6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

6.045 Final Exam

May 18, 2004

Name: ____

- Please write your name on each page.
- This exam is open book, open notes.
- There are two sheets of scratch paper at the end of this exam.
- Questions vary substantially in difficulty. Use your time accordingly.
- If you can not produce a full proof, clearly state partial results for partial credit.
- Good luck!

Problem	Points	Grade
1	40	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
Total	160	

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Problem 1: **Multiple Choice Questions. (40 points, 4 points for each question)**

For each question, any number of the listed answers may be correct. **Clearly** place an "X" in the box next to each of the answers that you are selecting.

1. Which of the following are known to be true?
   - ☐ If language A is recognized by an NFA, then the complement of A must also be recognizable by an NFA.
   - ☐ The complement of every Turing-decidable language is Turing-decidable.
   - ☐ NP = coNP.
   - ☐ NL = coNL.

2. Which of the following are true statements about the sizes of various kinds of representations of regular languages?
   - ☐ Every language recognizable by a DFA with n states is recognizable by some NFA with n states.
   - ☐ Every language recognizable by an NFA with n states is recognizable by some DFA with n states.
   - ☐ Every language describable by a length n regular expression is recognizable by an $O(n)$-state NFA.
   - ☐ If two languages A, B are recognized by two (potentially different) DFAs with n states, than the language $A \cup B$ can be recognized by a DFA with at most $2n + 1$ states.

3. Which of the following languages are Turing-recognizable?
   - ☐ $\{\langle M\rangle |$ M is a (deterministic) Turing machine and M accepts $010\}$.
   - ☐ $\{\langle M\rangle |$ M is a nondeterministic Turing machine and M accepts $010\}$.
   - ☐ $\{\langle M\rangle |$ M is a Turing machine and M does not accept $101\}$.
   - ☐ $\{\langle M\rangle |$ M is a Turing machine and $L(M) = \Sigma^*\}$.

4. Which of the following languages can be shown to be undecidable by a direct application of Rice's theorem?
   - ☐ $\{\langle M\rangle |$ M is a DFA and M accepts $010\}$.
   - ☐ $\{\langle M\rangle |$ M is a Turing machine and M accepts $010\}$.
   - ☐ $\{\langle M\rangle |$ M is a Turing machine and M accepts 010 and does not accept $101\}$.
   - ☐ $\{\langle M\rangle |$ M is a minimal Turing machine, that is, no Turing machine with a smaller representation recognizes the same language$\}$.

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5. Which of the following are decidable relative to the Post Correspondence Problem (PCP)? (That is, which are decidable by an oracle Turing machine that uses an oracle for PCP?)
   - ☐ The acceptance problem for oracle Turing machines relative to PCP.
   - ☐ A_{TM}, the acceptance problem for ordinary Turing machines.
   - ☐ The problem of whether $L(M)$ contains 010 and does not contain 101.
   - ☐ The emptiness problem (that is, does $L(M) = \emptyset$?) for ordinary Turing machines.

6. Which of the following are known to be true?
   - ☐ CLIQUE $\leq_P$ VERTEX-COVER.
   - ☐ CLIQUE $\leq_P$ 3SAT.
   - ☐ TQBF $\leq_P$ HAMCYCLE.
   - ☐ PATH $\leq_P$ $\{6045\}$.

7. Which of the following are known to be in NP?
   - ☐ $L_1 - L_2$, for all L_1, L_2 in NP.
   - ☐ $L_1 \cap L_2$, for all L_1, L_2 in NP.
   - ☐ $\overline{A_{TM}}$, the complement of the acceptance problem for Turing machines.
   - ☐ $\overline{\text{PATH}}$.

8. Which of the following are known to be true statements about log space reducibility?
   - ☐ Any log space transducer runs in polynomial time.
   - ☐ $\leq_L$ is transitive.
   - ☐ If $A \leq_L B$ and $B \in$ NL, then $A \in$ NL.
   - ☐ For all languages A and B, if $A \leq_P B$, then $A \leq_L B$.

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9. Which of the following are true statements about Savitch's theorem and its proof?
   - ☐ Savitch's theorem implies that NSPACE$(\log n)$ = SPACE$(\log n)$.
   - ☐ Savitch's theorem implies that NSPACE$(n^2) \subseteq$ SPACE(n^4).
   - ☐ In the proof of Savitch's theorem, when the simulating Turing machine computes CANYIELD recursively, it chooses the midpoint configuration nondeterministically.
   - ☐ When the simulating Turing machine computes CANYIELD recursively, it uses space approximately equal to the sum of the space bounds used by the two recursive calls to CANYIELD, plus space to record the midpoint configuration.

10. Consider a language L, and a probabilistic polynomial time Turing machine M such that M always accepts words in L, and for any word w not in L, M rejects w with probability at least 1/10. Which of the following must be true?
    - ☐ $L \in$ BPP
    - ☐ $L \in$ RP
    - ☐ $L \in$ coRP
    - ☐ $L \in$ NP

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Problem 2: **Regular Languages. (20 points)** Provide solutions with brief justifications.

1. Find regular languages L_1, L_2 over $\{a, b\}$ for which $L_1 \not\subseteq L_2$, $L_2 \not\subseteq L_1$ (i.e., they are not equal and neither is a subset of the other), and $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$.

2. Find a regular language L_1 and a non-regular language L_2 such that $L_1 \cap L_2$ is non-regular and yet $L_1 \cup L_2$ is regular.

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Problem 3: **Undecidability.** **(20 points)** Let $L = \{\langle M \rangle \mid M$ is a basic Turing machine that accepts 11 and does not accept 00$\}$. Use the Recursion Theorem to prove that L is undecidable. Fill in the blanks in the proof below.

Proof: For the sake of contradiction, assume that D is a decider Turing machine for L; that is, D accepts $\langle M \rangle$ if M accepts 11 and does not accept 00, and D rejects $\langle M \rangle$ if M does not accept 11 or does accept 00.

Then define a new Turing machine R, as follows:

R="On input w,

Obtain own description $\langle R \rangle$ via the Recursion Theorem.

Run D on input __________.

If this computation accepts then

accept w if ____ and

reject w if ____.

On the other hand, if this computation rejects then

accept w if ____ and

reject w if ____."

If R ____

then D ____,

which means R ____,

which is a contradiction.

On the other hand, if R ____

then D ____,

which means R ____,

which is again a contradiction. ■.

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Problem 4: **Solitaire. (20 points)** Consider the following solitaire game. You are given an $m \times k$ board where each one of the mk positions may be empty or occupied by either a red stone or a blue stone. Initially, some configuration of stones is placed on the board. Then, for each column you must remove either all of the red stones in that column or all of the blue stones in that column. (If a column already has only red stones or only blue stones in it then you do not have to remove any further stones from that column.) The objective is to leave at least one stone in each row. Finding a solution that achieves this objective may or may not be possible depending upon the initial configuration. Let

$$\text{SOLITAIRE} = \{\langle G \rangle \mid G \text{ is a game configuration with a solution}\}.$$

Prove that SOLITAIRE is in NP.

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Prove that SOLITAIRE is NP-hard.

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Problem 5: **NFA Equality. (20 points)** Define EQ_{NFA} to be the equivalence problem for NFAs, that is,

$$EQ_{NFA} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are NFAs and } L(M_1) = L(M_2)\}.$$

Show that EQ_{NFA} is in PSPACE.

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Problem 6: **Right on Target. (20 points)** We define the following language

$$\text{TARGET} = \{\langle G, t\rangle \mid G \text{ is a directed graph, } t \text{ is a node in } G\text{, and } t \text{ is reachable from every other node in } G \text{ via a directed path}\}.$$

Show that TARGET is NL-hard.

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Problem 7: **Random World. (20 points)** A language L has a probabilistic polynomial time Turing machine M that accepts words in L with probability at least $2/3$, rejects words not in L with probability at least $2/3$. Further, on any input w, M makes at most $\log_2(|w|)$ coin tosses (that is, in every computation path for input w, all but at most $\log_2(|w|)$ of the steps are deterministic).

Prove that $L \in \mathrm{P}$.

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END OF EXAM.

SCRATCH PAPER 1

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SCRATCH PAPER

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SCRATCH PAPER 2