min}\boldsymbol{e}_{S,min} \\ SOC_{min}\boldsymbol{e}_{S,min} \\ E_{G,min} \end{bmatrix} , \tag{5.14}$$

$$l_{max,a} = \begin{bmatrix} \boldsymbol{p}_{C,max(p)} \\ \boldsymbol{p}_{C,max(p)} \\ \boldsymbol{p}_{G,max(p)} \\ \boldsymbol{p}_{DG,max} \\ \boldsymbol{p}_{DG,max} \\ \boldsymbol{p}_{S,max} \\ \boldsymbol{p}_{S,max} \\ SOC_{max}\boldsymbol{e}_{S,max} \\ SOC_{max}\boldsymbol{e}_{S,max} \\ E_{G,max} \end{bmatrix} . \tag{5.15}$$

As can be seen, there is almost a one to one matching between the minimum and maximum constraints based on the component listed in $\boldsymbol{x_a}$ except for the first two vectors in $\boldsymbol{l_{max,a}}$ which have a charger constraint on the power drawn from the grid by each charger. This is to prevent the grid from charging both the PEV and the storage unit simultaneously. If this were to happen, it would violate the assumptions made earlier in the formulation with respect to the output power of the storage unit. This constraint was not necessary in the single charger case because the limit on the grid was lower than the power drawn by the charger.

5.1.2 Configuration B: Distributed DG and Centralized Storage

The difference between configuration B and configuration A is that the storage is now centralized. The centralized storage can be charged from either renewable DG. Configuration B is shown graphically in Fig. 5-2. Similar to the power from the grid, the power from the storage is divided between the two chargers:

$$P_{S1}(t) + P_{S2}(t) = P_S(t) \ . \tag{5.16}$$

To accommodate the change in the configuration, there is an addition of a storage unit power vector to represent the output power of the centralized storage unit and a deletion of a storage unit energy vector because there is only one storage unit and only one change in energy capacity of that storage unit:

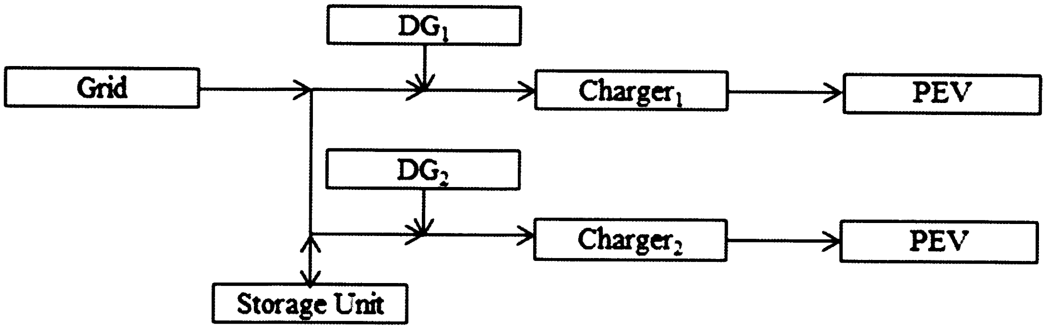


Figure 5-2: Configuration B for a two charger system consists of distributed renewable DG and a centralized storage unit.

$$\boldsymbol{x_b} = \begin{bmatrix} \mathbf{p}_{G1} \\ P_{G1,r} \\ \mathbf{p}_{G2} \\ P_{G2,r} \\ \mathbf{p}_{G} \\ P_{G,r} \\ \mathbf{p}_{2DG} \\ \mathbf{p}_{2S} \\ \mathbf{p}_{S} \\ \boldsymbol{\Delta e}_{S} \\ E_{G,r} \end{bmatrix}. \tag{5.17}$$

The corresponding $\boldsymbol{c_b}$ is:

$$\boldsymbol{c_b} = \begin{bmatrix} \mathbf{0} \\ 0 \\ \mathbf{0} \\ 0 \\ \mathbf{0} \\ C'_G \cdot \dfrac{T_{life,sys}}{T_{bp}} \\ \mathbf{c}_{2DG} \\ \mathbf{0} \\ 0 \\ \mathbf{0} \\ 0 \\ \mathbf{0} \\ C'_S \cdot \mathrm{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,S}}\right) \\ \mathbf{0} \\ C''_S \cdot \mathrm{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,S}}\right) \\ \mathbf{0} \\ C''_G \cdot \dfrac{T_{life,sys}}{T} \end{bmatrix}, \tag{5.18}$$

where each zero vector is of length N.

Due to the addition of a storage unit output power vector and the removal of an energy capacity vector in $\boldsymbol{x}_b$, $\boldsymbol{A}_b$ takes on a slightly different form:

$$\boldsymbol{A}_b = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_{3,1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_{3,2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_{3,3} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_4 & \boldsymbol{0} \end{bmatrix}, \tag{5.19}$$

where the difference is in the replacement of $\boldsymbol{A}_4$ in the second to last row with $\boldsymbol{A}_{3,3}$. This new submatrix will determine the rating of the centralized storage. The centralized storage is considered to discharge if either of the chargers are charging.

$\boldsymbol{D}$ is also slightly modified due to (5.16) and the centralized energy capacity:

$$\boldsymbol{D}_b = \begin{bmatrix} \boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{0} & \mathbf{D}_1 & \boldsymbol{0} & \boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{D}_1 & \boldsymbol{D}_1 & -\boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_1 & \boldsymbol{D}_1 & -\boldsymbol{D}_1 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_{2,1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_{2,2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_3 & \boldsymbol{D}_4 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{d}_5 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{1} \end{bmatrix}, \tag{5.20}$$

where (5.16) is mapped to the fourth row and there is only one row that calculates the change in the energy stored in the storage unit.

The constraint vectors for configuration B are almost the same as the vectors for configuration A except for the ninth subvector which relates to the storage unit power:

$$\boldsymbol{l}_{min,b} = \begin{bmatrix} -\boldsymbol{p}_{G,max(n)} \\ -\boldsymbol{p}_{G,max(n)} \\ -\boldsymbol{p}_{G,\max(n)} \\ \boldsymbol{p}_{DG,min} \\ \boldsymbol{p}_{DG,min} \\ \boldsymbol{p}_{S,min} \\ \boldsymbol{p}_{S,min} \\ \boldsymbol{p}_{S,min} \\ SOC_{min}\boldsymbol{e}_{S,min} \\ E_{G,min} \end{bmatrix}, \tag{5.21}$$

$$\boldsymbol{l}_{max,b} = \begin{bmatrix} \boldsymbol{p}_{C,max(p)} \\ \boldsymbol{p}_{C,max(p)} \\ \boldsymbol{p}_{G,\max(p)} \\ \boldsymbol{p}_{DG,max} \\ \boldsymbol{p}_{DG,max} \\ \boldsymbol{p}_{S,max} \\ \boldsymbol{p}_{S,max} \\ \boldsymbol{p}_{S,max} \\ SOC_{max}\boldsymbol{e}_{S,max} \\ E_{G,max} \end{bmatrix}. \qquad (5.22)$$

5.1.3 Configuration C: Centralized DG and Distributed Storage

In this last configuration, the storage is distributed as it was in configuration A but now the renewable DG is distributed. Configuration C is shown graphically in Fig. 5-3. Similar to the power from the grid, the power from the renewable DG is divided between the two chargers:

$$P_{DG1}(t) + P_{DG2}(t) = P_{DG}(t). \qquad (5.23)$$

To accommodate the change in configuration C from configuration A, there is an addition of a renewable DG power vector to represent the output power of the centralized renewable DG:

$$\boldsymbol{x}_c = \begin{bmatrix} \boldsymbol{p}_{G1} \\ P_{G1,r} \\ \boldsymbol{p}_{G2} \\ P_{G2,r} \\ \boldsymbol{p}_{G} \\ P_{G,r} \\ \boldsymbol{p}_{2DG} \\ \boldsymbol{p}_{DG} \\ \boldsymbol{p}_{2S} \\ \Delta\boldsymbol{e}_{2S} \\ E_{G,r} \end{bmatrix}. \qquad (5.24)$$

c_c will also see the addition of one more zero vector and term:

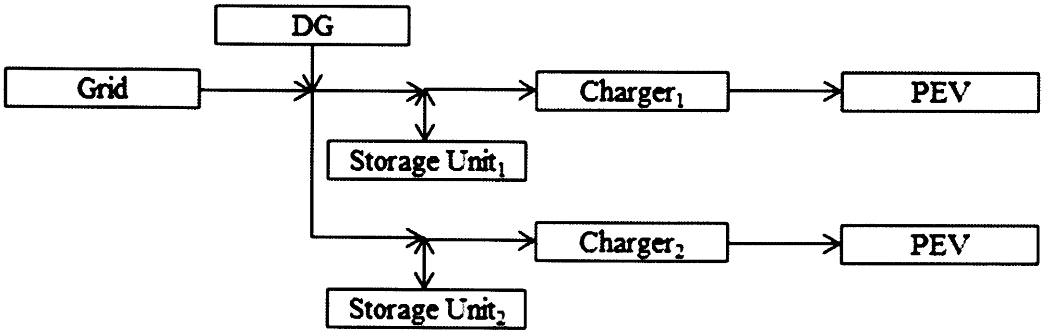


Figure 5-3: Configuration C for a two charger system consists of a centralized renewable DG distributed storage units.

$$\boldsymbol{c_c} = \begin{bmatrix} \mathbf{0} \\ 0 \\ \mathbf{0} \\ 0 \\ \mathbf{0} \\ C_G' \cdot \dfrac{T_{life,sys}}{T_{bp}} \\ \boldsymbol{c_{2DG}} \\ \mathbf{0} \\ C'_{DG} \cdot \mathrm{int}\left(1 + \dfrac{T_{life,sys}}{T_{life,DG}}\right) \\ \boldsymbol{c_{2S}} \\ \mathrm{C}_G'' \cdot \dfrac{T_{life,sys}}{T} \end{bmatrix}, \tag{5.25}$$

where each zero vector is of length *N*.

Due to the addition of a renewable output power vector in $\boldsymbol{x_c}$ and (5.23), $\boldsymbol{A_c}$ gains a row and a column as compared to $\boldsymbol{A_a}$:

$$\boldsymbol{A_c} = \begin{bmatrix} \boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{3,1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_{3,2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_4 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}_4 & \mathbf{0} \end{bmatrix}, \tag{5.26}$$

$\boldsymbol{D_c}$ gains a row and a column:

$$\boldsymbol{D_c} = \begin{bmatrix} \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{D}_1 & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{D}_1 & \boldsymbol{D}_1 & -\boldsymbol{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_1 & \mathbf{D}_1 & -\mathbf{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_{3,1} & \mathbf{0} & \boldsymbol{D}_4 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{D}_{3,2} & \mathbf{0} & \boldsymbol{D}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{d}_5 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1} \end{bmatrix}, \tag{5.27}$$

where the fourth row incorporates (5.23) and the fifth row is the only row that calculates the output power rating of the renewable DG.

The last part of configuration C is the constraint vectors:

$$l_{min,c} = \begin{bmatrix} -p_{G,max(n)} \\ -p_{G,max(n)} \\ -p_{G,max(n)} \\ p_{DG,min} \\ p_{DG,min} \\ p_{DG,min} \\ p_{S,min} \\ p_{S,min} \\ SOC_{min}\ e_{S,min} \\ SOC_{min}\ e_{S,min} \\ E_{G,min} \end{bmatrix}, \tag{5.28}$$

$$l_{max,c} = \begin{bmatrix} p_{C,max(p)} \\ p_{C,max(p)} \\ p_{G,max(p)} \\ p_{DG,max} \\ p_{DG,max} \\ p_{DG,max} \\ p_{S,max} \\ p_{S,max} \\ SOC_{max}\ e_{S,max} \\ SOC_{max}\ e_{S,max} \\ E_{G,max} \end{bmatrix}. \tag{5.29}$$

This linear programming formulation for each configuration is implemented in MATLAB.

5.2 Optimal Design Results and Discussion for Multiple Chargers

It is expected that many PEV owners will charge their vehicles at home but there will also be need for public PEV charging infrastructure for use when traveling or needing a quicker recharge than available at home. Public chargers may be installed in office parking lots, outside coffee shops and shopping centers or at traditional gas stations. The case study examined in this thesis is that of a public charging station outside a restaurant. This restaurant decides to install two 7.2 kW Level 2 PEV chargers in its parking lot. It expects heavy usage during lunch and dinner hours when customers come to eat. The average time a customer spends at the restaurant is approximately 30 minutes during lunch time and one hour during dinner time. At any given time, two PEVs can be charged and the restaurant expects roughly ten PEVs to be charged during a day - 5 during lunch and 5 at dinner. The expected drawn power profiles for the two chargers are shown in Fig. 5-4. The utility uses a separate meter for the PEV chargers. If the chargers draw more than 10 kW, the utility will charge the restaurant a demand charge for the peak power drawn during the month.

The restaurant is trying to decide between four possible configurations for the charging system. In each configuration, there is only one grid connection point through the separate meter from the utility. The first configuration is a fully distributed one. It has two individual solar PV generators above two parking spots (which provide shade for two cars) and two storage units. One solar PV generator and

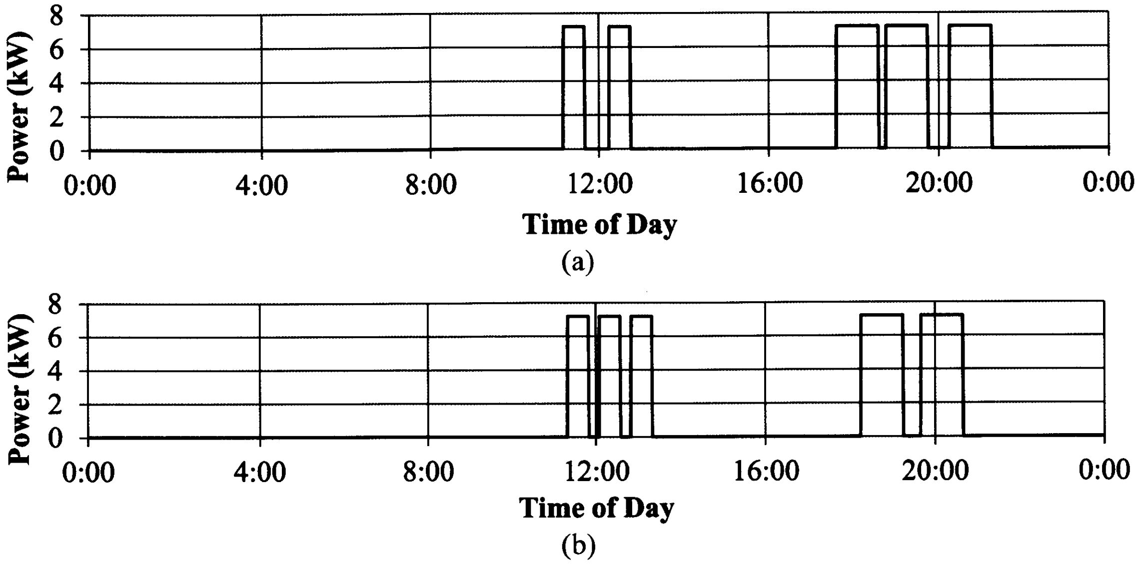


Figure 5-4: Expected average daily charge profiles for (a) charger 1 and (b) charger 2, for the public charging case.

battery is associated with each charger and only supply power to one PEV. The second configuration is similar except it has only one shared storage unit. The third configuration combines the two solar PV systems into a single renewable source but has separate storage units for each charger. The last configuration has one solar PV source and one shared storage unit. In all four configurations, there is only one connection with the grid and this is through the separate meter installed by the utility.

The first three configurations are the same as the three configurations developed in Section 5.1. The last configuration will use the residential configuration with a charger profile that is the sum of the two public charging profiles. Therefore, the three configurations are considered with $P_{G,max(p)}$ equal to 10 kW. The monthly connection charge is \$15 and there is no demand charge applied to the power drawn from the grid. It will be assumed that the restaurant has the same solar resource as Los Angeles, CA and therefore the median solar irradiation profile for Los Angeles, CA is used for this analysis.

5.2.1 Configuration A

This configuration has distributed solar PV generation and distributed storage units. Because the solar PV and storage unit are only meant to charge the PEV charger to which they are connected, their size depends on the power drawn by their respective charger. In this example, charger 2 has a longer period of power demand than charger 1 during lunch hours (i.e., at a time when the solar PV is generating power). As a result, the solar PV source and storage unit connected to charger 2 are larger than those connected to charger 1. This result can be explained by the fact that it is more cost effective over the life of the system to install a larger solar PV source to meet the needs of the PEV charger instead of purchasing energy from the grid. However, to prevent oversizing the storage unit, there is a limit to how large this solar PV system can be. Therefore, some power is still drawn from the grid during lunchtime. Fig. 5-5 shows the power and energy profiles of each charger, as well as the aggregate power profiles of the optimal design. The ratings and costs of each component of the optimal design are given in Table 5-2 and Table 5-3, respectively.

5.2.2 Configuration B

This configuration has distributed solar PV generation and a centralized storage unit. The advantage of this configuration is that surplus energy from one solar PV generator can be stored in the central storage unit and later used by either charger. Another advantage is a lower fixed cost for one storage unit than the fixed costs of two storage units. Table 5-3 shows that the cost of the storage unit for this configuration is less than the sum of the storage unit costs for configuration A. This can be attributed, in part, to the lower total energy capacity rating of the single storage unit than the sum of the energy capacity of the separate storage units. As seen in Table 5-2, the solar PV sources are equal in rating meaning that the parking lot shading canopies could have the same design (e.g., same area, same module layout, etc.). Fig. 5-6 shows the power profiles of each charger, as well as the aggregate power and energy profiles of the optimal design. The ratings and costs of each component of the optimal design are given in Table 5-2 and Table 5-3, respectively.

5.2.3 Configuration C

This next configuration has a centralized solar PV source but distributed storage units. In this configuration, the solar PV source can supply power to both of the chargers simultaneously. It can also charge either storage unit. As seen in Table 5-2, the two storage units have similar energy storage capacity. However, the cost of the solar PV source in this configuration is greater than it was in the previous two configurations due to a higher power rating. The total system lifecycle cost is slightly lower than that of configuration A but greater than that of configuration B. Fig. 5-7 shows the power profiles of each charger, as well as the aggregate power and energy profiles of the optimal design. The ratings and costs of each component of the optimal design are given in Table 5-2 and Table 5-3, respectively.

5.2.4 Configuration D

The last configuration has one solar PV source and one storage unit which can supply power to both chargers simultaneously. The ratings and costs of each component of the optimal design are given in Table 5-2 and Table 5-3, respectively. As seen in Table 5-3, this configuration yields the lowest cost solution out of all four configurations. However, it is interesting to note that the system lifecycle costs for configurations with separate storage units are similar and the system lifecycle costs for configurations with a single storage unit are similar. The restaurant should choose a design that has a single storage unit

Table 5-2: Optimal design ratings for public charging designs with solar PV system and lead-acid battery.

Option	$P_{DG1,r}$ (kW)	$P_{DG2,r}$ (kW)	$P_{DG,r}$ (kW)	$P_{S1,r}$ (kW)	$P_{S2,r}$ (kW)	$P_{S,r}$ (kW)	$P_{G,r}$ (kW)	$E_{S1,r}$ (kWh)	$E_{S2,r}$ (kWh)	$E_{S,r}$ (kWh)	$E_{G,r}$ (kWh)
A	0.667	1.194	N/A	2.8	2.461	N/A	10	7.478	11.124	N/A	45.183
B	0.645	0.644	N/A	N/A	N/A	4.4	10	N/A	N/A	17.354	47.101
C	N/A	N/A	2.059	2.8	2.462	N/A	10	9.163	9.439	N/A	44.067
D	N/A	N/A	1.92	N/A	N/A	4.4	10	N/A	N/A	17.347	44.336

Table 5-3: Optimal component costs for public charging designs with solar PV system and lead-acid battery.

Option	DG$_1$	DG$_2$	DG	Storage$_1$	Storage$_2$	Storage	Grid	System Lifecycle
A	\$3,135	\$5,454	N/A	\$9,261	\$12,199	N/A	\$49,777	\$79,826
B	\$3,039	\$3,035	N/A	N/A	N/A	\$19,232	\$51,738	\$77,044
C	N/A	N/A	\$9,259	\$10,743	\$10,715	N/A	\$48,636	\$79,354
D	N/A	N/A	\$8,648	N/A	N/A	\$19,224	\$48,912	\$76,785

but there is marginal difference between a design that employs separate solar PV sources or one that has a single solar PV source.

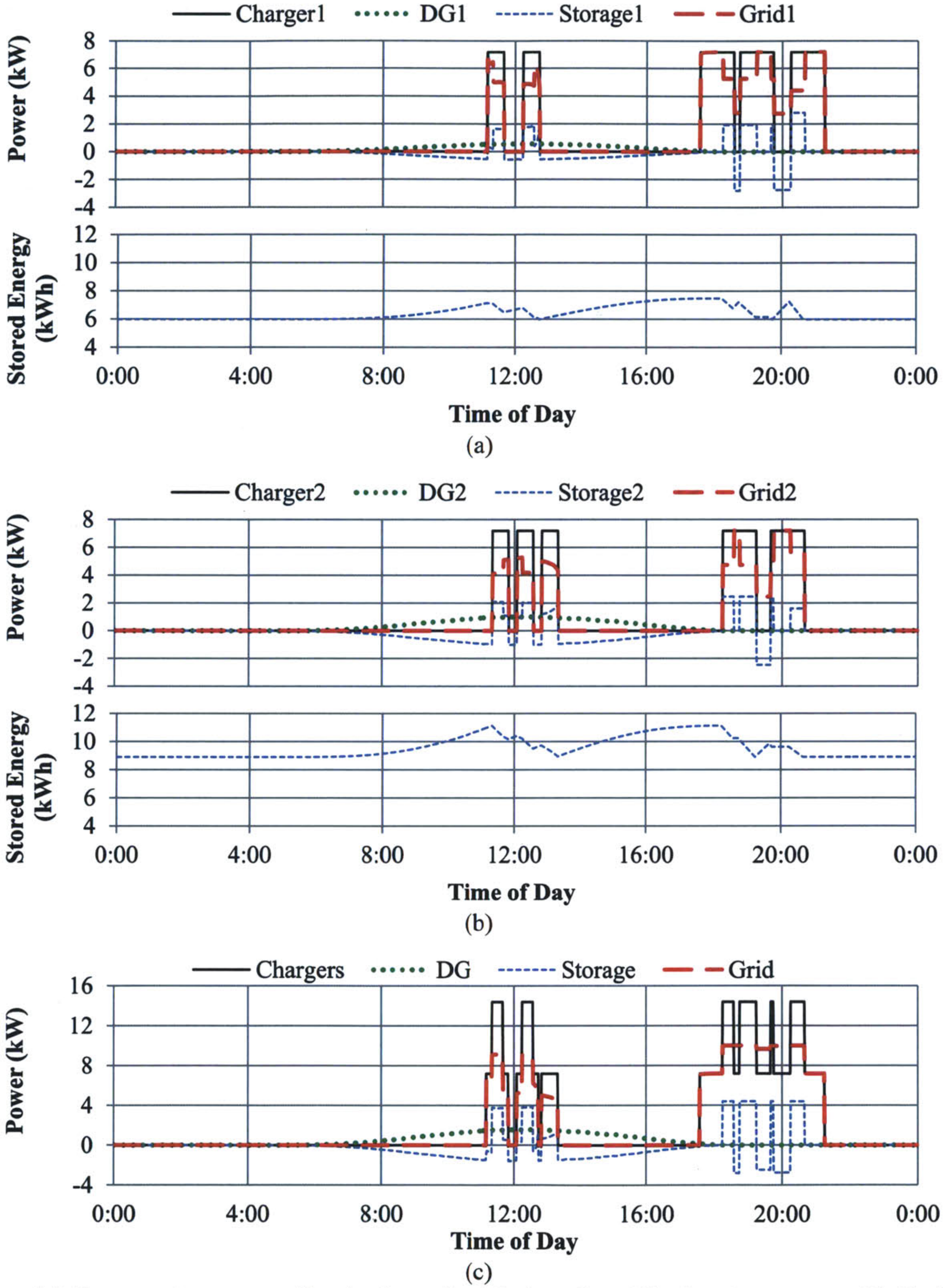


Figure 5-5: Power and energy profiles for the optimal design of a public charging system with distributed solar PV and distributed lead-acid battery as described in configuration A. The power and energy profiles are shown for (a) Charger1 and (b) Charger2 as well as (c) the aggregate power profiles of each component. Grid1 and Grid2 represent the power drawn from the grid by charger 1 and charger 2, respectively.

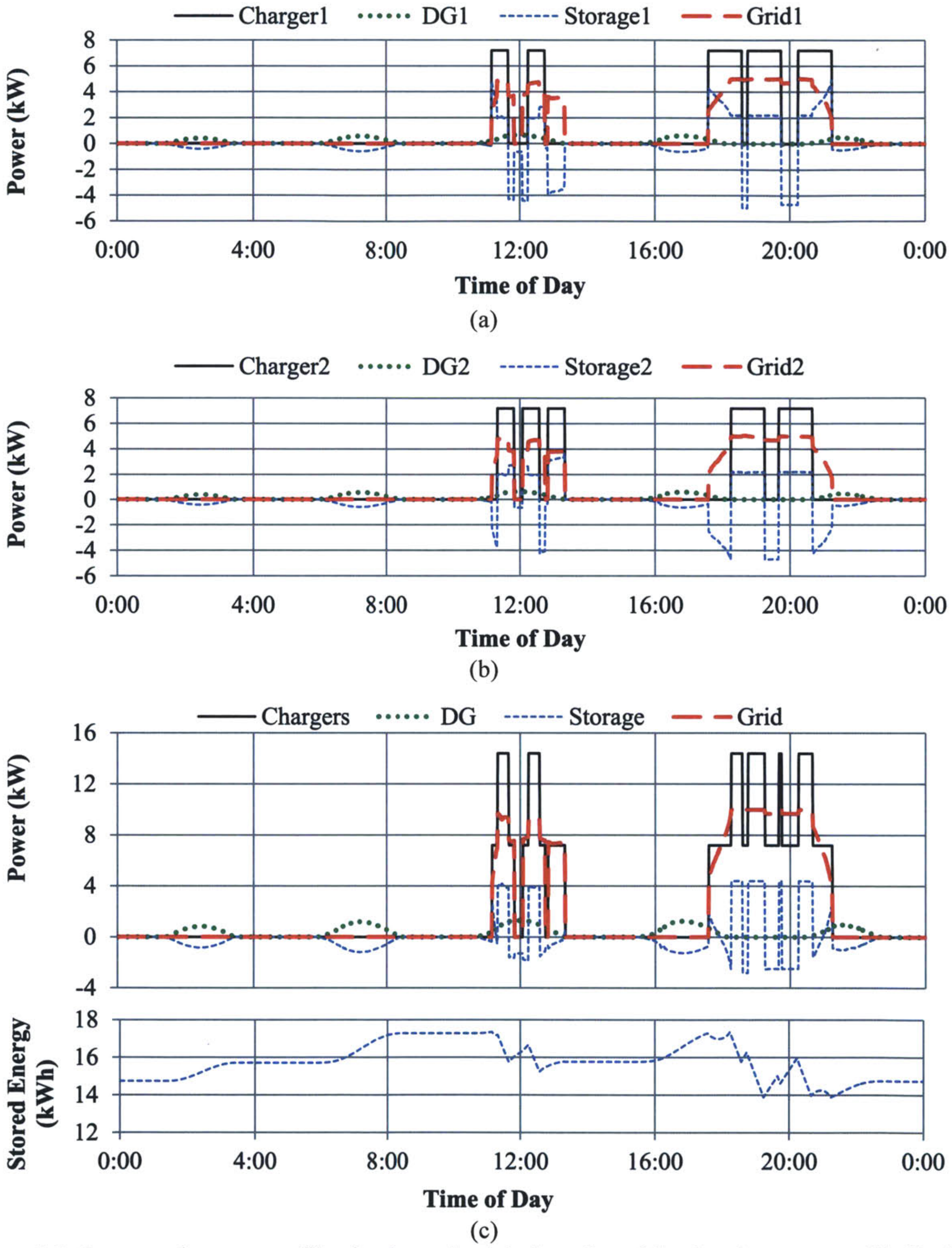


Figure 5-6: Power and energy profiles for the optimal design of a public charging system with distributed solar PV and centralized lead-acid battery as described in configuration B. The power profiles are shown for (a) Charger1 and (b) Charger2 as well as (c) the aggregate power and energy profiles of each component. Grid1 and Grid2 represent the power drawn from the grid by charger 1 and charger 2, respectively. Storage1 and Storage 2 represent the power drawn by the storage unit from charger 1 and charger 2, respectively.

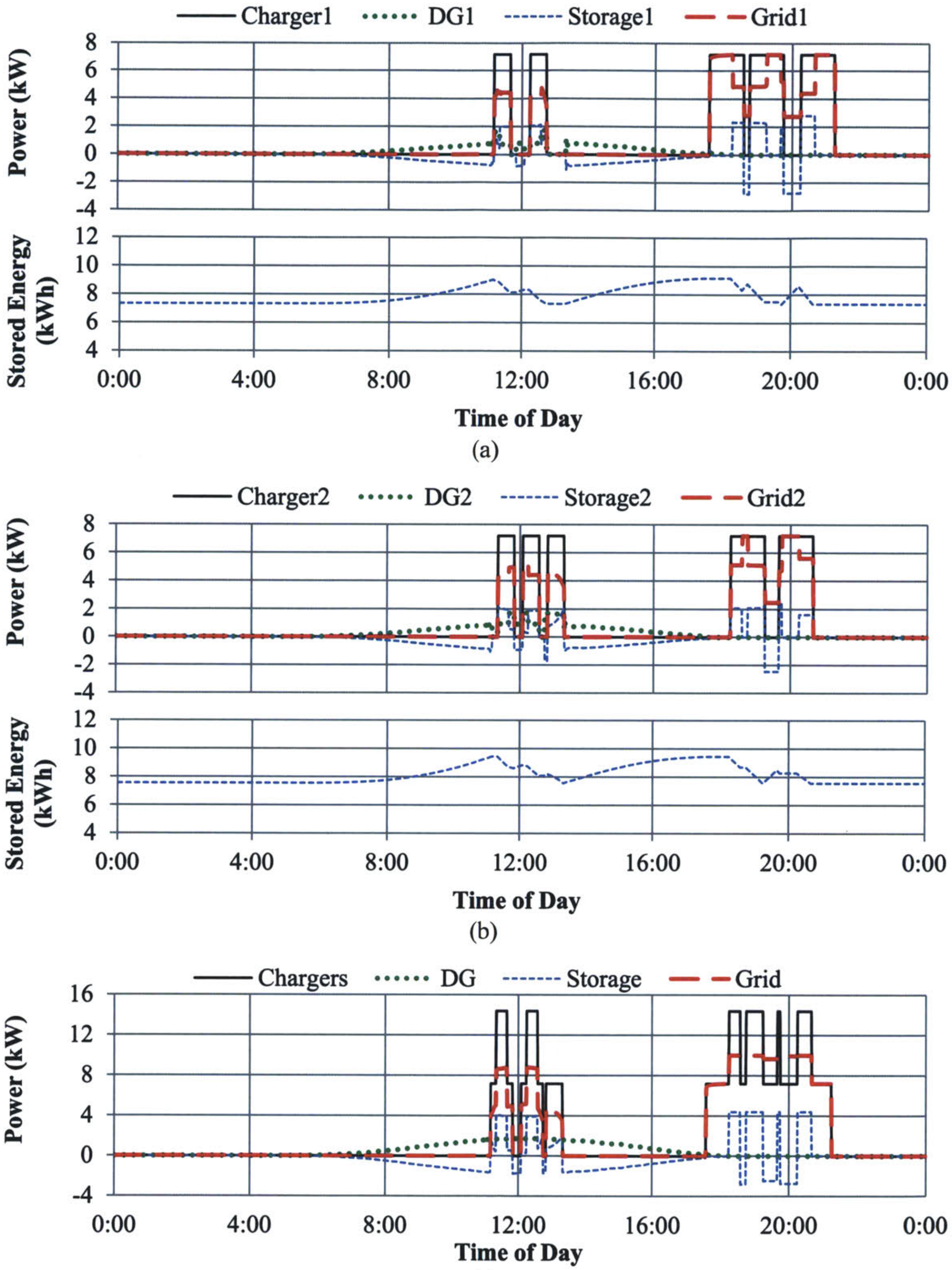


Figure 5-7: Power and energy profiles for the optimal design of a public charging system with centralized solar PV and distributed lead-acid battery as described in configuration C. The power and energy profiles are shown for (a) Charger1 and (b) Charger2 as well as (c) the aggregate power profiles of each component. Grid1 and Grid2 represent the power drawn from the grid by charger 1 and charger 2, respectively. DG1 and DG2 represent the power drawn from the DG from charger 1 and charger 2, respectively.

Chapter 6

Summary and Conclusions

Plug-in electric vehicles (PEVs) and renewable distributed generation (DG) are expected to have increasing penetration levels over the next few decades as they promise many societal benefits. The penetration density of these technologies in some geographic areas may be high enough to cause problems for the electric distribution system. This presents an opportunity to optimize the design of charging solutions that integrate renewable DG and/or storage with PEV chargers in order to mitigate their negative electric grid impacts.

6.1 Summary

This thesis presents a methodology and an optimization tool for the design of grid-interfaced PEV charging systems that integrate renewable generation and storage. The optimization tool is based on a linear programming approach which selects designs with minimum system lifecycle cost. A search-based optimization technique is also developed to validate the results.

A system lifecycle cost model is developed in Chapter 2 in order to compare alternate designs. The design with the lowest system lifecycle cost is considered optimal. Vendor data for renewable DG and storage is presented and used to determine the values of the cost and time duration parameters that are used in this thesis. Utility cost data is also compiled to determine appropriate values for electric grid related cost parameters. With this system lifecycle cost model (based on real data), a realistic analysis can be performed using the optimization tools.

Chapter 3 develops of the optimization methodology for a single charger system with centralized DG and storage. The methodology is formulated using two approaches. The first approach is based on linear programming, while the second is based on a limited search approach. In Chapter 4, these approaches are applied to a residential charging case in which the peak power drawn by the charger is greater than what can be supplied from the grid. Optimal designs are found for systems that included a lead-acid battery and a solar PV source or a wind turbine for two locations in the US. A sensitivity analysis is also performed to determine the cost competitiveness of lithium-ion battery technology relative to lead-acid batteries.

The linear programming formulation developed in Chapter 3 for a single charger is extended for multi-charger cases in Chapter 5. Four possible configurations of a two-PEV charger system are analyzed in this chapter using solar irradiation data from one US location. The configurations considered include combinations of centralized and distributed solar PV generation and lead-acid storage. The system is analyzed using the developed tool to find the optimal design configuration for a specified case study.

6.2 Conclusions

A number of conclusions have been drawn from this thesis. These include:

1) The proposed linear programming technique is an effective means to optimize charger systems with integrated distributed generation and/or storage.
2) Designs that draw the maximum available power from the grid have the lowest 20-year system lifecycle cost.
3) On a lifecycle basis, designs with a solar PV distributed source are better than those without one if the median noon-time irradiation of the location is greater than about 720 W/m^2.
4) Li-ion storage technology is not yet cost competitive with lead-acid storage technology but may become more attractive as its cost decreases over the next decade.
5) Available wind turbine technology is not cost effective for PEV chargers at least in locations with wind speeds similar to those of Los Angeles, CA and Boulder, CO.
6) When comparing configurations for a multiple PEV charging case, the one with centralized storage and generation has the lowest system lifecycle cost.

6.3 Directions for Future Work

In the future, the optimization tool can be further developed to make the analysis of multi-charger systems easier. For one, it needs to be extended to handle systems with more than two chargers, generation sources and storage units. Also, it would be valuable to reformulate it in such a way that it can handle the various configurations in a unified manner.

The system lifecycle cost model used in the optimization can also be improved. One aspect would be to gather additional cost data and come up with more accurate parameter values. Also, the lifecycle cost model could incorporate time-of-use electricity prices, so that designs that try to derive economic benefit from storing electricity when it is cheaper (e.g., during the night) and selling it back when it is more expensive can be evaluated.

In terms of analysis, it would be valuable to consider optimal designs for a residential neighborhood where renewable generation and storage resources are used in a cooperative manner by multiple houses.

Finally, the results of this work can be used to guide the development of the actual power electronic hardware needed for a grid-connected PEV charging system that integrates renewable generation and storage.

Appendix A

PLECS Simulation Model

In order to verify that the designs generated by the two formulations meet the system requirement of returning the storage unit to its original SOC at the end of the time period *T*, a simulation model for the PEV charging system was developed in PLECS – a switched-mode circuit and system simulation platform. A simulation model for the PEV charging system built in PLECS is shown in Figure A-1. In this simulation model, controllable current sources are used to model the power drawn from the grid, the renewable DG and the PEV charger. The energy stored in the storage unit is modeled using the voltage across a capacitor. The storage unit and renewable DG used in this simulation model are sized using the developed formulations. The simulation model also includes the control logic for controlling the power flow in the system. The power drawn by the PEV charger and the power available from the renewable DG are system inputs, while the power drawn from the storage and the grid are determined by the control logic. A switch in series with the storage unit is used to stop the flow of power from or to the storage unit when the energy stored in the storage unit goes below or above its minimum or maximum limits, respectively.

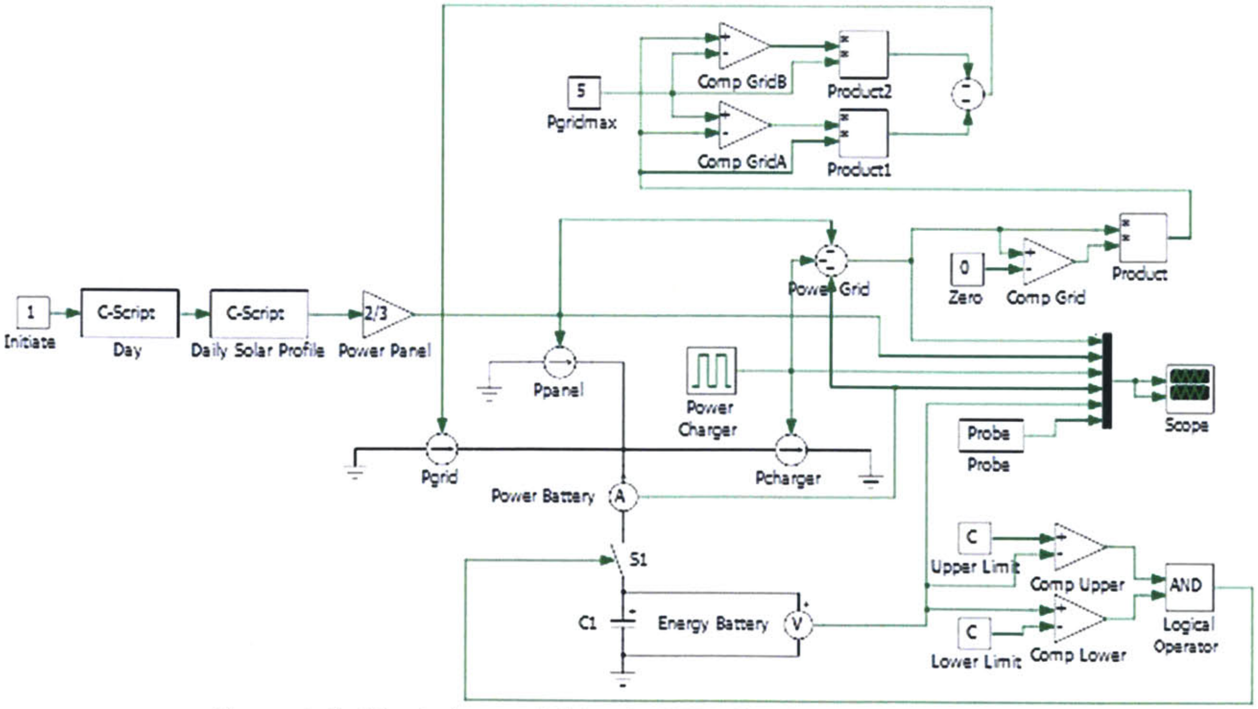


Figure A-1: Simulation model for the PEV charging system in PLECS.

Appendix B

MATLAB Code

This Appendix contains the MATLAB code that was used to generate the results that were presented in this thesis. There are a total of five files. The first file is the linear optimization tool for the single charger case. The second file is the search-based optimization tool for the single charger case. The third, fourth and fifth files are the multi-charger optimization tools for configurations A, B and C, respectively. Each file will require the necessary .csv input files for the charger and DG profiles. The remaining parameters and constraints can be manually set.

```
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% Linear Programming - Single Charger                                         %
% General equation to be solved: C = c' x                                     %
% Variables to solve for: Pdg, Ps, Pg, Es, Eg, each of length dt              %
% x =[Pg' Pg_rating Pdg' Pdg_rating Ps' Ps_rating Es' Es_rating Eg_rating]' %
% length of x is 4*dt + 5                                                     %
%                                                                             %
% To change between solar, wind, or no DG, you will have to vary which        %
% 'profile' to load.  Also, the prices will have to be adjusted.  To vary     %
% which architecture is displayed at the end, vary the Pc_max.                %
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

clear all
clc
close all

%Cost Parameters
T_lifesys = 20;          %system life (years)
T_lifedg = 25;           %DG life (years)
T_lifes = 6;             %storage unit life (years)
                             %6 for lead-acid
                             %12 for Li-ion
T_bp = 1/12;             %billing period (years)


C_dg_0 = 200;            %fixed cost of DG independent of rating ($)
                             %$200 for solar PV
                             %$3,750 for wind
C_s_0 = 110;              %fixed cost of storage unit independent of...
                             %rating ($)
                             %$110 for lead-acid
                             %$80 for Li-ion
C_g_0 = 15;               %fixed cost of grid connection applied to each...
                             %bill ($)
                             %$9 for residential
                             %$15 for general, no demand
                             %$60 for general, demand
Cp_dg = 4400;            %variable cost of DG ($/kW)
                             %$4400 for solar PV (includes inverter cost)
                             %$5500 for wind (includes power conditioning)
```

```
Cp_s = 200;               %variable power cost of storage unit ($/kW)
                              %$200 for charge controller
Cpp_s = 220;             %variable energy cost of storage unit ($/kWh)
                              %$220 for lead-acid
                              %$800 for Li-ion
Cp_g = 0;                %variable power cost of the grid ($/kW)
                              %$0 when no demand charge
                              %$10 when demand charge
Cpp_g = 0.14;            %variable energy cost of the grid ($/kWh)
                              %$0.14

% Establish constraints
Pc_max = 7.2;
Pg_max = 10;             %5 for residential
                         %10 for general, demand
Pdg_max = 50;            %10 for residential
                         %50 for general
Ps_max = 10;             %10 for residential
                         %50 for general
Es_max = 500;            %100 for residential
                         %500 for general
Eg_max = 1000;           %arbitrarily high value, no constraint

Pdg_min = 0;             %power cannot go into the DG
Ps_min = -Ps_max;        %high enough to charge
Pg_min = 0;              %no power into the grid
Es_min = 0;              %battery cannot reach zero state of charge
Eg_min = 0;              %no energy into the grid

n = 0.85;                %efficiency of the storage unit
SOC_max = 1;             %set maximum state of charge for the storage unit
SOC_min = 0.80;          %set minimum state of charge for the storage unit

d = 1;                   %days in time period
h = 24;                  %hours in time period
m = 60;                  %60/m is the resolution of the time vector
dt = d*h*m;              %time period in intervals
t = 1:1:dt;              %time vector in minutes for one full day

% EV charge profile
Pc_rating = 6;           %fixed, defined charge rating
Cprofile = load('Cprofile.csv');

Pc = Pc_rating*Cprofile';

% DG power profile
fdg = load('DGprofile.csv');

%% Develop c
%fill in c where appropriate
c = [zeros(1,dt), Cp_g*T_lifesys/T_bp, zeros(1,dt),...
     Cp_dg*floor(1+T_lifesys/T_lifedg), zeros(1,dt),...
     Cp_s*floor(1+T_lifesys/T_lifes), zeros(1,dt),...
     Cpp_s*floor(1+T_lifesys/T_lifes), Cpp_g*365/d*T_lifesys]';
```

```
%% Develop A and b to solve A*x <= b
% This is used to make sure that every value in Pdg' is less than or
% equal to Pdg_rating, every value in Ps' is less than or equal to
% Pdg_rating, etc.

% create A, start with zero because it will be a sparse matrix
A = zeros(length(c));

%create A1
A1 = zeros(dt, dt+1);
for i=1:dt
    A1(i,i) = 1;     %fill in 1 to compare a Pg value to Pg_rating
    A1(i,dt+1) = -1; %fill in -1 at Pg_rating to compare with Pg value
end

%create A2
A2=A1;

%create A3
A3 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
    %the maximum charge or discharge power for the storage unit
    if Pc(i) > 0   %vehicle is charging, storage will be discharging
                         %meaning Ps will be positive, Ps <= Ps_rating
        A3(i,i) = 1;     %fill in 1 at Ps value to compare to Ps_rating

    else                 %vehicle is not charging, storage can only charge
                         %meaning Ps will be negative, -Ps <= Ps_rating
        A3(i,i) = -1;    %fill in 1 at Psrating to compare with Ps value
    end

    A3(i,dt+1) = -1;   %fill in -1 at Psrating to compare with Ps value
end

%create A4
A4 = zeros(dt, dt+1);
for i=1:dt
    A4(i,i) = 1;         %fill in 1 at Es value to compare to Es_rating
    A4(i,dt+1) = -(SOC_max-SOC_min);   %fill in -(SOC_max-SOC_min) at
                                        %Es_rating to compare and scale
                                        %appropriately to Es value
end

%create A
Z = zeros(dt,dt+1);

A = [A1, Z , Z , Z , zeros(dt,1);
     Z , A2, Z , Z , zeros(dt,1);
     Z , Z , A3, Z , zeros(dt,1);
     Z , Z , Z , A4, zeros(dt,1)];

% create b which is just zero
b = zeros(size(A,1),1);
```

```
%% Develop D and g to solve D*x = g
% This relationship is used to make sure that the charger power
% requirements are met. Namely it makes sure that for every instance in
% time, Pc = Pdg + Ps + Pg.
% It will also be used to develop the shape of the panel power output and
% calculate Eb and Egrating.

%create D1
D1 = zeros(dt, dt+1);
for i=1:dt
    D1(i,i) = 1;            %fill in 1
end

%create D2
D2 = zeros(dt, dt+1);
for i=1:dt
    D2(i,i) = 1;              %fill in 1 for Pdg
    D2(i, dt+1) = -fdg(i);  %fill in corresponding -fdg in
                                    %Pdgrating position
end

%create D3
D3 = zeros(dt, dt+1);
for i=1:dt
    if Pc(i) > 0                             %Vehicle is charging
        D3(i,i) = d*h/dt/sqrt(n);            %Battery supplies less energy
                                             %than what was stored

    else
        D3(i,i) = d*h/dt*sqrt(n);            %Battery stores less energy than
                                             %what was supplied

    end
end

%create D4
D4 = zeros(dt, dt+1);
D4(1,1) = 1;
D4(1,dt) = -1;
for i=2:dt
   D4(i,i) = 1;
   D4(i,i-1) = -1;
end

%create d5
d5 = [d*h/dt*ones(1,dt),0];

%create D
z = zeros(1, dt+1);
D = [D1, D1, D1, Z , zeros(dt,1);
     Z , D2, Z , Z , zeros(dt,1);
     Z , Z , D3, D4, zeros(dt,1);
     d5, z , z , z , -1         ];

% create g
% we derive beq from the charging profile of the vehicle which is given
```

```
% fill in g
g = [Pc; zeros(size(D,1)-dt,1)];

%% Develop lmin and lmax for lmin < x < lmax
% the last relation to fulfill for the linearization tool is to define the
% upper and lower bounds for x such that lower bound < x < upper bound for
% all values within x

% Establish upper bounds based on maximum limits
lmax = [Pg_max*ones(dt+1,1); Pdg_max*ones(dt+1,1); Ps_max*ones(dt+1,1); ...
      Es_max*SOC_max*ones(dt+1,1); Eg_max];

% Establish lower bounds
lmin = [-Pg_min*ones(dt+1,1); Pdg_min*ones(dt+1,1); Ps_min*ones(dt+1,1); ...
      Es_min*SOC_min*ones(dt+1,1); Eg_min];

%% Solve using MATLAB's linear programming tool
% options = optimset('MaxIter',300);
[y, fval, exitflag, output]  = linprog(c,A,b,D,g,lmin,lmax);%,[],options);
if y(2*dt+2)>10^-6
    Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
        C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
else
    Cmin = C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
end

%% Display results
display(['The power rating of the DG is ', num2str(y(2*dt+2)),' kW'])
display(['The power rating of the storage is ', num2str(y(3*dt+3)),' kW'])
display(['The energy capacity of the storage is ',num2str(y(4*dt+4)),' kWh'])

display(['The maximum power drawn from the grid is ',num2str(y(dt+1)),' kW'])
display(['The total energy drawn from the grid is ',num2str(y(4*dt+5)),' kWh
per period'])

display(['The minimum lifetime cost is $', num2str(Cmin),'.'])
if y(2*dt+2)>10^-6
    display(['The DG cost is $',...
        num2str(C_dg_0*floor(1+T_lifesys/T_lifedg)+...
                Cp_dg*floor(1+T_lifesys/T_lifedg)*y(2*dt+2)),'.'])
else
    display(['The DG cost is $',...
        num2str(Cp_dg*floor(1+T_lifesys/T_lifedg)*y(2*dt+2)),'.'])
end
display(['The storage cost is $',...
    num2str(C_s_0*floor(1+T_lifesys/T_lifes)+...
            Cp_s*floor(1+T_lifesys/T_lifes)*y(3*dt+3)+...
            Cpp_s*floor(1+T_lifesys/T_lifes)*y(4*dt+4)),'.'])
display(['The grid cost is $',...
    num2str(C_g_0*T_lifesys/T_bp+Cp_g*T_lifesys/T_bp*y(dt+1)+...
    Cpp_g*365/d*T_lifesys*y(4*dt+5)),'.'])


t=1:dt;
```

```
Pg =[y(1:dt)];
Pdg =[y(dt+2:2*dt+1)];
Ps =[y(2*dt+3:3*dt+2)];

figure
plot(t,Pc, t, Pdg, t, Pg, t, Ps)
legend('Charger','DG','Grid','Storage Unit','location','best')
title(['Power Outputs'])
set(gca, 'XTick', [0:dt/24:dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('time of day (hours)')
ylabel('Power drawn (kW)')

Es = [y(3*dt+4:4*dt+3)]+y(4*dt+4)*SOC_min;

figure
plot(t,Es)
title(['Energy Stored in the Storage Unit'])
axis([0 dt 0 y(4*dt+4)*1.2])
set(gca, 'XTick', [0:dt/24:dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('time of day (hours)')
ylabel('Energy capacity (kWh)')
```

```
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% Search-Based Tool - Single Charger                                          %
% There is a limitation on the maximum power drawn from the grid             %
% we have assumed that the power flow in our system is controlled in such   %
% a way that the charger first takes power from the distributed renewable   %
% generation source.  When the distributed renewable generation source      %
% cannot meet the full power needs of the charger, the remaining power is   %
% first drawn from the grid.  Only when the grid power hits its limit do    %
% we draw power from the storage unit.                                       %
% No power from the DG will go into the grid                                 %
%                                                                            %
% To change between solar, wind, or no DG, you will have to vary which      %
% 'profile' to load.  Also, the prices will have to be adjusted.  To vary   %
% which architecture is displayed at the end, vary the Pg_rating vector.    %
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

clc
close all
clear all

%Cost Parameters
T_lifesys = 20;         %system life (years)
T_lifedg = 25;          %DG life (years)
T_lifes = 6;            %storage unit life (years)
                            %6 for lead-acid
                            %12 for Li-ion
T_bp = 1/12;            %billing period (years)


C_dg_0 = 200;           %fixed cost of DG independent of rating ($)
                            %$200 for solar PV
                            %$3,750 for wind
C_s_0 = 110;             %fixed cost of storage unit independent of...
                            %rating ($)
                            %$110 for lead-acid
                            %$80 for Li-ion
C_g_0 = 9;              %fixed cost of grid connection applied to each...
                            %bill ($)
                            %$9 for residential
                            %$15 for general, no demand
                            %$60 for general, demand
Cp_dg = 4400;           %variable cost of DG ($/kW)
                            %$4400 for solar PV (includes inverter cost)
                            %$5500 for wind (includes power conditioning)
Cp_s = 200;              %variable power cost of storage unit ($/kW)
                            %$200 for charge controller
Cpp_s = 220;            %variable energy cost of storage unit ($/kWh)
                            %$220 for lead-acid
                            %$800 for Li-ion
Cp_g = 0;               %variable power cost of the grid ($/kW)
                            %$0 when no demand charge
                            %$10 when demand charge
Cpp_g = 0.14;           %variable energy cost of the grid ($/kWh)
                            %$0.14
```

```
% Establish constraints
Pc_max = 7.2;
Pg_max = 5;               %5 for residential
                          %20 for general, demand
                          %50 for general, demand
Pdg_max = 10;             %10 for residential
                          %50 for general
Ps_max = 10               %10 for residential
                          %50 for general
Es_max = 100;             %100 for residential
                          %500 for general
Eg_max = 1000;            %arbitrarily high value, no constraint

Pdg_min = 0;              %power cannot go into the DG
Ps_min = -Ps_max;         %high enough to charge
Pg_min = 0;               %no power into the grid
Es_min = 0;               %battery cannot reach zero state of charge
Eg_min = 0;               %no energy into the grid

n = 0.85;                 %efficiency of the storage unit
SOC_max = 1;              %set maximum state of charge for the storage unit
SOC_min = 0.80;           %set minimum state of charge for the storage unit

d = 1;                    %days in time period
h = 24;                   %hours in time period
m = 60;                   %60/m is the resolution of the time vector
dt = d*h*m;               %time period in intervals
t = 1:1:dt;               %time vector in minutes for one full day

%          Unknown variables
% Pdg_rating = Max power rating of the DG (kW)
% Ps_rating = Max power rating of the storage unit (kW)
% Pg_rating = Max power drawn from the electric grid (kW)
% Es_rating = Max energy capacity of the storage unit (kWh)
% Eg_rating = Max energy drawn from the electric grid (kWh)

Pg_rating = 4.9:0.1:Pg_max;    %defines the range of architectures to explore
Pdg_rating = zeros(1,length(Pg_rating));
Ps_rating = zeros(1,length(Pg_rating));
Es_rating = zeros(1,length(Pg_rating));
Pg_minrating = zeros(1,length(Pg_rating));
Eg_rating = zeros(1,length(Pg_rating));

%           Time-based vectors
% Pdg = Output power output of the DG.  It will depend on the maximum power
%       output of the DG as well as the time of day (as defined by f
% Pg = Output power output of the grid. It will depend on the power output
%       of the DG and the input power required by the electric vehicle and
%       possibly storage
% Ps = Power output or input of the storage unit.  It will depend on the
%      power output of the DG, the grid and the input power required by
%      the electric vehicle
% Es = Energy delivered or absorbed by the storage unit

Pdg = zeros(1,dt);
```

```
Ps = zeros(1,dt);
Pg = zeros(1,dt);
Es = zeros(1,dt);

%% EV Model

%           Simple EV model
% Pc_rating = Power rating of the charger (kW)
% Eevb = Energy capacity of the electric vehicle (kWh)
% SOCi = Initial state of charge of the electric vehicle when it plugs in
%        somewhere between 0 and 1
% SOCf = Final state of charge of the electric vehicle
%        no more than 1
% Nev = Number of electric vehicles charging per day
% Pc = Charge profile of the electric vehicle

%Well-defined variables
Pc_rating = 6;      %fixed, defined charge rating: 6 kW
SOCi = 0.20;        %Start from a 20% SOC
SOCf = 1;           %Fully charged
Eevb = 30;          %battery capacity is 30 kWh
Nev = 1;            %flexible

% load Pc from a csv file.
% Right now, there is no time indicator in the file.  Data begins at 12:01
% am and there is data for every minute for 24 hours.

c = load('Cprofile.csv');

Pc = Pc_rating*c*Nev;

%% DG Model

%           Simple DG model
% f = The profile of the DG power output.  Varies from 0 to 1.

% load f from a csv file.
% Right now, there is no time indicator in the file.  Data begins at 12:01
% am and there is data for every minute for 24 hours.

f = load('DGprofile.csv');

%% Grid Model

%           Simple Grid model
% Delta_Es = The energy that has to be supplied to the charger from the
%              storage unit (kWh)
% Tc_off = the time period when the electric vehicle is not charging (h)

% Develop Pg

for j=1:length(Pg_rating)
    % need to solve for maximum grid power draw iteratively
    % initial guess
```

```
    Delta_Es = 1;
    Pdg_rating(j) = -0.001;
    Pg_rating(j)

    E_S2C = 0;
    Tc_off = 0;

    if norm(f) < 10e-6      %No DG unit
        for i=1:dt
            if Pc(i) > 0    %There is a vehicle charging
                E_S2C = E_S2C + (Pc(i) - Pg_rating(j))*d*h/dt; %Track kWh
                            %that goes from the storage unit to the charger
            else            %There is no vehicle charging
                Tc_off = Tc_off + d*h/dt;   %keep track of the hours
            end
        end

        for i=1:dt          %Now calculate Ps
            if Pc(i) > 0
                Ps(i) = Pc(i) - Pg_rating(j);
            else
                Ps(i) = -E_S2C/Tc_off/n;
            end
        end
    else                        %DG unit present
        Ps = zeros(1,dt);
    end


    %check if there was enough energy supplied to the charger
    % if not, guess a new Pdg_rating
    while Delta_Es > 0
        Es(1) = 0;
        Pdg_rating(j) = Pdg_rating(j)+0.001;       %increment by 10 W
        Pdg = Pdg_rating(j)*f;

   %calculate power from the grid
        for i=1:dt
            Pg(i) = min(max(Pg_min, Pc(i) - Pdg(i) - Ps(i)), Pg_rating(j));
        end

   %calculate storage unit energy profile
        for i=1:dt-1
            if sign(Pc(i)-Pdg(i)-Pg(i)) > 0
                Es(i+1) = Es(i) - (Pc(i) - Pdg(i) - Pg(i))*d*h/dt/sqrt(n);
            else
                Es(i+1) = Es(i) - (Pc(i) - Pdg(i) - Pg(i))*d*h/dt*sqrt(n);
            end
        end

    %calculate storage unit change in state-of-charge
        if sign(Pc(dt)-Pdg(dt)-Pg(dt)) > 0
            Delta_Es = -(Es(dt) - (Pc(dt) - Pdg(dt) -
Pg(dt))*d*h/dt/sqrt(n));
        else
```

```
            Delta_Es = -(Es(dt) - (Pc(dt) - Pdg(dt) -
Pg(dt))*d*h/dt*sqrt(n));
        end

        if Pdg_rating(j) > Pdg_max      %In case there is an infinite loop
            break                       %break the loop once constraint is
                                        %reached and move on
        end
    end

    Pg_minrating(j) = min(Pg);

    %find the average daily power from the grid
    Eg_rating(j) = trapz(t,Pg)*d*h/dt;

    %% Storage Model

    % Develop Ps
    Ps = Pc - Pdg - Pg;

    %calculate the power rating of the storage unit
    Ps_rating(j) = max(abs(min(Ps)),max(Ps));

    %calculate the energy rating of the storage unit
    Es_rating(j) = (max(Es)-min(Es))/(SOC_max-SOC_min);

end

%% Discard values that violate maximum constraints

Pdg_rating_trunc = [];
Ps_rating_trunc  = [];
Pg_rating_trunc  = [];
Es_rating_trunc  = [];
Eg_rating_trunc  = [];

for i=1:length(Pdg_rating)

    if ((Pdg_rating(i) <= Pdg_max) && (Ps_rating(i) <= Ps_max) && ...
            (Pg_rating(i) <= Pg_max) && (Es_rating(i) <= Es_max) && ...
            (Eg_rating(i) <= Eg_max) && (Pg_minrating(i) >=Pg_min))

        Pdg_rating_trunc = [Pdg_rating_trunc Pdg_rating(i)];
        Ps_rating_trunc  = [Ps_rating_trunc Ps_rating(i)];
        Pg_rating_trunc  = [Pg_rating_trunc Pg_rating(i)];
        Es_rating_trunc  = [Es_rating_trunc Es_rating(i)];
        Eg_rating_trunc  = [Eg_rating_trunc Eg_rating(i)];

    end
end
    Clength = length(Pdg_rating_trunc);
    Cdg = zeros(1, Clength);
    Cs  = zeros(1, Clength);
    Cm1 = zeros(1, Clength);
```

```
    Cm2 = zeros(1, Clength);
    Cm  = zeros(1, Clength);
    Cg  = zeros(1, Clength);
    Ctotal = zeros(1, Clength);

for j=1:Clength
    %% Cost Calculation

    %DG cost
    Cdg(j) = C_dg_0+Cp_dg*Pdg_rating_trunc(j);

    %Storage unit cost
    Cs(j) = C_s_0 + Cpp_s*Es_rating_trunc(j) + Cp_s*Ps_rating_trunc(j);

    %Maintenance cost
    Cm1(j) = floor(T_lifesys/T_lifedg)*Cdg(j);   %for DG
    Cm2(j) = floor(T_lifesys/T_lifes)*Cs(j);     %for storage unit
    Cm(j) = Cm1(j)+Cm2(j);

    %Grid cost
    Cg(j) = C_g_0*T_lifesys/T_bp + Cp_g*Pg_rating_trunc(j)*T_lifesys*12 + ...
        Cpp_g*Eg_rating_trunc(j)*T_lifesys*365/d;

    %Total cost
    Ctotal(j) = round((Cdg(j)+Cs(j)+Cm(j)+Cg(j))*100)/100;

end

%Find the lowest cost point
[Ctotal_min J]= min(Ctotal)

%% Display Results

display(['The power rating of the distributed generation is ',
num2str(Pdg_rating_trunc(J)),' kW'])
display(['The power rating of the storage is ', num2str(Ps_rating_trunc(J)),'
kW'])
display(['The energy capacity of the storage is ',
num2str(Es_rating_trunc(J)),' kWh'])

display(['The maximum power drawn from the grid is ',
num2str(Pg_rating_trunc(J)),' kW'])
display(['The total energy drawn from the grid is ',
num2str(Eg_rating_trunc(J)),' kWh per day'])

display(['The minimum lifetime cost is $', num2str(Ctotal_min),'.'])

if norm(f)>10^-6
    display(['The DG cost is $',num2str(Cdg(J)+Cm1(J)),'.'])
else
    display(['The DG cost is $', num2str(Cdg(J)+Cm1(J)-C_dg_0),'.'])
end
display(['The storage cost is $', num2str(Cs(J)+Cm2(J)),'.'])
display(['The grid cost is $', num2str(Cg(J)),'.'])
```

```
%% Plots

figure(1)
plot(Pg_rating_trunc, Cdg,Pg_rating_trunc, Cs, Pg_rating_trunc, Cm,
Pg_rating_trunc, Cg, Pg_rating_trunc, Ctotal)
legend('DG Cost','Storage Cost','Maintenance Cost','Grid Cost', 'Total System
Lifecycle Cost')
xlabel('Power Drawn from Grid')
ylabel('Cost ($)')
title('EV charging costs for 1 EV, 24 kwh, 6 kW each day for 20 years')

%recalculations of the time-based vectors for plotting purposes
E_S2C = 0;
Tc_off = 0;

for i=1:dt
    if Pc(i) > 0
        E_S2C = E_S2C + (Pc(i) - Pg_rating_trunc(J))*d*h/dt;
    else
        Tc_off = Tc_off + d*h/dt;
    end
end

if norm(f) > 10e-6;          %There is a DG unit
    Ps = zeros(1,dt);
else                         %There is no DG unit
    for i=1:dt
        if Pc(i) > 0
            Ps(i) = Pc(i) - Pg_rating_trunc(J);
        else
            Ps(i) = -E_S2C/Tc_off/n;
        end
    end
end

Pdg = Pdg_rating_trunc(J)*f;

for i=1:dt
    Pg(i) = min(max(0, Pc(i) - Pdg(i)-Ps(i)), Pg_rating_trunc(J));
end

Ps = Pc - Pdg - Pg;

Es(1) = 0;
for i=1:dt-1
    if sign(Ps(i)) < 0       %Storage is being charged
        Es(i+1) = Es(i) - sqrt(n)*Ps(i)*d*h/dt;
    else                     %Storage is supplying power
        Es(i+1) = Es(i) - Ps(i)/sqrt(n)*d*h/dt;
    end
end
min_Es = min(Es);
Es = (abs(min_Es) + Es + SOC_min*Es_rating_trunc(J));
```

```
figure
plot(t,Pc, t, Pdg, t, Pg, t, Ps)
legend('Charger','DG','Grid','Storage Unit','location','best')
title(['Power Profiles'])
set(gca, 'XTick', [0:120:dt])
set(gca, 'XTickLabel', [0:2:24])
xlabel('time of day (hours)')
ylabel('Power (kW)')

figure
plot(t,Es)
title(['Energy stored in the ',num2str(Ps_rating_trunc(J)),' kW, '...
    ,num2str(Es_rating_trunc(J)),' kWh Storage Unit'])
axis([0 dt 0 Es_rating_trunc(J)*1.2])
set(gca, 'XTick', [0:120:dt])
set(gca, 'XTickLabel', [0:2:24])
xlabel('time of day (hours)')
ylabel('Energy capacity (kWh)')

hold on
plot(t, SOC_min*Es_rating_trunc(J)*ones(1,length(t)),'--')
plot(t, Es_rating_trunc(J)*ones(1,length(t)),'--')
```

```
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% Linear Programming Tool - Configuration A                                 %
% Develop the variables to use matlab's linear optimization tool            %
% General equation to be solved: C = c' x                                   %
% Variables to solve for: Pdg, Ps, Pg, Es, Eg, each of length dt            %
% x =[Pg1' Pg1_rating                                                       %
%     Pg2' Pg2_rating                                                       %
%     Pg'  Pg_rating                                                        %
%     Pdg1'  Pdg1_rating                                                    %
%     Pdg2'  Pdg2_rating                                                    %
%     Ps1' Ps1_rating                                                       %
%     Ps2' Ps2_rating                                                       %
%     Es2' Es_rating                                                        %
%     Es2' Es_rating                                                        %
%     Eg_rating]'                                                           %
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

clear all
clc
close all

%Cost Parameters
T_lifesys = 20;         %system life (years)
T_lifedg = 25;          %DG life (years)
T_lifes = 6;            %storage unit life (years)
                            %6 for lead-acid
                            %12 for Li-ion
T_bp = 1/12;            %billing period (years)


C_dg_0 = 200;           %fixed cost of DG independent of rating ($)
                            %$200 for solar PV
                            %$3,750 for wind
C_s_0 = 110;             %fixed cost of storage unit independent of...
                            %rating ($)
                            %$110 for lead-acid
                            %$80 for Li-ion
C_g_0 = 15;              %fixed cost of grid connection applied to each...
                            %bill ($)
                            %$9 for residential
                            %$15 for general, no demand
                            %$60 for general, demand
Cp_dg = 4400;           %variable cost of DG ($/kW)
                            %$4400 for solar PV (includes inverter cost)
                            %$5500 for wind (includes power conditioning)
Cp_s = 200;              %variable power cost of storage unit ($/kW)
                            %$200 for charge controller
Cpp_s = 220;            %variable energy cost of storage unit ($/kWh)
                            %$220 for lead-acid
                            %$800 for Li-ion
Cp_g = 0;               %variable power cost of the grid ($/kW)
                            %$0 when no demand charge
                            %$10 when demand charge
Cpp_g = 0.14;           %variable energy cost of the grid ($/kWh)
                            %$0.14
```

```
% Establish constraints
Pc_max = 7.2;
Pg_max = 10;                %5 for residential
                            %10 for general, no demand
                            %50 for general, demand
Pdg_max = 50;               %10 for residential
                            %50 for general
Ps_max = 50                 %10 for residential
                            %50 for general
Es_max = 500;               %100 for residential
                            %500 for general
Eg_max = 1000;              %arbitrarily high value, no constraint

Pdg_min = 0;                %power cannot go into the DG
Ps_min = -Ps_max;           %high enough to charge
Pg_min = 0;                 %no power into the grid
Es_min = 0;                 %battery cannot reach zero state of charge
Eg_min = 0;                 %no energy into the grid

n = 0.85;                   %efficiency of the storage unit
SOC_max = 1;                %set maximum state of charge for the storage unit
SOC_min = 0.80;             %set minimum state of charge for the storage unit

d = 1;                      %days in time period
h = 24;                     %hours in time period
m = 60;                     %60/m is the resolution of the time vector
dt = d*h*m;                 %time period in intervals
t = 1:1:dt;                 %time vector in minutes for one full day

% EV charge profile
Pc_rating = 7.2;    %fixed, defined charge rating
Cprofile1 = load('Cprofile_public1.csv');
Cprofile2 = load('Cprofile_public2.csv');

Pc1 = Pc_rating*Cprofile1';
Pc2 = Pc_rating*Cprofile2';

% DG power profile
fdg1 = load('DGprofile.csv');
fdg2 = load('DGprofile.csv');

%% Develop c
%fill in c where appropriate
c = [zeros(1,dt), 0, ...                        %Pg1
     zeros(1,dt), 0, ...                        %Pg2
     zeros(1,dt), Cp_g*T_lifesys/T_bp, ...      %Pg
     zeros(1,dt), Cp_dg*floor(1+T_lifesys/T_lifedg), ...    %Pdg1
     zeros(1,dt), Cp_dg*floor(1+T_lifesys/T_lifedg), ...    %Pdg2
     zeros(1,dt), Cp_s*floor(1+T_lifesys/T_lifes), ...      %Ps1
     zeros(1,dt), Cp_s*floor(1+T_lifesys/T_lifes), ...      %Ps2
     zeros(1,dt), Cpp_s*floor(1+T_lifesys/T_lifes), ...     %Es1
     zeros(1,dt), Cpp_s*floor(1+T_lifesys/T_lifes), ...     %Es2
     Cpp_g*365/d*T_lifesys]';                               %Eg
```

```
%% Develop A and b to solve A*x <= b
% This is used to make sure that every value in Pdg' is less than or
% equal to Pdg_rating, every value in Ps' is less than or equal to
% Pdg_rating, etc.

% create A, start with zero because it will be a sparse matrix
A = zeros(length(c));

%create A1
A1 = zeros(dt, dt+1);
for i=1:dt
    A1(i,i) = 1;     %fill in 1 to compare a Pg value to Pg_rating
    A1(i,dt+1) = -1; %fill in -1 at Pg_rating to compare with Pg value
end

%create A2
A2=A1;

%create A3
A31 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
    %the maximum charge or discharge power for the storage unit
    if Pc1(i) > 0   %vehicle is charging, storage will be discharging
                        %meaning Ps will be positive, Ps <= Ps_rating
        A31(i,i) = 1;     %fill in 1 at Ps value to compare to Ps_rating

    else                %vehicle is not charging, storage can only charge
                        %meaning Ps will be negative, -Ps <= Ps_rating
        A31(i,i) = -1;    %fill in 1 at Psrating to compare with Ps value
    end

    A31(i,dt+1) = -1;   %fill in -1 at Psrating to compare with Ps value
end

%create A3
A32 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
    %the maximum charge or discharge power for the storage unit
    if Pc2(i) > 0   %vehicle is charging, storage will be discharging
                        %meaning Ps will be positive, Ps <= Ps_rating
        A32(i,i) = 1;     %fill in 1 at Ps value to compare to Ps_rating

    else                %vehicle is not charging, storage can only charge
                        %meaning Ps will be negative, -Ps <= Ps_rating
        A32(i,i) = -1;    %fill in 1 at Psrating to compare with Ps value
    end

    A32(i,dt+1) = -1;   %fill in -1 at Psrating to compare with Ps value
end

%create A4
A4 = zeros(dt, dt+1);
for i=1:dt
```

```
    A4(i,i) = 1;          %fill in 1 at Es value to compare to Es_rating
    A4(i,dt+1) = -(SOC_max-SOC_min);   %fill in -(SOC_max-SOC_min) at
                                        %Es_rating to compare and scale
                                        %appropriately to Es value
end

%create A
Z = zeros(dt,dt+1);

A = [A1, Z , Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , A1, Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , A1, Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , A2, Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , A2, Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z ,A31, Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z ,A32, Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z , A4, Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z , Z , A4, zeros(dt,1)];

% create b which is just zero
b = zeros(size(A,1),1);

%% Develop D and g to solve D*x = g
% This relationship is used to make sure that the charger power
% requirements are met. Namely it makes sure that for every instance in
% time, Pc = Pdg + Ps + Pg.
% It will also be used to develop the shape of the panel power output and
% calculate Eb and Egrating.

%create D1
D1 = zeros(dt, dt+1);
for i=1:dt
    D1(i,i) = 1;           %fill in 1
end

%create D2
D21 = zeros(dt, dt+1);
for i=1:dt
    D21(i,i) = 1;              %fill in 1 for Pdg
    D21(i, dt+1) = -fdg1(i);  %fill in corresponding -fdg in
                                    %Pdgrating position
end

%create D2
D22 = zeros(dt, dt+1);
for i=1:dt
    D22(i,i) = 1;              %fill in 1 for Pdg
    D22(i, dt+1) = -fdg2(i);  %fill in corresponding -fdg in
                                    %Pdgrating position
end

%create D3
D31 = zeros(dt, dt+1);
for i=1:dt
    if Pc1(i) > 0                              %Vehicle is charging
```

```
        D31(i,i) = d*h/dt/sqrt(n);           %Battery supplies less energy
                                        %than what was stored
    else
        D31(i,i) = d*h/dt*sqrt(n);  %Battery stores less energy than
                                        %what was supplied
    end
end

%create D3
D32 = zeros(dt, dt+1);
for i=1:dt
    if Pc2(i) > 0                            %Vehicle is charging
        D32(i,i) = d*h/dt/sqrt(n);           %Battery supplies less energy
                                        %than what was stored
    else
        D32(i,i) = d*h/dt*sqrt(n);  %Battery stores less energy than
                                        %what was supplied
    end
end

%create D4
D4 = zeros(dt, dt+1);
D4(1,1) = 1;
D4(1,dt) = -1;
for i=2:dt
   D4(i,i) = 1;
   D4(i,i-1) = -1;
end

%create d5
d5 = [d*h/dt*ones(1,dt),0];

%create D
z = zeros(1, dt+1);
D = [D1, Z , Z , D1, Z , D1, Z , Z , Z , zeros(dt,1);
     Z , D1, Z , Z , D1, Z , D1, Z , Z , zeros(dt,1);
     D1, D1,-D1, Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z ,D21, Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z ,D22, Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z ,D31, Z , D4, Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z ,D32, Z , D4, zeros(dt,1);
     z , z , d5, z , z , z , z , z , z , -1         ];

% create g
% we derive beq from the charging profile of the vehicle which is given

% fill in g
g = [Pc1; Pc2; zeros(size(D,1)-2*dt,1)];

%% Develop lmin and lmax for lmin < x < lmax
% the last relation to fulfill for the linearization tool is to define the
% upper and lower bounds for x such that lower bound < x < upper bound for
% all values within x

% Establish upper bounds based on maximum limits
```

```
lmax = [Pc_max*ones(dt+1,1);...
        Pc_max*ones(dt+1,1);...
        Pg_max*ones(dt+1,1);...
        Pdg_max*ones(dt+1,1);...
        Pdg_max*ones(dt+1,1);...
        Ps_max*ones(dt+1,1); ...
        Ps_max*ones(dt+1,1); ...
        Es_max*SOC_max*ones(dt+1,1);...
        Es_max*SOC_max*ones(dt+1,1);...
        Eg_max];

% Establish lower bounds
lmin = [-Pg_min*ones(dt+1,1);...
        -Pg_min*ones(dt+1,1);...
        -Pg_min*ones(dt+1,1);...
        Pdg_min*ones(dt+1,1);...
        Pdg_min*ones(dt+1,1);...
        Ps_min*ones(dt+1,1); ...
        Ps_min*ones(dt+1,1); ...
        Es_min*SOC_min*ones(dt+1,1);...
        Es_min*SOC_min*ones(dt+1,1);...
        Eg_min];

%% Solve using MATLAB's linear programming tool
% options = optimset('MaxIter',300);
[y, fval, exitflag, output]  = linprog(c,A,b,D,g,lmin,lmax);%,[],options);
if y(4*dt+4)>10^-6 && y(5*dt+5)>10^-6
    if y(7*dt+7)>10^-6 && y(9*dt+9)>10^-6
    Cmin = 2*C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
        2*C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    elseif y(7*dt+7)<10^-6 && y(9*dt+9)<10^-6
    Cmin = 2*C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
         C_g_0*T_lifesys/T_bp + c'*y;
    else
        Cmin = 2*C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
        C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    end
elseif y(4*dt+4)<10^-6 && y(5*dt+5)<10^-6
    if y(7*dt+7)>10^-6 && y(9*dt+9)>10^-6
        Cmin = 2*C_s_0*floor(1+T_lifesys/T_lifes) +...
            C_g_0*T_lifesys/T_bp + c'*y;
    elseif y(7*dt+7)<10^-6 && y(9*dt+9)<10^-6
        Cmin = C_g_0*T_lifesys/T_bp + c'*y;
    else
        Cmin = C_s_0*floor(1+T_lifesys/T_lifes) +...
            C_g_0*T_lifesys/T_bp + c'*y;
    end
else
    if y(7*dt+7)>10^-6 && y(9*dt+9)>10^-6
    Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
        2*C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    elseif y(7*dt+7)<10^-6 && y(9*dt+9)<10^-6
        Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
        C_g_0*T_lifesys/T_bp + c'*y;
    else
        Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
```

```
        C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    end
end

%% Display results
display(['The power rating of DG1 is ', num2str(y(4*dt+4)),' kW'])
display(['The power rating of storage1 is ', num2str(y(6*dt+6)),' kW'])
display(['The energy capacity of storage1 is ', num2str(y(8*dt+8)),' kWh'])

display(['The power rating of DG2 is ', num2str(y(5*dt+5)),' kW'])
display(['The power rating of storage2 is ', num2str(y(7*dt+7)),' kW'])
display(['The energy capacity of storage2 is ', num2str(y(9*dt+9)),' kWh'])

display(['The maximum power drawn from the grid is ', num2str(y(3*dt+3)),'
kW'])
display(['The total energy drawn from the grid is ', num2str(y(9*dt+10)),'
kWh per period'])

display(['The minimum lifetime cost is $', num2str(Cmin),'.'])
if y(4*dt+4)>10^-6
    display(['The DG1 cost is $',...
        num2str(C_dg_0*floor(1+T_lifesys/T_lifedg)+...
        Cp_dg*floor(1+T_lifesys/T_lifedg)*y(4*dt+4)),'.'])
else
    display(['The DG1 cost is $',...
        num2str(Cp_dg*floor(1+T_lifesys/T_lifedg)*y(4*dt+4)),'.'])
end

if y(5*dt+5)>10^-6
    display(['The DG2 cost is $',...
        num2str(C_dg_0*floor(1+T_lifesys/T_lifedg)+...
        Cp_dg*floor(1+T_lifesys/T_lifedg)*y(5*dt+5)),'.'])
else
    display(['The DG2 cost is $',...
        num2str(Cp_dg*floor(1+T_lifesys/T_lifedg)*y(5*dt+5)),'.'])
end

if y(8*dt+8)>10^-6
    display(['The storage1 cost is $',...
        num2str(C_s_0*floor(1+T_lifesys/T_lifes)+...
        Cp_s*floor(1+T_lifesys/T_lifes)*y(6*dt+6)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(8*dt+8)),'.'])
else
    display(['The storage1 cost is $',...
        num2str(Cp_s*floor(1+T_lifesys/T_lifes)*y(6*dt+6)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(8*dt+8)),'.'])
end

if y(9*dt+9)>10^-6
    display(['The storage2 cost is $',...
        num2str(C_s_0*floor(1+T_lifesys/T_lifes)+...
        Cp_s*floor(1+T_lifesys/T_lifes)*y(7*dt+7)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(9*dt+9)),'.'])
else
    display(['The storage2 cost is $',...
```

```
        num2str(Cp_s*floor(1+T_lifesys/T_lifes)*y(7*dt+7)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(9*dt+9)),'.'])
end

display(['The grid cost is $',...
    num2str(C_g_0*T_lifesys/T_bp+Cp_g*T_lifesys/T_bp*y(3*dt+3)+...
    Cpp_g*365/d*T_lifesys*y(9*dt+10)),'.'])


t=1:dt;

Pg1 =[y(1:dt)];
Pg2 =[y(dt+2:2*dt+1)];
Pg =[y(2*dt+3:3*dt+2)];
Pdg1 =[y(3*dt+4:4*dt+3)];
Ps1 =[y(5*dt+6:6*dt+5)];
Pdg2 =[y(4*dt+5:5*dt+4)];
Ps2 =[y(6*dt+7:7*dt+6)];

figure
plot(t,Pc1, t, Pdg1, t, Pg1, t, Ps1)
legend('Charger1','DG1','Grid1','Storage Unit1','location','best')
title(['Charger 1 Power Outputs'])
set(gca, 'XTick', [0:dt/(2*6):dt])
set(gca, 'XTickLabel', [0:.5:6])
xlabel('Day')
ylabel('Power drawn (kW)')

figure
plot(t,Pc2, t, Pdg2, t, Pg2, t, Ps2)
legend('Charger2','DG2','Grid2','Storage Unit2','location','best')
title(['Charger 2 Power Outputs'])
set(gca, 'XTick', [0:dt/(24):dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('Time of Day (hours)')
ylabel('Power drawn (kW)')

figure
plot(t,Pc2+Pc1, t, Pdg2+Pdg1, t, Pg, t, Ps1+Ps2)
legend('Charger','DG','Grid','Storage Unit','location','best')
title(['Sum of Each Component Power Outputs'])
set(gca, 'XTick', [0:dt/(24):dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('Time of Day (hours)')
ylabel('Power drawn (kW)')

figure
plot(t,Pc1, t, Pc2, t, Pdg1,t,Pdg2, t, Pg, t, Ps1,t,Ps2)
legend('Charger1','Charger2','DG1','DG2','Grid','Storage Unit1',...
    'Storage Unit2','location','best')
title(['Power Outputs'])
set(gca, 'XTick', [0:dt/(2*6):dt])
set(gca, 'XTickLabel', [0:.5:6])
xlabel('Day')
ylabel('Power drawn (kW)')
```

```
Es1 = [y(7*dt+8:8*dt+7)]+y(8*dt+8)*SOC_min;
Es2 = [y(8*dt+9:9*dt+8)]+y(9*dt+9)*SOC_min;

figure(5)
plot(t,Es1, t, Es2)
title(['Energy Stored in the Storage Units'])
axis([0 dt 0 max(y(8*dt+8),y(9*dt+9))*1.2])
set(gca, 'XTick', [0:dt/24:dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('time of day (hours)')
ylabel('Energy capacity (kWh)')
```

```
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% Linear Programming Tool - Configuration B                                   %
% Develop the variables to use matlab's linear optimization tool              %
% General equation to be solved: C = c' x                                     %
% Variables to solve for: Pdg, Ps, Pg, Es, Eg, each of length dt              %
% x =[Pg1' Pg1_rating                                                         %
%     Pg2' Pg2_rating                                                         %
%     Pg'  Pg_rating                                                          %
%     Pdg1'  Pdg1_rating                                                      %
%     Pdg2'  Pdg2_rating                                                      %
%     Ps1' Ps1_rating                                                         %
%     Ps2' Ps2_rating                                                         %
%     Ps'  Ps_rating                                                          %
%     Es'  Es_rating                                                          %
%     Eg_rating]'                                                             %
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

clear all
clc
close all

%Cost Parameters
T_lifesys = 20;         %system life (years)
T_lifedg = 25;          %DG life (years)
T_lifes = 6;            %storage unit life (years)
                            %6 for lead-acid
                            %12 for Li-ion
T_bp = 1/12;            %billing period (years)


C_dg_0 = 200;           %fixed cost of DG independent of rating ($)
                            %$200 for solar PV
                            %$3,750 for wind
C_s_0 = 110;             %fixed cost of storage unit independent of...
                            %rating ($)
                            %$110 for lead-acid
                            %$80 for Li-ion
C_g_0 = 15;              %fixed cost of grid connection applied to each...
                            %bill ($)
                            %$9 for residential
                            %$15 for general, no demand
                            %$60 for general, demand
Cp_dg = 4400;           %variable cost of DG ($/kW)
                            %$4400 for solar PV (includes inverter cost)
                            %$5500 for wind (includes power conditioning)
Cp_s = 200;              %variable power cost of storage unit ($/kW)
                            %$200 for charge controller
Cpp_s = 220;            %variable energy cost of storage unit ($/kWh)
                            %$220 for lead-acid
                            %$800 for Li-ion
Cp_g = 10;               %variable power cost of the grid ($/kW)
                            %$0 when no demand charge
                            %$10 when demand charge
Cpp_g = 0.14;           %variable energy cost of the grid ($/kWh)
                            %$0.14
```

```
% Establish constraints
Pc_max = 7.2;
Pg_max = 10;            %5 for residential
                        %10 for general, no demand
                        %50 for general, demand
Pdg_max = 50;           %10 for residential
                        %50 for general
Ps_max = 10;            %10 for residential
                        %50 for general
Es_max = 500;           %100 for residential
                        %500 for general
Eg_max = 1000;          %arbitrarily high value, no constraint

Pdg_min = 0;            %power cannot go into the DG
Ps_min = -Ps_max;       %high enough to charge
Pg_min = 0;             %no power into the grid
Es_min = 0;             %battery cannot reach zero state of charge
Eg_min = 0;             %no energy into the grid

n = 0.85;               %efficiency of the storage unit
SOC_max = 1;            %set maximum state of charge for the storage unit
SOC_min = 0.80;         %set minimum state of charge for the storage unit

d = 1;                  %days in time period
h = 24;                 %hours in time period
m = 60;                 %60/m is the resolution of the time vector
dt = d*h*m;             %time period in intervals
t = 1:1:dt;             %time vector in minutes for one full day

% EV charge profile
Pc_rating = 7.2;    %fixed, defined charge rating
Cprofile1 = load('Cprofile_public1.csv');
Cprofile2 = load('Cprofile_public2.csv');

Pc1 = Pc_rating*Cprofile1';
Pc2 = Pc_rating*Cprofile2';

% DG power profile
fdg1 = load('Dgprofile.csv');
fdg2 = load('DGprofile.csv');

%% Develop c
%fill in c where appropriate
c = [zeros(1,dt), 0, ...                        %Pg1
     zeros(1,dt), 0, ...                        %Pg2
     zeros(1,dt), Cp_g*T_lifesys/T_bp, ...      %Pg
     zeros(1,dt), Cp_dg*floor(1+T_lifesys/T_lifedg), ...    %Pdg1
     zeros(1,dt), Cp_dg*floor(1+T_lifesys/T_lifedg), ...    %Pdg2
     zeros(1,dt), 0, ...                        %Ps1
     zeros(1,dt), 0, ...                        %Ps2
     zeros(1,dt), Cp_s*floor(1+T_lifesys/T_lifes), ...      %Ps
     zeros(1,dt), Cpp_s*floor(1+T_lifesys/T_lifes), ...     %Es
     Cpp_g*365/d*T_lifesys]';                               %Eg

%% Develop A and b to solve A*x <= b
```

```
% This is used to make sure that every value in Pdg' is less than or
% equal to Pdg_rating, every value in Ps' is less than or equal to
% Pdg_rating, etc.

% create A, start with zero because it will be a sparse matrix
A = zeros(length(c));

%create A1
A1 = zeros(dt, dt+1);
for i=1:dt
    A1(i,i) = 1;     %fill in 1 to compare a Pg value to Pg_rating
    A1(i,dt+1) = -1; %fill in -1 at Pg_rating to compare with Pg value
end

%create A2
A2=A1;

%create A3
A31 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
    %the maximum charge or discharge power for the storage unit
    if Pc1(i) > 0   %vehicle is charging, storage will be discharging
                        %meaning Ps will be positive, Ps <= Ps_rating
        A31(i,i) = 1;     %fill in 1 at Ps value to compare to Ps_rating

    else                %vehicle is not charging, storage can only charge
                        %meaning Ps will be negative, -Ps <= Ps_rating
        A31(i,i) = -1;    %fill in 1 at Psrating to compare with Ps value
    end

    A31(i,dt+1) = -1;   %fill in -1 at Psrating to compare with Ps value
end

%create A3
A32 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
    %the maximum charge or discharge power for the storage unit
    if Pc2(i) > 0   %vehicle is charging, storage will be discharging
                        %meaning Ps will be positive, Ps <= Ps_rating
        A32(i,i) = 1;     %fill in 1 at Ps value to compare to Ps_rating

    else                %vehicle is not charging, storage can only charge
                        %meaning Ps will be negative, -Ps <= Ps_rating
        A32(i,i) = -1;    %fill in 1 at Psrating to compare with Ps value
    end

    A32(i,dt+1) = -1;   %fill in -1 at Psrating to compare with Ps value
end

%create A3
A33 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
```

```
    %the maximum charge or discharge power for the storage unit
    if (Pc1(i)+Pc2(i)) > 0   %vehicle is charging, storage will be
discharging
                            %meaning Ps will be positive, Ps <= Ps_rating
        A33(i,i) = 1;         %fill in 1 at Ps value to compare to Ps_rating

    else                    %vehicle is not charging, storage can only charge
                            %meaning Ps will be negative, -Ps <= Ps_rating
        A33(i,i) = -1;        %fill in 1 at Psrating to compare with Ps value
    end

    A33(i,dt+1) = -1;       %fill in -1 at Psrating to compare with Ps value
end

%create A4
A4 = zeros(dt, dt+1);
for i=1:dt
    A4(i,i) = 1;             %fill in 1 at Es value to compare to Es_rating
    A4(i,dt+1) = -(SOC_max-SOC_min);    %fill in -(SOC_max-SOC_min) at
                                         %Es_rating to compare and scale
                                         %appropriately to Es value
end

%create A
Z = zeros(dt,dt+1);

A = [A1, Z , Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , A1, Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , A1, Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , A2, Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , A2, Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z ,A31, Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z ,A32, Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z ,A33, Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z , Z , A4, zeros(dt,1)];

% create b which is just zero
b = zeros(size(A,1),1);

%% Develop D and g to solve D*x = g
% This relationship is used to make sure that the charger power
% requirements are met. Namely it makes sure that for every instance in
% time, Pc = Pdg + Ps + Pg.
% It will also be used to develop the shape of the panel power output and
% calculate Eb and Egrating.

%create D1
D1 = zeros(dt, dt+1);
for i=1:dt
    D1(i,i) = 1;             %fill in 1
end

%create D2
D21 = zeros(dt, dt+1);
for i=1:dt
```

```
    D21(i,i) = 1;            %fill in 1 for Pdg
    D21(i, dt+1) = -fdg1(i);  %fill in corresponding -fdg in
                                    %Pdgrating position
end

%create D2
D22 = zeros(dt, dt+1);
for i=1:dt
    D22(i,i) = 1;            %fill in 1 for Pdg
    D22(i, dt+1) = -fdg2(i);  %fill in corresponding -fdg in
                                    %Pdgrating position
end

%create D3
D3 = zeros(dt, dt+1);
for i=1:dt
    if (Pc1(i)+Pc2(i))> 0                           %Vehicle is charging
        D3(i,i) = d*h/dt/sqrt(n);           %Battery supplies less energy
                                        %than what was stored
    else
        D3(i,i) = d*h/dt*sqrt(n);  %Battery stores less energy than
                                        %what was supplied
    end
end

%create D4
D4 = zeros(dt, dt+1);
D4(1,1) = 1;
D4(1,dt) = -1;
for i=2:dt
   D4(i,i) = 1;
   D4(i,i-1) = -1;
end

%create d5
d5 = [d*h/dt*ones(1,dt),0];

%create D
z = zeros(1, dt+1);
D = [D1, Z , Z , D1, Z , D1, Z , Z , Z , zeros(dt,1);
     Z , D1, Z , Z , D1, Z , D1, Z , Z , zeros(dt,1);
     D1, D1,-D1, Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , D1, D1,-D1, Z , zeros(dt,1);
     Z , Z , Z ,D21, Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z ,D22, Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z , D3, D4, zeros(dt,1);
     z , z , d5, z , z , z , z , z , z , -1         ];

% create g
% we derive beq from the charging profile of the vehicle which is given

% fill in g
g = [Pc1; Pc2; zeros(size(D,1)-2*dt,1)];

%% Develop lmin and lmax for lmin < x < lmax
```

```
% the last relation to fulfill for the linearization tool is to define the
% upper and lower bounds for x such that lower bound < x < upper bound for
% all values within x

% Establish upper bounds based on maximum limits
lmax = [Pc_max*ones(dt+1,1);...
        Pc_max*ones(dt+1,1);...
        Pg_max*ones(dt+1,1);...
        Pdg_max*ones(dt+1,1);...
        Pdg_max*ones(dt+1,1);...
        Ps_max*ones(dt+1,1); ...
        Ps_max*ones(dt+1,1); ...
        Ps_max*SOC_max*ones(dt+1,1);...
        Es_max*SOC_max*ones(dt+1,1);...
        Eg_max];

% Establish lower bounds
lmin = [-Pg_min*ones(dt+1,1);...
        -Pg_min*ones(dt+1,1);...
        -Pg_min*ones(dt+1,1);...
        Pdg_min*ones(dt+1,1);...
        Pdg_min*ones(dt+1,1);...
        Ps_min*ones(dt+1,1); ...
        Ps_min*ones(dt+1,1); ...
        Ps_min*SOC_min*ones(dt+1,1);...
        Es_min*SOC_min*ones(dt+1,1);...
        Eg_min];

%% Solve using MATLAB's linear programming tool
% options = optimset('MaxIter',300);
[y, fval, exitflag, output] = linprog(c,A,b,D,g,lmin,lmax);%,[],options);
if y(4*dt+4)>10^-6 && y(5*dt+5)>10^-6
    if y(9*dt+9)>10^-6
        Cmin = 2*C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
            C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    else
        Cmin = 2*C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
            C_g_0*T_lifesys/T_bp + c'*y;
    end
elseif y(4*dt+4)<10^-6 && y(5*dt+5)<10^-6
    if y(9*dt+9)>10^-6
        Cmin = C_s_0*floor(1+T_lifesys/T_lifes) +...
            C_g_0*T_lifesys/T_bp + c'*y;
    else
        Cmin = C_g_0*T_lifesys/T_bp + c'*y;
    end
else
    if y(9*dt+9)
        Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
            C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    else
        Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
            C_g_0*T_lifesys/T_bp + c'*y;
    end
end
```

```
%% Display results
display(['The power rating of DG1 is ', num2str(y(4*dt+4)),' kW'])
display(['The power rating of DG2 is ', num2str(y(5*dt+5)),' kW'])

display(['The power rating of the storage unit is ', num2str(y(8*dt+8)),'
kW'])
display(['The energy capacity of storage unit is ', num2str(y(9*dt+9)),'
kWh'])

display(['The maximum power drawn from the grid is ', num2str(y(3*dt+3)),'
kW'])
display(['The total energy drawn from the grid is ', num2str(y(9*dt+10)),'
kWh per period'])

display(['The minimum lifetime cost is $', num2str(Cmin),'.'])
if y(4*dt+4)>10^-6
    display(['The DG1 cost is $',...
        num2str(C_dg_0*floor(1+T_lifesys/T_lifedg)+...
        Cp_dg*floor(1+T_lifesys/T_lifedg)*y(4*dt+4)),'.'])
else
    display(['The DG1 cost is $',...
        num2str(Cp_dg*floor(1+T_lifesys/T_lifedg)*y(4*dt+4)),'.'])
end

if y(5*dt+5)>10^-6
    display(['The DG2 cost is $',...
        num2str(C_dg_0*floor(1+T_lifesys/T_lifedg)+...
        Cp_dg*floor(1+T_lifesys/T_lifedg)*y(5*dt+5)),'.'])
else
    display(['The DG2 cost is $',...
        num2str(Cp_dg*floor(1+T_lifesys/T_lifedg)*y(5*dt+5)),'.'])
end

if y(8*dt+8)>10^-6
    display(['The storage cost is $',...
        num2str(C_s_0*floor(1+T_lifesys/T_lifes)+...
        Cp_s*floor(1+T_lifesys/T_lifes)*y(8*dt+8)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(9*dt+9)),'.'])
else
    display(['The storage cost is $',...
        num2str(Cp_s*floor(1+T_lifesys/T_lifes)*y(8*dt+8)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(9*dt+9)),'.'])
end

display(['The grid cost is $',...
    num2str(C_g_0*T_lifesys/T_bp+Cp_g*T_lifesys/T_bp*y(3*dt+3)+...
    Cpp_g*365/d*T_lifesys*y(9*dt+10)),'.'])


t=1:dt;

Pg1 =[y(1:dt)];
Pg2 =[y(dt+2:2*dt+1)];
Pg =[y(2*dt+3:3*dt+2)];
Pdg1 =[y(3*dt+4:4*dt+3)];
```

```
Ps1 =[y(5*dt+6:6*dt+5)];
Pdg2 =[y(4*dt+5:5*dt+4)];
Ps2 =[y(6*dt+7:7*dt+6)];
Ps =[y(7*dt+8:8*dt+7)];

figure
plot(t,Pc1, t, Pdg1, t, Pg1, t, Ps1)
legend('Charger1','DG1','Grid1','Storage Unit1','location','best')
title(['Charger 1 Power Outputs'])
set(gca, 'XTick', [0:dt/(2*6):dt])
set(gca, 'XTickLabel', [0:.5:6])
xlabel('Day')
ylabel('Power drawn (kW)')

figure
plot(t,Pc2, t, Pdg2, t, Pg2, t, Ps2)
legend('Charger2','DG2','Grid2','Storage Unit2','location','best')
title(['Charger 2 Power Outputs'])
set(gca, 'XTick', [0:dt/(24):dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('Time of Day (hours)')
ylabel('Power drawn (kW)')

figure
plot(t,Pc2+Pc1, t, Pdg2+Pdg1, t, Pg, t, Ps)
legend('Charger','DG','Grid','Storage Unit','location','best')
title(['Sum of Component Power Outputs'])
set(gca, 'XTick', [0:dt/(24):dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('Time of Day (hours)')
ylabel('Power drawn (kW)')

figure
plot(t,Pc1, t, Pc2, t, Pdg1,t,Pdg2, t, Pg, t, Ps1,t,Ps2)
legend('Charger1','Charger2','DG1','DG2','Grid','Storage Unit1',...
    'Storage Unit2','location','best')
title(['Power Outputs'])
set(gca, 'XTick', [0:dt/(2*6):dt])
set(gca, 'XTickLabel', [0:.5:6])
xlabel('Day')
ylabel('Power drawn (kW)')

Es = [y(8*dt+9:9*dt+8)]+y(9*dt+9)*SOC_min;

figure(5)
plot(t,Es)
title(['Energy Stored in the Storage Unit'])
axis([0 dt 0 y(9*dt+9)*1.2])
set(gca, 'XTick', [0:dt/24:dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('time of day (hours)')
ylabel('Energy capacity (kWh)')
```

```
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% Linear Programming Tool - Configuration C                                   %
% Develop the variables to use matlab's linear optimization tool              %
% General equation to be solved: C = c' x                                     %
% Variables to solve for: Pdg, Ps, Pg, Es, Eg, each of length dt              %
% x =[Pg1' Pg1_rating                                                         %
%     Pg2' Pg2_rating                                                         %
%     Pg'  Pg_rating                                                          %
%     Pdg1'  Pdg1_rating                                                      %
%     Pdg2'  Pdg2_rating                                                      %
%     Pdg'  Pdg_rating                                                        %
%     Ps1' Ps1_rating                                                         %
%     Ps2' Ps2_rating                                                         %
%     Es2' Es_rating                                                          %
%     Es2' Es_rating                                                          %
%     Eg_rating]'                                                             %
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

clear all
clc
close all

%Cost Parameters
T_lifesys = 20;         %system life (years)
T_lifedg = 25;          %DG life (years)
T_lifes = 6;            %storage unit life (years)
                            %6 for lead-acid
                            %12 for Li-ion
T_bp = 1/12;            %billing period (years)


C_dg_0 = 200;           %fixed cost of DG independent of rating ($)
                            %$200 for solar PV
                            %$3,750 for wind
C_s_0 = 110;             %fixed cost of storage unit independent of...
                            %rating ($)
                            %$110 for lead-acid
                            %$80 for Li-ion
C_g_0 = 15;              %fixed cost of grid connection applied to each...
                            %bill ($)
                            %$9 for residential
                            %$15 for general, no demand
                            %$60 for general, demand
Cp_dg = 4400;           %variable cost of DG ($/kW)
                            %$4400 for solar PV (includes inverter cost)
                            %$5500 for wind (includes power conditioning)
Cp_s = 200;              %variable power cost of storage unit ($/kW)
                            %$200 for charge controller
Cpp_s = 220;            %variable energy cost of storage unit ($/kWh)
                            %$220 for lead-acid
                            %$800 for Li-ion
Cp_g = 0;               %variable power cost of the grid ($/kW)
                            %$0 when no demand charge
                            %$10 when demand charge
Cpp_g = 0.14;           %variable energy cost of the grid ($/kWh)
                            %$0.14
```

```
% Establish constraints
Pc_max = 7.2;
Pg_max = 10;            %5 for residential
                        %10 for general, no demand
                        %50 for general, demand
Pdg_max = 50;           %10 for residential
                        %50 for general
Ps_max = 50;            %10 for residential
                        %50 for general
Es_max = 500;           %100 for residential
                        %500 for general
Eg_max = 1000;          %arbitrarily high value, no constraint

Pdg_min = 0;            %power cannot go into the DG
Ps_min = -Ps_max;       %high enough to charge
Pg_min = 0;             %no power into the grid
Es_min = 0;             %battery cannot reach zero state of charge
Eg_min = 0;             %no energy into the grid

n = 0.85;               %efficiency of the storage unit
SOC_max = 1;            %set maximum state of charge for the storage unit
SOC_min = 0.80;         %set minimum state of charge for the storage unit

d = 1;                  %days in time period
h = 24;                 %hours in time period
m = 60;                 %60/m is the resolution of the time vector
dt = d*h*m;             %time period in intervals
t = 1:1:dt;             %time vector in minutes for one full day

% EV charge profile
Pc_rating = 7.2;    %fixed, defined charge rating
Cprofile1 = load('Cprofile_public1.csv');
Cprofile2 = load('Cprofile_public2.csv');

Pc1 = Pc_rating*Cprofile1';
Pc2 = Pc_rating*Cprofile2';

% DG power profile
fdg = load('DGprofile.csv');

%% Develop c
%fill in c where appropriate
c = [zeros(1,dt), 0, ...                        %Pg1
     zeros(1,dt), 0, ...                        %Pg2
     zeros(1,dt), Cp_g*T_lifesys/T_bp, ...      %Pg
     zeros(1,dt), 0, ...                        %Pdg1
     zeros(1,dt), 0, ...                        %Pdg2
     zeros(1,dt), Cp_dg*floor(1+T_lifesys/T_lifedg), ...    %Pdg
     zeros(1,dt), Cp_s*floor(1+T_lifesys/T_lifes), ...      %Ps1
     zeros(1,dt), Cp_s*floor(1+T_lifesys/T_lifes), ...      %Ps2
     zeros(1,dt), Cpp_s*floor(1+T_lifesys/T_lifes), ...     %Es1
     zeros(1,dt), Cpp_s*floor(1+T_lifesys/T_lifes), ...     %Es2
     Cpp_g*365/d*T_lifesys]';                               %Eg
```

```
%% Develop A and b to solve A*x <= b
% This is used to make sure that every value in Pdg' is less than or
% equal to Pdg_rating, every value in Ps' is less than or equal to
% Pdg_rating, etc.

% create A, start with zero because it will be a sparse matrix
A = zeros(length(c));

%create A1
A1 = zeros(dt, dt+1);
for i=1:dt
    A1(i,i) = 1;     %fill in 1 to compare a Pg value to Pg_rating
    A1(i,dt+1) = -1; %fill in -1 at Pg_rating to compare with Pg value
end

%create A2
A2=A1;

%create A3
A31 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
    %the maximum charge or discharge power for the storage unit
    if Pc1(i) > 0   %vehicle is charging, storage will be discharging
                        %meaning Ps will be positive, Ps <= Ps_rating
        A31(i,i) = 1;     %fill in 1 at Ps value to compare to Ps_rating

    else                %vehicle is not charging, storage can only charge
                        %meaning Ps will be negative, -Ps <= Ps_rating
        A31(i,i) = -1;    %fill in 1 at Psrating to compare with Ps value
    end

    A31(i,dt+1) = -1;   %fill in -1 at Psrating to compare with Ps value
end

%create A3
A32 = zeros(dt, dt+1);
for i=1:dt
    %this is a trick to make Ps_rating equal the to absolute value of
    %the maximum charge or discharge power for the storage unit
    if Pc2(i) > 0   %vehicle is charging, storage will be discharging
                        %meaning Ps will be positive, Ps <= Ps_rating
        A32(i,i) = 1;     %fill in 1 at Ps value to compare to Ps_rating

    else                %vehicle is not charging, storage can only charge
                        %meaning Ps will be negative, -Ps <= Ps_rating
        A32(i,i) = -1;    %fill in 1 at Psrating to compare with Ps value
    end

    A32(i,dt+1) = -1;   %fill in -1 at Psrating to compare with Ps value
end

%create A4
A4 = zeros(dt, dt+1);
for i=1:dt
```

```
    A4(i,i) = 1;         %fill in 1 at Es value to compare to Es_rating
    A4(i,dt+1) = -(SOC_max-SOC_min);   %fill in -(SOC_max-SOC_min) at
                                        %Es_rating to compare and scale
                                        %appropriately to Es value
end

%create A
Z = zeros(dt,dt+1);

A = [A1, Z , Z , Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , A1, Z , Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , A1, Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , A2, Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , A2, Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , A2, Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z ,A31, Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z ,A32, Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z , Z , A4, Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z , Z , Z , A4, zeros(dt,1)];

% create b which is just zero
b = zeros(size(A,1),1);

%% Develop D and g to solve D*x = g
% This relationship is used to make sure that the charger power
% requirements are met. Namely it makes sure that for every instance in
% time, Pc = Pdg + Ps + Pg.
% It will also be used to develop the shape of the panel power output and
% calculate Eb and Egrating.

%create D1
D1 = zeros(dt, dt+1);
for i=1:dt
    D1(i,i) = 1;           %fill in 1
end

%create D2
D2 = zeros(dt, dt+1);
for i=1:dt
    D2(i,i) = 1;            %fill in 1 for Pdg
    D2(i, dt+1) = -fdg(i);  %fill in corresponding -fdg in
                                    %Pdgrating position
end

%create D3
D31 = zeros(dt, dt+1);
for i=1:dt
    if Pc1(i)> 0                               %Vehicle is charging
        D31(i,i) = d*h/dt/sqrt(n);              %Battery supplies less energy
                                        %than what was stored
    else
        D31(i,i) = d*h/dt*sqrt(n);  %Battery stores less energy than
                                        %what was supplied
    end
end
```

```
%create D3
D32 = zeros(dt, dt+1);
for i=1:dt
    if Pc2(i)> 0                             %Vehicle is charging
        D32(i,i) = d*h/dt/sqrt(n);           %Battery supplies less energy
                                         %than what was stored
    else
        D32(i,i) = d*h/dt*sqrt(n);  %Battery stores less energy than
                                         %what was supplied
    end
end

%create D4
D4 = zeros(dt, dt+1);
D4(1,1) = 1;
D4(1,dt) = -1;
for i=2:dt
   D4(i,i) = 1;
   D4(i,i-1) = -1;
end

%create d5
d5 = [d*h/dt*ones(1,dt),0];

%create D
z = zeros(1, dt+1);
D = [D1, Z , Z , D1, Z , Z , D1, Z , Z , Z , zeros(dt,1);
     Z , D1, Z , Z , D1, Z , Z , D1, Z , Z , zeros(dt,1);
     D1, D1,-D1, Z , Z , Z , Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , D1, D1,-D1, Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , D2, Z , Z , Z , Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z ,D31, Z , D4, Z , zeros(dt,1);
     Z , Z , Z , Z , Z , Z , Z ,D32, Z , D4, zeros(dt,1);
     z , z , d5, z , z , z , z , z , z , z , -1         ];

% create g
% we derive beq from the charging profile of the vehicle which is given

% fill in g
g = [Pc1; Pc2; zeros(size(D,1)-2*dt,1)];

%% Develop lmin and lmax for lmin < x < lmax
% the last relation to fulfill for the linearization tool is to define the
% upper and lower bounds for x such that lower bound < x < upper bound for
% all values within x

% Establish upper bounds based on maximum limits
lmax = [Pc_max*ones(dt+1,1);...
        Pc_max*ones(dt+1,1);...
        Pg_max*ones(dt+1,1);...
        Pc_max*ones(dt+1,1);...
        Pc_max*ones(dt+1,1);...
        Pdg_max*ones(dt+1,1); ...
        Ps_max*ones(dt+1,1); ...
```

```
        Ps_max*ones(dt+1,1);...
        Es_max*SOC_max*ones(dt+1,1);...
        Es_max*SOC_max*ones(dt+1,1);...
        Eg_max];

% Establish lower bounds
lmin = [-Pg_min*ones(dt+1,1);...
        -Pg_min*ones(dt+1,1);...
        -Pg_min*ones(dt+1,1);...
        Pdg_min*ones(dt+1,1);...
        Pdg_min*ones(dt+1,1);...
        Pdg_min*ones(dt+1,1); ...
        Ps_min*ones(dt+1,1); ...
        Ps_min*ones(dt+1,1);...
        Es_min*SOC_min*ones(dt+1,1);...
        Es_min*SOC_min*ones(dt+1,1);...
        Eg_min];

%% Solve using MATLAB's linear programming tool
% options = optimset('MaxIter',300);
[y, fval, exitflag, output]  = linprog(c,A,b,D,g,lmin,lmax);%,[],options);
if y(6*dt+6)>10^-6
    if y(9*dt+9)>10^-66 && y(10*dt+10)>10^-6
        Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
            2*C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    elseif y(9*dt+9)<10^-66 && y(10*dt+10)<10^-6
        Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
            C_g_0*T_lifesys/T_bp + c'*y;
    else
        Cmin = C_dg_0*floor(1+T_lifesys/T_lifedg) + ...
            C_s_0*floor(1+T_lifesys/T_lifes) + C_g_0*T_lifesys/T_bp + c'*y;
    end
else
    if y(9*dt+9)>10^-66 && y(10*dt+10)>10^-6
        Cmin = 2*C_s_0*floor(1+T_lifesys/T_lifes) +...
            C_g_0*T_lifesys/T_bp + c'*y;
    elseif y(9*dt+9)<10^-66 && y(10*dt+10)<10^-6
         Cmin = C_g_0*T_lifesys/T_bp + c'*y;
    else
         Cmin = C_s_0*floor(1+T_lifesys/T_lifes) +...
            C_g_0*T_lifesys/T_bp + c'*y;
    end
end

%% Display results
display(['The power rating of the DG is ', num2str(y(6*dt+6)),' kW'])

display(['The power rating of storage unit1 is ', num2str(y(7*dt+7)),' kW'])
display(['The energy capacity of storage unit1 is ', num2str(y(9*dt+9)),'
kWh'])

display(['The power rating of storage unit2 is ', num2str(y(8*dt+8)),' kW'])
display(['The energy capacity of storage unit2 is ', num2str(y(10*dt+10)),'
kWh'])
```

```
display(['The maximum power drawn from the grid is ', num2str(y(3*dt+3)),'
kW'])
display(['The total energy drawn from the grid is ', num2str(y(10*dt+11)),'
kWh per period'])

display(['The minimum lifetime cost is $', num2str(Cmin),'.'])
if y(6*dt+6)>10^-6
    display(['The DG cost is $',...
        num2str(C_dg_0*floor(1+T_lifesys/T_lifedg)+...
                Cp_dg*floor(1+T_lifesys/T_lifedg)*y(6*dt+6)),'.'])
else
    display(['The DG cost is $',...
        num2str(Cp_dg*floor(1+T_lifesys/T_lifedg)*y(6*dt+6)),'.'])
end

if y(8*dt+8)>10^-6
    display(['The storage1 cost is $',...
        num2str(C_s_0*floor(1+T_lifesys/T_lifes)+...
        Cp_s*floor(1+T_lifesys/T_lifes)*y(7*dt+7)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(9*dt+9)),'.'])
else
    display(['The storage1 cost is $',...
        num2str(Cp_s*floor(1+T_lifesys/T_lifes)*y(7*dt+7)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(9*dt+9)),'.'])
end

if y(9*dt+9)>10^-6
    display(['The storage2 cost is $',...
        num2str(C_s_0*floor(1+T_lifesys/T_lifes)+...
        Cp_s*floor(1+T_lifesys/T_lifes)*y(8*dt+8)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(10*dt+10)),'.'])
else
    display(['The storage2 cost is $',...
        num2str(Cp_s*floor(1+T_lifesys/T_lifes)*y(8*dt+8)+...
        Cpp_s*floor(1+T_lifesys/T_lifes)*y(10*dt+10)),'.'])
end

display(['The grid cost is $',...
    num2str(C_g_0*T_lifesys/T_bp+Cp_g*T_lifesys/T_bp*y(3*dt+3)+...
    Cpp_g*365/d*T_lifesys*y(10*dt+11)),'.'])

t=1:dt;

Pg1 =[y(1:dt)];
Pg2 =[y(dt+2:2*dt+1)];
Pg =[y(2*dt+3:3*dt+2)];
Pdg1 =[y(3*dt+4:4*dt+3)];
Pdg2 =[y(4*dt+5:5*dt+4)];
Pdg =[y(5*dt+6:6*dt+5)];
Ps1 =[y(6*dt+7:7*dt+6)];
Ps2 =[y(7*dt+8:8*dt+7)];

figure
plot(t,Pc1, t, Pdg1, t, Pg1, t, Ps1)
legend('Charger1','DG1','Grid1','Storage Unit1','location','best')
```

```
title(['Charger 1 Power Outputs'])
set(gca, 'XTick', [0:dt/(2*6):dt])
set(gca, 'XTickLabel', [0:.5:6])
xlabel('Day')
ylabel('Power drawn (kW)')

figure
plot(t,Pc2, t, Pdg2, t, Pg2, t, Ps2)
legend('Charger2','DG2','Grid2','Storage Unit2','location','best')
title(['Charger 2 Power Outputs'])
set(gca, 'XTick', [0:dt/(24):dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('Time of Day (hours)')
ylabel('Power drawn (kW)')

figure
plot(t,Pc2+Pc1, t, Pdg, t, Pg, t, (Ps1+Ps2))
legend('Charger','DG','Grid','Storage Unit','location','best')
title(['Sum of Component Power Outputs'])
set(gca, 'XTick', [0:dt/(24):dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('Time of Day (hours)')
ylabel('Power drawn (kW)')

figure
plot(t,Pc1, t, Pc2, t, Pdg,t, Pg, t, Ps1,t,Ps2)
legend('Charger1','Charger2','DG',' Grid','Storage Unit1',...
    'Storage Unit2','location','best')
title(['Power Outputs'])
set(gca, 'XTick', [0:dt/(2*6):dt])
set(gca, 'XTickLabel', [0:.5:6])
xlabel('Day')
ylabel('Power drawn (kW)')

Es1 = [y(8*dt+9:9*dt+8)]+y(9*dt+9)*SOC_min;
Es2 = [y(9*dt+10:10*dt+9)]+y(10*dt+10)*SOC_min;

figure(5)
plot(t,Es1, t, Es2)
title(['Energy Stored in the Storage Unit'])
axis([0 dt 0 y(9*dt+9)*1.2])
set(gca, 'XTick', [0:dt/24:dt])
set(gca, 'XTickLabel', [0:1:24])
xlabel('time of day (hours)')
ylabel('Energy capacity (kWh)')
```

References

[1] "Electrification Roadmap – Revolutionizing Transportation and Achieving Energy Security," Electrification Coalition, November, 2009.

[2] S. W. Hadley and A. Tsvetkova, "Potential Impacts of Plug-in Hybrid Electric Vehicles on Regional Power Generation," Oak Ridge National Laboratory, Oak Ridge, TN, Rep. ORNL/TM-2007/150, January, 2008.

[3] "Environmental Assessment of Plug-in Hybrid Electric Vehicles, Volume 1: Nationwide Greenhouse Gas Emissions," EPRI, Palo Alto, CA, Tech. Rep. 1015325, July 14, 2007.

[4] Database of State Incentives for Renewables & Efficiency, North Carolina State University and the Interstate Renewable Energy Council, viewed on July 29, 2011 at: http://www.dsireusa.org

[5] F. Hacker, R. Harthan, F. Matthes, and W. Zimmer, "Environmental Impacts and Impact on Electricity Market of a Large Scale Introduction of Electric Cars in Europe - Critical Review of Literature," The European Topic Centre on Air and Climate Change, July, 2009.

[6] "Transitions to Alternative Transportation Technologies – Plug-in Hybrid Electric Vehicles," National Research Council, The National Academies Press, Washington, D.C., 2010.

[7] Annual Energy Outlook 2010 Year-by-Year Reference Case Tables (2007-2035), US Energy Information Administration, viewed on May 30, 2010 at:

http://www.eia.gov/oiaf/archive/aeo10/aeoref_tab.html

[8] F. Oduyemi, Plug-in Electric Vehicles and the Dynamic Smart Grid, Southern California Edison, viewed on January 11, 2011 at:

http://www.valleyconnect.com/Libary/2010.02.03_SCE_AdvancedTechnology.pdf

[9] *SAE Electric Vehicle and Plug in Hybrid Electric Vehicle Conductive Charge Coupler*, SAE J1772, 2010.

[10] J. Motavalli, "Killer App for E.V.'s: 30-Minute Recharges," *The New York Times*, Automobiles, pp. 2, August 8, 2010.

[11] ChargePoint Networked Charging Stations, Coulomb Technologies, viewed on December 20, 2010 at: http://www.coulombtech.com/solutions-commercial-products-charging-stations.php

[12] California Electricity Profile 2008 Edition, US Energy Information Administration, viewed on May 7, 2010 at: http://www.eia.gov/cneaf/electricity/st_profiles/california.html

[13] Californa ISO Peak Load History 1998 through 2009, California ISO, viewed on May 7, 2010: http://www.caiso.com/1fb4/1fb4af6c73260.pdf

[14] Highway Statistics 2008, US Department of Transportation Federal Highway Administration, viewed on May 7, 2010 at:

http://www.fhwa.dot.gov/policyinformation/statistics/2008/index.cfm

[15] P. Richardson, D. Flynn, and A. Keane, "Impact Assessment of Varying Penetrations of Electric Vehicles on Low Voltage Distribution Systems," *Proceedings of the IEEE Power and Energy Society General Meeting*, Minneapolis, MN, July, 2010.

[16] J. A. Peças Lopes, F. J. Soares, and P. M. Rocha Almeida, "Integration of Electric Vehicles in the Electric Power System," *Proceedings of the IEEE*, vol. 99, pp. 168-183, 2011.

[17] C. Gerkensmeyer, J. G. DeSteese, and M. C. W. Kintner-Meyer, "Technical Challenges of Plug-In Hybrid Electric Vehicles and Impacts to the US Power System: Distribution System Analysis," Pacific Northwest National Laboratory, Richland, WA, Rep. PNNL-19165, January, 2010.

[18] P. Fairley, "Speed Bumps Ahead for Electric-Vehicle Charging," *IEEE Spectrum*, vol. 47, no. 1, pp. 13-14, January, 2010.

[19] P. P. Barker and R. W. De Mello, "Determining the Impact of Distributed Generation on Power Systems: Part 1 - Radial Distribution Systems," *IEEE Power Engineering Society Summer Meeting*, vol. 3, pp. 1645-1656, Seattle, WA, 2000.

[20] "Integrating Distributed Resources into the Electric Utility Distribution Systems: EPRI White Paper," EPRI, Palo Alto, CA, Tech. Rep. 1004061, 2001.

[21] *IEEE Standard for Interconnecting Distributed Resources with Electric Power Systems*, IEEE Std 1547-2003, 2008.

[22] R. A. Walling and K. Clark, "Grid Support Functions Implemented in Utility-Scale PV Systems," *2010 IEEE PES Transmission and Distribution Conference and Exposition*, New Orleans, LA, 2010.

[23] ParkSolar™: Solar Integrated Infrastructure and Building System Planning, Design, Construction, Evision Solar, viewed on October 1, 2010 at: http://envisionsolar.com/project-portfolio/parksolar

[24] Solar EV Dock, Adrian Smith and Gordon Gill Architecture, viewed on October 1, 2010 at: http://smithgill.com/#/work/solar_ev_dock

[25] Eric Wesoff, "Solar Frontier CIS PV Panels Charging the Nissan Leaf," *Greentech Media*, July 11, 2011.

[26] UGE Products, Urban Green Energy Inc., viewed on August 11, 2011 at: http://www.urbangreenenergy.com/products

[27] D. Xu, L. Kang, L. Chang, and B. Cao, "Optimal Sizing of Standalone Hybrid Wind/PV Power Systems using Genetic Algorithms," *Canadian Conference on Electrical and Computer Engineering*, pp. 1722-1725, Saskatoon, SK Canada, 2005.

[28] B. S. Borowy and Z. M. Salameh, "Methodology for Optimally Sizing the Combination of a Battery Bank and PV Array in a Wind/PV Hybrid System," *IEEE Transactions on Energy Conversion*, vol. 11, no. 2, pp. 367-375, June, 1996.

[29] A. Gupta, R. P. Saini, and M. P. Sharma, "Hybrid Energy System Sizing Incorporating Battery Storage: An Analysis via Simulation Calculation," *2009 Third International Conference on Power Systems*, Kharagpur, India, 2009.

[30] *IEEE Recommended Practice for Sizing Lead-Acid Batteries for Stand-Alone Photovoltaic (PV) Systems*, IEEE Std 1013-2007, 2007.

[31] A. Nagarajan and W. Shireen, "Grid Connected Residential Photovoltaic Energy Systems with Plug-in Hybrid Electric Vehicles (PHEV) as Energy Storage," *2010 IEEE Power and Energy Society General Meeting*, Minneapolis, MN, 2010.

[32] I. Cvetkovic et al., "Future Home Uninterruptible Renewable Energy System with Vehicle-to-Grid Technology," *2009 Energy Conversion Congress and Exposition*, pp. 2675-2681, San Jose, CA, 2009.

[33] 2011 Chevy Volt Specs and Features, General Motors, viewed on August 1, 2011 at: http://www.chevrolet.com/volt/features-specs/

[34] Nissan Leaf Electric Car: 100% electric. Zero gas. Zero tailpipe., Nissan Motor Company Ltd., viewed online on August 1, 2011 at: http://www.nissanusa.com/leaf-electric-car/index

[35] Samuel H. Williamson, "Seven Ways to Compute the Relative Value of a U.S. Dollar Amount, 1774 to present," MeasuringWorth, viewed on August 8, 2011 at: http://www.measuringworth.com/uscompare/

[36] R. Wiser, G. Barbose, and C. Peterman, "Tracking the Sun: The Installed Cost of Photovoltaics in the U.S. from 1998-2007," Lawrence Berkeley National Laboratory, Berkeley, CA, Rep. LBNL-1516E, February, 2009.

[37] "Solar Energy Technologies Program Multi-Year Program Plan, 2007-2011," U.S. Department of Energy, Washington, D.C., January, 2006.

[38] "2008 Solar Technologies Market Report," U.S. Department of Energy, Washington, D.C., January, 2010.

[39] Solarbuzz: Solar Market Research and Analysis, The NPD Group, viewed on August 8, 2011 at: http://www.solarbuzz.com/

[40] L. Bony, S. Doig, C. Hart, E. Maurer, and S. Newman, "Achieving Low-Cost Solar PV: Industry Workshop Recommendations for Near-Term Balance of System Cost Reductions," Rocky Mountain Institute, Snowmass, CO, September, 2010.

[41] BP Solar Product Families, BP Solar International Inc., viewed on June 24, 2011 at: http://www.bpsolar.us/solar-products/products

[42] Renewable Energy, Eco Friendly & Energy Efficient Products, EcoDirect, viewed on August 22, 2011 at: http://www.ecodirect.com/

[43] BD Batteries - Nationwide AGM Battery Distribution, BD Batteries, viewed on August 22, 2011 at: http://www.bdbatteries.com/

[44] Solar Panels, Wholesale Solar, Inverters, Charge Controllers, Affordable Solar, viewed on August 22, 2011 at: http://www.affordable-solar.com/

[45] BP Solar Panels, Solar Home, viewed on August 22, 2011 at: http://www.solarhome.org/bpsolarpanels.aspx

[46] MrSolar.com, Online Solar, Inc., viewed on August 22, 2011 at: http://www.mrsolar.com/

[47] AltE: Solar Panels & Solar Energy Gear, Alternative Energy Store, viewed on August, 22, 2011 at: http://www.altestore.com/store/

[48] Microinverters, Enphase Energy, viewed on August 20, 2011 at: http://enphase.com/products/microinverters/

[49] "AWEA Small Wind Turbine Global Market Study 2008," American Wind Energy Association, Washington, D.C., 2008.

[50] "AWEA Small Wind Turbine Global Market Study Year Ending 2008," American Wind Energy Association, Washington, D.C., 2009.

[51] "AWEA Small Wind Turbine Global Market Study Year Ending 2009," American Wind Energy Association, Washington, D.C., 2010.

[52] SoleVento Wind and Solar Solutions, SoleVento, LLC., viewed on August 22, 2011 at: http://soleventollc.com/store/

[53] Wind Turbines & Generators Solar Power - Discount Wholesale Prices, Applied Magnets, viewed on August 22, 2011 at: http://www.magnet4less.com/

[54] Order Online Solar Photovoltaic (PV), Wind Generation and Micro-hydro Renewable Energy Products and Information, Solar Conduit, viewed on August 22, 2011 at: https://solarconduit.com/shop/

[55] Empower your World with Green Energy, Solar Sphere, viewed on August 22, 2011 at: http://www.spheralsolar.com/

[56] Portable Solar Panels, Solar Energy Kits and Roof Mount, Million Solar Roofs, viewed on August 22, 2011 at: http://www.millionsolarroofs.com/

[57] Your Guide to Everything Solar, Solar Guide, viewed on August 22, 2011 at: http://solarguide.com/

[58] Home Wind Turbines, HomeWind.net, viewed on August 22, 2011 at: http://homewind.net/

[59] SMA Solar Technology, SMA America, LLC, viewed on August 22, 2011 at: http://www.sma-america.com/en_US.html

[60] Get Started with Green Energy, GoGreenSolar.com, viewed on August 22, 2011 at: http://www.gogreensolar.com/

[61] Southwest Windpower Website, Southwest Windpower, viewed on April 2, 2011 at: http://www.windenergy.com/

[62] Small Wind Turbines for Homes, Farms, and Small Businesses, Bergey Windpower, viewed on April 2, 2011 at: http://www.bergey.com/

[63] Hummer Small Wind Turbine, Hummer Wind Power, LLC., viewed on August 22, 2011 at: http://www.hummerwind.com/

[64] Creating and Delivering Clean Energy Solutions, WePOWER, LLC., viewed on August 22, 2011 at: http://wepower.us/

[65] Kestrel Small Wind Turbines for Renewable Energy, Eveready (Pty) Ltd., viewed on August 22, 2011 at: http://www.kestrelwind.co.za/

[66] Earth and Air Technologies Renewable Alternative Energy, Earth and Air Technologies, LLC., viewed on August 22, 2011 at: http://www.earthandairtech.com/

[67] Charity Solar Website, Charity Solar, Inc., viewed on August 22, 2011 at: http://charitysolar.com/

[68] Chris Ladd, "How Centuries-Old Flywheels Can Improve the Electric Grid," *Popular Mechanics*, Science, December, 2009.

[69] Solar Electric Power Systems for On & Off Grid, Northern Arizona Wind & Sun, viewed on April 15, 2011 at: http://www.solar-electric.com/

[70] Trojan Battery Company Website, Trojan Battery Company, viewed on April 15, 2011 at: http://www.trojanbatteryre.com/index.html

[71] Crown Battery Website, Crown Battery, viewed on April 15, 2011 at: http://www.crownbattery.com/

[72] The Sealed Battery Specialists, MK Battery, viewed on April 15, 2011 at: http://www.mkbattery.com/index.php

[73] Rolls Battery. One Mean Battery., Surrette Battery Company, Ltd., viewed on April 15, 2011 at: http://www.rollsbattery.com/

[74] Sun Xtender AGM Solar Batteries, Concorde Battery Corp., viewed on April 15, 2011 at: http://www.sunxtender.com/

[75] "Electric Energy Storage Technology Options: A White Paper Primer on Applications, Costs, and Benefits," EPRI, Palo Alto, CA, Tech. Rep. 1020676, December, 2010.

[76] P. Sankey, D. T. Clark, and S. Micheloto, "The End of the Oil Age 2011 and Beyond: A Reality Check," Deutsche Bank Securities Inc., December, 2010.

[77] IOTA Engineering Emergency Fluorescent Ballasts, Emergency Lighting, Inverters, and AC/DC Power Converters, IOTA Engineering, LLC., viewed on August 22, 2011 at: http://www.iotaengineering.com/

[78] Power Inverter, Inverter Charger, Battery Charger Manufacturer, Xantrex Technology USA Inc., viewed on August 22, 2011 at: http://www.xantrex.com/

[79] Power Inverters, Modified and Pure Sine Wave Inverter, Samlex America Inc., viewed on August 22, 2011 at: http://www.samlexamerica.com/

[80] Off-Grid Photovoltaic Systems, Apollo Solar, Inc., viewed on August 22, 2011 at: http://apollosolar.com/

[81] Industry Leaders in Solar Boost Charge Controllers and Maximum Power Point Tracking (MPPT), Blue Sky Energy Inc., viewed on August 22, 2011 at: http://blueskyenergyinc.com/

[82] Morningstar Corporation Website, Morningstar Corporation, viewed on August 22, 2011 at: http://www.morningstarcorp.com/

[83] OutBack Power Website, OutBack Power Technologies, Inc., viewed on August 22, 2011 at: http://www.outbackpower.com/

[84] MidNite Solar Website, MidNite Solar, viewed on August 22, 2011 at: http://www.midnitesolar.com/

[85] Regulatory Information - SCE Tariff Books, Southern California Edison, viewed on August 22, 2011 at: http://www.sce.com/AboutSCE/Regulatory/tariffbooks/ratespricing/default.htm

[86] Electric Rates, Pacific Gas and Electric Company, viewed on August 22, 2011 at: http://www.pge.com/nots/rates/tariffs/electric.shtml

[87] Schedule of Rates for Electric Service, Commonwealth Edison Company, viewed on August 22, 2011 at: https://www.comed.com/Documents/CustomerService_RateInfo/Ratebook.pdf

[88] Rate Schedules, Florida Power and Light Company, viewed on August 22, 2011 at: http://www.fpl.com/rates/pdf/2011%20Jul%2026%20-%20Section%208%20ebook.pdf

[89] Rates and Tariffs, Consolidated Edison, Inc., viewed on August 22, 2011 at: http://www.coned.com/rates/

[90] Electric Power Monthly, U.S. Energy Information Administration, viewed on August 22, 2011 at: http://www.eia.gov/cneaf/electricity/epm/epm_sum.html

[91] HybridCars Website, HybridCars.com, viewed on August 22, 2011 at: http://hybridcars.com/hybrid-market-dashboard/november-2008-dashboard-25328.html

[92] Alternative Fuels & Advanced Vehicles Data Center, U.S. Department of Energy, viewed on August 22, 2011 at: http://www.afdc.energy.gov/afdc/fuels/electricity.html

[93] 1SolTech Website, 1SolTech Corporation, viewed on August 22, 2011 at: http://www.1soltech.com/

[94] Advanced Soalr Photonics Website, Advanced Solar Photonics, LLC., viewed on August 22, 2011 at: http://www.advancedsolarphotonics.com/

[95] Canadian Solar Website, Canadian Solar, viewed on April 15, 2011 at: http://www.canadiansolar.com/

[96] Crown Renewable Energy Website, Crown Renewable Energy, viewed on April 15, 2011 at:

http://www.crownsolarco.com/

[97] EcoSolargy Website, ecoSolargy, viewed on April 15, 2011 at: http://www.ecosolargy.com/

[98] Pacific SolarTech Website, Pacific SolarTech, viewed on April 15, 2011 at: http://www.pacificsolartech.com/

[99] Sonali Solar Website, Sonali Energees, viewed on April 15, 2011 at: http://www.sonalisolar.com/

[100] Evergreen Solar Website, Evergreen Solar, Inc., viewed on April 15, 2011 at: http://evergreensolar.com/

[101] J. Cole, Nissan Leaf Electric Car News, Discussion, Blog and Forum, viewed on August 22, 2011 at: http://nissan-leaf.net/2010/08/04/leaf-depth-of-discharge/

[102] D. Bertsimas and J.N. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, Belmont, MA, 1997.

[103] UO Solar Radiation Monitoring Laboratory, University of Oregon Solar Radiation Monitoring Laboratory, viewed on June 21, 2011 at: http://solardat.uoregon.edu/

[104] Solar Resource & Meteorological Assessment Project (SOLRMAP), Loyola Marymount University, viewed on July 4, 2011 at: http://www.nrel.gov/midc/lmu/