6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 7: Counter and Stack Machines, Reducibility, Rice's Theorem

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Problem 1: These are the key concepts from lecture this week:

1. Counter and Stack Machines - We can simulate a Turing machine computation with two stacks, can simulate two stacks with three counters and three counters with two. Reference for this material : Hopcroft, Motwani and Ullman.
2. Mapping Reducibility - pages 189-194 of old edition of Sipser (make sure you understand Theorems 5.16, 5.17, 5.22, and 5.23)
3. Rice's Theorem

Problem 2:(Rice's Theorem)
Classify each of the following problems as either

- **(D)** decidable,
- **(R)** recognizable but not decidable,

and indicate which undecidable examples follow from **Rice's Theorem**. [1]

1. EQ_{NFA}, the Equivalence problem for NFA's.
2. $\{\langle M\rangle |\ M$ is a Turing machine that accepts at least 42 different strings $\}$.
3. $\{\langle M\rangle |\ M$ is a Turing Machine that has at least 42 states $\}$.
4. $\{\langle M\rangle |\ M$ is a Turing Machine that runs for at least 42 steps when started with a blank input tape $\}$.
5. $\{\langle M\rangle |\ M$ accepts the string 01 in a perfect square number of steps $\}$.
6. $\{\langle M\rangle |\ L(M)$ is recognized by a Turing Machine that has an even number of states$\}$.

Problem 3:(Mapping Reducibility)
Answer the following True or False:

1. E_{TM} is mapping reducible to A_{TM}.
2. $A_{TM} \leq_m 0^*1^*$.

Problem 4:(Pebble Machine)
A "pebble machine" is a TM with two tapes – an input tape and a work tape. The input tape contains the input string, and is read-only. The machine cannot write on the work-tape either !! But, it has *three distinguishable pebbles* which can be placed anywhere on the work tape, and moved around. The machine can determine when two pebbles are adjacent to each other. How powerful is this machine ?

[1] Check your answers from the back of this handout

Problem 2 Solutions:

1. D; recall the EQ_{DFA} algorithm.
2. R; Undecidable by Rice's Theorem; Recognizable by running the TM in parallel using a dove-tailing kind of trick.
3. D; This is a simple check, given a machine's description. Rice doesnt apply because this is not a language property.
4. D; just simulate M for up to 42 steps. Rice doesnt apply because this is not a language property.
5. R; Undecidable, but not by Rice's theorem. Recognizable, trivially. Just run the machine on input 01.
6. D; This is a trivial language property.

Problem 3 Solutions:

1. False; A_{TM} is recognizable, E_{TM} is not. See Corollary 5.17.
2. False; 0^*1^* is decidable, A_{TM} is not. See Theorem 5.16.