6.045J/18.400J: Automata, Computability and Complexity   Prof. Nancy Lynch

Quiz 2

March 31, 2004

Please write your name in the upper corner of each page.

Problem 1:

True or False (15 points). Full credit will be given for correct answers. If you include justification for your answers, you may obtain partial credit for incorrect answers.

1. True or False: The Post Correspondence Problem for a 3-letter alphabet is decidable.

2. True or False: The set A of all finite subsets of $\{0,1\}^*$ (that is, $A = \{S \subseteq \{0,1\}^* \mid S \text{ is finite}\}$) is a countable set. (You can think of A as the set of all *finite* languages.)

3. True or False: Rice's Theorem implies that $\{\langle M \rangle \mid M \text{ is a Turing machine and } M \text{ accepts the string } 01$ and M does not accept the string $10\}$ is undecidable.
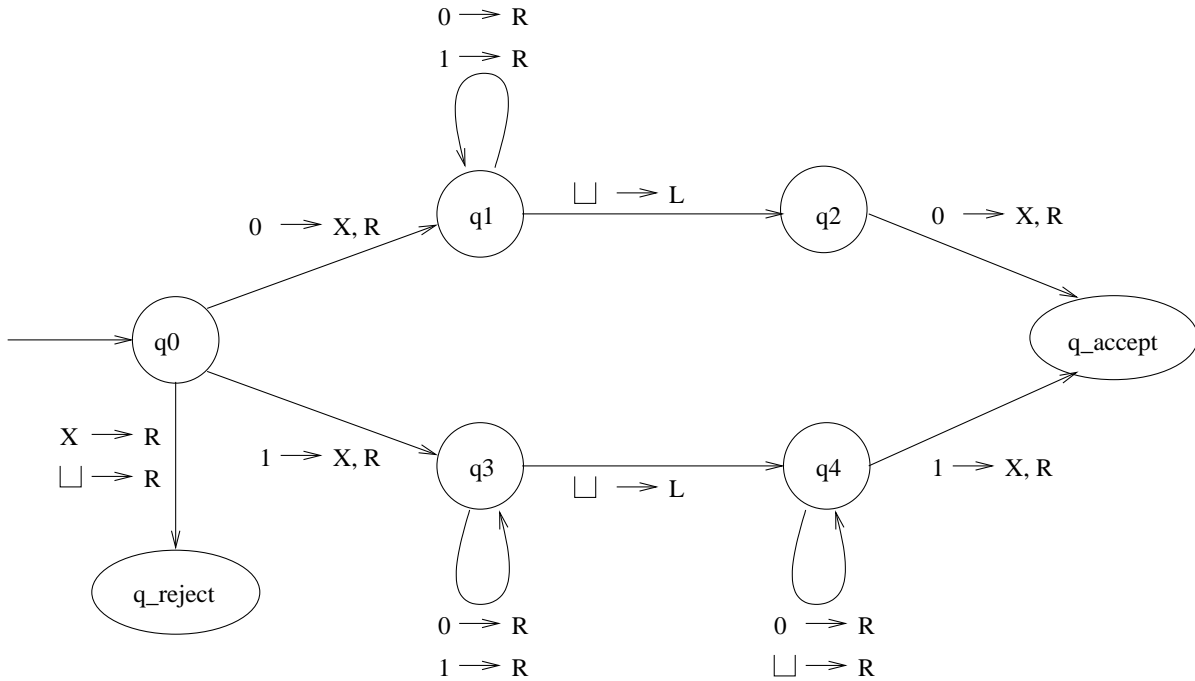
4. True or False: The intersection of two Turing-recognizable languages must be Turing-recognizable.

5. True or False: If a language L is recognized by a Turing machine, then L is recognized by a one-stack, one-counter machine (i.e., a machine that has one stack and one counter).

Problem 2: **(20 points)** Consider the following formal description of a Turing Machine M, where $Q = \{q_0, q_1, q_2, q_3, q_4, q_{reject}, q_{accept}\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, X, \sqcup\}$. Assume that any unspecified transitions go to q_{reject}.

$0 \rightarrow R$

$1 \rightarrow R$

$0 \rightarrow X, R$ q1 $\sqcup \rightarrow L$ q2 $0 \rightarrow X, R$

q0

q_accept

$X \rightarrow R$

$\sqcup \rightarrow R$ $1 \rightarrow X, R$ q3 $\sqcup \rightarrow L$ q4 $1 \rightarrow X, R$

q_reject

$0 \rightarrow R$ $0 \rightarrow R$

$1 \rightarrow R$ $\sqcup \rightarrow R$

1. **(6 points)** Write out the accepting computation history of M on input 00.

2. **(6 points)** Describe in a few words the behavior of M on input 10.

3. **(8 points)** What language does M recognize?

Problem 3: (**25 points**) Let $FIN = \{\langle M \rangle \mid M$ accepts only a finite number of strings$\}$.
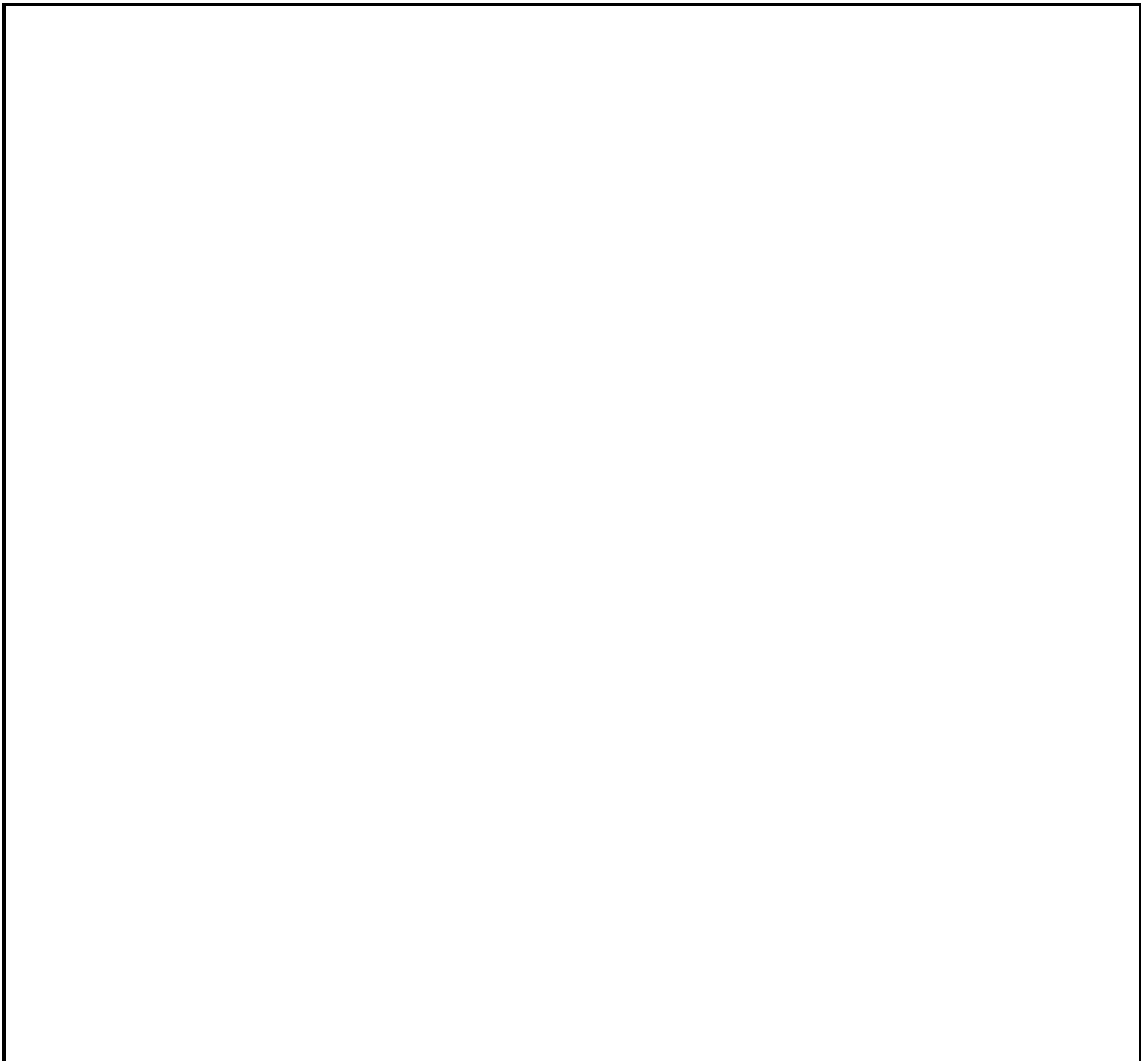
1. (**5 points**) Does Rice's Theorem apply to FIN? Why or why not?

Prove the following two results about FIN. You may use any results proved in class or in Sipser's book, but if you do, then cite the results explicitly.

2. (**10 points**) FIN is not Turing-recognizable. (Hint: Use mapping reducibility.)

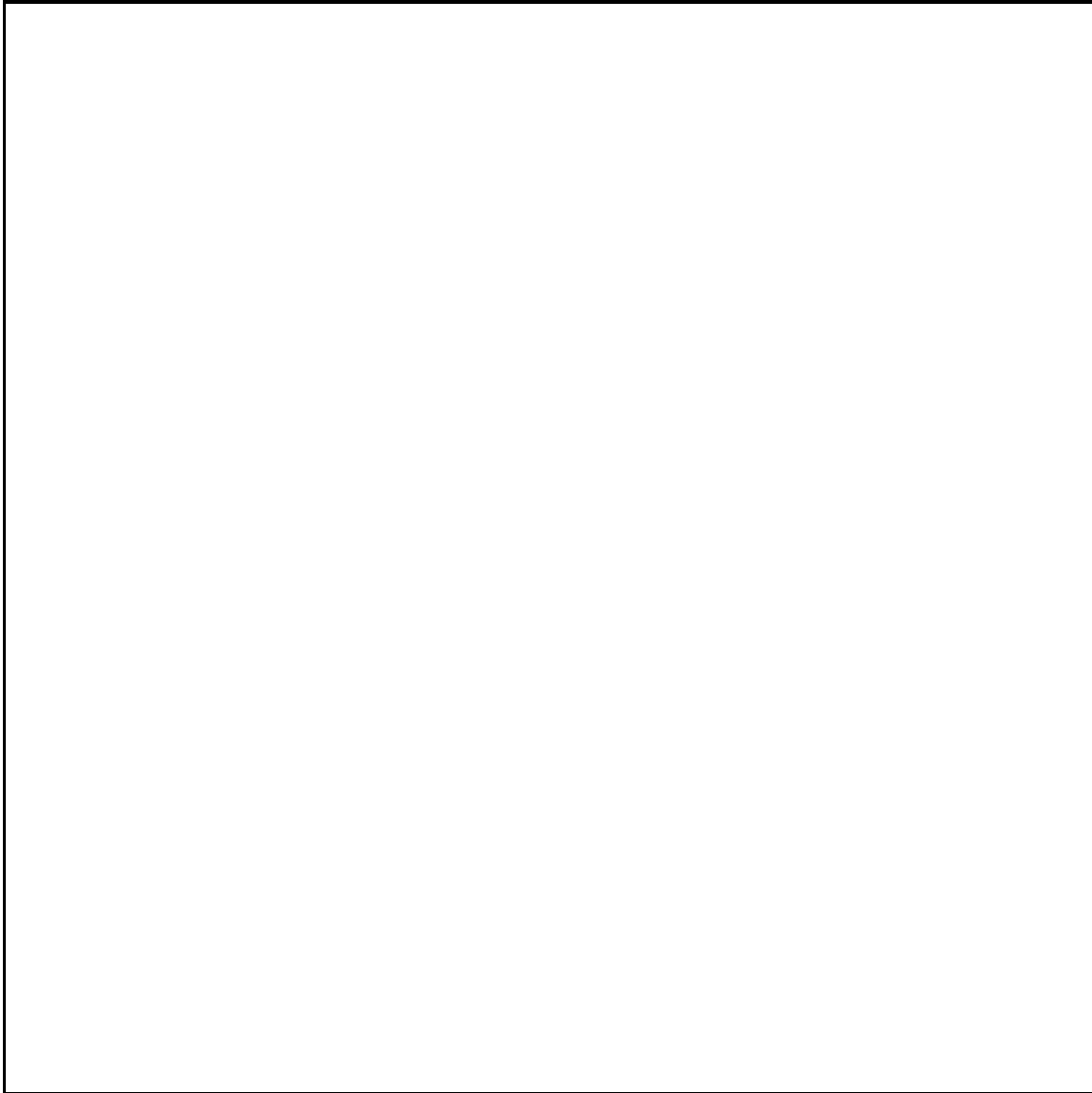3. **(10 points)** $\overline{FIN}$ is not Turing-recognizable, that is, FIN is not co-recognizable. (Hint: Use mapping reducibility.)

Problem 4: **(20 points)** Define a Turing machine M to be *almost-minimal* if there does not exist another Turing machine M' such that $L(M') = L(M)$ and $|\langle M' \rangle| < \frac{1}{2}|\langle M \rangle|$. That is, there is no machine that recognizes the same language and has an encoding whose size is less than half of the size of $\langle M \rangle$.

Prove that there is no enumerator that outputs a set S consisting of an infinite number of almost-minimal machine descriptions. (Hint: Use the recursion theorem.)

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FLIP PAGE FOR PROBLEM 5.

Problem 5: **(20 points)** On your first day on your new job at SmartCompilers.com your boss assigns you the task of adding the following new feature to the company's development suite:

"Analyze the source code of a function, and verify that no array will ever be accessed out of bounds. Output TRUE if the code is safe in this respect, and FALSE if for some inputs, the function will access an array out of bounds."

For example, the following program should fail the verification:

```
void sloppy_code(int k) {
    int array[10];
    for (int i=0; i<k; i++) { array[i]=i; }
}
```

Program *sloppy_code* should fail, because on input $k = 15$, the array will be accessed at position 11 (i.e., $array[11] = 11;$), which is outside its declared range of 10.

In your answers, you may assume that the behavior of the function is fully determined by its source code and its inputs (i.e., it does not make any outside calls to libraries or other functions). Also assume for simplicity that the only variables a function uses are (possibly unbounded) integers, and integer arrays.

1. **(5 points)** State your boss' request as a language decision problem.
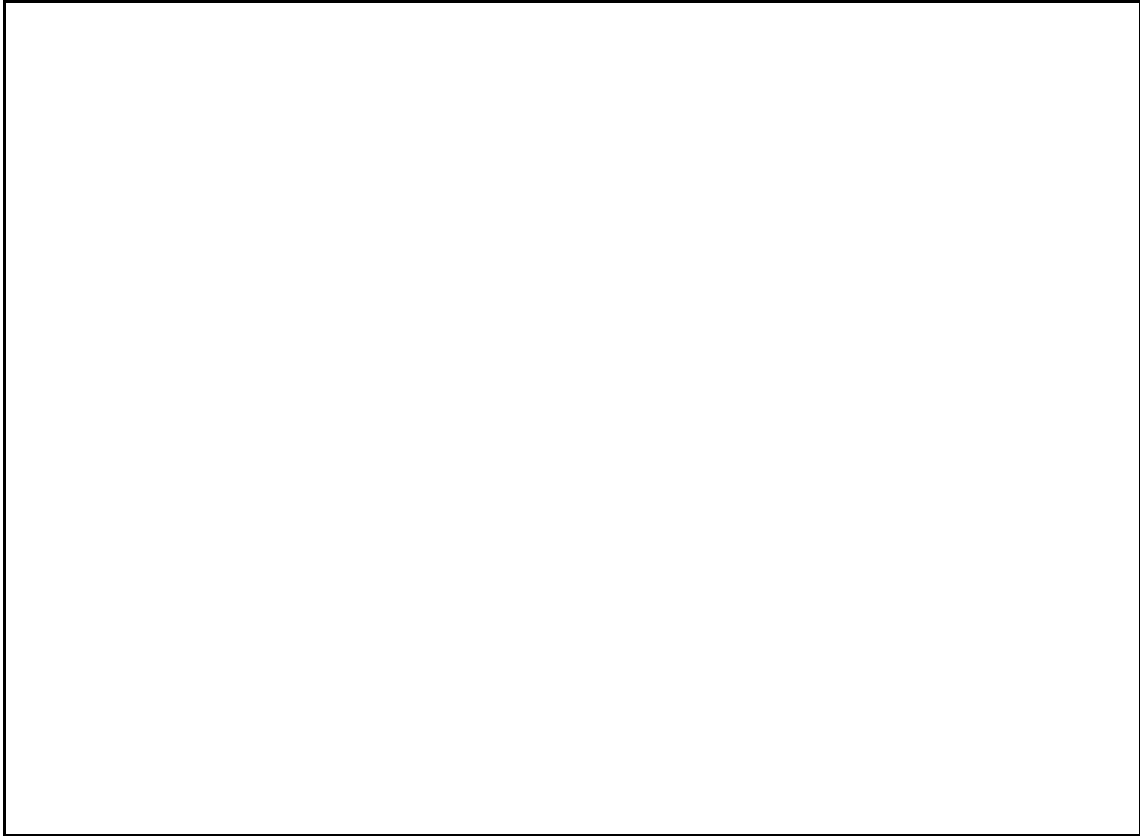
$SAFE = \{\langle F \rangle |$

2. **(7 points)** In order to prove to your boss that this task is impossible, show a mapping reduction from some language we know is undecidable to $SAFE$ (Hint: consider using $E_{TM} = \{\langle M \rangle | L(M) = \emptyset\}$, or if you prefer you can even use E_{2CM}, the analogous language for 2-counter machines):

We will reduce _____ to $SAFE$ by mapping each input of the form _____ to input of the form $\langle R \rangle$, where the function R is described below:

R="On input x,

3. **(8 points)** Is $SAFE$ recognizable? Co-recognizable? Neither? Both? Explain.

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Scratch Paper.