6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 2: DFAs and NFAs

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Problem 1: Define the following words, phrases and symbols.

1. Finite state machine, finite automaton
2. Sipser page 49. Determinism vs Nondeterminism
3. Sipser page 54. DFA vs NFA
4. Regular Language
5. Sipser page 53. $(Q, \Sigma, \delta, q_0, F)$
6. ϵ
7. Epsilon Transition
8. Sipser page 36,40. A machine can accept many strings, but only a single language. To avoid confusion, we will usually say a machine accepts a string and recognizes a language.

Problem 2: Are the following statements true or false?

1. It is possible for a finite automaton to recognize an infinite language.
2. Every deterministic finite automaton is also a nondeterministic finite automaton.
3. For every nondeterministic finite automaton, there is an equivalent nondeterministic finite automaton that has no epsilon transitions.
4. For every nondeterministic finite automaton, there is an equivalent nondeterministic finite automaton that has a single accept state.
5. If you swap the accepting states and the non-accepting states on ANY finite automaton, the new machine will recognize the complement of the original language.
6. The class of languages recognized by non-deterministic finite automata is closed under complementation.
7. Let L be a language recognized by a deterministic finite automaton. Define a language

$$L' = \{x_2x_1x_4x_3 \ldots x_{2n}x_{2n-1} \mid x_1x_2x_3x_4 \ldots x_{2n-1}x_{2n} \in L\}$$

   L' is recognized by some deterministic finite automaton.

Problem 3:

1. Create DFAs for each of the following languages over the alphabet $\Sigma = \{0, 1\}$.
   (i) The language of strings that contain both a '0' and a '1'.
   (ii) The language of strings that contain '0110' and end in '1'.
   (iii) The language of strings that do not contain an odd number of 1s.

(iv) The language of part (iii) star.

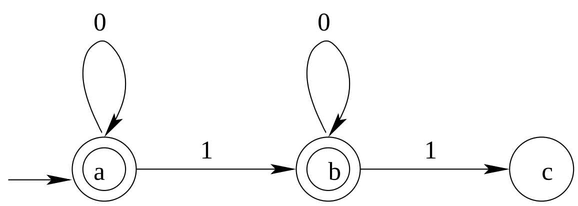
(v) $L = \{w \mid w$ contains at least two 0s and at most one 1 $\}$.

2. An NFA with 3 states for the language $L = \{w \mid w$ ends with $00\}$.

Problem 4: Let $C_n = \{x \mid x$ is a binary number divisible by $n\}$. Show that for each n, the language C_n is regular. (Assume for simplicity that the string represented by ϵ is divisible by n for any n).

Problem 5: Let L be any language. Prove that $L \subseteq L^2$ if and only if ϵ is in L.

Problem 6: What language L does the following automaton recognize ? Prove that the automaton indeed recognizes the language you think it recognizes. To do this, we will need to prove that our FA (1) accepts all strings in L and (2) does not accept any string not in L.



1. Characterize each state.
2. Forward direction (accepts all strings in L). Proof by Induction?
3. Reverse direction (does not accept any string outside of L). Proof by Contradiction?

Problem 7: **Optional** (if we have enough time)
An *all-paths-NFA* M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if and only if *every* possible state that M could be in after reading x is a state from F. Prove that all-NFAs recognize exactly the regular languages. (Notice the contrast with NFAs)