6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Quiz 1 (from Spring 2004)

Please write your name in the upper corner of each page. (2 Points)

Problem 1:

True or False (18 points) Full credit will be given for correct answers. If you include justification for your answers, you may obtain partial credit for incorrect answers.

In all parts of this question, the alphabet Σ is $\{0, 1\}$.

1. True or False: If L is a regular language and F is a finite language (i.e., a language with a finite number of words), then $L \cup F$ must be a regular language.

2. True or False: If L is a regular language, then $\{ww^R : w \in L\}$ must be a regular language. (Here, w^R denotes the reverse of string w.)

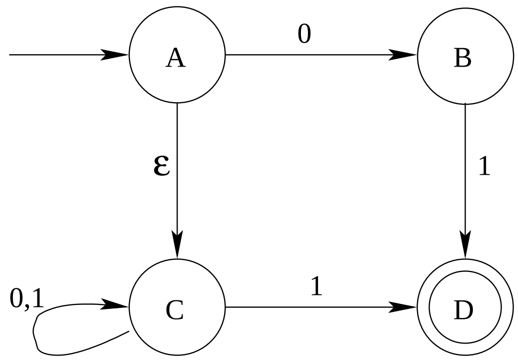
3. True or False: Regular expressions that do not contain the star operator can represent only finite languages.

4. True or False: Define $EVEN(w)$, for a finite string w, to be the string consisting of the symbols of w in even-numbered positions. For example, $EVEN(1011010) = 011$.
   If L is a regular language, then $\{EVEN(w) : w \in L\}$ must be regular.

5. True or False: For every pair of regular expressions R and S, the languages denoted by $R(SR)^*$ and $(RS)^*R$ are the same.

6. True or False: If L_1 and L_2 are languages such that L_2, L_1L_2, and L_2L_1 are all regular, then L_1 must be regular.

Problem 2: **(20 points)** Consider the following NFA:

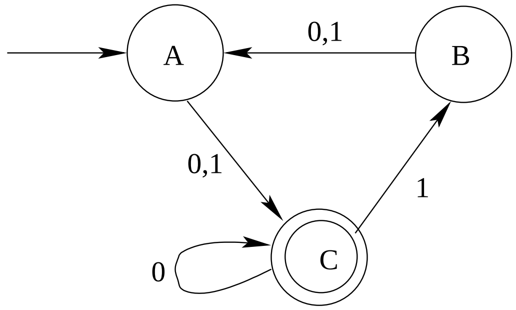


1. **(16 points)** Convert this NFA into an equivalent DFA using the procedure we studied in class. **Your answer should be the state diagram of a DFA. Your diagram should include only the states that are reachable from the start state.** (Note: There are not more than a half-dozen states in the resulting DFA). Please label your states in some meaningful way. You may explain your work, to receive more credit for an incorrect answer.

2. **(3 points)** What language is recognized by your DFA? Your answer may be either a regular expression or an explicit description of the set.

3. **(3 points)** Give a DFA with two states that recognizes the same language.

Problem 3: **(20 points)** Find a regular expression for the language recognized by this machine, using the procedure we have studied in class: Show all your work, in particular, the state diagrams after the removal



of each successive state. You may omit $\emptyset$-transitions from your diagrams. Please start by giving the GNFA before state removal and then remove states in the order C, B, A.

GNFA before state removal:

After removing state C:

After removing state B:

After removing state A:

Regular expression representing the language recognized by the original DFA:

Problem 4: **(20 points)** Regular expressions are defined using three operators: union, concatenation, and star. Suppose we define "Extended Regular Expressions" in the same way as regular expressions, with the addition of the *set intersection* operator. For example, $((0 \cup 1)^*1) \cap (1(0 \cup 1)^*)$ is an extended regular expression, which denotes the set of words that both begin and end with 1.

Show that adding the intersection operator does not extend the power of ordinary regular expressions. Do this by describing a *procedure* that, given an extended regular expression α, produces an ordinary regular expression β that represents the same language. You may use procedures described in class and in Sipser's book without saying how they work, e.g., you may say things like "convert the NFA to a DFA". The description of your procedure should be concise, but the procedure need not be the most efficient one possible.

Input: Extended Regular Expression α

Procedure:

Output: Regular Expression β, which is equivalent to α.

Problem 5: **(20 points)** Prove that the following language L over the alphabet $\{a, b, c\}$ is not regular:

$$L = \{wcx : w, x \in \{a, b\}^* \text{ and the number of } a's \text{ in } w \text{ is equal to the number of } b's \text{ in } x\}.$$

For example, the word $ababab c bbb$ is in L.

Claim: L is not regular.
Proof: