## **Recitation 8: Oracle Turing Machines**

March 31, 2005

**Reading:** Section 6.3

Problem 1: Quiz Questions? Quiz Statistics ...

Problem 2: Define the following new terms and concepts:

- 1. An **oracle** for a language.
- 2. An oracle Turing machine.
- 3. Turing reducibility.
- 4. A language A is **decidable relative to** a language B.
- 5.  $T^{A_{TM}}$ .

**Summary:** If  $T^B$  decides A, then we say that A is decidable relative to B. A language A is Turing reducible to language B, written  $A \leq_T B$ , if A is decidable relative to B. If  $A \leq_T B$  and B is decidable, then A is decidable.

How is this different from mapping reducibility? Turing reducibility is a generalization of mapping reducibility. If  $A \leq_m B$ , then  $A \leq_T B$ , but there is no reverse implication.

**Problem 3**: Let's explore a case where  $A \leq_T B$ , but  $A \not\leq_m B$ . First, recall that  $E_{TM} \not\leq_m A_{TM}$ . Why? We know that  $A_{TM}$  is recognizable. If  $E_{TM} \leq_m A_{TM}$ , then by Thm 5.22  $E_{TM}$  would also be recognizable. However, we know that  $E_{TM}$  is *not* recognizable.

Now, let's see why  $E_{TM} \leq_T A_{TM}$ . Given an oracle for  $A_{TM}$  we show how to build an oracle TM to decide  $E_{TM}$ . (Details on the board; see Sipser page 212.)

**Problem 4**: Show  $HALT_{TM}$  is decidable relative to  $A_{TM}$ .