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Supergravity computations without gravity complications

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The conformal compensator formalism is a convenient and versatile representation of supergravity (SUGRA) obtained by gauge-fixing conformal SUGRA. Unfortunately, practical calculations often require cumbersome manipulations of component field terms involving the full gravity multiplet. In this paper, we derive an alternative gauge fixing for conformal SUGRA which decouples these gravity complications from SUGRA computations. This yields a simplified tree-level action for the matter fields in SUGRA which can be expressed compactly in terms of superfields and a modified conformal compensator. Phenomenologically relevant quantities such as the scalar potential and fermion mass matrix are then straightforwardly obtained by expanding the action in superspace.

I. INTRODUCTION

Supersymmetry (SUSY) is a well-studied and highly motivated extension of the standard model. While some aspects of SUSY phenomenology may be understood purely in the limit of global SUSY, others require the full machinery of supergravity (SUGRA). For example, SUGRA plays an essential role in the super-Higgs mechanism, whereby the goldstino of spontaneous SUSY breaking is eaten to become the longitudinal mode of the gravitino [1,2]. This induces a nonzero mass for the gravitino $m_{3/2}$, which plays a crucial role in SUSY cosmology and collider phenomenology [3,4]. Furthermore, the full SUGRA formalism is required for a proper description of "no-scale" SUSY breaking [1,5,6], which arises when moduli mix directly with the gravity multiplet.

Despite its clear phenomenological significance, not all of the SUGRA formalism is actually relevant for practical calculations. For instance, couplings to the graviton are more or less unimportant for SUSY phenomenology, and in any case are fixed by general covariance. Likewise, interactions with the goldstino and the transverse modes of the gravitino are dictated by supercurrent conservation in the underlying theory. For the purpose of understanding phenomenology at colliders and in cosmology, one's main concern is to ascertain the effects of SUGRA on the vacuum structure and particle spectrum of a given SUSY model. In this case, the full machinery of SUGRA can obfuscate rather than illuminate the physics.

In this paper, we show that SUGRA and its many complications can be dramatically simplified by applying an appropriately chosen gauge fixing or, equivalently, a prescient Kähler transformation. Our starting point will be the so-called conformal compensator formalism [7], which is well-suited to some but not all practical calculations. In the conformal compensator formalism, one accounts for the most important SUGRA effects by augmenting the usual superspace formalism of global SUSY with a conformal compensator superfield $\boldsymbol{\Phi}$. In the literature, the standard gauge fixing yields

$$\boldsymbol{\Phi} = 1 + \theta^2 F_{\Phi}, \tag{1}$$

where F_{Φ} is the scalar auxiliary field of SUGRA.[1] The conformal compensator couples to chiral superfields X^i and vector superfields V^a via the SUGRA action

$$\mathcal{L}_{\mathrm{SUGRA}} = -3 \int d^4\theta \boldsymbol{\Phi}^{\dagger}\boldsymbol{\Phi} e^{-K/3} + \int d^2\theta \boldsymbol{\Phi}^3 W + \mathrm{H.c.}$$
$$+ \frac{1}{4}\int d^2\theta f_{ab} W^{a\alpha} W^b_{\alpha} + \mathrm{H.c.} + \dots, \tag{2}$$

where the Kähler potential K is a (Yang-Mills) gauge invariant function of chiral and vector superfields, and the superpotential W and gauge kinetic function f_{ab} are holomorphic functions of chiral superfields. Here, the ellipsis ($\dots$) denotes terms involving the graviton, gravitino, and vector auxiliary field, and we work in natural units, where $M_{\mathrm{Pl}} = 1$.

Famously, anomaly mediation [8] is most easily understood via conformal compensator methods. More recently, this formalism has been applied to the case where multiple sequestered SUSY breaking sectors give rise to a corresponding multiplicity of goldstini [9,10]. The conformal compensator offers a simple way of understanding how goldstini obtain a universal tree-level mass of $2m_{3/2}$ [9] in theories of F-term SUSY breaking, and illuminates modifications which can arise in certain "goldstini variations" [11] and theories with imperfect sequestering [12].

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[1]Throughout this work, boldface and regular typeface will denote superfields and component fields, respectively. Moreover, given a superfield X, we will denote its lowest component by X.

Despite the utility of the conformal compensator method, there are many situations where a naive application of $\mathbf{\Phi}$ from Eq. (1) leads to incorrect answers, in particular, if the terms denoted by the ellipsis in Eq. (2) are improperly ignored. For example, the ellipsis contains nonminimal couplings between matter fields and the graviton, so to properly calculate the spectrum and couplings for matter fields, one must first Weyl rescale the metric to canonically normalize the Einstein-Hilbert action. Likewise, in the presence of no-scale SUSY breaking, Eq. (2) simply yields the *wrong* fermion mass matrix unless kinetic mixing terms between matter fermions and the gravitino are properly included. Given these complications, it is perhaps unsurprising that much of the SUGRA literature simply abandons the superfield version of SUGRA in Eq. (2) altogether and simply works in the component form.

In this work, we present an alternative gauge fixing in which Eq. (2) can be employed while entirely *ignoring* additional terms involving the gravity multiplet. Hence, the mixing terms are eliminated, analogous to the R_ξ gauges of spontaneously broken Yang-Mills theory. In this gauge fixing, the conformal compensator is written as

$$\mathbf{\Phi} = e^{\mathbf{Z}/3}(1 + \theta^2 F_\Phi), \tag{3}$$

$$\mathbf{Z} = \langle K/2 - i\mathrm{Arg}W \rangle + \langle K_i \rangle X^i, \tag{4}$$

where hereafter $\langle \rangle$ will denote a vacuum expectation value (vev) and i and $\bar{i}$ subscripts will denote differentiation with respect to X^i and $X^{\bar{i}\dagger}$, respectively. Note that we are working in a "zero vev" basis in which X^i has been appropriately shifted such that $\langle X^i \rangle = 0$.[2]

Given this choice of $\mathbf{\Phi}$, one recovers the correct tree-level spectrum and couplings for matter and gauge fields to leading order in $1/M_{\mathrm{Pl}}$, including all effects proportional to $m_{3/2}$ without needing to perform any component manipulations. Thus, we can effectively decouple any complications posed by the gravity multiplet from calculations involving the matter fields alone.

Alternatively, the above gauge choice can be interpreted as a well-chosen Kähler transformation. In particular, we use the fact that tree-level supergravity is invariant under

$$K \to K - Z - Z^\dagger, \qquad W \to e^Z W, \tag{5}$$

where Z is any chiral multiplet. The gauge fixing in Eq. (3) is equivalent to a prescient choice for Z which simply cancels the problematic terms in the Kähler potential that

are linear in superfields, allowing one to use the standard form of $\mathbf{\Phi}$ from Eq. (1). Linear terms in the Kähler potential typically imply no-scale SUSY breaking, so this Kähler transformation effectively converts no-scale SUSY breaking into more familiar F-term breaking. As an added bonus, in this Kähler basis the tree-level equation of motion for F_Φ always yields

$$\langle F_\Phi \rangle = m_{3/2}, \tag{6}$$

after adjusting the cosmological constant (c.c.) to zero, so it is straightforward to identify SUGRA effects proportional to $m_{3/2}$.

Our proposed gauge fixing is related to an "improved gauge fixing" discussed many years ago by Kugo and Uehara [13].[3] In that context, the improved gauge fixing was used merely as means to more efficiently calculate the component SUGRA action. Moreover, the gauge fixing of Ref. [13] has no simple superfield realization like Eq. (3), and residual manipulations of the component SUGRA action were required to obtain the matter spectrum and couplings. Here, we avoid those complications at the expense of ignoring higher order $1/M_{\mathrm{Pl}}$ suppressed effects.

The remainder of the paper is organized as follows. In Sec. II, we review the formalism of conformal supergravity and show why the standard gauge fixing is suboptimal. We then derive our preferred gauge choice in Sec. III, and present a number of consistency checks. We conclude in Sec. IV, leaving further calculational details to the appendices. In a companion paper [15], we will use our novel gauge fixing to properly calculate the mass spectrum of goldstini and modulini in general theories of F-term, D-term, and no-scale SUSY breaking.

II. STRUCTURE OF SUPERGRAVITY

We begin with a brief review of conformal SUGRA, highlighting the subtleties and deficiencies of the standard gauge fixing.

A. Conformal Supergravity

As is well known, the SUGRA action can be derived by reinterpreting minimal SUGRA as a gauge fixing of conformal SUGRA [7]. The purpose of the conformal compensator is to realize this gauge fixing in superspace.

The gauge redundancies of minimal SUGRA are diffeomorphisms, local Lorentz transformations, and local supersymmetry. The additional gauge redundancies of conformal SUGRA are local dilatations $\hat{D}$, local $U(1)_R$ chiral transformations $\hat{A}$, conformal supersymmetry $\hat{S}_\alpha$, and special conformal transformations $\hat{K}_\mu$. The special conformal transformations can be fixed by setting the dilatation gauge

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[2]To work in an arbitrary vev basis, on can simply replace X^i with $(X^i - \langle X^i \rangle)$. The presence of terms in Eq. (3) which depend explicitly on vevs may be unintuitive. After all, the values of these vevs are unknown without computing the vacuum structure of the theory, which is in turn dependent on the vevs. However, we will show in Sec. III C that one can solve for these vevs self-consistently.

[3]For a recent study of SUGRA gauge fixing in the context of inflation, see Ref. [14].

field to zero,[4] but this leaves two real gauge freedoms, $\hat{D}$ and $\hat{A}$, and a Weyl spinor gauge freedom, $\hat{S}_\alpha$.

In order to fully gauge fix conformal SUGRA to ordinary SUGRA, one introduces the conformal compensator $\mathbf{\Phi}$, which is a chiral superfield with conformal weight 1.[5] Before gauge fixing, the components of $\mathbf{\Phi}$ are given by

$$\mathbf{\Phi} = \{\sigma, \sigma\zeta_\alpha, \sigma F_\Phi\}, \tag{7}$$

where the overall factor of σ is unconventional, but convenient for later purposes. A description of how to express these components in superspace is given in Appendix A, where the main subtlety is that a multiplet with nonzero conformal weight has additional couplings to the vector auxiliary field b_μ in the gravity multiplet. In particular, Eq. (1) is secretly hiding relevant terms involving b_μ.

Under the dilatation and chiral transformations parametrized by a complex number λ, and the superconformal transformation parametrized by ρ_α, the components of $\mathbf{\Phi}$ transform as:

$$\sigma \rightarrow e^\lambda \sigma, \qquad \zeta_\alpha \rightarrow \zeta_\alpha + \rho_\alpha. \tag{8}$$

Thus, the lowest and fermionic components of $\mathbf{\Phi}$ are pure gauge modes, and one can use the remaining extra gauge freedoms to set

$$\hat{D}: |\sigma| = 1, \qquad \hat{A}: \mathrm{Arg}\sigma = 0, \qquad \hat{S}_\alpha: \zeta_\alpha = 0. \tag{9}$$

This is the conventional gauge choice which results in Eq. (1). As we will soon see, the SUGRA action suggests a more convenient gauge fixing for practical computations.

B. SUGRA Action

To construct a valid SUGRA action, we must know the conformal weights of all fields in the theory. Besides the conformal compensator, all other chiral superfields have conformal weight 0, and we will denote their components by

$$X^i = \{X^i, \chi^i_\alpha, F^i\}. \tag{10}$$

Vector multiplets also have conformal weight 0, but since SUSY-covariant derivatives D_α and $\bar{D}_{\dot{\alpha}}$ have conformal weight 1/2, the gauge field strengths W^a_α have conformal weight 3/2.

In a superconformal theory, the only objects which can be consistently coupled to conformal gravity are real multiplets $\mathbf{\Xi}$ with conformal weight 2 and chiral multiplets $\mathbf{\Sigma}$

with conformal weight 3.[6] In general, $\mathbf{\Xi}$ and $\mathbf{\Sigma}$ will be composite multiplets, and we indicate their components by

$$\mathbf{\Xi}_{(w=2)} = \{C, \xi_\alpha, M, A_\mu, \lambda_\alpha, D\}, \tag{11}$$

$$\mathbf{\Sigma}_{(w=3)} = \{z, \chi_\alpha, F\}. \tag{12}$$

As argued in Appendix A, once we choose the appropriate gauge fixing for $\mathbf{\Phi}$, we can regard $\mathbf{\Xi}$ and $\mathbf{\Sigma}$ as ordinary *global* superfields (with the corresponding expansion in θ and $\bar{\theta}$) for any calculations involving matter fields alone.

From $\mathbf{\Xi}$ and $\mathbf{\Sigma}$, one can construct superconformally invariant D-term and F-terms, respectively[7]:

$$\mathbf{\Xi}_D = \frac{1}{2}eD - \frac{1}{2}e(\lambda\sigma^\mu\bar{\psi}_\mu - i\xi\sigma^{\mu\nu}D^c_\mu\psi_\nu + \mathrm{H.c.}) + \frac{C}{3}\left(\frac{1}{2}eR - \mathcal{L}_{\mathrm{RS}}\right) + \ldots, \tag{13}$$

$$[\mathbf{\Sigma}]_F = e(F - i\sqrt{2}\chi\sigma^\mu\bar{\psi}_\mu - z\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu), \tag{14}$$

where e is the determinant of the metric, R is the Ricci scalar, $\mathcal{L}_{\mathrm{RS}} = \epsilon^{\mu\nu\alpha\beta}\bar{\psi}_\mu\bar{\sigma}_\mu\partial_\alpha\psi_\beta + \ldots$ is the massless Rarita-Schwinger action for the gravitino ψ_μ, and $D^c_\mu = \partial_\mu + \ldots$ is the SUGRA-covariant derivative. The ellipsis in $[\mathbf{\Xi}]_D$ represents terms that are quadratic in the gravitino, but do not contribute to the gravitino mass or kinetic term. While it is possible to rewrite these D-term and F-term invariants as superspace integrals involving the gravity supermultiplet, it is more convenient to express only the matter part of the action in (global) superspace, leaving couplings to the gravity multiplet in component form.

Since ordinary matter multiplets have vanishing conformal weight and $\mathbf{\Xi}$ ($\mathbf{\Sigma}$) has conformal weight 2 (3), the conformal compensator $\mathbf{\Phi}$ is necessarily present in any SUGRA action containing matter. Given the Kähler potential K, superpotential W, and gauge kinetic function f_{ab}, we can construct the following fully superconformally invariant action at tree-level[8]:

$$\mathcal{L}_{\mathrm{SUGRA}} = [-3\mathbf{\Phi}^\dagger\mathbf{\Phi}e^{-K/3}]_D + [\mathbf{\Phi}^3 W]_F + \mathrm{H.c.} + \left[\frac{1}{4}f_{ab}W^{a\alpha}W^b_\alpha\right]_F + \mathrm{H.c.} \tag{15}$$

By expanding out Eq. (15), we recover Eq. (2) as desired. Note that all interactions with the gravity multiplet, as well as the graviton and gravitino kinetic terms, come from the

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[4] Given the upcoming discussion in Sec. III A, one might wonder whether alternative gauge fixings for $\hat{K}_\mu$ might simplify other aspects of the SUGRA action. We were unable to find any such simplifications.

[5] There are of course alternative choices for how to introduce compensator multiplets. This particular choice is referred to in the literature as "$n = -1/3$."

[6] In some of the SUGRA literature, the $w = 2$ vector multiplet $\mathbf{\Xi}$ is expressed as a $w = 3$ chiral multiplet, namely, the object $\bar{D}^2\mathbf{\Xi}$ from global superspace. We find that using $\mathbf{\Xi}$ directly is more transparent for practical calculations.

[7] These expressions differ from those in Ref. [13] because we use the two-component fermion notation of Ref. [1]. In addition, we employ different minus sign and factor of 2 conventions.

[8] With radiative corrections, one needs to account for a conformally violating regulator, which introduces additional dependence on $\mathbf{\Phi}$.

covariant forms of the corresponding D- and F-term expressions in Eqs. (13) and (14). While in general the action can include additional terms involving SUSY-covariant derivatives D_α and $\bar{D}_{\dot\alpha}$, we will neglect this complication in the present work.

C. Problematic Terms

The standard approach in the existing SUGRA literature is to expand Eq. (15) in components using the definition of Φ in Eq. (1). However, one sees immediately from Eqs. (13) and (14) that this gauge choice yields problematic terms that mix gravity and matter fields and must be carefully accounted for in any actual calculation. These problematic terms pertain to:

(i) *graviton normalization and kinetic mixing*:

$$\frac{C}{3}\frac{R}{2}. \tag{16}$$

At the very minimum, $\langle C/3\rangle$ must be set equal to $-M_{\rm Pl}^2$ in order to canonically normalize the Einstein-Hilbert action R. In addition, if C depends linearly on the matter multiplets, then there are undesirable kinetic mixing terms between the matter fields and the graviton proportional to $\langle C_i\rangle$ and $\langle C_{\bar\imath}\rangle$. Note that $\langle R\rangle = 0$ in the flat-space vacuum, so one need not worry about mass corrections arising if C depends quadratically on matter fields. Also note that if the Einstein-Hilbert term is canonically normalized, then so is the Rarita-Schwinger action.

(ii) *gravitino kinetic mixing*:

$$i\xi\sigma^{\mu\nu}\partial_\mu\psi_\nu + {\rm H.c.} \tag{17}$$

A kinetic mixing term between the gravitino and matter fields implies a noncanonical gravitino multiplet with the wrong Rarita-Schwinger action. In contrast, mass mixings of the form $\eta\sigma^\mu\bar\psi_\mu$ are perfectly healthy, since η can be identified as the goldstino from spontaneous SUSY breaking which is eaten by the gravitino in unitary gauge. [See Eq. (35)].

(iii) *gravitino mass phase*:

$$-z^\dagger\psi_\mu\sigma^{\mu\nu}\psi_\nu + {\rm H.c.} \tag{18}$$

The usual gravitino mass parameter is defined as a real number, so $\mathrm{Arg}\langle z\rangle$ should be set equal to zero.

In the standard approach, these three problem terms are eliminated by (i) performing a field-dependent Weyl rescaling of the metric, (ii) applying a shift to the gravitino which is distinct from going to unitary gauge, and (iii) performing a chiral rotation of the fermions. While these manipulations are perfectly well-defined in terms of component fields, no simple interpretation exists in terms of superfields. That is to say, the terms in the ellipsis of

Eq. (2) hide relevant mixing terms between the gravity multiplet and the matter multiplets.

In addition to the above three complications, there is a less apparent fourth one—a marginal operator mixing the vector auxiliary field b_μ with scalars

$$b_\mu\partial^\mu\phi. \tag{19}$$

This term is not readily visible in Eq. (13), and arises because the conformal compensator has conformal weight 1. (See Appendix A). Because of the quadratic pieces in this interaction, integrating out the vector auxiliary field generates additional scalar kinetic terms, again mixing the gravity and matter multiplets.

III. A NOVEL GAUGE FIXING

Next, let us demonstrate how complications from mixing with gravitational modes can be straightforwardly removed by an appropriate choice of gauge.

A. Exploiting Gauge Freedoms

From Eq. (8), we see that the $\hat{D}$, $\hat{A}$, and $\hat{S}_\alpha$ gauge freedoms can be spent to fix the components σ and ζ_α in Φ equal to *any* desired values. In particular, we can even set σ and ζ_α to field-dependent functions of the matter fields, and we will exploit this freedom to manifestly resolve the three complications discussed in Sec. II C.

In Wess-Zumino gauge for the gauge fields, but not yet gauge fixing Φ, Eq. (15) implies that

$$C = -3\sigma^\dagger\sigma e^{-K/3}, \tag{20}$$

$$\xi_\alpha = 3i\sqrt{2}\sigma^\dagger\sigma e^{-K/3}\left(\zeta_\alpha - \frac{K_i}{3}\chi_\alpha^i\right), \tag{21}$$

$$\mathrm{Arg}[z] = \mathrm{Arg}[\sigma^3 W]. \tag{22}$$

Thus, we see that there is sufficient freedom in σ and ζ_α to set $C = -3$, $\xi_\alpha = 0$, and $\mathrm{Arg}z = 0$ to all orders in the fields. This is essentially equivalent to the gauge choice advocated in Ref. [13], which is reviewed in Appendix B. However, in this gauge, one is forced to integrate out the vector auxiliary field of SUGRA in components, which is at odds with our aim to describe SUGRA solely in the language of superfields.

For the purposes of calculating the spectrum and couplings of matter fields, it is actually more convenient to impose a less stringent gauge choice where we only impose the conditions $C = -3$, $\xi_\alpha = 0$, and $\mathrm{Arg}z = 0$ to leading order in field fluctuations:

$$\sigma = \exp\left[\frac{1}{3}(\langle K/2 - i\mathrm{Arg}W\rangle + \langle K_i\rangle X^i)\right], \tag{23}$$

$$\zeta_\alpha = \frac{1}{3}\langle K_i\rangle\chi_\alpha^i, \tag{24}$$

where again we are working in a "zero vev" basis where $\langle X^i \rangle = 0$. This gauge fixing is sufficient to ensure that there are no linear mixing terms between matter field fluctuations and the graviton/gravitino. In addition, as long as one is working in flat space, one can ignore additional scalar mass terms from the remaining quadratic nonminimal couplings to the Einstein-Hilbert term.

B. Going to Superspace

While the gauge choice in Eq. (24) successfully decouples the matter fields from the gravity multiplet, it is rather inconveniently written in terms of components. Fortunately, modulo a field redefinition of the auxiliary field F_Φ, this gauge choice can be rewritten compactly in terms of superfields as Eqs. (3) and (4), repeated here for convenience:

$$\mathbf{\Phi} = e^{\mathbf{Z}/3}(1 + \theta^2 F_\Phi),$$

$$\mathbf{Z} = \langle K/2 - i\mathrm{Arg}W \rangle + \langle K_i \rangle X^i.$$

Were $\mathbf{\Phi}$ an ordinary superfield, then the mapping from Eq. (24) to Eq. (3) would occur without complication, but since $\mathbf{\Phi}$ has conformal weight 1, there is an additional subtlety.

Recall that matter fields participate in additional interactions with the gravity multiplet beyond those explicitly shown in Eqs. (13) and (14). These couplings are dictated by the full SUGRA invariance of the theory. For example, when expanding the composite multiplets $\mathbf{\Xi}$ and $\mathbf{\Sigma}$, one encounters SUGRA-covariant derivatives D_μ^c containing additional couplings to the graviton and gravitino. However, for calculations involving these matter fields alone, these additional terms are irrelevant.[9] Similarly, for matter fields of conformal weight 0, integrating out the vector auxiliary field only generates $1/M_{\mathrm{Pl}}^2$ suppressed dimension six operators, which can also be ignored.

Nevertheless, a complication still arises because the conformal compensator has conformal weight 1, and as shown in Appendix A, it couples nonminimally to the vector auxiliary field. A SUGRA-covariant derivative acting on σ yields

$$D_\mu^c \sigma = \partial_\mu \sigma - \frac{i}{2} b_\mu \sigma + \dots, \qquad (25)$$

and since $\langle \sigma \rangle \neq 0$, linear terms involving the vector auxiliary field b_μ appear in the action.

Fortunately, one can prove via explicit computation (see Appendix C) that in our gauge fixing in Eq. (24), $b_\mu = 0$ to leading order in M_{Pl} and can thus be ignored. As advertised, our gauge fixing can indeed be written in the form of Eq. (3) up to irrelevant $1/M_{\mathrm{Pl}}$-suppressed operators.

It is instructive to understand why b_μ can be completely ignored in our gauge but in contrast must be carefully included in the gauge proposed by the authors of Ref. [13]. In Ref. [13], σ was chosen to give $C = -3$ to all orders in the field expansion, which implied that σ was a function of both chiral and antichiral fields. However, σ itself is the lowest component of a chiral multiplet, so $D_\mu^c \sigma$ should also be a chiral object. Since spacetime derivatives on σ give derivatives on both chiral and antichiral fields, the b_μ equation of motions must cancel off the derivatives on the antichiral fields in order to preserve the known holomorphic structure of SUSY. In our gauge fixing, σ is only dependent on chiral fields since vevs are just constant complex numbers—thus, holomorphicity is manifest, and b_μ can be set to 0 at leading order.

C. Understanding VEVs

Equation (3) is the primary result of this paper, and it is worth understanding why explicit vevs are appearing in our gauge choice.

Two of the three problematic terms in Sec. II C are associated with linear terms in the Kähler potential. In particular, if K_i were equal to zero, then there would be no quadratic mixing terms in Eq. (16) or Eq. (17). In addition, as shown in Appendix C, the problematic mixing with b_μ in Eq. (19) would vanish if $K_i = 0$. For a general theory, K_i will not equal zero everywhere in field space, so the best we can hope for is that $\langle K_i \rangle = 0$ at the minimum of the potential. Interpreting our gauge fixing as the Kähler transformation in Eq. (5), we can indeed remove such linear terms by the appropriate choice of $\mathbf{Z}$, but that choice will explicitly depend on $\langle K_i \rangle$. In this sense, the appearance of vevs in our gauge fixing is unavoidable.

Crucially, classical equations of motion are not affected by the appearance of vevs. For a general function $f(x)$, the solution to $\langle \partial f/\partial x \rangle = 0$ is the same for f as it is for the first-order Taylor expansion

$$\tilde{f}(x) = \langle f(x) \rangle + \langle f'(x) \rangle (x - \langle x \rangle), \qquad (26)$$

or even any linear combination of f and $\tilde{f}$. Thus, one can self-consistently solve for the $\langle K_i \rangle$ terms in Eq. (3) by treating $\langle K_i \rangle$ as numbers whose values are determined by the scalar equations of motion. Of course, f and $\tilde{f}$ have different quadratic terms, and we will verify in the next subsection that scalar masses take on the expected SUGRA values.

Finally, the appearance of vevs in the gauge fixing means that Eq. (3) is not manifestly (Yang-Mills) gauge invariant if there is a charged field X^i that gets a vev.[10] This is not really an issue, of course, since a charged field getting a vev implies spontaneous gauge symmetry breaking. However, one should remember that calculations using

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[9]One might worry about additional contributions to the problematic terms in Eq. (16) and (17), or Eq. (18), but these are absent.

[10]In addition, Eq. (3) is expressed in a "zero vev" basis where $\langle X^i \rangle = 0$, further obscuring (Yang-Mills) gauge invariance.

Eq. (3) are only correct up to $1/M_{\mathrm{Pl}}^2$ suppressed dimension six operators, so there will in general be (Yang-Mills) noninvariance in the dimension six interactions. Note that the form of Eq. (3) already assumed Wess-Zumino gauge for the gauge multiplets, and one would need to redo the analysis of Sec. III A to find the best SUGRA gauge fixing if one wanted to use, say, unitary gauge for the massive vector multiplets.

D. Consistency Checks

To finish our discussion, we wish to show that our gauge choice satisfies a number of consistency checks. For simplicity, we will ignore the contributions from vector multiplets for this discussion, but one can verify that gauge interactions also turn out as expected. Since SUGRA is known to be Kähler invariant, the final results should be written in terms of the invariant Kähler potential

$$G \equiv K + \log W + \log W^*. \tag{27}$$

Note, however, that Kähler invariance is not manifest in our gauge choice, since the gauge fixing is equivalent to picking a preferred Kähler basis where gravity can be decoupled. (See Appendix B for a gauge choice with manifest Kähler invariance but other complications).

From Eqs. (14) and (15), we see that the gravitino mass is given by

$$m_{3/2} = \langle \mathbf{\Phi}^3 W|_{\theta^0} \rangle = e^{\langle G \rangle/2}. \tag{28}$$

This is the familiar Kähler invariant form of the gravitino mass, as desired.

If one ignores the ellipsis in Eq. (2), then familiar global SUSY techniques can be used to derive the spectrum and couplings of matter fields. As derived in Appendix D, the scalar and fermion kinetic terms are proportional to the Kähler metric $\langle G_{i\bar{j}} \rangle$, leading to

$$-\langle G_{i\bar{j}} \rangle \partial_\mu X^i \partial^\mu X^{\dagger \bar{j}}, \tag{29}$$

and

$$-\langle G_{i\bar{j}} \rangle \bar{\chi}^{\bar{j}} i \bar{\sigma}^\mu \partial_\mu \chi^i, \tag{30}$$

for the scalars and the fermions, respectively. Note the appearance of vevs in these expressions, as higher-order field couplings to the kinetic terms differ from the exact SUGRA results at order $1/M_{\mathrm{Pl}}^2$.

The scalar potential is derived in Appendix E. After solving the F_Φ and F^i equations of motion, we find

$$V = e^G (G^i G_i - 3) e^{-2K_{\mathrm{quad}}/3}, \tag{31}$$

which depends on the Kähler potential starting at quadratic order in the fields

$$K_{\mathrm{quad}} \equiv K - \langle K \rangle - \langle K_i \rangle X^i - \langle K_{\bar{j}} \rangle X^{\dagger \bar{j}}. \tag{32}$$

In Eq. (31), indices are raised and lowered with the inverse metric $G^{i\bar{j}}$ (without a vev), such that $G^i \equiv G^{i\bar{j}} G_{\bar{j}}$.

We see that the condition for vanishing c.c. is the same as in exact SUGRA

$$\langle G_i G^i \rangle = 3, \tag{33}$$

so the vacuum structure is maintained. After tuning the c.c. to zero, the equation of motion for F_Φ yields

$$\langle F_\phi \rangle = m_{3/2}, \tag{34}$$

as advertised in Eq. (6). Note that Eq. (31) matches the SUGRA scalar potential to leading order in $1/M_{\mathrm{Pl}}$, and includes all the effects proportional to $m_{3/2}$. In particular, the K_{quad} term would appear to give corrections at quadratic order in fields, but these vanish once the c.c. is tuned to zero.

Most important for our companion paper [15], we recover the SUGRA results for the fermion spectrum. Before going to unitary gauge for the gravitino, the (normalized) goldstino mode η_{eaten} couples the gravitino as

$$- i \frac{m_{3/2}\sqrt{3}}{\sqrt{2}} \eta_{\mathrm{eaten}} \sigma^\mu \bar{\psi}_\mu + \mathrm{H.c.} \tag{35}$$

From Eqs. (13)–(15), and using the F^i equation of motion, we can identify the goldstino mode as

$$\eta_{\mathrm{eaten}} = \frac{1}{\sqrt{3}} \langle G_i \rangle \chi^i. \tag{36}$$

As shown in Appendix F, the complete fermion spectrum (including the goldstino) is

$$-\frac{1}{2} m_{ij} \chi^i \chi^j + \mathrm{H.c.}, \quad m_{ij} = m_{3/2} \langle \nabla_i G_j + G_i G_j \rangle, \tag{37}$$

where $\nabla_i G_j \equiv \partial_i G_j - \Gamma_{ij}^k G_j$ depends on the Christoffel symbol Γ_{ij}^k derived from the Kähler metric. This mass matrix is not particularly illuminating in and of itself, but the fact that it can be derived entirely within the superspace formalism will allow us to easily compute the spectrum of fermion masses in a companion paper [15] on goldstini from F-term, D-term, and no-scale SUSY breaking.

IV. CONCLUSIONS

In this paper, we have derived a novel gauge choice for conformal SUGRA which results in a simplified version of the tree-level SUGRA Lagrangian. Our improved gauge is manifestly better suited for phenomenological applications, and moreover is an improvement over the gauge choice of Ref. [13] since it decouples matter modes not only from the graviton and gravitino but also from the vector auxiliary field. Hence, SUGRA calculations involving matter fields alone can be performed directly in (global) superspace, including effects proportional to $m_{3/2}$.

This gauge fixing can also be understood as the presciently chosen Kähler transformation in Eq. (5). This is not surprising, since a Kähler transformation

$$K \rightarrow K - Z - Z^\dagger, \qquad W \rightarrow e^Z W$$

has the same effect as a gauge transformation on the conformal compensator (plus a field redefinition of F_ϕ)

$$\Phi \rightarrow e^{Z/3}\Phi. \qquad (38)$$

In this paper, we have exploited this Kähler redundancy to simplify SUGRA calculations.

This formulation makes certain aspects of SUGRA manifest while obscuring others. By expanding Eq. (15) as Eq. (2), we have emphasized the spectrum and couplings of matter fields but have hidden gravitational interactions. On the other hand, the couplings of the graviton and gravitino are given at leading order by the well-known stress-energy tensor and supercurrent, respectively, so little is lost by hiding them. Crucially, Eq. (2) still includes all couplings to the goldstino degree of freedom identified in Eq. (36), which is phenomenologically more relevant than the transverse gravitino modes in the goldstino equivalence limit.

In a companion paper [15], we will use this novel gauge fixing to calculate the spectrum of goldstini in general theories of F-term, D-term, and "almost no-scale" SUSY breaking. With standard component SUGRA methods, such calculations would be tedious and obscure, but in superspace, they become straightforward and transparent. Obviously, one would like to generalize this gauge-fixing procedure beyond minimal SUGRA models, by properly including radiative corrections and SUSY-covariant derivatives in the action. Ultimately, one hopes that this alternative gauge fixing will shed light on the more general properties of SUGRA beyond phenomenology.

ACKNOWLEDGMENTS

APPENDIX A: SUPERFIELDS IN SUGRA

In order to describe matter multiplets in terms of the ordinary global superspace variables θ and $\bar{\theta}$, we need to know how to package the components of a multiplet into θ-dependent superfields, including any relevant SUGRA effects. Here, we follow the logic of Ref. [13,16], though we use two-component fermion notation.

For a chiral multiplet X with components

$$X = \{X, \chi_\alpha, F\}, \qquad (A1)$$

we can construct a familiar looking superfield in terms of the usual global superspace variables as

$$\mathbf{X} = X + \sqrt{2}\theta\chi + \theta^2 F + i\theta\sigma^\mu\bar{\theta}D_\mu^c X$$
$$- \frac{i}{\sqrt{2}}\theta\theta D_\mu^c \chi \sigma^\mu \bar{\theta} + \frac{1}{4}\theta^4 D_\mu^c D^{c\mu} X. \qquad (A2)$$

Compared to the expressions from global SUSY, we have simply replaced the ordinary derivative ∂_μ with the SUGRA-covariant derivative D_μ^c.

The SUGRA-covariant derivative depends on the graviton, gravitino, and vector auxiliary field, with full expressions given in Ref. [16]. For our purposes, we are mainly interested in anomalous couplings to the vector auxiliary field b_μ, which arise because the conformal compensator has conformal weight 1. The SUGRA-covariant derivative for a chiral multiplet of conformal weight w is

$$D_\mu^c X = \left(\partial_\mu - i\frac{w}{2}b_\mu\right)X + \dots, \qquad (A3)$$

$$D_\mu^c \chi_\alpha = \left(\partial_\mu - i\left(\frac{3}{4} - \frac{w}{2}\right)b_\mu\right)\chi_\alpha + \dots, \qquad (A4)$$

where the ellipsis indicates additional terms involving the graviton and gravitino.

Immediately, we see that the expression in Eq. (1) for the naïve conformal compensator is incomplete. Since the conformal compensator has conformal weight 1, there should be additional terms involving b_μ in the $\theta\bar{\theta}$ and θ^4 components. As discussed more in Appendix C, we can avoid that complication by ensuring that the equations of motion fix $b_\mu = 0$ to leading order in $1/M_{\mathrm{Pl}}$.

We also see that for ordinary chiral multiplets with $w = 0$, there are no complications posed by D_μ^c. While there are residual couplings of b_μ to fermions, they only generate $1/M_{\mathrm{Pl}}^2$ suppressed dimension six operators. Similarly, it is straightforward to show that the graviton and gravitino terms elided in Eq. (A4) do not introduce any of the problematic mixing terms discussed in Sec. II C. A similar analysis holds for vector multiplets, with the same conclusions.

Thus, for calculations involving matter fields alone, we are free to use ordinary *global* superfields for calculational purposes, as long as we ensure that $b_\mu = 0$ to leading order in $1/M_{\mathrm{Pl}}$. The condition $b_\mu = 0$ is a key feature of the gauge choice in Eq. (3), a feature not present in Ref. [13], as discussed in the next appendix.

APPENDIX B: THE KUGO-UEHARA GAUGE

The gauge choice in this paper is closely related to a gauge choice presented by Kugo and Uehara in Ref. [13], which we review in this appendix. The Kugo-Uehara gauge choice starts with essentially the same logic as in Sec. III A, but differs from our gauge fixing by setting

$$C = -3, \qquad \xi_\alpha = 0, \qquad \text{Arg} z = 0, \qquad \text{(B1)}$$

to all orders in the fields. This results in the conformal compensator having components

$$\mathbf{\Phi} = \exp\left[\frac{1}{3}(K/2 - i\text{Arg}W)\right] \times \left\{1, \frac{K_i\chi^i}{3}, F_\phi\right\}, \quad \text{(B2)}$$

where K and W include their full field dependence. By doing a field redefinition of F_Φ, this is more conveniently expressed in term of the superpotential W and invariant Kähler potential G from Eq. (27) as[11]

$$\mathbf{\Phi} = W^{-1/3}\tilde{\mathbf{\Phi}}, \qquad \tilde{\mathbf{\Phi}} = e^{G/6} \times \left\{1, \frac{G_i\chi^i}{3}, F_\phi\right\}. \quad \text{(B3)}$$

Here, W is a full chiral superfield, but G is only a function of the scalar fields.

This gauge choice has a number of nice features. First, this gauge choice makes Kähler invariance manifest. Looking at Eq. (2), we see that the $W^{-1/3}$ term in $\mathbf{\Phi}$ precisely cancels against the superpotential W in the action, and the very same $W^{-1/3}$ term combines with $e^{-K/3}$ to yield

$$(WW^*)^{-1/3}e^{-K/3} \equiv -3e^{-G/3}. \qquad \text{(B4)}$$

Thus, the SUGRA action depends only on G, which can be thought of as the invariant Kähler potential lifted into superspace. Second, this gauge choice eliminates the problematic mixing terms everywhere in moduli space, such that no matter what the field vevs are, none of the three terms from Sec. II C ever arise. Finally, the fermionic component of $\tilde{\mathbf{\Phi}}$ is proportional to the eaten goldstino $G_i\chi^i/\sqrt{3}$, and setting this component to zero automatically results in unitary gauge for the gravitino.

Despite the apparent simplicity of this Kugo-Uehara gauge, it suffers from a hidden problem, namely, problematic mixing with the vector auxiliary field b_μ described in Eq. (19). This means that $\mathbf{\Phi}$ cannot simply be lifted into global superspace, since its components have residual dependence on the SUGRA-covariant derivatives from Eq. (A4). Integrating out b_μ gives additional terms in the action which spoil the simplicity of Eq. (2). In addition, $\mathbf{\Phi}$ does not have the expected holomorphy properties of a usual chiral multiplet, since $\tilde{\mathbf{\Phi}}$ depends on the full Kähler potential G, which is a function of both chiral and antichiral fields.

Thus, we prefer Eq. (3) for practical calculations. That said, the gauge choice in Eq. (B3) is convenient for

zero-momentum calculations where the problematic term in Eq. (19) is irrelevant. Alternatively, if one wanted to keep track of the most important effects of b_μ using a superfield language, one could introduce a new auxiliary field multiplet

$$\mathbf{B} = -\frac{1}{2}\theta\sigma^\mu\bar{\theta}b_\mu, \qquad \text{(B5)}$$

and make the replacement

$$\mathbf{\Phi}^\dagger\mathbf{\Phi} \rightarrow \mathbf{\Phi}^\dagger e^{\mathbf{B}}\mathbf{\Phi} \qquad \text{(B6)}$$

in Eq. (2). Interpreting b_μ as the gauge field for local $U(1)_R$ chiral transformations, we see that $\mathbf{\Phi}$ indeed has conformal weight 1.[12]

APPENDIX C: THE VECTOR AUXILIARY FIELD

As described in Appendix A, in order to convert global superfields into SUGRA superfields, all spacetime derivatives ∂_μ must be replaced by covariant derivatives D_μ^c which are covariant under the complete set of conformal SUGRA gauge redundancies. This covariant derivative D_μ^c contains couplings to the gravity multiplet, which are largely irrelevant for our phenomenological purposes. The one exception is covariant derivatives acting on the conformal compensator itself, since the compensator has conformal weight 1.

Since we have parametrized $\mathbf{\Phi}$ in Eq. (7) with an overall factor of σ, we can treat σ as the only field with conformal weight 1, with ζ_α and F_Φ having the usual conformal weight 0. Thus, the only nontrivial covariant derivative is

$$D_\mu^c\sigma = \partial_\mu\sigma - \frac{i}{2}wb_\mu\sigma + \ldots, \qquad \text{(C1)}$$

where the conformal weight $w = 1$ for σ, b_μ is the vector auxiliary field, and we have elided the graviton and gravitino terms for simplicity.

We can expand out Eq. (2) to isolate terms depending on the vector auxiliary field:

$$3\sigma^\dagger\sigma e^{-K/3}\left[\frac{b^\mu b_\mu}{4} + b^\mu\text{Im}\left(\frac{1}{3}K_i\partial_\mu x^i - \frac{\partial_\mu\sigma}{\sigma}\right)\right], \quad \text{(C2)}$$

where we have elided additional terms that are quadratic in fermion fields. Using the gauge fixing from Eq. (23), we see that

$$\frac{\partial_\mu\sigma}{\sigma} = \frac{1}{3}\langle K_i\rangle\partial_\mu x^i, \qquad \text{(C3)}$$

so there are no kinetic mixing terms of the form of Eq. (19), only higher order terms in the scalar field expansion. Since

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[11]The derivation in Ref. [13] is slightly different from the one presented in this appendix. There, the conformal compensator was first rescaled by the superpotential $\mathbf{\Phi} = W^{-1/3}\tilde{\mathbf{\Phi}}$ and then the gauge fixing was applied to $\tilde{\mathbf{\Phi}}$ itself. Obviously, the two methods differ only by a Kähler transformation, and the two final results are the same. The practical difference is that Ref. [13] did not have to impose the condition from Eq. (18).

[12]More precisely, a chiral multiplet with conformal weight w has charge $\{w, w\}$ under $\hat{D}$ and $\hat{A}$, while an antichiral multiplet with weight w has charge $\{w, -w\}$. Recall, though, that we have used the $\hat{K}_\mu$ gauge freedom to eliminate the dilatation gauge field.

b_μ has a mass of order M_{Pl}^2, the only effect of the auxiliary fields is to generate $1/M_{\text{Pl}}^2$ suppressed dimension six operators involving matter fields, which are irrelevant for our phenomenological purposes. Therefore we are free to set $b_\mu = 0$ to leading order for this gauge choice.

It is now clear why the Kugo-Uehara gauge choice in Eq. (B2) has residual dependence on the vector auxiliary field. In that gauge, σ is a function of both chiral and antichiral fields, so the leading order cancellation seen in Eq. (C3) does not persist. In this sense, our gauge choice is unique, since it is the minimal (local) gauge fixing that eliminates Eq. (19).

APPENDIX D: SCALAR AND FERMION KINETIC TERMS

Using Eq. (2), it is straightforward to check that our gauge fixing results in the expected scalar and fermion kinetic terms in SUGRA. In particular, Eq. (2) can be expanded using standard superspace methods, up to corrections at order $1/M_{\text{Pl}}$.

The kinetic operators can only come from the first term in Eq. (2), and we parameterize the chiral supermultiplets as in Eq. (10). To simplify the notation, it is convenient to introduce the superfield

$$\boldsymbol{T} = -3\boldsymbol{\Phi}^\dagger \boldsymbol{\Phi} e^{-K/3}, \tag{D1}$$

resulting immediately in the kinetic terms

$$- T_{i\bar{j}} \partial_\mu X^i \partial^\mu X^{\dagger \bar{j}}, \tag{D2}$$

and

$$- T_{i\bar{j}} \bar{\chi}^{\bar{j}} i \bar{\sigma}^\mu \partial_\mu \chi^i, \tag{D3}$$

for the scalars and the fermions, respectively.

Using the gauge fixing for $\boldsymbol{\Phi}$ in Eq. (3), it is straightforward to find $T_{i\bar{j}}$ from Eq. (D1). Since $\boldsymbol{\Phi}$ is itself expressed as a function of the chiral multiplets, there is no complications in taking field derivatives. We find

$$T = -3e^{-K_{\text{quad}}/3}, \tag{D4}$$

$$T_i = \delta K_i e^{-K_{\text{quad}}/3}, \tag{D5}$$

$$T_{i\bar{j}} = \left(G_{i\bar{j}} - \frac{\delta K_i \delta K_{\bar{j}}}{3}\right) e^{-K_{\text{quad}}/3}, \tag{D6}$$

where $\delta K_i \equiv K_i - \langle K_i \rangle$ and K_{quad} is defined in Eq. (32).

Because of the relationship between the vevs

$$\langle T_{i\bar{j}} \rangle = \langle G_{i\bar{j}} \rangle, \tag{D7}$$

we indeed recover the correct SUGRA kinetic terms. At higher orders in the field expansion of Eqs. (D2) and (D3), there will be deviations from the SUGRA predictions at order $1/M_{\text{Pl}}^2$.

APPENDIX E: SCALAR POTENTIAL AND AUXILIARY VEVS

Another check of the gauge fixing is to make sure that the vacuum structure of the theory matches the exact results from SUGRA. Again, we can use superspace methods to analyze Eq. (2). It is convenient to define the superfield

$$\boldsymbol{P} = \boldsymbol{\Phi}^3 \boldsymbol{W}, \tag{E1}$$

and we will continue to use the notation $\boldsymbol{T}$ from Eq. (D1).

The scalar potential derived from Eq. (2) is

$$-V = T_{i\bar{j}} F^i F^{\dagger \bar{j}} + T_i F^i F_\Phi^\dagger + T_{\bar{j}} F^{\dagger \bar{j}} F_\Phi + T F_\Phi F_\Phi^\dagger$$
$$+ P_i F^i + 3 F_\Phi P + P_{\bar{j}}^\dagger F^{\dagger \bar{j}} + 3 F_\Phi^\dagger P^\dagger. \tag{E2}$$

The expression for F_Φ is obtained from its equation of motion

$$F_\Phi = -\frac{T_i F^i + 3 P^\dagger}{T}. \tag{E3}$$

Our gauge choice has $\langle T_i \rangle = 0$ and $\langle T \rangle = -3$ by construction, and $\langle P \rangle = m_{3/2}$ from Eq. (28), so

$$\langle F_\Phi \rangle = m_{3/2} \tag{E4}$$

as advertised.

Substituting F_Φ back into Eq. (E2), we have

$$-V = \left(T_{i\bar{j}} - \frac{T_i T_{\bar{j}}}{T}\right) F^i F^{\dagger \bar{j}} + \left(P_i - 3P \frac{T_i}{T}\right) F^i$$
$$+ \left(P_{\bar{j}}^\dagger - 3P^\dagger \frac{T_{\bar{j}}}{T}\right) F^{\dagger \bar{j}} - 9\frac{PP^\dagger}{T}. \tag{E5}$$

Using the fact that

$$T_{i\bar{j}} - \frac{T_i T_{\bar{j}}}{T} = G_{i\bar{j}} e^{-K_{\text{quad}}/3}, \tag{E6}$$

$$P_i - 3P \frac{T_i}{T} = G_i P, \tag{E7}$$

$$PP^\dagger = e^G e^{-K_{\text{quad}}}, \tag{E8}$$

we can simplify the potential and solve for F^i

$$F^i = -P^\dagger G^i e^{K_{\text{quad}}/3}. \tag{E9}$$

Here, $G^i \equiv G^{i\bar{j}} G_{\bar{j}}$ is defined in terms of the inverse Kähler metric. The final expression for the scalar potential is

$$V = e^G (G^i G_i - 3) e^{-2K_{\text{quad}}/3}, \tag{E10}$$

which agrees with the SUGRA scalar potential at leading order in $1/M_{\text{Pl}}^2$ when expanded around flat space.

APPENDIX F: GOLDSTINO MODE AND FERMION SPECTRUM

The final check of our gauge fixing is to verify that the fermion structure matches the SUGRA expectation. For simplicity, we will ignore gauginos for this discussion, and we will again use the notation of T from Eq. (D1) and P from Eq. (E1).

The goldstino mode couples to the gravitino as in Eq. (35). Using Eqs. (13) and (14), we can identify the goldstino direction η_{eaten} as

$$\eta_{\text{eaten}} = \frac{\sqrt{2}}{\sqrt{3}m_{3/2}}\left(\frac{1}{2}T|_{\bar{\theta}\bar{\theta}\theta} + P|_{\theta}\right). \qquad \text{(F1)}$$

Focusing only on the minimum of the scalar potential, and using the auxiliary field equations of motion, we find

$$\begin{aligned}
\eta_{\text{eaten}} &= \frac{1}{\sqrt{3}m_{3/2}}\langle(F_{\Phi}^{\dagger}T_i + T_{i\bar{j}}F^{\dagger\bar{j}}) + 2P_i\rangle\chi^i \\
&= \frac{1}{\sqrt{3}}\langle G_i\rangle\chi^i, \qquad \text{(F2)}
\end{aligned}$$

as desired. With D-terms turned on, η_{eaten} will pick up an additional contribution from the gaugino in $T|_{\bar{\theta}\bar{\theta}\theta}$ as well as $W^{\alpha}W_{\alpha}|_{\theta}$.

To check the fermion masses, we expand out Eq. (2), looking for the operators $\chi^i\chi^j$ in the Lagrangian:

$$-\frac{1}{2}\langle T_{ij\bar{k}}F^{\dagger\bar{k}} + T_{ij}F_{\Phi}^{\dagger} + P_{ij}\rangle\chi^i\chi^j + \text{H.c.}, \qquad \text{(F3)}$$

where again we have only considered the vevs of these expressions. Using the definitions of T and P, we can extract

$$\langle T_{ij\bar{k}}F^{\dagger\bar{k}}\rangle = -m_{3/2}\langle G_{ij\bar{k}}G^{\bar{k}}\rangle, \qquad \text{(F4)}$$

$$\langle P_{ij} + T_{ij}F_{\Phi}^{\dagger}\rangle = m_{3/2}\langle G_{ij} + G_iG_j\rangle. \qquad \text{(F5)}$$

Thus, the fermion mass matrix is

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}m_{3/2}\langle\nabla_iG_j + G_iG_j\rangle\chi^i\chi^j + \text{H.c.}, \qquad \text{(F6)}$$

where $\nabla_iG_j \equiv \partial_iG_j - \Gamma_{ij}^kG_k$, and the Christoffel symbol Γ_{ij}^k is derived from the Kähler metric $G_{ij\bar{k}} = G_{m\bar{k}}\Gamma_{ij}^m$. This is the expected SUGRA result.

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