

Supply Chain Responsiveness for a Large Retailer

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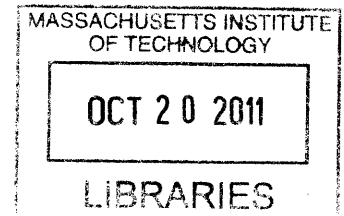
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ABSTRACT

A large U.S. based retailer underwent a large, complex multi-year supply chain network transformation. This transformation resulted in significant savings in logistics costs. Additionally, the regional distribution center that was introduced as part of this transformation as a new node between the supplier and the store became the decision making center for placing purchase orders with suppliers and for receiving and shipping the purchase order to individual stores. This resulted in longer lead times causing a change in the in-store units held and therefore, directly impacting the net sales. This thesis focuses on establishing the relationship between the stores performance and lead-time, review-time combinations in both supply chain networks, the original direct to store and the new regional distribution based networks.

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1 Background

In the retail industry, there are several ways for a retailer to distribute products from vendors to individual stores. The three primary models are:

- a) Direct to Store: In this model stores receive goods directly shipped by vendors from their own facilities. It is a key method of selling and distributing products for many industries, such as food, personal care products, and consumer packaged goods.
- b) Cross dock Distribution Center: Shipments from inbound suppliers are moved directly to outbound vehicles, with very little or no storage in between. This may be done to change the type of conveyance, to sort material intended for different destination, or to combine material from different origins into transport vehicles with the same or similar destinations. It is dependent on continuous communication between vendors, distribution center and all point of sales.
- c) Traditional Warehouse and Distribution Center: Vendors ship goods to retail distribution centers where the goods are stored until store orders need to be fulfilled. The warehouse or distribution center is a principle part of the entire order fulfillment process. A large retailer might sell tens of thousands of products from multiple vendors; it would be inefficient to ship all products directly from each vendor to stores. Large distribution centers for companies such as Walmart serve 50-125 stores. Vendors ship truckloads of products to the distribution center, which stores the product until needed by the retail location and ships the proper quantity.

Each method has advantages and disadvantages. For example, the DTS model incurs a high cost of transportation and receiving at stores while any DC based system will increase lead time and holding costs. In reality, most retailers use some type of hybrid system running both cross dock and traditional distribution operations in a single facility. The home product retailer that we will study in this thesis, for example, traditionally had 75% of its goods delivered by vendors direct to its stores and 25% from its own distribution center to the point of sales. Over the last several years, however, it has reversed these percentages so that now only a small percentage of goods are moved by DTS. This is discussed in greater detail later in this thesis.

In the highly competitive retail market, companies need to increase their responsiveness towards variable customer demand. Kesen, Kanchanapiboon & Das (2010) state that under stochastic demand, “the buyer may prefer to use supply flexibility, as opposed to an inventory holding strategy, to counter the demand drop.”. However, in most cases the lead-time as set by the supplier is inflexible. Kesen et al. (2010) further state that “Das and Abdel-Malek (2003) observe that supply relationships are prone to deteriorate as demand uncertainty increases, since one or both parties attempt to violate an inflexible contract. They identify two common cases of buyer requests when demand is uncertain: (i) a short lead time order shipment following a demand surge, and (ii) a smaller than normal order quantity following a demand drop.

In scenarios where lead-time is inflexible and stores facing stochastic demand, often stores aim to increase redundancy (in terms of time, inventory, service or a hybrid of the above) in the supply chain to adjust to variable demand; however increased inventory level also means higher inventory holding cost and inefficient use of limited working capital. BigRetailer, the company used as case study in this paper, has gone through multi-year project to transform its supply chain model from a direct-to-store model to a centralized distribution model. This gives BigRetailer several edges. Centralized distribution model performs as cross docking distribution center for all product deliveries to stores which enables maximization of economics of scales in outbound product flows and minimization of costly inventory function of warehousing. BigRetailer collaborates with downstream stores on latest demand information and coordinate order placement with vendors for all stores in the network. As a further development, BigRetailer builds in more intelligence into the channel by providing last minute reallocation functionality to synchronize product allocation with real-time demand in stores. BigRetailer's new RDC facilities will coordinate information flow across downstream network in the following several ways:

1. Aggregate the demand for all SKU across all stores
2. Place a common order with the supplier for the SKU
3. Receive the purchase order from the vendors
4. Receive product inflow from vendors
5. Dis-aggregate this purchase order across all the stores in the RDC's geographic limits and reallocate among stores based on latest demand information
6. Dispatch products to individual stores

This thesis concerns itself mainly with a large retailer that shifted its distribution from a Direct To Store (DTS) model to a cross-dock flow using Regional Distribution Centers (RDC). We will refer to the firm as BigRetailer from here out. Each model is discussed in turn.

In the DTS flow model, individual stores place orders every fixed period of time with vendors directly. Product replenishment happens the equivalent of lead-time (can be weeks or days depending on the service level provided by the supplier) after supplier receives the order and are delivered to stores directly. In some cases, BigRetailer has 4-5 stores in the same region, all within a radius of 20 miles, but there is very limited coordination among stores in terms of product ordering and delivery as each store is managed under different logistics processes and timelines. Stores are subject to minimum ordering quantity (MOQ) under the DTS model and each ordering quantity needs to be rounded up to a buy pack quantity as requested by suppliers. This prevents stores from ordering when demand arises if the ordering quantity doesn't meet up with MOQ. A buy-pack represents the minimum number of units of any given SKU that the supplier is willing to produce and sell in a purchase order, such that the quantity of units in the order is an integer multiple of the buy-pack quantity. MOQ represents the minimum value either in units or in Dollars for any SKU or a combination of SKUs that the purchase order must meet or exceed so that the purchase order is acceptable to the supplier. MOQ is set by the supplier. For example, if store A wants to order 92 widgets and the supplier MOQ is 100 units with 6 units per one pack, store A will either "kill" the order by ordering none, or "fill" the order by over ordering 10 units to bump up the total ordering quantity to be 102 units (6 units/buy-pack* 17 buy-packs).

Under the newer RDC model, individual stores review their inventories in the same period of time (periodical review) and signal demand information back to RDC for consolidation. The RDC aggregates the demand from all stores in its network and place one single common order with the vendors. During this process, the RDC will not add any additional changes to the order, as the demand information is trusted to be the most accurate available information in the system. When the product arrives at the RDC, it will be held for the time required to disburse the product in appropriate quantity to its stores. This time period is usually one to two days as RDC favors timely distribution of products.

BigRetailer has further modified the RDC model by including a re-allocation step at the RDC prior to final distribution to the stores. It builds in more intelligence into the product distribution process by empowering RDC with last-minute reallocation authority. This means that now the products that stores receive are not necessarily exactly the same that they ordered with RDC. Demand may have changed since the stores placed their last order and the re-allocation logic might have shifted some product to more demanding stores.

In the old DTS model, stores need to bear with such discrepancy by taking what they have ordered. Now with the last-minute reallocation option, stores can signal to the RDC with what their latest demand looks like and get the benefit of synchronizing their orders with demand by having RDC matching supply with demand in the best possible way for the entire channel. Under the RDC model (with and without reallocation), the MOQ constraint is no longer applicable. In the RDC model aggregated demand from all stores will exceed supplier MOQ and since the

RDC will place one single order on behalf of all stores with the supplier. When the supplier fulfills the purchase order, the RDC will receive the product delivery in a consolidated manner on behalf of the stores. Therefore, individual stores have the flexibility to place orders of required quantity at each review period and will not have to worry about supplier MOQ constraints. This helps each store bridge the gap between what they have forecasted as upcoming demand and inventory replenishment decisions. Stores also benefit from higher ordering frequency and lower resources dedicated to order management and handling cost.

1.1 Problem Statement

The lead-time for suppliers to ship products to RDC as compared to stores will not change from the supplier's perspective. However, in the new model the products go to RDC first and are briefly held in the RDC for one or two days before they get shipped out to stores. This intermediate node created by the RDC increases the lead-time for certain products to reach their respective stores compared to that under the DTS model. With longer lead-time, the product's replenishment cycle gets extended and hence the stores need to keep a relatively higher inventory leading to a higher operational cost. This increase in the lead-time affects other supply chain network input parameters like review time, service quality etc. and impacts the stores' output performance in terms of sales and stock-outs.

However, with aggregate ordering from the RDC, the stores can effectively order more frequently. This means that instead of being able to order only every other week, for example,

due to MOQ requirements, a store could order from this vendor every week. More frequent review periods allows for more responsive inventory replenishment.

The trade-off between the DTS and the RDC distribution models, then, boils down to a trade-off between a longer L (lead time) and a shorter R (Review period) in a periodic inventory review system. Thus there arises a need to develop a framework to help BigRetailer to measure lead-time and review time in context of the supply chain network responsiveness in the centralized distribution model and the stores performance in this new network vis-à-vis the stores performance in the DTS model.

This thesis will look into several dimensions of the new RDC model and focus on finding what the thresholds are such that the benefits of increased ordering frequency and RDC reallocation are outweighed by increased lead-time and how much value do reallocation processes contribute to the overall distribution system in terms of establishing an integrated inventory management system across the entire downstream store network.

The remainder of the thesis is organized as follows:

Chapter 2 discusses the research approach undertaken to establish periodic review models for the three product flows: DTS, RDC and RDC with re-allocation. Chapter 3 discusses the model mechanics, data simulation methodology and sensitivity analysis. Chapter 4 discusses data simulation and reallocation logic. Chapter 5 provides a deep analysis for different scenarios of

demand patterns, lead-time and review time for DTS, RDC and RDC with reallocation. Chapter 6 discusses the conclusions and suggestions that the author reached at the end of the research.

2 Research Approach:

The research for this thesis looks at three different channels, each from supplier to store, for a larger North America home goods retailer that we will refer to as BigRetailer. Each channel was modeled and subject to the common demand pattern for a given set of store-SKU combinations. The primary points of difference amongst these three channels are a) the combination of lead-time and review time, b) MOQ constraints, and c) allocation scheme. The three models to be discussed subsequently are Model One: Direct-to-Store Mode (DTS), Model Two: Regional Distribution Center Model (RDC), and Model Three: Regional Distribution Center Model with Allocation (RDC with re-allocation).

2.1 MODEL One - Direct-to-Store (DTS):

Under the Direct-to-store supply chain model, BigRetailer's stores place orders with suppliers individually. The stores review inventory after every pre-fixed time range and take a decision on whether to place orders with suppliers based on demand forecast and order up to level. Each order is subject to minimum ordering quantity (MOQ) and buy-pack number. This increases redundancy in the inventory pipeline (in-transit units and on-order units) and constrains the resources.

In the DTS network, the stores place a purchase order directly with the supplier. This can be seen in Figure 1, where each store places a purchase order with the supplier individually depicted by

the dotted arrow. After processing the purchase order the supplier ships out the goods to each store depicted by the truck on the full arrow from supplier to store in Figure 1. The individual shipment of the purchase orders to each store, results in potentially multiple less-than-truckloads (LTL) shipments resulting in high logistics cost to BigRetailer. For the purpose of this research, the time between when the store places the purchase order and when the stores receive the goods against the purchase order is considered the lead-time. The time between two such successive purchase orders for the same SKU is considered the review time.

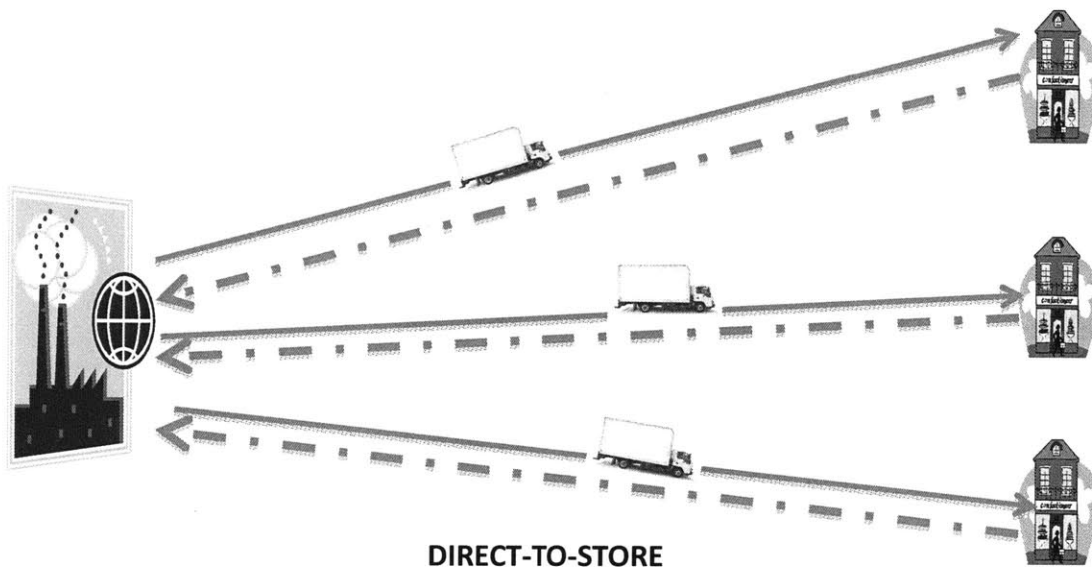


Figure 1 Direct to Store (DTS)

The key input parameters are as follows:

- a. Annual demand,
- b. Forecasting error,
- c. Safety factor (k value),
- d. Lead-time, and

e. Review-time

The DTS model will be constructed with the above variables as inputs to the model. In addition to the above list, one year's worth of sales data will also be input to the model. The data has been rolled up weekly sales as provided by BigRetailer.

Note: The MOQ constraint affects the review cycle by acting as a deterrence factor as shown below:

- a) A store performs a review,
- b) Identifies a need to place a purchase order,
- c) Determines that the purchase order is short of the MOQ and
- d) Consequently does not place the purchase order and has to deal with the gap using alternative means (if any) or risk stock out(likely).

This “killing” of the purchase order implies that the review period was missed. Each time the review period is missed, it increases the average length of the review cycle in that year. Thus, the “implied review period” in the DTS model with MOQ constraint is much longer than the actual review period. A longer review period implies less number of reviews per year and as will be shown in subsequent sections, higher number of units per order. The longer implied review period affects both the stores and the supplier as they have to deal with their respective demand variability.

The Cycle Service Level (CSL) is an input variable in the safety stock equation, used to determine the different inventory levels and influence the store performance. Changes in safety stock will also impact the actual store service levels. To analyze the correlation between CSL and Safety Stock, and the implications of changes in lead time and review time to the average inventory level, the analysis will be based on actual sales numbers from the stores identified by BigRetailer.

2.2 MODEL Two – Regional Distribution Center Model (RDC):

In Model two, this paper discusses the mechanics in which the Regional Distribution Center reduces the risk exposure that a single storefront is subject to, by sharing and integrating downstream and upstream information. In the Regional Distribution Center supply chain network, a new node is introduced as an intermediary between the supplier and the stores. This new node is the Regional distribution center (RDC). The RDC coordinates the information sharing and product distribution between the supplier and the stores. The RDC acts as an aggregating and dis-aggregating agent between the stores and the supplier. BigRetailer's stores share their current needs with the RDC. On a pre-fixed time, the RDC adds up the SKU demand across all the stores and places one large purchase order with the supplier on behalf of the stores. This purchase order is not subject to the MOQ criteria discussed earlier in the DTS network section. The RDC adds up the units required in all the stores under its network. Therefore, each purchase order by far exceeds the suppliers MOQ requirement. Additionally, by placing the purchase order by accumulating the needs of the stores, the RDC provides the supplier with a large order pool enabling the supplier to send out smaller number of Full truck-load shipments directly to RDC

(instead of many stores) thus reducing the logistics cost. After processing the purchase order the supplier ships out the goods to the RDC depicted by the truck on the full arrow from supplier to RDC in Figure 2. The RDC performs the following activity on receiving the purchase order:

- a) Identify the individual stores needs
- b) Break up the purchase order into individual shipments in buy-pack multiples to meet the needs in step (a)
- c) Ship out the individual stores' shipments

RDC acts as a pure cross-dock facility, such that at the end of the “shipping the goods to stores” activity there is no inventory left in the RDC. The time between when the store places the purchase order and when the stores receive the goods against the purchase order is considered the lead-time. The time between two such successive purchase order for the same SKU is considered the review time.

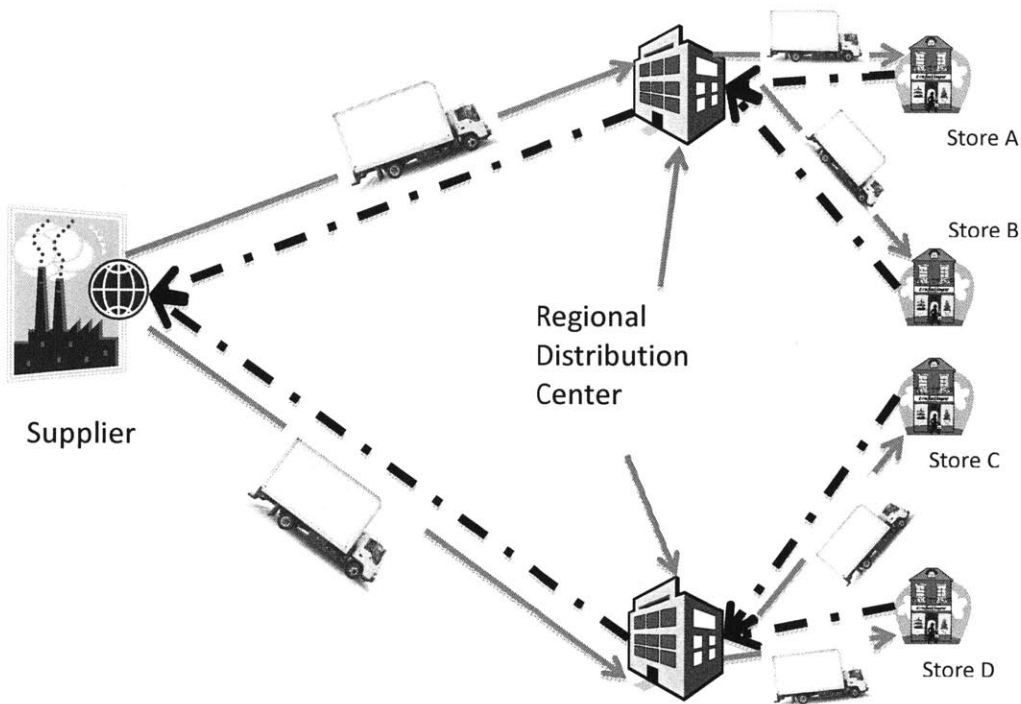


Figure 2 - Regional Distribution Center (RDC)

Similar to the approach taken in Model One, the authors will aggregate the stores' required units on a weekly basis and input into RDC simulation model to get the cumulative product replenishment details for a given SKU every week. Here the model is designed such that as and when a purchase order is received, it is disbursed to the stores in the quantity they requested. The lead-time and review-time in this model will be modified to reflect the new supply chain network configuration (lead time will be longer and review time shorter in the new network). The output of model two will be compared against the individual store ordering details in model one. The objective is to investigate how demand aggregation and ordering coordination will influence store inventory decision, store performance in terms of weeks and units stocked-out and associated inventory cost.

2.3 Model Three - RDC with Re-allocation model:

Model Three builds up on model two by adding additional logic in the disbursement of the received purchase order to the store. The only difference between model two and model three is that model three uses additional information (current demand signals from stores) and re-allocation logic to break up the received purchase orders in a manner that helps improve the service level of the stores with equal or less inventory as compared to model two. This can be seen in Figure 3.

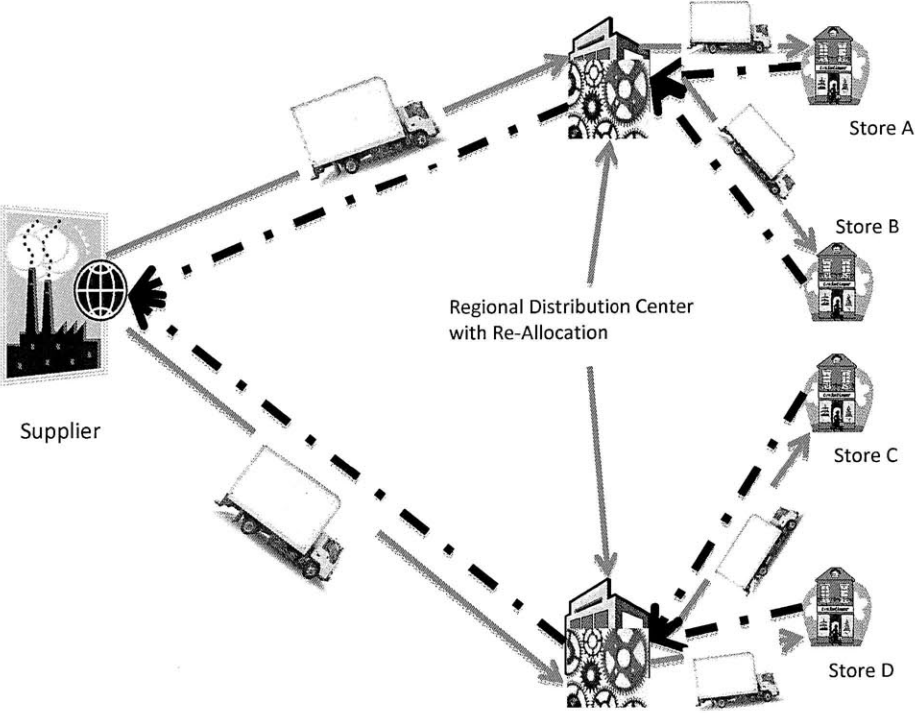


Figure 3 – RDC with re-allocation

Model three also involves taking all the variables that were evaluated and analyzed in model one and two. Model three will have an additional step of re-allocation into the RDC simulation model

situation where DC will allocate received orders based on latest demand updates from stores. This re-allocation logic will take into account the fact that demand may have changed from the time when the order was initially placed, thus, adding a layer of variability of how many units a store is actually getting from RDC because under reallocation model, priority has shifted from individual store optimization to entire downstream network optimization. Individual stores may be forced to take more units (in buy-pack multiples) than what they actually need; they are also subject to situations when they couldn't get fully replenished due to sudden demand upsurge from other stores. This thesis will look at reallocation model as a risk sharing system where supply is fluid and is being shaped to meet demand. Each individual in the system shares opportunities and risks to optimize the allocation for the entire RDC network.

2.4 Model Summary – DTS, RDC and RDC with Re-allocation:

Figure 4 represents a summary picture of the three models depicted earlier in Figure 1, Figure 2, and Figure 3. The dotted lines represent the information sent by stores to suppliers either directly or via RDC. In Figure 4, it can be seen that DTS has a lead-time of two weeks. In case of RDC (without reallocation), the lead time is longer by one week, to account for the time spent by the received purchase order in the RDC while it is being broken up into smaller buy-pack multiples needed by the individual stores. Note: In reality the product disbursement takes less than one week. This depiction is only for the purposes of visualization of all three models simultaneously. Finally, in case of RDC with Re-allocation, the time to deliver the goods is slightly longer than that seen in case of RDC without re-allocation. The longer lead-time is due to the fact that the RDC receives the latest demand signals from the stores to evaluate how much quantity is really

required by the stores. This additional information is depicted in Figure 4 by the dotted line from Week 3 to the RDC line. The RDC makes a decision based on its judgment of the demand requirement in the entire network of stores and then allocates the inventory such that it can divert the flow where the need is the highest.

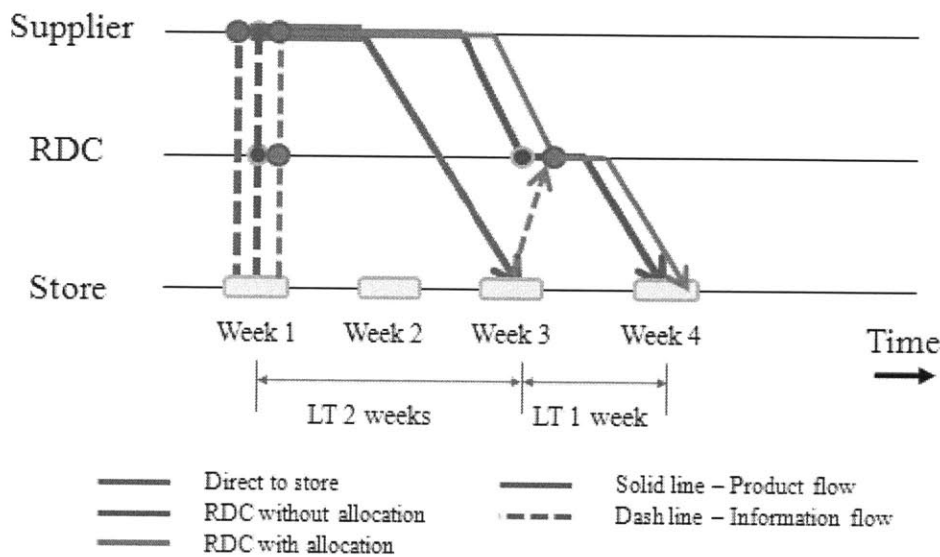


Figure 4 - Summary flow of DTS, RDC and RDC with re-allocation models

Thus, it can be seen that DTS model has the shortest lead-time, RDC without reallocation model and RDC with re-allocation model have relatively longer lead-times. DTS model is constrained by the MOQ limits which makes the actual review period longer than pre-decided review time whenever the stores needs do not meet or exceed the MOQ requirement by the supplier. For example, in a (3,7) (Lead time, Review time) scenario, store reviews inventory level once every 7 weeks, total 7 reviews in a year assuming 52 weeks in a year. Assuming store needs to order for all 7 review occasions, however in two such review periods the stores required less than the

MOQ level and hence had to “kill” the purchase order. Therefore, the store ends up ordering only five times in a year. This translates into a ten week actual review time, which is three weeks longer than what has been defined in the store’s inventory management system. This MOQ constraint decreases the frequency of order placement resulting in more units per order but fewer orders per year, consequently, raising the variability as seen by the stores in their pipeline inventory positions and as seen by the suppliers in the demand via less frequent but bigger purchase orders.

This MOQ constraint is no longer applicable in the RDC and RDC with re-allocation models. Thus, the review period in the RDC and RDC with re-allocation model is much shorter as compared to that of the DTS model. Shorter review periods lead to more frequent ordering with relatively less units per order being successfully placed with the supplier in both the RDC models. The subsequent sections of this thesis will investigate how the lead-time and review time changes, impact various inventory buckets like in-transit units, in-store units, on-order units and safety stock in each of the three models.

	Lead-Time	Review time	MOQ	Allocation Scheme
DTS	X weeks	Every Y weeks (but longer in practice due to MOQ)	Applicable	None.
RDC without Re-allocation	Longer than X weeks = X_1 weeks	Shorter than Y weeks and not impacted by any ordering constraint	Not Applicable	Simple. Get what you ordered.
RDC with Re-allocation	X_1 weeks + few hours (due to re-allocation)	Shorter than Y weeks and not impacted by any ordering constraint	Not Applicable	Sophisticated. Deals with Overage receipts and Underage receipts differently

Table 1 Summary of DTS, RDC and RDC w/ Re-allocation

3 Data and Model Mechanics:

This section describes the inventory management policies that will be simulated in the thesis and the mechanics under which the input data will be passed through each of the three models discussed in earlier sections.

3.1 Model Mechanics - Periodic Review

BigRetailer uses a periodic review model, (R,S) system to manage its inventories. Periodic review model relies on the inventory position for the SKU in question. The inventory position is defined as the combination of in-store units + in-transit units + on-order units. The procedure is that every R units of time BigRetailer orders to raise the inventory position to the level S. The (R,S) system offers a regular opportunity to adjust the order up to level (OUTL) if the demand pattern is changing with time.

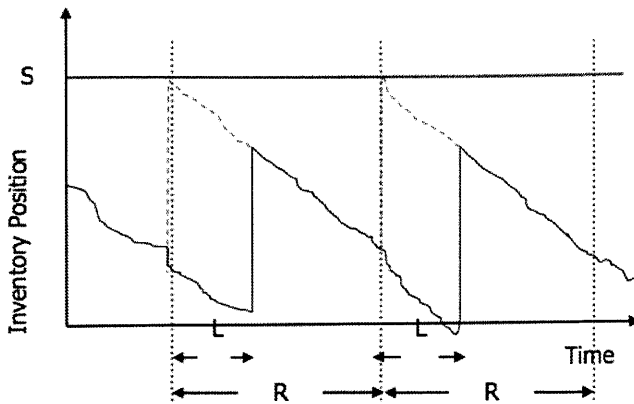


Figure 5 - Periodic Review (R,S) model1 (© Chris Caplice, MIT)

Equation 1 – order upto level

$$S \text{ (order upto level)} = X_{l+r} + \text{Safety Stock}$$

Where, X_{l+r} is demand over lead-time and review time.

Under the periodic review policy, each SKU has a Order-up-to-level (OUTL) that is defined in **Error! Reference source not found.** and depicted by 'S' in Figure 5. Each store that manages the SKU has a pre-defined length of time, at the end of which it performs a review of the inventory position of the SKU. This pre-defined time is known as the review period and is depicted by R in Figure 5. The OUTL is depicted by Equation 1.

¹ MIT Course lecture notes, ESD.260, Lecture 10, Fall 2010

3.2 MOQ ordering constraint

In the DTS scenario, stores are subject to a Minimum Ordering Quantity (MOQ) constraint specified by suppliers. MOQ is defined at either individual SKU level or for a combination of multiple SKUs by the supplier. MOQ can be in Dollars or in units. The MOQ in model one is a Dollar amount. This constraint is imposed by the supplier on BigRetailer's stores. This is done to ensure that a certain minimum volume is included in the order by BigRetailer's stores. This enables to supplier to keeps its production levels profitable. The model follows the ordering rules as mentioned below:

- a. "Fill" or place order greater than MOQ if current demand \geq MOQ
- b. "Kill" or cancel order if current demand $<$ MOQ

3.3 Key parameters

This thesis will focus on several key performance indicators in the model analysis. These are as listed below:

1. Safety stock (SS),
2. Order up to level (OUTL)
3. Inventory holding cost,
4. Cycle service level (CSL),
5. Weeks stocked-out,
6. Demand fill rate and
7. Total ordering quantity.

Safety stock is a function of demand variability (σ) and desired cycle service level (CSL),

Equation 2² – Safety Stock

$$SS = \sigma_{L+R} * k, \text{ (Silver, Pyke \& Peterson 1998)}$$

Where, k value refers to safety factor equivalent to normal probability distribution based on the given CSL value ($1 - p_{u \geq (k)}$, p_u is the probability of stock out occurring),

L = Lead-time,

R = Review time, and

σ_{L+R} is the standard deviation of demand over period equal to lead-time + review time.

Equation 3 – Std Dev over lead time and review time

$$\sigma_{L+R}^3 = RMSE / \sqrt{\left(\text{total selling period} / (\Sigma(\text{lead-time, review time})) \right)} \text{ (Chris Caplice 2010)}$$

Where RMSE = Root mean square error

Equation 4 – Demand fill rate

$$\text{Demand fill rate} = 1 - (\text{unsatisfied demand in units} / \text{total demand in units})$$

Equation 5 – Weeks stocked out

$$\text{Weeks Stocked-out} = \text{Total \# of weeks stocked out} / \text{Total selling period}$$

Equation 6 – Inventory holding cost

$$\text{Inventory holding cost} = \frac{Q}{2} * v * r$$

Where, v = cost of product, r = inventory holding cost

² Silver, E. A., Pike, D. F., & Peterson, R., (1998). Inventory Management and Production Planning and Scheduling John Wiley & Sons. p 279.

³ MIT Course lecture notes, ESD.260, Lecture 10, Fall 2010

3.4 Key performance metrics

The key parameters in the last section are used as input for the inventory model for simulation purpose. Results of the simulation are captured in Table 2.

From the entire list of performance metrics, the authors focus mainly on four indicators. They are

- a) Percentage weeks stocked out,
- b) First pass fill rate service level (IFR),
- c) Total weekly inventory in Dollars and
- d) Inventory position in Dollars.

BigRetailer's supply chain is biased towards service level with high importance and priority given to having the right products on shelf for customers at any point of time. Therefore, IFR is one of the key service level metrics that the authors put into comparison among the three product flows – DTS, RDC and RDC with reallocation. Similarly, identifying the percentage of weeks stocked out is another key metric that BigRetailer tracks to evaluate the performance of store inventory management.

Inventory position, as a function of service level, lead time and review time, represents how many products are in the pipeline in total including both in store and in transit stock. It provides an overall picture of the total inventory level for BigRetailer's channel network and helps RDC to make ordering decisions. As BigRetailer doesn't take ownership of products until they reach its facilities, in-store inventory becomes more relevant in evaluating the impact of different product flows to the stores as in-store inventory incurs real cost to stores including cost of good

and inventory holding cost. Hence, in-store inventory is one of the key metrics that is used as performance indicator for evaluating the three product flows for BigRetailer.

Total Annual SALES per store in units	Cumulative annual sales
%Week STOCKOUT per store	Total weeks stocked out as a percentage of Total selling period in weeks
Annual Stockout Weeks Per Store	Total weeks stocked out annually
Annual Units Stockout Per store (5skus)	Total units stocked out per store (for five SKUs)
First Pass Fill rate service per store	Total Units serviced as a percentage of total units in demand
Total Annual INVEOH per store	Total units physically in store
Total Annual INVEOH per store \$\$\$	Total Dollar value of units physically in store
Total Annual INV Ordered Units	Total units ordered per store
Total Annual INV ordered \$\$\$	Total Dollar value of units ordered
Total Annual INV position Units	Total Inventory position in units
Total Annual Inv Position in \$\$\$	Total Dollar value of Inventory position
Total Annual Inv Intransit Units	Total units in transit
Total Annual INV Arrived	Total units received at RDC
Total Annual INV Intransit+Arrived Units	Total units in transit and received at RDC
Total Annual Total Inv intransit + arrived \$\$\$	Dollar value of total units in transit and received at RDC
Total Annual INV Allocated Units	Total units allocated to stores after receiving at RDC

Table 2 – Performance Metrics

3.5 Sensitivity analysis for single SKU single store

Increasing lead-time (L) can cause stores to carry more stock or drop CSL and similarly, changes in review time (R) can impact the inventory carried by the stores Review time can be shortened by increasing inventory review frequency to counter the impact from a longer lead-time on safety stock and OUTL. As shown in the safety stock chart in Figure 6, the value of safety stock increases as lead-time increases. Consider the following scenarios for RMSE of 1200 units per year over a year and CSL of 99%:

Scenario1: For lead-time of 7 weeks and review time of 7 weeks, the safety stock = 1448 units.

Scenario2: For lead-time of 10 weeks and review time of 7 weeks, the safety stock = 1596 units.

Scenario3: For lead-time of 10 weeks and review time of 4 weeks, the safety stock = 1448 units.

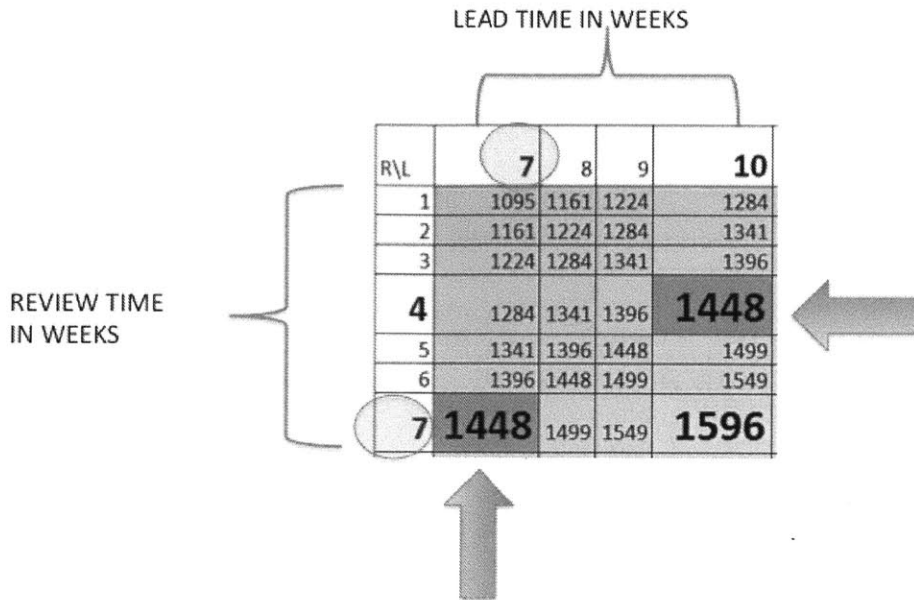


Figure 6 - Impact of lead-time and review time on Safety stock

As can be noted above, it is the combination of $L+R$ that governs the movement of safety stock, all else being fixed. This is evident from Equation 2 and Equation 3 above. Thus in Scenario1 the value of $L+R = 14$ which when increased to $L+R=17$ causing safety stock to increase from 1448 units to 1596 units. In Scenario3 the value is moved back to $L+R=14$ causing safety stock to come back to 1448 units. At this point it is important to note that even though $L+R = 14$ days in both scenario1 and scenario3, lead-time is 3 days longer (longer wait for the store to receive purchase order) and review-time is 3 days shorter (more frequent ordering potentially) in scenario3. This will impact the ordering patterns, in-store units, in-transit units, and the store stock out rate. NOTE: The MOQ discussion earlier brought out the fact that even though review period might be set to 'x' weeks, but if the store manager cannot place an order during any given review cycle due to MOQ barrier, the implied review period automatically becomes longer than

'x' weeks. However, Safety stock continues to be calculated using $R = 'x'$ weeks as can be seen in Equation 2 & Equation 3. Since the Safety Stock is set up-front at the start of the selling season, it is necessary for the store managers to identify a relevant fixed review period.

Following is a 52 week simulation of sales using a Periodic (R,S) model as described earlier in section "3.1 Model Mechanics - Periodic Review ". The sales data is rolled up to a weekly level consistent with the inputs provided by BigRetailer. Below is a simulation of ordering patterns depicted by in-transit units (every order placed in transit for the equivalent of "lead-time" weeks).

The inputs to the model are given below:

1. Normally distributed annual demand
2. Total Annual demand = ~13500 units
3. Average Annual demand = ~260 units/week
4. Low demand variability (low COV) per week of sales
5. Lead-time and review time as described below

This paper evaluates fast moving products with primarily high co-efficient of variance as provided by BigRetailer. Co-efficient of variance is calculated as the ratio of average demand and standard deviation of the demand. Walkenhorst 2007 investigated the appropriateness of using normally distributed data when lead-time and demand are stochastic. He states "Several published articles discussed using a gamma distribution rather than a normal distribution to model demand when the lead time and demand per unit time are both stochastic. But even as

Mark Keaton⁴ suggests this, he acknowledges that a normal distribution ,may be a reasonable approximation for fast moving ‘Type A’ items, but it is not suitable for slower moving items.”

Walkenhorst further investigates the mechanism to quantify the effects of reduced lead-time and increased delivery frequency for a large supplier company. Referring to the input demand data, he notes that “Since this study only focused on fast moving items, the normal distribution was deemed to be appropriate.” and from his discussion with industry practitioners “As further practical evidence of this, a Program Manager from HP named Barrett Crane commented during a presentation on supply chain improvements at HP over the past six years that they have always used the normal approximation and found it to work very well.”

Thus, it can be seen normally distributed data appropriately represents the sales data required for simulation in the periodic inventory review model as discussed earlier. For all subsequent analysis and discussion in this paper will be based on the assumption that the sales data is normally distributed.

As described earlier in Scenario1 above, the model is given an input of 52 weeks of normally distributed data and Lead-time = 7 weeks and review time = 7 weeks. Figure 7 – Direct to Store model below depicts this situation.

⁴ Keaton, Mark, Using the gamma distribution to model demand when lead time is stochastic, *Journal of Business Logistics*; 1995; 16, 1; ABI/INFORM Global, pg. 107

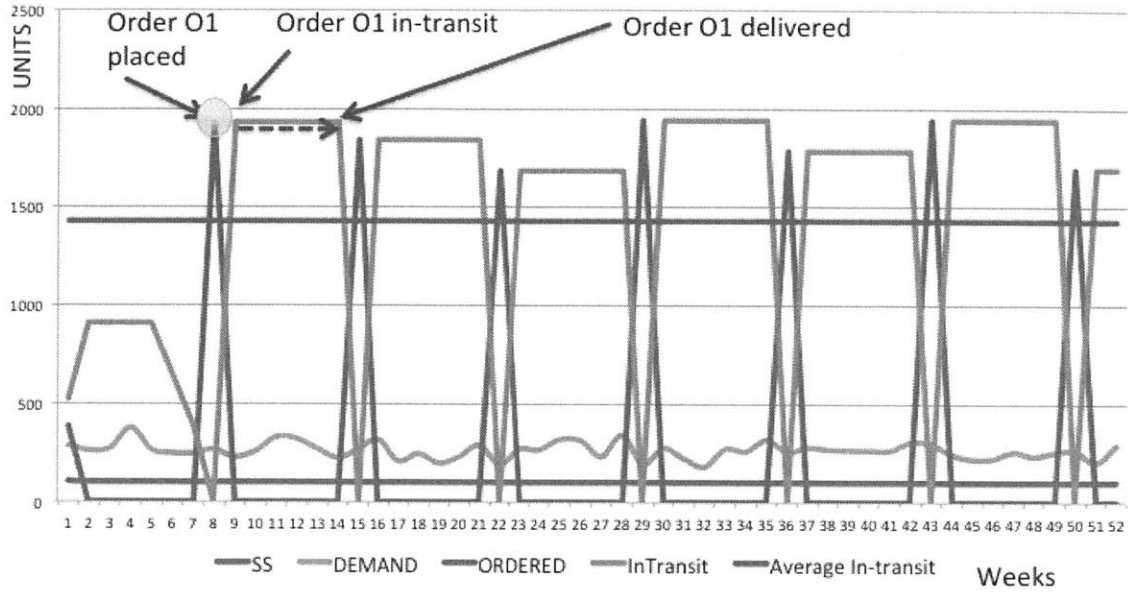


Figure 7 – Direct to Store model with Lead-time = 7 weeks and Review time = 7 weeks

It can be seen that an order is placed every seven weeks and each order travels for seven subsequent weeks. A review happens on the seventh week and an order O_1 is placed on the seventh week. Order O_1 travels for subsequent seven weeks (as depicted by the dotted double arrow line) till the fourteenth week. O_1 is delivered on the fourteenth week and another review happens and another order O_2 is placed. Average order size is approx. 1800 units in Figure 7.

T = Total selling period

R = Review time (time period after which a review is performed)

L = Lead-time, given the values of T , R and L , the number of reviews can be calculated as

N_r = Number of reviews in T ,

N_{dlvrs} = Number of deliveries in T ,

Equation 7 - Number of reviews

$$N_r = T/R$$

and

Equation 8 – Number of deliveries

$$N_{dlvrs} = (T-L)/R$$

Thus in a year, there will be approx. $N_r =$ seven reviews (total period=52 divided by review period=7) and $N_{dlvrs} =$ seven deliveries ((total period=52 minus lead time=7) divided by review time=7).

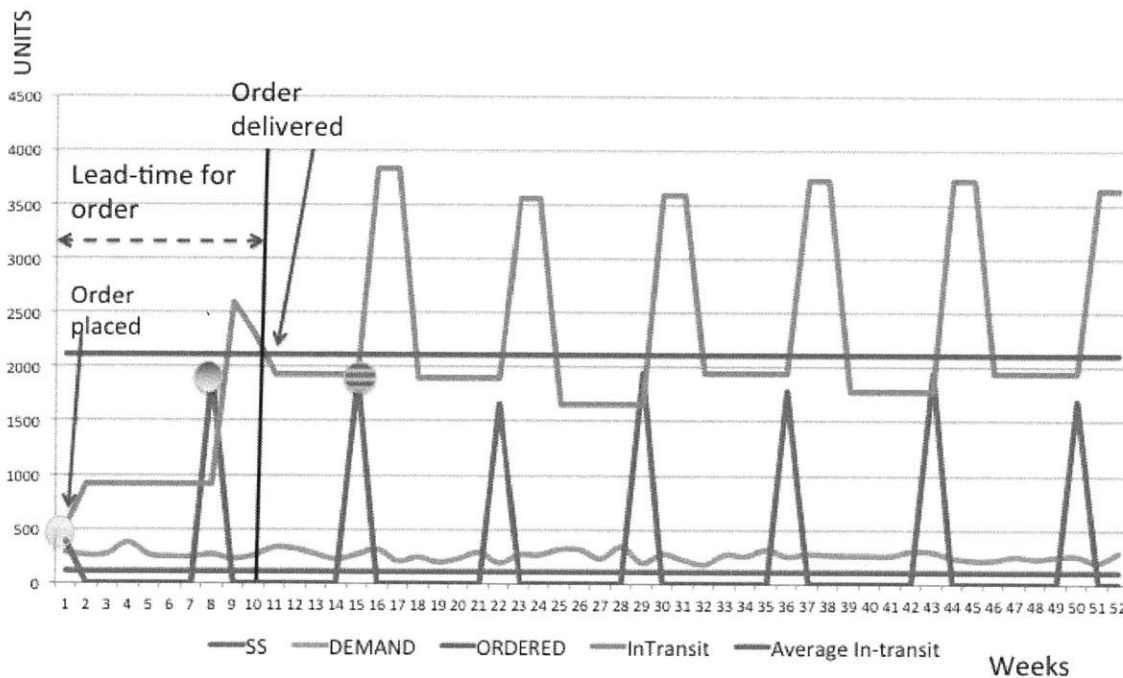


Figure 8 – Direct to store model with lead time = 10 weeks and review time = 7 weeks

Similar to the order and lead-time pattern in Figure 7, it can be seen in Figure 8 that an order is placed every seven weeks and each order travels for ten subsequent weeks. Thus each order overlaps the placement of a subsequent order (if there are no constraints prohibiting the placement of a subsequent order). A review happens on the seventh week. Thus an order O3 is placed on the first week and another order O4 is placed on the seventh week. O3 travels for subsequent ten

weeks (as depicted by the double arrow line) till the tenth week. O_3 is delivered on the tenth week. Meanwhile, as O_3 was travelling, O_4 was placed on the seventh week. This is seen in the increase in the in-transit value from approx. 950 units to approximately 2650 units for 3-4 weeks after O_4 is placed. When O_3 is delivered on the tenth week, the in-transit value falls down to approx. 1800 units and O_4 travels for the next ten weeks and is delivered on the seventeenth week. Another review happens on the fourteenth week, which travels till the twenty fourth week. This pattern continues cyclically and it can be noted above that the in-transit units consistently falls down to approx. 2000 units every time an order is delivered (every ten weeks). Average order size is approx. 1800 units in Figure 8 . Thus, in a full year approx. $N_r =$ seven reviews happen (total period=52/review period=7) and $N_{dlvrs} =$ six deliveries ((total period=52 minus lead time=10)/review time=7) are made. In-transit value jumps up to approx. 3600 units every time an order is placed (every seven weeks).

It can be seen that compared to Figure 7 where the average in-transit units is approx. 1400 units , the average in-transit units in Figure 8 is approx. 2100 units. The only difference between the two figures is that the lead-time is increased from 7 weeks to 10 weeks. Review time is seven weeks in both the figures. Approx. 42% increase in lead-time leads to approx. 50% increase in the average in-transit units.

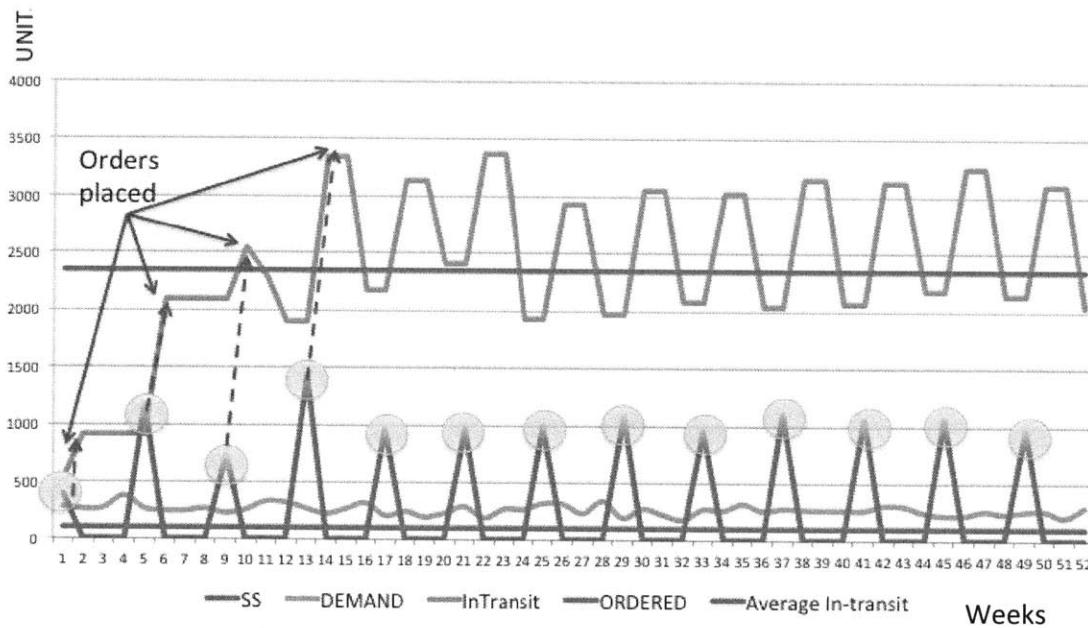


Figure 9 – Direct to Store model with Lead-time = 10 weeks and Review time = 4 weeks

As shown above in Figure 9, the order and in-transit patterns though similar to those in Figure 8 have different levels of units carried in each bucket. Here the lead-time is ten weeks and the review time is four weeks. Hence, more frequent review of each order happens as compared to the review period of seven weeks in Figure 7 & Figure 8. An order is placed every review period (four weeks). Each order travels for a lead-time period (ten weeks) and is delivered at the end of this period. In Figure 9, an order O_5 is placed on week four. This order O_5 travels for next ten weeks and is delivered on the fourteenth week. This is seen above as the period marked “Lead-

time marked as O_5 . The in-transit value rises every time an order is placed and falls every time an order is delivered. Also, while order O_5 is in-transit, on week eight (four weeks after order O_5 is placed), order O_6 is placed. This leads to a bump in the in-transit value. Compared to Figure 8, the lead-time in Figure 9 is the same (ten weeks) but the review time is much less (four weeks). This represents a 42% decrease in the review time and hence results in more frequent reviews (total period=52/review period=4) or approx. 13 weeks and eleven deliveries ((total period=52 minus lead time=10)/review period=4). The average in-transit units are slightly elevated at approx. 2300 units compared to 2100 units in Figure 8, an increase of approx. 10%. The big-ticket item here is the average size of the order placed is reduced to approximately half in Figure 9. The average order size in Figure 9 is approx. 1000 units, compared to 2100 units in Figure 8 a decrease of approx. 52%. Refer to Table 3 for summary information this discussion. Compared to Figure 7 where the lead-time, review time combination was 7 weeks and 7 weeks respectively, in Figure 9 the lead-time, review time combination is ten weeks and four weeks respectively. This represents a 42% increase in lead-time and a 42 % decrease in the review time. The average in-transit unit in Figure 7 is 1400 units while that in Figure 9 is 2400 units. The average order size in Figure 7 is 2100 units while that in Figure 9 is 1000 units. This represents a 71% increase in the average in-transit units and an approx. 52% decrease in the average order size.

Lead time + Review Time	LEAD TIME	REVIEW TIME	AVG. IN-TRANSIT UNITS	AVG. ORDERED UNITS
14 WEEKS	7 WEEKS	7 WEEKS	1400 units	2000 units
17 WEEKS	10 WEEKS (INC)	7 WEEKS (No Change)	2150 units (INC)	1900 units (slight DEC)
14 WEEKS	10 WEEKS (No Change)	4 WEEKS (DEC)	2350 units (INC)	1000 units (DEC)

Table 3 – Summary of Lead-time, review time impact on pipeline inv

Note: The above discussion does not take into account the impact of MOQ (in case of DTS) or Re-allocation (in case of RDC with re-allocation). It is assumed that every time the store manager is successfully able to place a purchase order as per the store's needs. In such a scenario, lead-time= 7 weeks and review time = 7 weeks represents DTS model and Lead-time=10 weeks and review time = 4 weeks represents RDC models. Additionally, when the lead-time is constant at 10 weeks, the average in-transit units also change from 2150 units to 2350 units. The reason In-transit change in this case is because the model assumed lost sales. The demand flowing through the system is lower based on rounding off of the years and assuming that the demand in that period is not backordered. Otherwise, the in-transit units during constant lead-time period would be identical.

In summary, as BigRetailer introduces the RDC model, increase in the lead time for products to be delivered to store facilities is inevitable; however the new RDC model brings the benefit of a higher ordering frequency due to order coordination. Is the new RDC model helping BigRetailer to achieve a lower safety stock level and to reduce inventory holding cost? What is the potential impact to individual store under this new RDC model? These are the questions to be analyzed

and solved. This paper will start with single SKU single store analysis to set the stage for more complex multiple SKU and store scenario analysis.

The initial step is to obtain weekly sales demand of a single SKU under one vendor to test the sensitivity of safety stock to changes in lead-time and review time. As lead-time and review time change, order size and frequency and in-transit units variability changes. Next step is to change lead-time and review time respectively to track changes in safety stock, in-store stock, in-transit stock, and inventory position assuming no changes in expected store CSL. This helps to determine where the breaking point is for BigRetailer is to achieve economic balance among lead-time, CSL and safety stock.

Usually ordering cost is also a key component in the total inventory management costing structure. However in the case of BigRetailer, the company considers ordering cost as insignificant as compared to the large volume and scale of products that it purchase from the suppliers. Therefore ordering cost is not included in calculation and analysis.

3.6 Multiple SKU and multiple stores

To make this study sample more representative, the authors analyzed and identified five SKU and six stores under one RDC by extending the analysis coverage and comparison with what was obtained from a “single SKU-single store” analysis. As discussed earlier, under DTS model each of the six stores review inventory positions on different fixed period of time and place orders with suppliers directly and individually. Single orders are subject to buy pack quantity and MOQ

ordering constraint. Under centralized RDC model, MOQ is no longer a relevant consideration as BigRetailer management assumes aggregated demand of 'n' stores (n being a high double digit number) will always exceed the MOQ which is the same value for both individual store and RDC. The model accumulates each individual SKU's weekly demand and aggregate them at RDC level. Using aggregated demand, as input to the simulation model will help determine the ordering quantity that RDC should place with suppliers. In the meanwhile, the model will run simulation for all 5 stores individually and capture their ordering mechanics as well. Finally, the results of DTS and RDC model are brought together and compared for benefits and risks.

3.7 Initialization and formula in the model

In order to identify appropriate amount of units in stores physically and in transit, to meet subsequent demand from the beginning of the selling period certain inventory initialization decisions had to be made. It was decided that the demand in initial weeks will be successfully met with adequate quantity of units in store and simultaneously order some units in advance such that the stores can expect to receive more units as the lead-time progresses with each selling week. In the model, the store takes over the ordering policy from week one of the selling period. The in-store units for the beginning-on-hand (BOH) inventory is the average demand for over 'x' weeks ($x = \text{lead-time} + \text{review time} - 2$) and inventory in transit for the year beginning is two weeks of average weekly demand. The Order up to level⁵ is as defined in Equation 1. The Standard deviation of annual sales is as defined in Equation 3.

⁵ Silver, E. A., Pike, D. F., & Peterson, R., (1998). Inventory Management and Production Planning and Scheduling. John Wiley & Sons. p 276.

The Inventory holding cost = $n\%$ of cost of good (where n is provided by BigRetailer). The model ensures that no back orders are placed and any unsatisfied demand is regarded as lost sales. As supplier MOQ is in dollars, all the product units are converted into dollar value based on the cost to BigRetailer to acquire the products (acquisition cost). Truckload capacity is not a major concern here as long as supplier MOQ is met. BigRetailer and suppliers will work to optimize transportation flow based on product nature, quantity and expected delivery time.

Several buckets were created to capture the different stages of product flow. Inventory end on hand (EOH), inventory beginning on hand (BOH), in-transit inventory, delivered inventory and Inventory position.

Below are some of the guidelines:

Orders are placed only at the end of a week; same logic is applied to product delivery, which happens at the end of a week. For example, products get delivered on week 2 only become “real” inventory on week 3 and are counted as EOH for week 2 and BOH for week 3. Any given week that has a negative inventory end on hand or negative ordering quantity will be normalized to zero units. However, the model captures the weeks with negative inventory as weeks with stock out and the negative inventory as the # of units not sold and it will directly impact actual CSL and demand fill rate. Though BigRetailer doesn’t specify what are the costs associated with lost sales, lost sales will be treated as opportunity cost and built into the total operation cost.

4 Setting up Inventory Simulation Model

The following guidelines and assumptions are followed in the simulation model; these are incorporated in the subsequent mathematical decision rules:

1. Inventory category: It is assumed that products are delivered at the end of the week, therefore, the products are only counted as actual inventory for the following week. Hence, the “beginning-of-week” inventory equals inventory carried over from last week plus new inventory received at the end of the last week. Every order placed by the store will go through stages including ordered, in transit, arrived before it’s finally captured under the EOH bucket.
2. Known demand pattern: For the purpose of this thesis, the demand forecast details, for a store-SKU combination were unavailable. Actual sales numbers were used in the simulation model to determine the required qty in the store such that the Inventory position of the store is equal to the OUTL level. The main purpose of the study is to understand tradeoff between lead-time, review time and inventory level. Therefore, the nature of data forecast and associated variances won’t be a significant factor as long as the same data is used in analyzing other scenarios. The forecast variance has been parameterized in the model.
3. Ordering policy: As BigRetailer doesn’t change suppliers on a frequent basis and has products shipped in the same transportation mode, periodic review is particularly appealing for replenishment coordination as all items in a coordinated group can be given the same review interval. Disadvantages of periodic review could be a high safety stock

and a high order-up-to-level, but BigRetailer believes that lower ordering and handling cost for more than ten thousands of products in BigRetailer product portfolio justify it.

4. Ordering quantity: Typically, the inventory position for all the SKUs in a store will be reviewed once in every fixed length review period. Inventory position is the sum of actual inventory on hand and orders in pipeline including products in transit and arrived in the same week. Hence, the authors have defined “order-up-to-level” (OUTL) as the equivalent of demand forecast of lead-time and review time plus safety stock for the same period of time. This is a straightforward method for store managers to calculate the current inventory level positioned in front of demand forecast by calculating the gap between inventory gap and order up to level and that is the inventory quantity need to be replenished. However, this is not a constraint free situation as suppliers impose MOQ and buy-pack quantity requests, store managers need to go one step further to measure the quantity they are supposed to order against MOQ and decide whether the purchase order meets with the requirement. As the model follows the “kill-order” policy if the required-quantity is below the ceiling set by the supplier’s MOQ policy, store manager will not place orders if the overall quantity doesn’t meet up with MOQ and wait till next review period to review demand again. If required-quantity exceeds MOQ, store manager needs to round it up or down to the nearest buy pack units for ordering placing purpose. Here the model will follow the rule of half adjusting, decimal less than half will be round down, more than half will be rounded up. Again, the objective here is not to find out what is the optimal ordering quantity under MOQ and buy-pack constraint situation, but rather to understand how periodical review model responds under different product flow scenarios.

5. Product category: As BigRetailer has more than 'n' stores (n is a large two digit number) dealing with thousands of products under one regional DC, the authors will pick five stores and six SKUs as study sample in the simulation model. To ensure this sample data is representative of other stores and products under one DC, the authors pick five SKUs from high velocity/sales stores with normally distributed sales record.

To aid the analysis, BIGRETAILER's weekly sales figures have been employed to simulate the three different scenarios (DTS model, RDC model, RDC model with reallocation) and thus, compare how lead time, review time and reallocation is changing the overall ordering quantity, safety stock and service levels under one RDC.

In the new RDC centric supply chain model, BigRetailer has one RDC covering around 'n' (n is in the range of a high two digit number to a low three digit number) store fronts in the region and act as a consolidation hub for the stores to place order and ship products. Immediate benefit to stores is that MOQ constraint is no longer in the picture as they don't place orders with suppliers directly but through RDC. RDC will consolidate demand from all store fronts in the region and place a single order with the vendor. For stores that have relatively low velocity, it will also benefit from higher ordering frequency. But for stores that have higher velocity and faster inventory turnover, they may find certain SKUs are ordered less frequently in the new model. In the DTS model, they may replenish the inventory every one-week but under RDC model they order once every two weeks. Because RDC model is make every store review in the same inventory review period, some high velocity stores lose the flexibility to place orders as they

wish. This is one of the constraints that can be observed under the new RDC model. However, as BigRetailer targets to maintain the same CSL in the RDC model as it has in the DTS model, with a longer lead time BigRetailer has to shorten the overall review days to maintain a high CSL expectation. In other word, ordering frequency must be enhanced at overall level.

The authors use the simulation model to gain more insights to the RDC model from below two perspectives:

1. Assuming the RDC model has the same CSL as in the DTS model, what ordering frequency does the RDC have in order to justify increased lead time due to extra cross-dock time at RDC facility.
2. What is the optimal safety stock level that BigRetailer wants to carry in order to maintain the expected CSL.

The thesis uses the same logic in simulation model for DTS model as stores follow the same periodical review ordering policy. But in the RDC model, RDC places one large “umbrella” order based on aggregated demand signal from stores instead of stores placing orders with suppliers individually, therefore demand patterns in RDC looks significantly different from DTS model. As forecasting error and safety stock are directly correlated to annual demand and demand variability even these values are impacted in the conversion from DTS to RDC.

The RDC model follows the same guidelines and assumptions as DTS model. The same SKU and Stores dataset are re-used so that the analysis result can be compared with DTS model

directly. It's straightforward to quantify the dynamics that RDC, an integrated ordering system, bring into the system. The model will track key parameters for each individual SKU under each store to evaluate the impact of RDC.

For the third model, this thesis discusses the concept of reallocation into the RDC model. The ordering pattern in this model is basically the same with RDC model where RDC aggregates demand signals from stores and send to suppliers as ordering information. As RDC receives deliveries from suppliers, it doesn't simply break down the packs and give out the exact same quantity it ordered on behalf of the stores. Instead, RDC will apply judgment in the reallocation process to re-evaluate and rationalize the allocation quantity to stores. RDC will refer to several factors for decision making: new demand, sales forecast and past week sales performance. The objective of reallocation is to align supply with demand, reduce redundancy and enhance system reactivity through information sharing with downstream stores and share risks across the network.

In the simulation model, the authors assume the cost of products are still the same so that any improvements in the system are purely driven by order coordination and demand reallocation. In the reallocation model, the authors assume all the parameters are exactly the same with RDC model without reallocation in terms of lead-time, demand, ordering quantity until products are in the RDC facility. Due to a relatively larger lead time between ordering time and actual delivery time, stores' demand pattern are very likely to be different from the time when RDC placed the order. New demand can be higher or lower than what have been ordered before. Is the reallocation method is being fair to all stores in the network?

The authors will look into situations where new demand is either lower or higher than ordered quantity respectively to work out the allocation plan for stores.

4.1 Model 3 – cumulative current need is lower than cumulative received quantity

In the case where current demand from stores is lower than what RDC has ordered with supplier “lead-time equivalent” weeks ago, then, RDC observes a surplus in inventory on hand. Since RDC doesn’t carry any inventory for stores, it will allocate the excess inventories into stores with every store sharing part of the over ordered amount. The authors worked with BigRetailer to identify the in-practice guidelines for this situation.

To deal with this situation the RDC observes the following steps:

Step1: From the entire received units, allocate to each store what they currently need

Step2: Allocate the remaining amount to the stores, in proportion of the stores’ expected average weekly sales.

In the reallocation model, some stores may be “penalized” to take more than what they asked for even though they are being accurate with their demand forecasting. But they will also benefit from a larger replenishment volume if they are in short of supply and there is downside to other stores. Under the ideal normally distributed sales data scenario, the authors believe chances for a

store to “cover” for others and “getting covered” by others are equi-likely in a multiyear sales scenario. This is considered a fair risk sharing reallocation model.

4.1.1 Allocation logic for overage in received units

$D_{i,k}$ - demand for week i of store k ,

$F_{i,k}$ - Sales forecast as of week i of store k ,

$R_{i,k}$ - Total received allocation for week i of store k ,

L - Lead time

Week	Store 1		Store 2		Store 3		...	Store k	
1	$D_{1,1}$	$F_{1,1}$	$D_{1,2}$	$F_{1,2}$	$D_{1,3}$	$F_{1,3}$...	$D_{1,k}$	$F_{1,k}$
2	$D_{2,1}$	$F_{2,1}$	$D_{2,2}$	$F_{2,2}$	$D_{2,3}$	$F_{2,3}$...	$D_{2,k}$	$F_{2,k}$
3	$D_{3,1}$	$F_{3,1}$	$D_{3,2}$	$F_{3,2}$	$D_{3,3}$	$F_{3,3}$...	$D_{3,k}$	$F_{3,k}$
...
i	$D_{i,1}$	$F_{i,1}$	$D_{i,2}$	$F_{i,2}$	$D_{i,3}$	$F_{i,3}$...	$D_{i,k}$	$F_{i,k}$

Equation 9 – Reallocation scheme (overage)

$$R_{i,k} = D_{i,k} + (\sum D_{i-L,k} - \sum D_{i,k}) * \frac{F_{i,k}}{\sum F_{i,k}}$$

Equation 9 – Reallocation scheme (overage), each store first receives the equivalent of its current demand. This is depicted by $D_{i,k}$. At this point, the RDC has access to expected sales forecast for each store-SKU in its network depicted by $F_{i,k}$.

After allocating to each store their respective demanded quantity, the remaining quantity of units is depicted by $\sum D_{i-L,k} - \sum D_{i,k}$. This delta quantity is allocated to the stores in proportion of their

expected sales by $\frac{F_{i,k}}{\sum F_{i,k}}$. Thus the allocation scheme ensures that each store receive with they currently need and additionally any excess amount is allocated in the proportion of what the stores' are expected to sell.

4.2 Model 3 – cumulative current need is higher than cumulative received quantity

In the case where current demand from stores is higher than what RDC has ordered with suppliers, then, RDC observes a shortage in inventory on hand. This means not all the stores could be fully replenished due to demand upside in the network. Under this situation, the model will introduce an allocation scheme that looks at current needs and allocates inventory intelligently such that units flow to where they are needed most.

This allocation model reflects BigRetailer's key focus on order fill rate and CSL. As the allocation model is biased towards stores with low inventory level by granting appropriate allocations to minimize chances of stock out and maintain high CSL. As mentioned earlier that MOQ, one of the ordering constraints, is not a concern in the new RDC model, buy-pack quantity still stays as key requirement. Therefore, when working on allocation plan, RDC will need to either round up or down the allocation units to the nearest buy pack quantity before releasing the numbers to stores. The model currently follows the rule of half adjusting, decimal less than half will be round down, more than half will be rounded up. For stores that end up with

the same decimal quantity in the allocation plan, the model uses sales forecasting to decide which store to round up or down.

4.2.1 Allocation logic for underage in received units

$D_{i,k}$ - demand for week i of store k ,

$F_{i,k}$ - Sales forecast as of week i of store k ,

$R_{i,k}$ - Total received allocation for week i of store k ,

L - Lead timeWeek	Store 1	Store 2	Store 3	...	Store k
1	$D_{1,1}$	$D_{1,2}$	$D_{1,3}$...	$D_{1,k}$
2	$D_{2,1}$	$D_{2,2}$	$D_{2,3}$...	$D_{2,k}$
3	$D_{3,1}$	$D_{3,2}$	$D_{3,3}$...	$D_{3,k}$
...
i	$D_{i,1}$	$D_{i,2}$	$D_{i,3}$...	$D_{i,k}$

Equation 10 - Re-allocation Scheme (Underage)

$$R_{i,k} = P_{i,k} + \left(\frac{D_{i,k}}{\sum D_{i,k}} \right) * (D_{i-L} - \sum P_{i,k})$$

In the underage allocation scheme, the RDC observes the following steps:

Step 1: Identify all the stores with less-than-buy-pack units in their in-store inventory buckets

Step 2: Allocate each store identified in step1 the equivalent of one buy-pack worth of inventory

Step 3: Allocate the remaining units to the stores, in proportion of their current demand

Based on analyses for RDC model with reallocation, it can be observed that the value created by having RDC in between supplier and individual stores doesn't limit only to reducing stock out chances and higher ordering frequency, it also enables the downstream network to optimize inventory position and enhance reactivity to demand variability.

4.3 Data simulation

The model was input with BigRetailer's sales data for all three network flow analysis – DTS, RDC, and RDC with reallocation. Conclusions were drawn from analyzing the ordering patterns and inventory management of the three network flows by comparing the mechanics among each other are as below, which could provide some insight for BigRetailer to what the thresholds are where the benefits of increased ordering frequency and RDC reallocation are outweighed by increased lead time.

To test the resiliency of the simulation model and to solidify the conclusions, the authors used average sales number and standard deviation of annual demand to simulate five years of sales a.k.a. demand data to expand our analysis coverage to a wider timeframe. Apply sensitivity analysis, run data through the three simulation models and evaluate changes and discrepancies.

The model captures key parameters in below chart and highlights the areas where discrepancies are observed from the analysis based on simulation data and the one based on 2009 sales data.

5 Analysis

Each of the vendors-sku-store combinations that were supplied by BigRetailer for the purpose of analysis in this thesis were analyzed for the following characteristic:

1. Sales volume
2. Consistency of the same five or six SKUs across a six stores
3. Common characteristics/attributes in terms of COV/MOQ

Two vendors were selected for deeper analysis using the three models as discussed in the earlier chapters. Finally, one vendor was selected (D28V1) from the eleven vendors under a particular geographic RDC to compare the three product flow situations and evaluate the differences in inventory, fill rate, service level due to change in lead time, review time and ordering patterns in detail.

D28V1 carries a 'n' (where n is a mid to high two digit number) SKU product line and has an annual sales amount at 's' million units (where s is a low single digit number). Average sales per week is 't' thousand units (where t is a low two digit number), average cost of good is 'x'\$ per unit. All three supply chain network flow models that were developed for this thesis can take the above mentioned values of 'n', 's', 't', 'x' as a input variable and process them consistently. The average co-efficient of variance (COV) of the demand and average weekly sales is above one which means demand variance for the product in stores over weeks is fairly high. In order to select a representative sample data to analyze the outcome of the simulation model, the top five

SKUs and six stores out of 'p' stores (where p is a high double digit value) under D28V1 were chosen as representative for data analysis. These five SKUs are being sold across all the six stores and are ranked amongst the top eight SKUs in terms of sales volume. To begin with the analysis, the model takes as input the thirty sets of store and SKU data (six stores * five SKUs) combination into the three supply chain flow model and creates the same metrics in the output as discussed in section "3.3" to facilitate an apples-to-apples comparison. The model is built on the conclusions of research topics on the appropriateness of data distribution (normal or gamma or poisson etc), that the sales data is normally distributed (Walkenhorst 2008). To get the best result out of the simulation model the authors simulate normally distributed data from the average and standard deviation of the actual sales data for the period of 52 weeks. The analysis was conducted for three different scenarios, scenario 1) high co-efficient of variance scenario without MOQ impact, scenario 2) high co-efficient of variance with MOQ impact, and scenario 3) low co-efficient of variance scenario with MOQ impact. The authors will also use different lead-time and review time combination to test the sensitivity of inventory levels and service levels towards the changes in time.

BigRetailer's storefronts review their inventory once every week. However due to MOQ constraint, they actually won't be able to order every time as they wish because the stores won't be able to hit the MOQ every time. So there is a gap between review time and actual ordering time due to delay in ordering.

5.1 Analysis of weekly sales with High COV without MOQ impact

Inputs to the Model:

1. Model 1 - DTS (lt,rt) = (3,7) in weeks,
2. Model 2 – RDC (lt,rt) = (4,3) in weeks,
3. Model 3 - RDC with reallocation (lt,rt) = (5,3) in weeks,

It is evident above that model 1 has shorter lead-time and longer review time compared to the remaining two model. Model 3 has a longer lead-time and a shorter review time compared to the first model. Model 2 represents an intermediate position in terms of lead-time and review time combinations.

To analyze the performance of the stores with the above combination of lead-time and review time across model 1, a low MOQ number was used to simulate the scenario of high COV without MOQ impact. It represents the situation when the vendor MOQ constraint is so low or the stores' sale volumes are so high that MOQ constraint has no real impact on the store's ordering patterns. The stores are able to order every time as they have gap between inventory position and order up to level. See Figure 10 below.

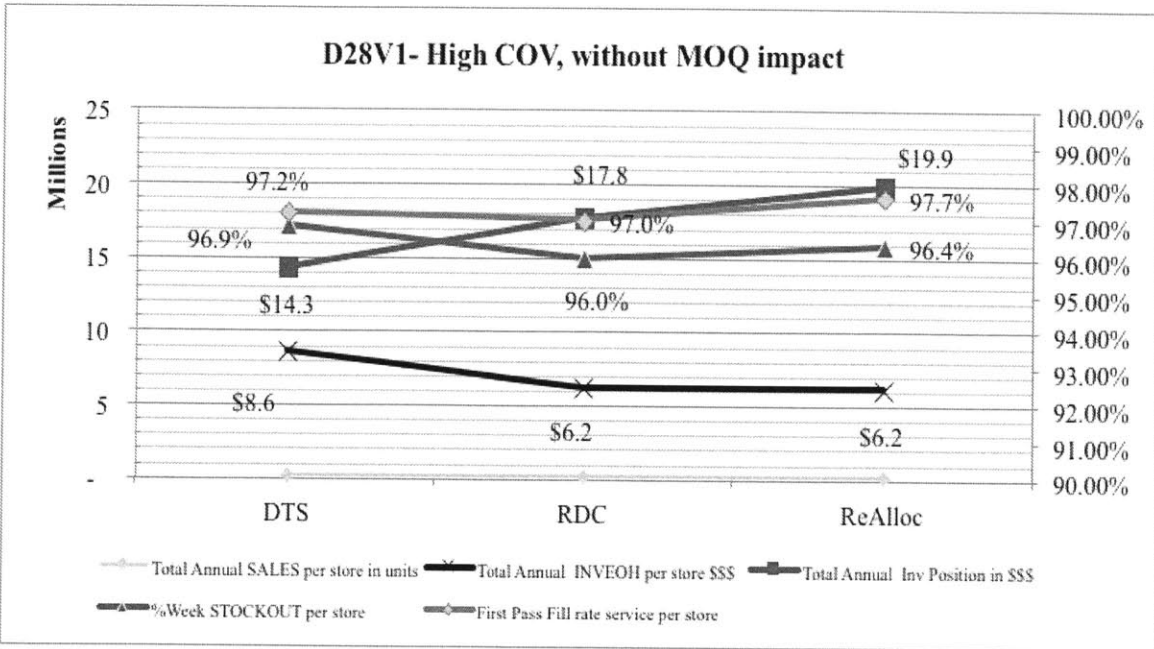


Figure 10 – High COV without MOQ constraint

Under similar first pass fill rate (~97-97.6%) and weeks not-stocked-out rate (~96-97%), RDC with reallocation has the lowest cumulative inventory in store among the three types of product flows. Lead-time under RDC with reallocation model at 5 weeks is significantly longer than DTS model at 3 weeks (67% longer). Review time under RDC with reallocation model at 3 weeks is significantly shorter than DTS model at 7 weeks (42% shorter). With a shorter review period stores order more frequently under RDC with reallocation model and hence a higher inventory position which include products ordered, in transit and in store inventory. But BigRetailer doesn't take ownership of the products until they are delivered to its store or RDC facility, therefore products in pipeline which are not physically located at BigRetailer premises are not considered as real inventory and do not incur inventory holding costs to BigRetailer.

5.2 Analysis of weekly sales with High COV With MOQ impact

Inputs to the Model:

1. Model 1 - DTS (lt,rt) = (3,7) in weeks,
2. Model 2 - RDC (lt,rt) = (4,3) in weeks,
3. Model 3 - RDC with reallocation (lt,rt) = (5,3) in weeks,
4. MOQ constraint large enough to cause the stores to not order 10-40% of the total number of review periods

In this scenario, the model implements a high MOQ value so that stores won't be able to hit ordering quantity every time. The weeks not ordered due to MOQ constraint are in the range of 1 week to 2.67 weeks out of total possible 7 weeks (total period=52/review period=7) of ordering cycle. As can be seen in

Figure 11, both inventory position and inventory on hand under "DTS model with MOQ" increase significantly compared to "DTS model with no MOQ" scenario. This can be attributed to extended ordering cycle. Though stores are still reviewing inventory every seven weeks, the stores' actual ordering cycle is as long as 10 weeks due to the limit placed in overcoming the MOQ.

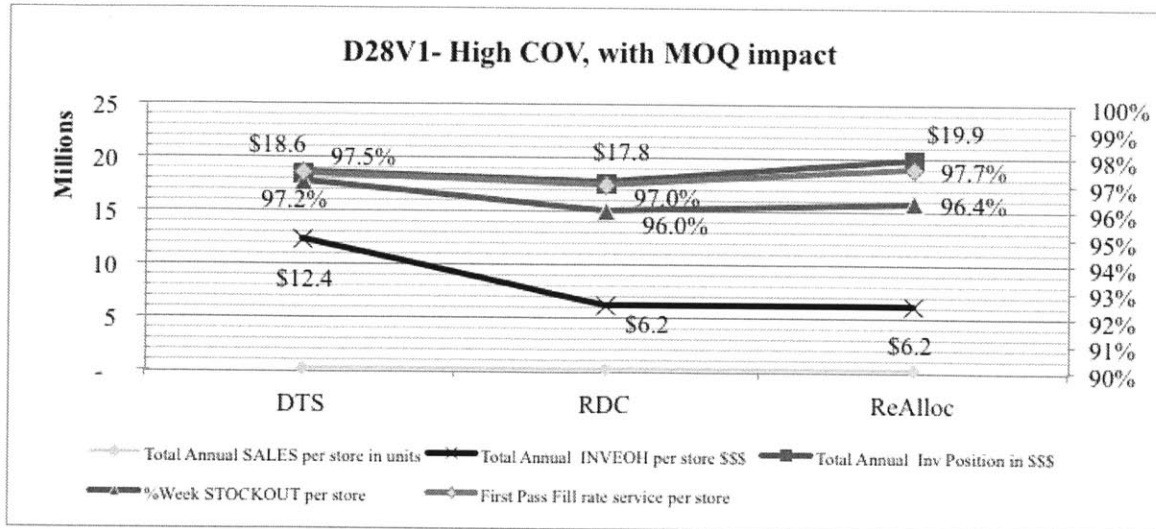


Figure 11 – High COV with MOQ constraint

5.3 Analysis of weekly sales with Low COV With MOQ impact

Inputs to the Model:

1. Model 1 - DTS (lt,rt) = (3,7) in weeks,
2. Model 2 - RDC (lt,rt) = (4,3) in weeks,
3. Model 3 - RDC with reallocation (lt,rt) = (5,3) in weeks,
4. MOQ constraint large enough to cause the stores to not order 10-40% of the total number of review periods

In this scenario, the model uses a high value MOQ value so that stores won't be able to hit ordering quantity every time. The weeks not ordered due to MOQ constraint are in the range of 1.2 weeks to 2.17 weeks out of total possible 7 weeks (total period=52/review period=7) of ordering cycle. As can be seen in

Figure 12, inventory position for “DTS model with MOQ” is nearly 55% of that in the “RDC with Reallocation” model. This can be attributed to longer lead-time.

In the high COV scenario (see

Figure 11), demand variance is high; therefore the possibility of inventory on hand falling below safety stock is higher compared to that in the low COV scenario (see

Figure 12). As inventory falls below safety stock, it’s usually followed by several weeks of a steep upward trend in the inventory level as stores continue to fill up the inventory gap to ordering up to level. High demand variances will cause stores to stock high amounts of in-store inventory level and make the fixed valued OUTL less effective in determining the optimal inventory position for stores. The stores’ inventory positions in High COV sales are much higher than that held by stores in the Low COV sales (upto 40% less). Similarly the in-store units in the High COV sales are much higher than those in the Low COV sales (upto 50% less).

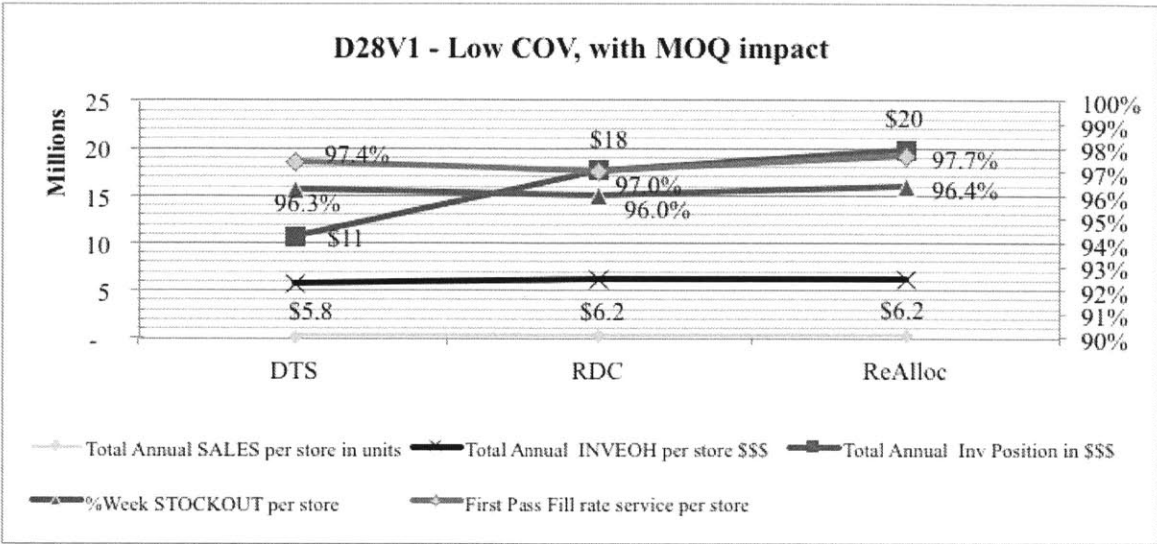


Figure 12 - LOW COV with MOQ constraint

Below is a comparison of the inventory situation under High COV with that of Low COV scenario of same stores and same SKU. With a lower demand variance, Low COV scenario demands a lower safety stock level and OUTL due to lower demand variance. In-store inventory is consistent week over week and has no stock out weeks. High COV scenario needs almost 6 times more safety stock and 3 times more in store inventory to cover demand between each review period and lead-time variances. Refer to

Figure 13 &

Figure 14 below.

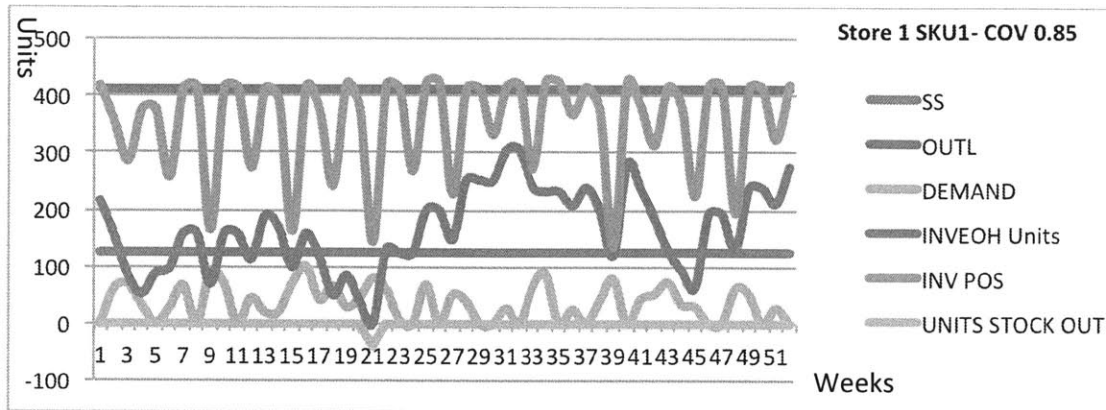


Figure 13 – SINGLE STORE SINGLE SKU HIGH COV sales

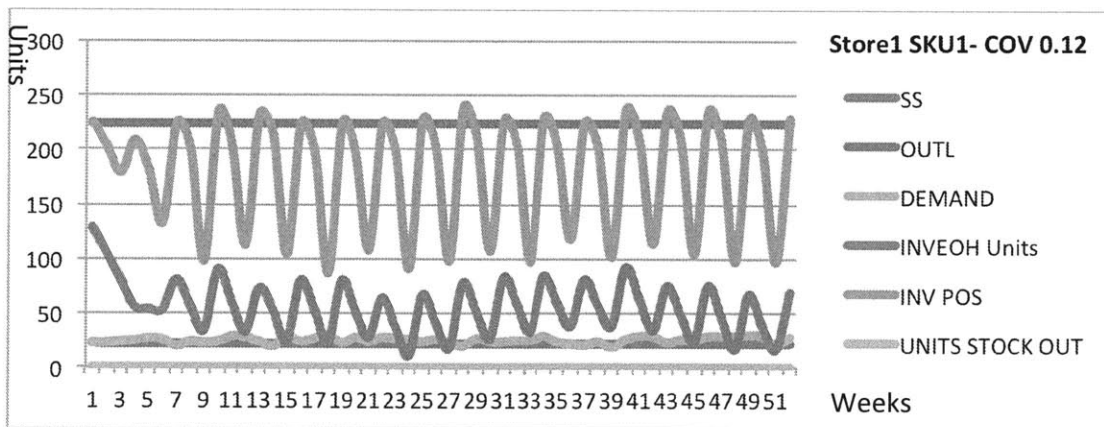


Figure 14 SINGLE STORE SINGLE SKU LOW COV sales

5.4 RDC and RDC Re-allocation with the same lead-time/review time

Inputs to the Model:

1. Model 1 - DTS (lt,rt) = (3,7) in weeks,
2. Model 2 - RDC (lt,rt) = (4,3) in weeks,

3. Model 3 - RDC with reallocation (lt,rt) = (4,3) in weeks,
4. MOQ constraint large enough to cause the stores to not order 10-40% of the total number of review periods

Here the authors assume that RDC will take not take one extra week to finalize the reallocation plan before actually delivering the products to stores. The total number of weeks used in the simulation model is 52 weeks and the analysis is based on six stores and five SKU. As such scale of analysis, variance of 1 week in the lead time won't make big difference to the inventory situation to the entire network of stores.

Outcome of the new lead-time input is shown in the chart below. With lead-time reduced by one week, neither inventory on hand nor inventory position has significantly decreased and demonstrates similar pattern to the longer lead-time scenario. Therefore, it is confirmed that RDC with reallocation model works well for normally distributed sales trend regardless of high or low co-efficiency between demand deviation and demand average. It enables BigRetailer to deliver the same or better cycle service level and item fill rate with a lower (~70%) in store inventory presentation.

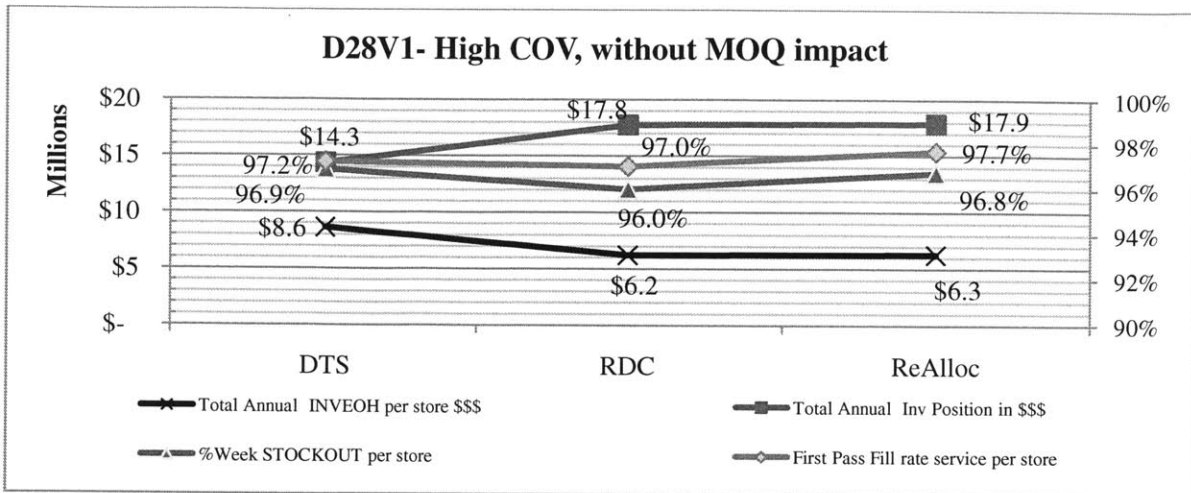


Figure 15 High COV without MOQ impact

5.5 Analysis of daily sales with High COV Without MOQ impact

In order to validate the relationship and impact of lead-time, review time on the sales in the section from 5.1 to 5.3, the data analysis was performed on a “weekly sales” worth of data for 52 weeks. This was primarily due to the nature of the data provided to the authors by the BigRetailer. BigRetailer was not able to provide any further granular level of data (daily sales etc.). This constraint on the analysis meant that the model was designed to change lead-time and review time in steps of one week or more integral multiples of a week.

This caused a strong impact on the model as seen above in the sections 5.1 to 5.3, a movement of lead-time from four weeks in the RDC model (model2) to five weeks in RDC with reallocation (model3), meant a 25% increase in the time to re-allocate the received purchase order. This is unrealistic in the real world scenario of BigRetailer’s operations. Usually once a purchase order is received by the RDC, it is essentially a break up and cross-dock operations that takes between

20 hours and 46 hours, on a lead-time (supplier to RDC) that can range from 5 days to 21 days.

Note: The lead-time has no impact on the processing time taken by RDC to re-allocate the goods in a pre-decided manner.

To overcome this challenge, the authors re-created a daily sales scenario worth of 260 days of selling period (assuming 5 days a week for 52 weeks = $5 \times 52 = 260$ = one year worth of daily sales data). The daily sales data was normally distributed around the actual mean and standard deviation as provided by BigRetailer. The annual sales data was divided by 260 days to calculate the mean value and COV from section 5.1 was used to calculate the equivalent standard deviation. Thus, daily sales data was derived using BigRetailer's weekly sales data. This daily sales data (High COV) was passed through the three models with and without MOQ constraints. Below is a summary of the analysis and findings.

Inputs to the Model:

1. Model 1 - $DTS(lt,rt) = (3,7)$ in days,
2. Model 2 - $RDC(lt,rt) = (4,3)$ in days,
3. Model 3 - RDC with re-allocation(lt,rt) = (5,3) in days
4. MOQ constraint large enough to cause the stores to not order 10-40% of the total number of review periods (only
5. Figure 17 - High COV Daily Sales with MOQ constraint), not applicable for
6. Figure 16

The findings here are consistent with those in sections 5.1 and 5.2. In

Figure 16, the daily sales with High COV and no MOQ constraints are recreated. It can be noted that under the same levels of store performance (units on shelf and days stocked out), the in-stores units are uniform in Model 1 and Model 3, while the inventory position in Model 3 is higher by 25% compared to that in Model 1.

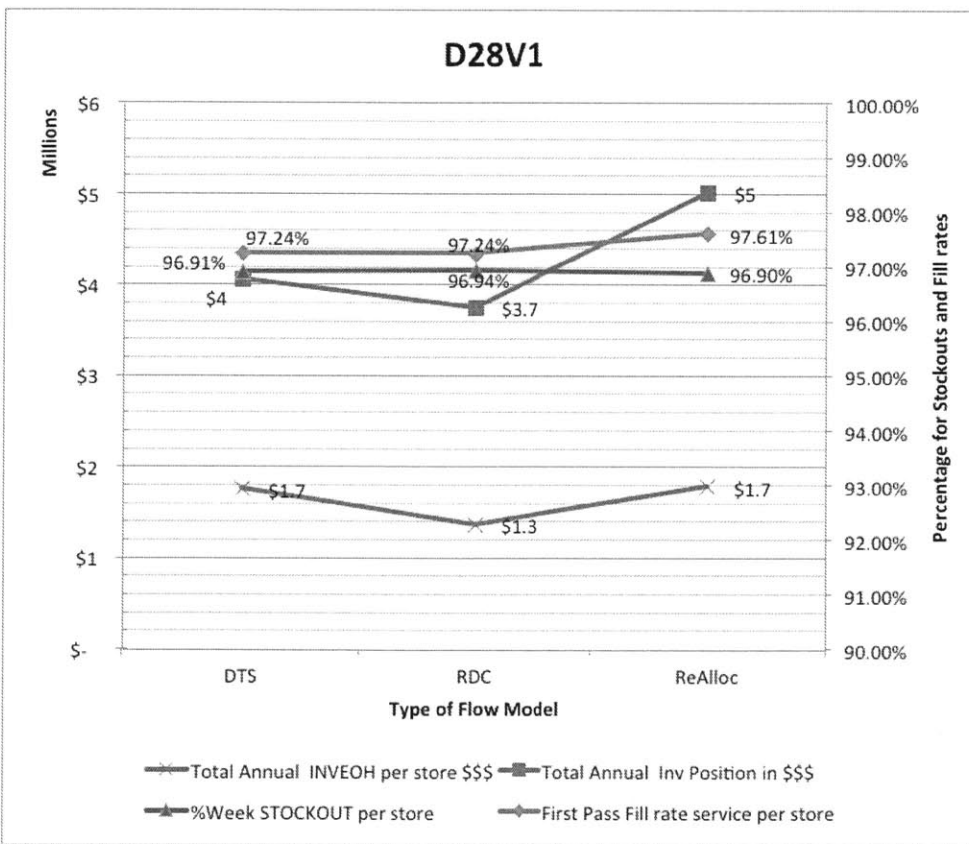


Figure 16 High COV Daily Sales with no MOQ constraint

Similarly, in

Figure 17 the daily sales with High COV and With MOQ constraint are recreated. The results are comparable to the equivalent sales in section 5.2. In

Figure 17, the in-store units increase as the MOQ constraint causes the stores to miss a few (between 10% and 40%) of the pre-decided weeks of review as they cannot place an order that is large enough to breach the MOQ ceiling. The in-store units for the RDC model with re-allocation is a slightly lower number, as in the RDC with re-allocation model an intelligent decision making algorithm manages the allocation of units where they are most needed thus optimizing the inventory. This is consistent with the weekly sales with High COV and with MOQ constraint as seen in

Figure 11 – High COV with MOQ constraint. The inventory position in

Figure 17 shows that DTS with MOQ constraint carries a larger amount as compared to that in DTS without MOQ constraint (see

Figure 16) and this value is closer the inventory position held in Model 3 for daily sales with High COV and with MOQ constraint.

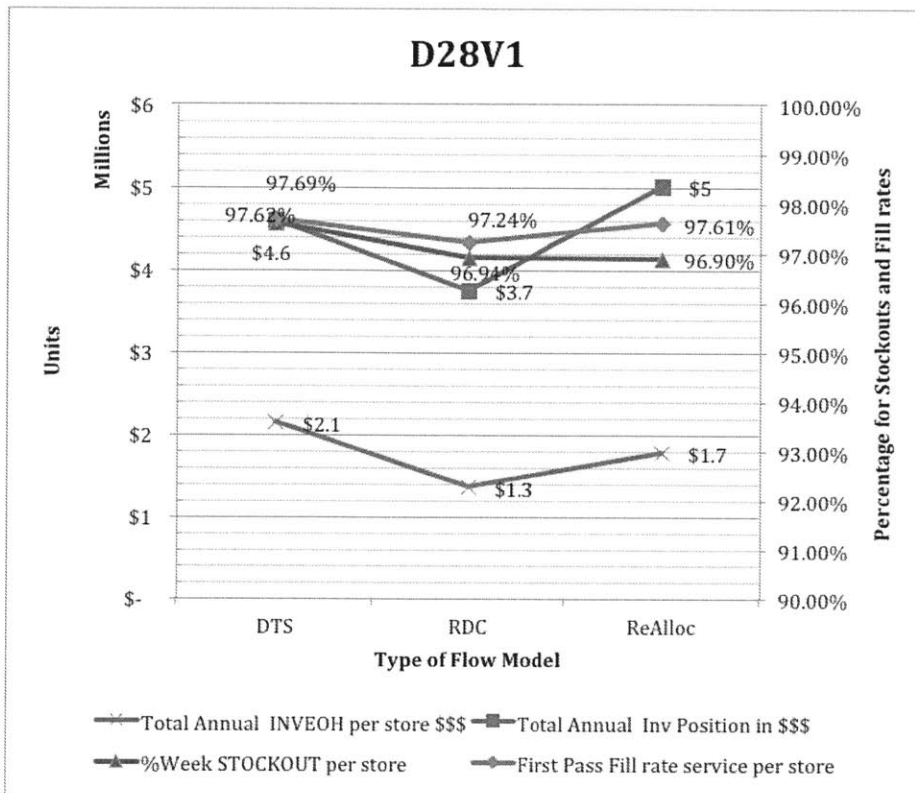


Figure 17 - High COV Daily Sales with MOQ constraint

The results of the daily sales analysis are consistent with those of the weekly sales analysis for sales with High COV.

6 Conclusion

In the periodic review model followed by BigRetailer for managing its inventory, it can be seen that lead-time and in-transit units are directly correlated. Similarly, review period and the size of a purchase order are directly correlated. The safety stock and order up to level are directly correlated with the sum of lead-time and review time.

The transition from a DTS to a RDC supply chain network increases the lead-time causing the in-transit units to rise significantly. The reallocation logic helps mitigate some of this increase by introducing intelligent allocation scheme such that stores can perform competitively with fewer units in store. RDC model integrates channel demand by creating a communication platform with individual stores. It leverages the system transparency and instant data availability to coordinate ordering system for the entire channel network. Overall in-store inventory is reduced under RDC model compared to DTS model with CSL and Item Fill Rate (IFR) are still retained at same level or in some cases better.

The last minute reallocation functionality gives BigRetailer the benefit of countering demand variance in stores by reallocating products among stores on need basis. RDC performs as a coordination center providing shield to all the stores covered under its network against unexpected demand changes and supply disruptions. Aggregated demand evens out the impact from demand variance to stores' inventory position. Stores subject to lower exposure under RDC model compared to DTS model and are less volatile to demand variance.

Most of the stores benefit from the higher ordering frequency allowed in the RDC model as there is no MOQ constraint under such situation. Under RDC model, the stores may order smaller quantities, which would not have passed the MOQ constraint under DTS model. This helps stores achieve a better in store presentation and subsequently better CSL and IFR.

One disadvantage of RDC model is longer lead-time for stores to receive product delivery due to the additional step of reallocation. However, according to the above analysis, 1-week variance in lead-time has minimal impact on stores' inventory level compared to over 52 weeks of sales. The extended lead-time was offset by the shorter review time and demand reallocation.

Further research in identifying the impact of lead-time variability on store-SKU performance and deeper analysis on the product segmentation from BigRetailer's perspective will sharpen the understanding of the dynamics of RDC with re-allocation, clearly delineating the cut-off threshold where a store is better off with DTS over the benefits of RDC with / without reallocation.

Availability of forecast data with smaller variance error will also aid in making better ordering decisions every review period. The ordering method in this paper is based on current demand, however, a better sense of expected demand "lead-time" period after the order is placed, through high quality forecasts, will help mitigate the stock scenarios and reduced the in-transit inventory which is consistently the highest in the new network.

The numbers generated from the analysis in the various graphs used in this paper are subject to the many assumptions made in the design of all three models. Generating a hundred percent accurate number is however not the goal. The goal of this analysis is to identify and explain the correlation of the various input parameters such as lead time, review time, safety factor with the output metrics such as weeks stock out, item fill rate, total in-store inventory and total inventory positions.

Additionally, Simchi-Levi et al. (2008) state that organizations with RDC centric supply chain network might want to investigate the lead-time and transportation cost tradeoff. If the supplier is able to access the demand information from the RDC (sort of look-ahead), just as RDC is able to access this information from the stores, the supplier can (make-to-stock) manufacture to a probable 80% required quantity in advance and ship it as soon as the order is placed while keeping the delta amount to be manufactured against direct order. This will allow BigRetailer to shave off some of the lead-time and bring it closer to a DTS level value and the supplier can move from being a reactive partner to being a responsive partner. Thus Big Retailer will benefit from the efficiencies of the reallocation model with more frequent reviews and possibly shorter lead-time.

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