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A BID-RENT MODEL OF URBAN RESIDENTIAL LOCATION

BY

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Submitted in Partial Fulfillment
of the Requirements for the
Degree of Bachelor of Science
at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1975

Signature of Author ....
Department of Urban Studies & Planning,
May 9, 1975

Certified by ....
Thesis Supervisor

Accepted by ....
Chairman, Departmental Committee on Theses

ABSTRACT

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A bid-rent model of residential allocation is formulated on the basis of treating housing as a hedonic good and incorporating this into Alonso's theory of the urban land market. Utility functions for housing and "non-locational expenditures" are estimated. A simulation model of the Boston SMSA housing market is built. The model is made operational and the results of a simulation run are compared to the actual location pattern to determine the reliability of the model, as well as used to provide a basis of comparison for subsequent runs.

In four policy runs, various alterations are made in the supply and demand characteristics of the model. Two runs make alterations in the supply--one adds low-income housing in the suburbs, while the other renovates the downtown area. The first of the demand runs examines the effects of a percent-of-rent transfer payment, while the second looks at a straight income transfer to low-income groups. Implications for both policy and the model are discussed. Conclusions detail possible uses of the model, its reliability in its present form, and recommendations for future improvements.

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Introduction

Life in a metropolitan area is a blend of an imposing number of urban activities. Yet one of these activities manages to be responsible for the overwhelming majority of the land use in that area. This activity is housing. It is one of the few activities (or products) in which everyone, in one way or another, participates. In general, consumption of housing services accounts for about 20 percent of a household's budget. Yet we still know very little about how the housing decision is made, that is, what causes people to live where they do.

The narrowing of this knowledge gap is one of the prime aims of this dissertation. A solid, internally consistent theory of the urban housing and location decision has been operationalized in an effort to test the theoretical specification against the reality. If we can begin to understand the nature and magnitudes of the various inputs to the housing decision, then we can hope to influence these choices in desirable directions. Some policies have been simulated; not only do they give us information as to their effects on the urban pattern, but they also provide feedback on the modelling process, allowing the model maker to revise his tools. This exercise has three goals, then: to develop a theory of the urban residential location process, to test the applicability of this theory in the real world, and finally to test frequently-encountered urban policies in this newly-developed laboratory.

In accordance with these goals, this paper has been divided into five parts: (1) an explanation of the theory and the model, (2) and (3) the fitting of the model to the residential situation in the Boston SMSA, and (4) and (5) tests of various policies. Hopefully, this research has initiated the development of a fruitful branch of investigation and a powerful planning tool.

THE THEORY OF HOUSING AND ITS APPLICATION

We have already established the need for studying the behavior of the housing market in the urban situation. However, we have also implied that there is something unique about the housing market that differentiates it from other markets. Why can we not study it as we would a market for cars, or even more difficult, why is it different from the other urban location markets, such as the markets for commercial and industrial space? Answering these questions should give us some important clues as to how our model should be handled.

The bulk of our answer lies in two major factors - the durability of housing and its heterogeneous nature. Housing is a unique good - the overwhelming majority of the population owns or rents only one housing unit, and almost none have more than one unit in the same urban area. Therefore, the choice of a housing unit takes on an added importance in the present period. Housing lasts considerably more than one period, however. In fact, it is the most durable of all consumer goods. The consumption decision in one period can influence the consumption decision for years to come, in that the costs of rectifying a wrong decision (or even one that has become less practical over time) can be very great.

Also, one household's consumption decision can influence another's, in that the secondary market in the good provides most of the market activity, so that the characteristics of the housing unit are determined by the tastes of the initial occupant of that unit.

Much more important as a differentiator of the housing market is the heterogeneous nature of the good. In commercial and industrial location theory, the location decision can generally be reduced to one of maximizing expected profitability, generally measured in commonly accepted monetary terms. Expected market size, costs of providing a work force, costs of land, transportation costs, and all the rest of the inputs to a rational industrial location decision can be collapsed into the one-dimensional world of dollars and cents. Households are not in business; their decisions cannot be collapsed into decisions of profitability. Thus arose the definition and treatment of housing as a hedonic good, that is, one that gives pleasure. Housing is viewed as a composite package of goods, each of which gives, in combination with the rest of the package, a certain utility to the household. Our problem in analyzing the housing market lies in the difficulty in measuring the "quantity" of housing; the hedonic approach to housing gets around this problem by measuring the amount of utility that a particular household may derive from that package. There is very little meaning in the traditional concept of price of housing versus quantity until we infuse the "quantity" measure with its multi-dimensional nature; we can then assume that a household will pay more for "more" housing.

Our model of the urban housing market combines this approach to housing with a generalization of Alonso's approach to the urban land market. (Alonso,1964.) Alonso's residential market involves tradeoffs among three goods — land, distance from the center, and a composite "other good". He postulates a utility function which relates the tradeoffs among these goods to the amount of happiness (utility) that the household derives from them. Using this function in conjunction with a budget constraint and the traditional marginality conditions, he arrives at a "bid-price" curve which gives the bid of a household for a parcel of land as a function of the utility level, the attributes of the parcel, and the income of the household. Each landlord behaves as an auctioneer, and sells his parcel to the household which bids the highest. Thus, residential parcels are allocated to households.

We have extended Alonso's model so that it includes the structures and the neighborhood attributes of given parcels, rather than the unbuilt featureless plain of his model. (See Wheaton, 1974 for further discussion.) This implies a much shorter time horizon than the Alonso model. In the long run, all structures are variable, and therefore a long-run equilibrium solution will approximate the featureless plain, since possible buildings for any location enter into profitability considerations. This model, then, is one of a short-run equilibrium. Consider a vector of housing attributes X (these may be attributes of the structure, the lot, the neighborhood, or the location of the house within the metropolitan area). Then we may postulate the existence of a utility function (of as yet unspecified form) such that $U_o = U(X,M)$ where M is the consumption of all other goods. We may also assume that these functions are different for each individual, in which case the above becomes

(1) $$U_{o_i} = U_i \, (X_i, M_i).$$

Furthermore, each individual is subject to a budget constraint

(2) $$y_i = R_i + p_m M_i$$

where y is income, R is the total expenditure on housing, and p_m and M are the price and quantity of consumption of a composite good representing all non-locational expenditure. If we assume p_m equal to 1, we get (without loss of generality)

(3) $$Y_i = R_i + M_i.$$

Finally, if we assume that U_i is invertable, we can write

(4) $$M_i = U_i^{-1} \, (U_{o_i}, X_i)$$

Combining (3) and (4), that is, constraining the household's utility function by its budget, we can get our basic "bid-rent" equation, that is,

$$(5) \qquad Y_i - R_i = U_i^{-1}(U_{o_i}, X_i), \quad \text{or}$$

$$(6) \qquad R_i = Y_i - U_i^{-1}(U_{o_i}, X_i).$$

The intuitive appeal of this bid-rent formulation is clear. Let us assume that one component of our housing package has positive utility, $dU/dx_j > 0$, or in other words, an additional amount of an attribute with positive utility will increase the amount that a household is willing to bid for a housing package, given a constant level of utility. This last, the assumption of a constant level of utility across all housing packages, is a crucial one for our model, but one that is easily explainable in first-year economics. If we had a number of different households, all with identical utility functions and incomes, and yet at different utility levels, those households at the lower utility level would be willing to move into the houses of those at the higher level. This would put an upward pressure on the rents of those at the higher level, which would continue until utilities were equalized and no one would be made better off by moving. It should be emphasized that this applies only to households with identical utility functions and incomes.

The bid-rent function enables us to find a bid-rent for each housing package for each household, given some level of utility U_{0i} and income Y_i. Our allocation procedure is identical to Alonso's, in that the parcel goes to the highest bidder. This merely says that the landlord is maximizing his profits, certainly a reasonable assumption.

Up to this point, I have alluded to the allocation mechanism only so far as to say that the housing packages go to the highest bidder. However, it should be readily apparent that with arbitrarily selected levels of U_0, there will be many households which bid successfully on more than one house, and many which bid successfully on none. Since we have earlier established that very few households will command more than one housing package in reality, some sort of adjustment mechanism is necessary. It is this mechanism which makes the model work.

First, let me establish one very important point. The model being described here is an equilibrium model. Therefore, the mechanismsused to reach this equilibrium are designed not so much as accurate representations of real world processes as they are intuitively reasonable means for reaching a goal. We are not claiming that every household bids for every housing unit, nor are we claiming that in actuality the landlord opens a series of sealed bids to determine his tenants; rather, we believe that this is a reasonable abstraction of the actual processes. Having established this, I may move on to the adjustment processes.

After the auction as described above, each household has been allocated a certain number of houses, either more or less than it needs. The model compares the number allocated to the number needed. If there is an excess of houses, the utility level for that household is revised upward (the equivalent of a downward shifting of the bid-rent schedule), and vice versa for a deficiency. The bid-rent calculations and allocation are then repeated, the entire process being rerun until demand just equals supply for each household. Again, we must remember that this iterative approach is a model representation. It could be analogized to the real world by saying that people enter the market with certain expectations about the availability of units and the price structure and revise their expectations on the basis of new information, but the analogy has only limited application.

A further theoretical justification of this approach is its duality with a utility-maximization approach. In equilibrium, the model produces an envelope of bid-rents which represents the actual rent gradient (it is made up of the bid-rents of successfully bidding households). Given this rent gradient, allowing each household to maximize its utility would result in exactly the same location pattern as our model produces. This theorem is proved in Wheaton (July, 1974); there is no reason to reproduce it here. It is sufficient to say that the existence of this duality makes the solution of the equilibrium much easier, since the manipulation of $\underline{n}$ utility levels is much easier than $\underline{m}$ prices, when $\underline{m}$ is much greater than $\underline{n}$ (which, as we shall see later, is the case here).

I have described a method by which the existing housing stock is allocated. Although it forms a large part of the housing market, it is clearly not the entire market, in that new housing should also be considered. In fact, we have built into the model a mechanism for the development of vacant land, one which closely parallels Alonso's model. In our model, vacant locations are characterized only by neighborhood characteristics, and a number of possible housing types (of varying density) are postulated. Bid rents are calculated for all combinations of housing types and households for each location. That combination of household and house type which yields the most profit per acre is selected by the developer for development. The units acquired in this way also enter into the utility adjustment calculations.

The model has been presented as one in which each household has a separately identified utility function and bids on a number of differentiated units. However, even modern data-processing techniques cannot handle the manipulation of 900,000 households and houses with any reasonable cost. Even if it were possible, the utility functions can only be determined by revealed preference, and revealed preference can only be used if there are a number of observations on the same decision-maker. Since we have only one observation on each household, we are forced to aggregate households into strata, using a method to be described in the next section. By the same token, using data on each housing unit in the metropolitan area would be prohibitively expensive to use, so it was necessary to aggregate into groups of houses with common characteristics.

THE TIE MECHANISM

The aggregation into zones of housing units necessitated a mechanism for dividing up the zones among different strata, in order to introduce some locational heterogeneity into the model and enable it to reach equilibrium within a reasonable amount of time. It was postulated that the various imperfections of the market -- imperfect information, costs of search and moving, et cetera -- make getting the absolute maximum possible rent on any given unit very unlikely. Therefore, we introduced a tie range into the model; that is, any stratum that bid within T1 of the maximum bid on a housing unit, or offered a profit per acre within T2 of the maximum, was considered to have been successful in the auction. The zone was then divided equally among all successful bidders. Larger tie ranges imply greater market imperfections, and result in more diversified housing patterns.

RESERVATION PRICES

In a bid-rent model where the number of available units just equals the number of households, relative rents will be determined but the absolute level will be indeterminate. In our model, the number of units generally exceeds the number of households, meaning that in equilibrium, some of the units must be vacant. This was incorporated by setting a market reservation price for built-up units and one for vacant land. If the maximum bid for a unit is below this price, it is presumed that the rent cannot cover the variable costs of renting the unit, and it will be held off the market. This gives us a numeraire, as the marginally rented unit will rent for just above the reservation price. Without such a reservation price to vary the supply, the model as constructed could never reach equilibrium.

UTILITY ADJUSTMENT MECHANISM

In our formulation of the utility function, we made the explicit assumption that the utility function is separable. This means that the term $U^{-1}(U_{o_i},X)$ may be broken into $f(U_{o_i})*g(X)$, that is, the rent structure can be varied by varying only $f(U_{o_i})$ while all other terms remain constant over all iterations. This had very important implications in terms of the cost of running the model. The adjustment mechanism is tied to both excess demand within the strata and excess demand in the market as a whole. If total supply exceeds total demand, it indicates that the rent structure is too high relative to the reservation price, and the adjustment mechanism raises all utilities, thus lowering the rent structure. If a particular stratum commands more units than it needs, its bid-rent curve is too high compared to other strata, and its utility is raised. This mechanism tends to drive the market to equilibrium. Although we have been unable to develop a theoretical proof that the process will reach equilibrium, the model has generally tended to converge (within the limits imposed by the imperfect divisibility of the model) within 100 iterations.

ASSUMPTIONS

Our specification of an urban residential location model contains a number of important assumptions, both implicit and explicit. In order to fully understand and be able to use the output of such a model, we must be well grounded in its theoretical underpinning. This section of the paper is designed to present some of the assumptions and analyze their implications for the results.

A. Perhaps most important is our supposition that a given population can be stratified by its socio-economic characteristics into a manageable number of groups, that the resulting agglomerations of people will have very similar preference structures within the group, and that different groups will exhibit very dissimilar behavior. This presumption allows us to estimate the utility functions, for, as mentioned above, we need many observations on behavior of the same household before we can specify its preference function. If our assumption is not valid, that is, is socio-economic variables are not important in determining behavior, then our estimates of the utility functions will be the same for all strata, and should differ only by a stochastic term. It was noted earlier that the bid-rent solution is the dual of the utility-maximization solution. Estimation of the bid-rent functions assumes that each rent lies on the bid-rent curve of the strata located there, and furthermore that any two zones in which a strata is located lie on the same bid-rent (or indifference) curve. These must be utility-maximizing points; otherwise the revealed preferences are not truly preferences.

B. Second, and a rather trivial assumption in theory but one that may be often violated in reality, is the fact that the model constrains identical units to rent for the same price. This applies only to units where each of the x_j has the same value; it does not mean that two units which have the same utility (and thus the same bid-rent) for stratum 1 will have the same utility and rent for stratum 2. In fact, the existence of geographically and informationally segregated submarkets may allow differences in rent to exist.

C. Throughout the early part of this paper, I have implied that all households make bids on all units. This argues for perfect information on both the part of the household, in knowing what units are available, and the landlord, in that there is no element of risk that a better offer will come along. The need for this assumption is partially mitigated by the tie mechanism, which provides a range of acceptable bids. This does not necessarily define the proper submarkets, however, in that a household's search may be limited by geographical area or other non-random influence; the need for perfect information, then, might somewhat bias the model.

D. Preference structures, or utility functions, once estimated, are assumed to be relatively constant through time. Especially if we maintain that the market is tending towards an equilibrium, this assumption is necessary, as it needs a constant goal to tend toward. As we have formulated the vacant land mechanism, this assumption greatly eases computation, since it is assumed that the profit-maximizing solution in period 1 will be the profit-maximizing solution in all periods to come. Were preference structures to change with any rapidity over time, this would no longer be valid. Note that this does not imply that a particular household may not change its preferences as its situation in the life-cycle changes, but rather that a family at a certain position in the life cycle in period 1 will have the same preferences as a different family at the same point in the life cycle in period N.

E. This is a residential allocation model. As such, the location of industrial and commercial activity, even population-serving commercial activity, is presumed to be independent of the pattern of residential location. Insofar as residential and population-serving retail location are interdependent, the model will be biased in its location patterns. However, the implicit assumption is that these secondary effects on the location pattern are inconsequential.

F. Household utility functions include only predetermined characteristics of the structure and the neighborhood. Since the model locates all households simultaneously, considerations of the makeup of the neighborhood in terms of other households present are not important; in other words, household location decisions are independent. This could be removed by calculating the ethnic makeup of a given neighborhood after each iteration and using it as an input to the next iteration (as an item in the preference function). However, this approach has the conceptual disadvantage of making the final equilibrium dependent on the path used to obtain it, which is undesirable as the iterations themselves have no conceptual significance.

G. A strong assumption, and one that lies at the heart of the model, is that there does exist an equilibrium towards which the market is tending at any one time. This is in effect saying that the behavior of the market is purposive rather than random. If we do not make this assumption, our model can have no use, since the effects of any policies we may introduce will not have predictable results, in that the market will make no effort to reestablish equilibrium.

H. Maximum bid-rent is the sole determinant of the successful bidder in a particular zone. In the real world, many other factors enter - certain units require no unrelated individuals of opposite sexes, some require no pets, and, most importantly, many discriminate because of color. There is no indication of this built into the model; builders and landlords are motivated only by profit considerations. Except for discrimination by color, which will be discussed later, it does not appear that this will introduce important discrepancies into the model.

UTILITY FUNCTION FORMULATION AND ESTIMATION

The utility function lies at the core of our model, since it is this function which is the major determinant of all bid-rents, and thus the final location pattern and rent structure. The formulation and estimation of this function, then, is a crucial part of the modelling process and, as it is the focus of many of our assumptions, one that deserves to be treated at some length. This section will be devoted to an explication of the utility function, the data used to estimate it, and the methods and results of estimations.

FORMULATION OF THE FUNCTION

As we have noted before, households derive utility from both the characteristics of the housing unit in which they locate as well as the characteristics of the neighborhood. We have also stipulated in the previous section that the function be separable, which eases the operation of the model with no great loss in theoretical flexibility. Finally, we shall make the fairly evident assumption that extremely low levels of some components of housing (e.g. number of rooms, quality of the unit, ease of transportation) entail very severe disutilities. A function which satisfies these characteristics, and one that is still very easy to estimate, is the Cobb-Douglas form,

$$U = \prod_i X_i^{a_i}.$$

This formulation has certain disadvantages, namely that the elasticities of substitution between any two components are all equal and unitary, but the savings over estimating a more complex function where this is not true (e.g. a generalized C. E. S. function) more than make up for this restriction. The estimating form of the equation was

$$\log(Y-R) = U - \sum_i a_i \log X_i + \varepsilon$$

which is simply

$$U = M^{a_M} \prod_i X_i^{a_i}$$

transformed for ease of estimation, with

M = Y-R = all non-locational expenditure

U = level of utility

X = vector of neighborhood and housing attributes

a = vector of estimated coefficients

$a_M \equiv 1.$

Marginal rates of substitution, which provide one of the few bases for comparing utility functions in this model, are found to be

$$dX_i \;/\; dX_j = \frac{a_i X_j}{a_j X_i}.$$

An assumption of the model reported earlier is that the market tends toward an equilibrium in the long run, or the model would have no significance. While important, this is in atuality a weak assumption about equilibrium. In order to estimate the utility functions, we must make a much stronger assumption. At any point in time, the market is assumed to be approximately in a short-term equilibrium. This is necessary to validate our estimation, since I earlier pointed out that estimation could only be made possible by numerous observations along the same utility curve, which implies that all households must be at the same utility level at the time of estimation. Ideally, these estimations should have been done on the basis of individual household data. However, such data was not available, and census-tract level aggregations had to be used, thus eliminating much of the richness of the data. Problems with this technique are discussed in a following section. The error term ε in our specification is in part designed to account for fluctuations from the equilibrium utility level due to frictional costs in the market, such as moving or transaction costs. This assumption about ε, and the assumption that it enters additively in the log-linear form, are necessary for simplicity of estimation. Assumptions about the distribution of ε will be discussed later.

This same aggregation, while making the estimation possible, also leads to some serious problems. Grouping such diverse units as households into strata of identically-behaving decision makers eliminates much of the richness and explanatory power of the model. While we hope to have captured much of the variance in individual behavior, there are so many influences on the housing decision that large prediction errors on the micro-scale will be unavoidable. All we can really hope to predict is gneral patterns. Many of the aggregation problems were inherent in the data; these will be discussed in the next section.

DATA AND SOURCES

The bulk of the data used in the calibration of the model was drawn from the 1970 Census of Housing. Using the data contained in this source, we aggregated individual households into ninety-six household groups, or strata. These strata were defined by two races, six median family sizes, and eight median income levels. From this point on, strata will be referred to by index numbers, where the first number represents race, the second family size, and the third income, according to the following table:

FIRST DIGIT: RACE		SECOND DIGIT: SIZE		THIRD DIGIT: INCOME (median)	
1	White	1	One person	1	$ 1,800
2	Non-white	2	Two "	2	2,600
		3	Three "	3	3,600
		4	Four "	4	4,600
		5	Five "	5	8,000
		6	Six + "	6	12,500
				7	20,000
				8	28,000

For instance, Stratum 128 refers to white, two-person households with median income $28,000.

The specification of the utility function included four housing attributes and six neighborhood attributes. The dependent variable in all regressions was taken to be the log of median income less the mean annual value of housing units in the tract. The latter quantity was available by income and race.

Of the structural characteristics that we used, only one, mean age of the unit (AGE), varied by stratum as well as tract, being available by race and family size. This variation, combined with the mean annual value variation, was sufficient to allow us to independently estimate the ninety-six strata. The other three structural variables were ROOMS, LOTSIZE and PLUMBING. The first, ROOMS, is the sum of the mean number of bedrooms and mean number of bathrooms for each tract. LOTSIZE is a measure of the average lot size per unit, calculated as the total land devoted to residential use divided by the total number of units in the tract. The land use data came from the data bank created for the EMPIRIC model. Thus, our model combines 1963 land use data with 1970 census data. This could create certain problems, but the assumption was made that as far as residential use was concerned, the 1963 data was a good approximation. The final structural variable, PLUMBING, is the percent of the units in the tract that have bad plumbing, and was taken directly from the census.

Six neighborhood variables were used, representing both the physical and fiscal quality of life. One of the most important of these is TRAVEL COSTS, which represents the costs of an "average" trip from the zone in question. It represents a weighted average of activities in all other zones. Since travel costs play such an important role in all location theory, we would expect accessibility to be very significant in our estimations.

Also included in the utility function are two land use categories, which are used to represent the quality of the neighborhood. LANDUSE1 is the percentage of the total land in the tract devoted to noxious uses, that is, industrial use. LANDUSE2 is similarly the percentage of land in recreational uses. These three variables are calculated at various aggregation levels; travel costs being based on BRA districts within the cities and towns outside, while LANDUSE1 and LANDUSE2 correspond to EMPIRIC districts, which are on a much finer scale.

The last three neighborhood variables relate to the level of services provided by the neighborhood. They are the pupil-teacher ratio (PUPIL RATIO), per capita expenditures on crime prevention (CRIMEEXP), and the property tax ratio (PROPTAX - defined as total poll and property tax revenues divided by income). These variables are available only by town, the model thereby suffering the severe misfortune of having one value for all of the 156 census tracts in the city of Boston.

This last comment leads us to a very important area, that is, the possible breakdown of the model due not to theoretical difficulties but to deficiencies in the data. Such implicit assumptions as one crime prevention level for all of Boston, or equal quality schools in Roxbury and Hyde Park, tend to mask many of the important differences in these areas. Within individual tracts, using the mean characteristics may have much the same effect, in that one hundred two-room efficiency apartments and one hundred ten-room houses average out to two hundred six-room units. Neither the inhabitants of the efficiency nor those of the house will choose the six-room unit, and the model will therefore incorrectly predict the location pattern. We see, then, that an implicit assumption in the model is that the variation within census tracts as far as housing and neighborhood characteristics is small compared with the variation among tracts. Only as far as this expectation is fulfilled can we expect the model to accurately reproduce reality.

ESTIMATION

The basis for the estimation was the 446 census tracts of 1960 which make up the Boston Standard Metropolitan Statistical Area. Census data from 1970 was converted to agree with the 1960 tracts in order to have agreement between census and land use data. Certain tracts were dropped from the sample due to the unavailability of certain data points. Furthermore, in the individual estimations, only those tracts in which there were more than three households in the stratum in question were used. In the white strata, this generally did not lead to any problems, but for the non-white strata there were often very few such observations. Finally, of the ninety-six possible strata, only fifty-four were sufficiently large to allow meaningful estimations. Of these, thirty-one were white and twenty-three black. Complete listings of both zones and zonal characteristics and strata and strata coefficients are found in the Appendices.

Two major problems had to be overcome in the estimations, problems that invalidated the normal least-squares approach. The first of these is heteroscedasticity in the error term, due to the grouping of data. The theoretical formulation of our problem deals with the individual household, and states that

$$\log(Y_i - R_i) = U - \sum_i a_i X_i + U_i.$$

However, our data is of the form

$$\log(\bar{Y}_i - \bar{R}_i) = U - \sum_i a_i \bar{X}_i + \bar{U}.$$

where a bar denotes the mean. In Johnston (1963), we find that if the variance of the dependent variable is i.i.d. and equal to σ^2 in the first case, then the variance of the dependent variable in the second case will be $E(\bar{u}\bar{u}') = \sigma^2 GG'$, where G is a grouping matrix, and $(GG')^{-1}$ is an m by m matrix (m equals number of tracts) with the number of observations on the diagonal and all off-diagonal terms equal to zero. It can be shown that using a matrix with the square roots of the number of observations on the diagonal and zeros on the off-diagonals, that is,

$$A = \begin{bmatrix} \sqrt{n_1} & & 0 \\ & \sqrt{n_2} & \\ 0 & & \ddots \end{bmatrix}$$

will reduce this error variance to a constant σ^2. Therefore, weighting our observations by the square root of the number of households of that particular stratum in the zone will eliminate the heteroscedasticity in the error term.

The second problem is one of errors in variables. The structural characteristics that we use in the estimations are not the true means of the strata being estimated. For instance, in zone i, we use the mean age of unit for large black families as a whole for each of the strata 261, 262,... 268. This introduces a bias into the data, for if we assume that unit age has negative utility, and that more money therefore allows one to buy less of it, then the true mean X_i will be consistently less than the mean for all of the eight strata, which is the variable that we are using. In other words, by using these bias means, we no longer have independence of he regressor and the error term.

An instrumental variable (IV) approach was used to alleviate this problem. It was assumed that if the mean characteristics were ranked by zone, and these rankings grouped into three ranks (high, medium, low), then these final rankings would be uncorrelated with the measurement bias of the census means, but highly correlated with the means themselves, thereby fulfilling the requirements of an IV. Since IV gave more intuitively appealing results than OLSQ, the results from the former were used, and are reported here.

RESULTS

Results of the estimations were generally quite good, with coefficients generally having quite reasonable values, as will be discussed later in the section. However, the estimation did point up some ambiguities in our variables, as well as accentuating the problems of aggregation that were discussed earlier. For instance, our AGE variable is presumed *a priori* to have negative utility, in that all other things such as condition being equal, people will prefer new houses to old ones. However, twenty-two of the fifty-four strata had positive utility for age, primarily among the poor and blacks. One could hypothesize that this is not a revealed preference, but a necessity, in that the poor are willing to live in older units in order to afford other things. Or, artificially high rents in poor units could be the result of market imperfections (e.g. racial discrimination or imperfect information) that prevents the market from reaching a true equilibrium. The same argument could be applied to LOTSIZE, since the estimations tell us that blacks do not like to have land. In fact, the results of the black estimations generally showed the effects of these imperfections, in that there were many wrong signs. The existence of wrong signs is not in itself justification for discarding those strata, for in the absence of a discrimination mechanism in the model, they may help to more accurately reproduce reality. In fact, were we to include racial composition of the tract as a variable, many of these wrong signs would be reversed.

USES OF THE MODEL - THE BASE RUN

The first test of any model is its ability to reproduce reality. In this case, the reality which we are attempting to recreate is the location pattern existing in Boston at the time of the estimations. The results of such a run would be useful in two major areas. First, they allow us to test the reasonableness of our assumptions about the market and equilibrium conditions. If the fit between the model and the reality is extremely poor, there is no reason to suspect that any policy implications that may be drawn from it are any better. Second, the base run provides us with a reference point for future runs -- the impacts of a policy can only be examined within the framework that it was formulated. That is, we are only looking at how much a policy <u>changes</u> a given situation, and in order to do this, we need that given situation.

As mentioned in the last section, only fifty-four of the ninety-six stratum were estimated. Similarly, not all of the zones in the Boston area could be used due to the unavailability of data. Ten census tracts were eliminated for the base run, those being the Harbor Islands, Manchester, Hamilton, two tracts in Danvers, one tract in Lexington, Ashland, Duxbury, and two tracts in Chelsea. Even with these eliminations, however, there would still be many more housing units than households. Rather than selectively eliminating housing units, it was decided that the least distorting method of adjusting the model would be to increase all stratum sizes by six percent. This had the effect of leaving only about one percent of the stock vacant at equilibrium, rather than the seven percent that would occur otherwise. Strata were thus forced to live in undesirable central-city tracts as well as more desirable suburban ones, and any policy that we might try would have more visible effects.

The only parameters of the model that affect the final equilibrium, and thus are worth reporting here, are the reservation price and the tie range. As the reservation price, we chose a figure in the neighborhood of the minimum mean annual value for any of our tracts. There was a cluster of tracts in the $900 per year range, so we chose this as the figure below which units would go vacant. The tie range is a much more nebulous idea, in that a representative figure is not so easily available. We found that a tie range of $40, which works out to only $3.33 per month, had good convergence properties, while still being small enough that landlords would be able to discriminate between moderately differing bids. Both the reservation price and tie range were kept constant throughout all the policy runs in order to have a basis for comparison.

OPERATION OF THE MODEL

A word about the feasibility of using this model is appropriate before a discussion of the results. As yet, we have no theoretical proof that such a large and complex model will converge to a stable supply-demand equilibrium. However, in the many runs that were done, this model showed very good convergence properties. In general, given a reasonable set of initial utilities (the solution of the base run was used as a starting point for the policy runs) the model converged to near-equilibrium (lumpiness prevents perfect equilibrium in this case) within two or three runs of forty to fifty iterations each. Without the vacant land mechanism included, fifty iterations generally took 25 to 40 seconds of machine time, the cost of a run (given that compilation was done beforehand) being in the neighborhood of ten dollars. It is far from being an expensive planning tool, then, except for any expenses that are incurred in the data collection and estimation.

RESULTS OF THE BASE RUN

The results of the base run are not perfect, but they are good enough to impart some validity to the form and operation of the model. The fit was very good on the macro scale. The total number of households in the model was 838053; the model allocated 838214 houses to these households, allowing 9181 to go vacant. Given sufficient time, the allocation procedure probably could have gotten an even closer fit, but experience shows that the location and rent patterns would only be minutely changed. As far as individual strata were concerned, the model varies in its capability to handle them efficiently. For large strata, such as 111 (46336 households) or 125 (48072 households), the lumpiness of the model is not a problem. In fact, the model allocations of 46437 and 47645 houses represent only .2 and .8 percent errors respectively. However, for the smaller, generally black, strata, there are much larger errors. For instance, stratum 243 (939 households) was allocated 1554 units, while 265 (1227 households) was allocated 778 units. The explanation for this phenomenon is rather simple, yet its remedy is rather obscure. Most of the zones in this model have upwards of one or two thousand units in them. Therefore, if a black stratum is only slightly short of having enough houses, and revises its utilities only enough to capture part of one extra zone, this still may result in the addition of five hundred or so houses, which would mean excess supply of 50%, while it would mean only a few tenths of one percent for a stratum such as 111. However, without dramatically increasing the costs of the model by increasing the number and thereby decreasing the size of zones, it is difficult to see how this problem can be averted.

The location pattern defined by the model is quite reasonable and gives a fair reflection of reality, again within the severe limitations imposed by the aggregation both in the estimation and simulation. Five zones, representing 6260 units, went completely vacant, while eight other zones (2921 units) went partially vacant (that is, the maximum rent was between $860 and $940). The vacant zones were all in what would be termed central Boston -- one in the Central Business District (a negligible number of units), one in the South End, two in the Huntington Avenue-Symphony Hall area, and one in the Massachusetts Avenue-Harrison Avenue part of Roxbury (to be specific, zones 25, 32, 44, 45, and 57). All of these zones were characterized by a very small mean number of rooms, small mean lot size, and a high percentage of bad plumbing. When comparing these results with the existing reality, it is important to remember that a good area and a poor area are represented in the model as a mediocre area, and that we therefore might not find that which we expect.

Partially vacant areas tend to show the same characteristics, zones 54 and 56 flanking zone 57 in Roxbury, while 29 (200 units) represents the area around the financial district. Three of the partially vacant zones -- 11 (adjacent to Logan Airport), 58 (industrial part of South Boston), and 432 (Chelsea) -- all have both poor plumbing (indicating a general deterioration of the housing stock) and a high percentage of land in a noxious category. The two partially vacant districts in Lynn (194, 195) are similar to the former in their small rooms and low lotsize averages.

Before proceeding with the locational analysis, it is necessary first to issue some sweeping generalities. The most important is that overall, the more representative the averages are of the entire zone, the more likely it is that the model will reproduce reality. It is this fact of life which injected some large discrepancies into our model. In the data set used for this model, the neighborhood characteristics dealing with governmental functions (specifically CRIMEEXP, PUPIL RATIO, and PROPTAX) were available only by city or town. In this matter, the city of Boston is treated as a single entity, and we have only one number for each variable, and that number is the same for all of Boston's 156 census tracts. While this may have introduced some bias into the estimations, it seems to be much more important when trying to determine the locational pattern of groups within the city. To a certain extent, this problem affects many of the other cities and towns, particularly Cambridge (30 tracts), Lynn (19 tracts), and Quincy (11 tracts). Even for towns with only one tract, if the town is large and the actual pattern of expenditures is variable, there will be quite a discrepancy. Again, it is simply a question of how the variances within zones compare to the variances among zones.

When we separate the two largest towns, Boston and Cambridge, from the rest of the SMSA, we find strong evidence for the above argument. The base run was compared to the actuality by obtaining a listing of the ten most heavily represented strata in each zone, and then seeing how well the model predicted these strata. In general, the model does better in the non-Boston-Cambridge (herein referred to as non-central) zones than it does in the central zones. Of the most populous stratum in each zone, the model successfully predicts their presence in 33 of the 185 central zones, and 83 of the 251 non-central zones, or 17.8% and 33.1%. For predicting either the first of the second most populous strata, the figures are 25.9% for the central and 46.6% for the non-central. Finally, the percentage of zones in which the model predicted none of the top five strata was 48.6% for the central and 34.2% for the non-central. This is significant only in that it points up the difficulties inherent in using greatly aggregated data and trying to predict fairly disaggregate behavior from it.

Perhaps the best way to examine the fit with reality is to look at specific areas, see how well the model predicts the actual location pattern, and attempt to understand why it does or does not agree. A good place to start is with East Boston, which is represented by zones 1-12. Here we have a good fit in zones 2, 3, 4, 5, and 8, and a rather poor fit in the others. One reason for the poor agreement is that the model places large numbers of blacks, specifically strata 211, 223, 224, 234, and 264 (that is, fairly low income blacks) in this area, particularly in those areas that are high in noxious land use due to the presence of various Massport facilities. It is not unreasonable to suspent that, given a simultaneous relocation of all households in the metropolitan area, East Boston might well become a black ghetto, if our measurement of the utility parameters may be believed. If we look at East Boston as a whole, we see that the model predicted strata 125, 135, and 145 in fairly large numbers and, while they might not agree on the zonal level, there are certainly large numbers of these household types in the general area. This leads toward the conclusion that aside from questions of race, which will be discussed later, the model does a good job of predicting location on a multiple-tract level.

In South Boston, another area interesting to look at because of its easy physical definition, we find similar behavior. In this case, I would say that we got a reasonable fit in nine of the twelve tracts. On the area level, the fit was very good -- in reality, the most predominant strata in this area are 111, 112, 114, 125, 126, 136, and others of similar size and income range. The model predicts a majority of the households will be 111, 115, 125 and 126. The worst fit of any zone in South Boston is zone 60, for which the model predicts five household types, none of whom are in the top ten in actuality. It is not clear exactly what causes this, but the very low age variable for this zone leads me to suspect that there is a low-income housing project in that area, which would bias the income range of the residents downward from what the equilibrium solution would predict.

Some areas, even those within the central area, are predicted very well by the model. For instance, an area in North Dorchester, defined as zones 72-74, 77, and 92-96, fits well in all but one zone. It is especially interesting in that both the real and predicted situations are mixes of low-income (111) and higher income, larger (135, 136, 145, 146) strata. In fact, in four of these nine zones, the model predicts the most populous stratum, a most impressive average.

As mentioned earlier, the model tends to predict better in the smaller, richer (and perhaps more homogeneous?) suburbs. For instance, in the ten zones (372-381) that make up the towns of Needham, Dedham, Westwood and Dover, the model predicts the first or second most populous stratum in eight of them. In Weston, the model predicts three of the top four strata, and does not put any strata into the zone which are <u>not</u> there in reality. This last is an important point. Through any number of processes or random occurrences, many strata may decide to locate in a particular zone, whether or not it is an equilibrium solution. However, one would think that if a stratum elects <u>not</u> to locate in a particular zone, that zone is dominated by other choices. In such a situation, our model should also not locate that stratum in that zone. It is understandbale why it fails this test in many places, such as in Dedham, where it locates strata 123 and 124 in two zones each, while in reality they are in the top ten in only one zone each. Due to the uniformity of the neighborhood variables over the town, it is difficult to differentiate one Dedham zone from another. Since travel costs are also figured by town, we lose 40% of the variability when we get down to the tract level, and maybe even more in terms of explanatory power.

A WORD ABOUT RENTS

Up to this point, I have discussed almost entirely locational patterns produced by that model. Let me depart for a while from my locational wanderings and interject a brief word about rents. For each zone in the model, the maximum bid-rent, which is in terms of an equivalent annual rent, is output. When I compare these rents to the mean rents for the zones in the real world, I am surprised to find that in many areas the fit is very good, and in some it is absolutely incredible.

First, two generalizations:

1) The model predicts the rent structure better in rich communities than in poor communities.

2) The model predicts rents better outside of Boston than within Boston.

Now, let me try to support these sweeping generalizations with facts. The Table on the following page, which details rents for the twelve highest-rent zones in the real world, is supportive of the first statement. We see that the zones which are ranked 1, 2, 3 in the real world are ranked 3, 1, 2

BASE RUN RENTS

ZONE	ACTUAL RENT	ACTUAL RANK	MODEL RENT	MODEL RANK
343	$ 6516,93	1	$ 4308.49	3
381	6177.19	2	5413.56	1
366	5628.49	3	4391.00	2
230	5544.72	4	3890.27	9
306	5516.52	5	3635.09	12
322	5338.92	6	4302.16	4
341	5270.63	7	3614.28	13
325	5254.16	8	3997.18	7
369	5195.45	9	3946.38	8
354	4777.69	10	4018.73	6
326	4538.99	11	4121.74	5
392	4157.28	12	3723.57	11

by the model, an amazingly close correspondence given the major assumptions inherent in this model. We also see that the twelve stratum are among the thirteen highest in the model, although the one-to-one correspondence of rank order is not quite as good. On the other side of the coin, we have the low rent zones, those overage, rundown hovels which offend the sensibilities of all strata. Of the top thirteen lowest-renting zones in reality (all but one of which are in Boston), the model predicts exactly none. But wait, one says, could this not be due to the fact that the data for Boston is very aggregate, and that what we are showing here is actually a result of the second assumption? To test this, I looked at the lowest-renting non-Boston zones. Of the top ten of these, the model predicts only five. We see, then, that the model predicts the richer rents much better than the poorer ones. We also see that the second assumption is true, since within Boston the model has difficulty predicting relative rent levels even among the richer zones.

There are some areas that were predicted so well that they deserve special mention here, whether to vindicate the model or just to show something positive. For instance, zones 380-386 are shown on the following page.

ZONE	ACTUAL RENT	MODEL RENT
380	$ 3627.13	$ 3607.05
381	6177.19	5413.56
382	3394.15	3599.45
383	2464.15	2497.29
384	2676.56	2678.99
385	2449.25	2393.70
386	2111.46	2112.87

There is no clear pattern that indicates why these particular towns should fit so well, however, so I shall have to stick with my original conclusions. Not much more can be said about the rent pattern; clearly, the reservation price has much to do with both the level of rents and the relations thereof, and since it was arbitrarily chosen, there is little to be gained from extracting further conclusions.

In summary, though, one would have to say that the response of the model to reality is quite good. On the tract level there is considerable discrepancy, but on the town or BRA district level, it does quite well. Many of the difficulties are with the data, and could be resolved without violating the integrity of the model framework. It certainly fits well enough that we can trust it to correctly indicate the direction and relative magnitude of the impact that various policies, either stock- or household-oriented, might have. Before I go on to discuss the various policy simulations, however, I believe that a fairly detailed examination of the reasons that the model deviates from reality is in order, for only by understanding these can we competently analyze and understand the results of subsequent runs.

BASE RUN VERSUS REALITY - WHY ARE THEY DIFFERENT?

As mentioned at the very start of this paper, this model falls squarely in the realm of equilibrium models; it makes no pretense of simulating dynamic processes. This implies a number of basic assumptions about the nature of the housing market, none of which are inviolably true in the real world. The city is not at a static equilibrium; it is a dynamic system, in which only the smallest fraction of the potential decision-makers are in the market at any one time, and only a somewhat larger fraction even reevaluate their situations in any one period. The decision-maker, when making his choice among a number of housing packages, is choosing so as to maximize some expected utility function over time, since he expects to consume that housing package over a long period (this is more true in the owner market than the rental market, since the latter is more fluid). It is not at all clear that we can say that at any one point in time, then, present utility is maximized by the current consumption package. Rather, of the packages that faced the decision-maker at some time in the past, this package was the one that maximized his expected utility function. For instance, a young couple with two children might search for a five- or six-bedroom house in anticipation of another child, when a four-bedroom house would maximize their present utility.

However, this in itself does not present any conceptual difficulties. Since we are attempting to control for position in the life cycle (although granted that in the present data aggregation we are not), we could say that the choices revealed, and thus the utility function that we measure, is as expected utility function over time, and that households still act to maximize their utility or, in our formulation, make bids such as to equalize all expected utilities. This argument would be valid so long as the anticipated pattern of events for each household was the actual pattern. However, chance usually takes a hand, and alters events such that the utility maximizing bundle at time t_o is no longer the utility maximizing bundle at time $t_o + t$. Once again, so long as we make a certain assumption, this poses no difficulties. The assumption is that movement from one bundle to another is frictionless, or at worst such costs are not significant compared to the total utility function. In such a case, as expected utilities change, the location pattern will change, and the system will always be in equilibrium.

This does not seem to be the case in the real world, however. Moving costs can be very significant, or the household may not even be aware of the existence of better alternatives, that is, there are search costs involved. I stated earlier that these deviations from the optimal package would be accounted for by the tie range mechanism. This might be true if we had some a priori knowledge about the magnitude of these costs and what an appropriate tie range might be. In fact, increasing the tie range to $60 ($5 per month) would no doubt result in the model correctly locating many more strata. It would also, unfortunately, result in more mislocated strata, and without more information, we cannot really justify a particular value of the tie mechanism.

If these costs are not minimal, then many households will not be in their optimum packages. Due to the nature of an optimum, they can only deviate to the lower side. Therefore, when we estimate the utility functions, we might get biased estimates of the true parameters, that is, the ε's are not spherical. Tracing it through, then, we can see that an assumption of an equilibrium might lead to a model that produces very different results from the reality.

The model as postulated lacks many of the characteristics of the true housing market, some of which play a very important role in distorting our results. Some of these problems have been alluded to previously; they will be examined in more depth here.

A. As we saw earlier, the base run allocated almost as many blacks as whites to the zones in East Boston, while it left most areas in Roxbury almost completely white. It is clear why this has occurred -- there is no explicit segregation built into the model. To phrase this another way, we could say that there is positive utility to living in close proximity to similar people, and a strong disutility to dissimilar people, especially those of another race. Some of this might be implicit in the estimated parameters;

that is, the fact that blacks are relegated to high-density, low quality units might show up as a liking of such units, meaning that lot size, rooms, and similar normally positive-utility variables would have negative utility. In fact, the estimations showed that all but five of the twenty-three black strata had negative utility for LOTSIZE, while fourteen had positive utility for travel costs (which was uniformly negative for white strata). In general, the preponderance of wrong signs was in the black strata. This did not result in a segregated housing pattern. Rather, the tendency seemed to be that blacks of a certain income group would locate in neighborhoods dominated by whites of lower income groups. Poor blacks, however, were generally lumped together. It is clear that if we wanted to reproduce the actual housing pattern, we would have to introduce some concept of segregation, to limit the choices of certain subgroups.

A number of solutions present themselves, none of them very satisfactory. First, one could include in the utility function a measure of the percent of a stratum's representation in the tract; and then use such a measure in the simulations. The major problem with this is that with each iteration, the makeup of the tract is different, and each stratum would thus have changing utility for the same zone.

Clearly, this would make the equilibrium solution dependent on both the starting point and the path used to approach it. Since we have admitted that the iterations have no real-world counterparts, but are merely a means to an end, having that end dependent on the means is clearly undesirable.

A second approach would be to assert that there is some constant monetary disutility associated with blacks. For instance, if we could say that blacks are worth $10 per month less than whites, we could subtract $120 from all black bids. This is a rather simplistic view, and avoids the argument that different areas discriminate to different extents against different ethnic groups. Any attempt to include these last considerations would, under any plan that I could envision, lead to a predetermination of the results, thus rendering the model insignificant. Before we could include a phenomenon as complex as explicit segregation, then, considerable study would be called for.

B. A problem with the data rather than the structure of the model is that of using census tract means. I strongly pointed out in the beginning of this paper that housing is among the most heterogeneous of goods available on the market. How, then, can we assume that the one to eight thousand housing units in a census tract may be adequately represented by the means of that tract? Also, as pointed out before, if we average a very fine housing unit and a very poor housing unit, we will get two very mediocre housing units. If the poor unit would have been occupied by stratum 111, and the good one by stratum 168, we will not get these same strata locating in the averages, but maybe two households of type 134. This is a major reason why the model fits the reality no better than it does, and is also a reason why the more homogeneous suburban zones are predicted better than the heterogeneous urban zones.

The necessity for using means (or some other 'representative' value) might seem to invalidate the model if we claim that heterogeneity is the order of the day. However this problem was more than adequately solved with a concept espoused in the Arthur D. Little study of the San Francisco area. This is the concept of a "fract", a homogeneous group of houses located within the borders of a heterogeneous zone. The fract has no particular location within the zone, it is simply the group of all similar housing types. Breaking the 436 zones of the model into 1500 or so fracts would, while significantly increasing the cost, also overwhelmingly increase the predictive power thereof. Costs of data acquisition would be quite high, however, since one would either have to resort to original Census data or construct a survey especially for the study. Carried to an extreme, the fracts could be individual housing units, and the ultimate in disaggregation would have been purchased, albeit at very great cost. Some work in this direction, however, does seem to be very desirable.

C. Aggregation of the strata seems to be at least as big a problem as that of the census means. By saying that all white, single persons earning 10 to 14 thousand dollars a year will behave similarly is an heroic assumption. Additionally, we have no household age variable, and we therefore lose more valuable life-cycle differentiation. Lumping many dissimilar households in to the same stratum would lead to estimations that produce very questionable parameters, since we would be trying to fit one curve to points that lie on many different curves. For instance, we have only three income categories for the entire $10,000 plus bracket, and yet it is very difficult to believe that there are only three types of preference above this figure. Once again, we are faced with an economic decision -- the formulation of the model is good in theory, but we are forced to resort to unnatural levels of aggregation in order to hold down costs. It would be preferable to increase the number of strata, but it is necessary to have a degree of aggregation sufficient to allow meaningful estimations. The aggregation which we have used might very well lead to differences between model and reality, since we may be trying to predict the behavior of a group that does not behave as a group.

An associated problem is the question of whether the observed consumption patterns represent revealed preferences over all possible alternatives, or merely over a limited submarket. This returns us to our argument about segregation and other market imprefections, in that if a decision-maker's choice set is limited to certain packages, we cannot say anything about his preferences in relation to opportunities outside that choice set. Making the assumption that the choice set consists of all available alternatives is equivalent to reinforcing the existing location pattern, since it says that the presently consumed unit is preferable to <u>all</u> other packages, while in reality it may be the optimum only within a certain available subset. This type of bias will not be so evident in our model, since it would tend to produce a model solution very close to the actual solution, rather than causing divergence. It is also unclear how one could define the available choice sets without again predetermining the outcome of the model.

D. A large part of the diversity of the housing market has not even been included in the model. This includes such features as the myriad architectural characteristics of the unit, the "status" of the neighborhood, microclimate, and dozens of other features that are either unquantifiable or would cost too much to include in the model. Most of these features were deliberately left out; what the model tries to do is predict general trends of locational behavior given as few salient characteristics of the housing stock as possible. However, the abovementioned qualities go a long way towards explaining both prices of housing and locational behavior on the individual scale, and such non-normal disturbances as a "high-status" neighborhood could easily bias the results.

Finally, there is another question of data which has not been treated before. For each zone, only one accessibility measure is computed. However, it should be fairly obvious that accessibility for a rich family with two cars is not the same as for a poor individual with no car. This is not a question of how much the household values accessibility, it is more a question of how to define it. Similarly, accessibility from Newton is not the same for individuals who work in Newton as for those who work downtown. If we assume that employment among people in a particular stratum is distributed the same as that in the parent population, then we can use one accessibility measure for all stratum, at least as far as concerns this particular problem. As long as we are assuming that employment is distributed independently of housing, it might be interesting to see the effects of using a different accessibility matrix for each stratum, based on both the modal split common to each stratum and its particular distribution of employment. Once again, it is somewhat unclear what kind of bias would be introduced into the model by using such an aggregate accessibility measure.

I have, in these pages, pointed up a number of imperfections that exist and compromises that were made in the model. In light of these faults, is this model useful for anything, and can we draw any meaningful conclusions from it? My answer is an emphatic yes. Simplifications have been made, but few of these simplifications actually disrupt the theory behind the model, rather, they are made for cost considerations. If we accept the results of the base run as our test city, and apply policies to that city, we should get the same sort of results as if we were to apply these same policies to the Boston SMSA, holding all factors not in the model constant. What the model gives us is a laboratory in which we can assess the impacts of social actions at low cost and *ceteris paribus*. This is what we want to know - what is the impact of a particular policy, not what will be the state of the city twenty years from now. The former is controllable by the policy maker, the latter is a sum of both policies and random influences. Since this model is an equilibrium model, and the city is rarely allowed to reach equilibrium after a particular perturbation, we will *not* be able to predict total system responses. We *can* however, make meaningful statements about the direction and magnitude of impacts on the urban pattern of policies, independent of other effects. It is in this spirit that we proceed to the next section, in which we analyze a number of possible market interventions.

POLICY RUNS - ALTERATIONS IN THE HOUSING STOCK

In this section, I begin to examine what can be done with this model of the urban residential location decision in terms of testing various public and private policies. This chapter is devoted to an examination of two policies that deal exclusively with alterations or additions to the existing housing stock, seeking to discover the market response. The two strategies that were tried were renovation of the inner-city housing stock and construction of low-income housing in the suburban towns.

URBAN RENEWAL

For much of the past decade-and-a-half, the bulldozer and crane, used on a massive scale, have been important tools in the planner's repertoire. The prevailing view was that the only way to save the cities is by upgrading the housing stock in the downtown area and thereby attract a 'higher class' of people back into the city. Naturally, this view is contingent on viewing the city as more than a daytime place to transact business. However, "urban renewal" often came to be an obscene word when it referred to wholesale destruction of inner-city areas and the subsequent dislocation of thousands of families who could ill-afford to move.

What I have attempted to simulate in this policy run is a sane renewal program consisting of both construction of new high-rise, luxury buildings and renovation of many existing units. Moderation has two aims -- the social aim of not uprooting an impossible number of people and thereby creating more ill than good, and the modelling aim of being able to trace the impact of the policy (that is, not have too many factors changing at once). In fact, some of the changes that were made in the housing stock are changes that are either currently occurring or are slated to occur in the near future.

A brief verbal discussion of the changes being made is important here, although a complete listing can be found on the page following. We slated only three areas for high-rise, luxury apartments. These are zone 31 (southwest of Boston Common, future home of part of Park Plaza), zone 33 (currently the home of Charles River Park, some of which was not included in the base run) and zone 44 (along Massachusetts Avenue at the site of the Christian Science development). These units were designed with small lot sizes, exceptional plumbing, a fairly good room size for the city, and relatively low mean ages (since the original estimations only used ages between 30 and 50 years in general, it was felt that the

RENOVATION OF THE HOUSING STOCK

NEW ZONE (OLD ZONE)	AGE	ROOMS	LOTSIZE	PLUMBING	# OF UNITS
6	40.0 (49.999)	5.399	.021	.002 (.139)	907
8	40.0 (48.122)	5.357	.019	.002 (.141)	587
11	40.0 (48.916)	5.103	.016	.002 (.337)	577
12	40.0 (48.147)	5.507	.018	.002 (.152)	1026
31	20.0 (34.498)	6.0 (2.803)	.005 (.002)	.002 (.109)	1200 (73)
32	37.0 (47.435)	4.5 (3.541)	.01 (.001)	.02 (.166)	265 (535)
33	20.0 (49.999)	6.0 (3.507)	.005 (.016)	.002 (.369)	1200 (58)
44	30.0 (48.042)	6.0 (3.909)	.005	.002 (.192)	1345
45	35.0 (47.911)	6.0 (3.485)	.02 (.005)	.002 (.082)	3000 (4131)
49	40.0 (47.834)	5.0 (3.602)	.01 (.003)	.002 (.146)	1800 (2.35)
50	42.0 (48.027)	3.455	.006	.02 (.048)	4527
53	40.0 (49.545)	4.483	.006	.02 (.232)	1124
54	40.0 (49.183)	4.5 (4.139)	.007	.02 (.291)	946
56	35.0 (49.324)	5.0 (3.194)	.015 (.011)	.02 (.459)	280 (461)
57	35.0 (49.999)	5.5 (3.649)	.01 (.004)	.02 (.376)	250 (396)
58	40.0 (49.999)	5.636	.019	.02 (.137)	371

introduction of units with ages of less than 20 years would introduce severe distortions into the model because of large prediction errors in that range). The other changes in the housing stock, also in or around the downtown area, consist mainly of renovation and facelifting. The zones selected were those that commanded rents of less than $1000 in the base run and had a significant number of units (thus eliminating the CBD zones 25 and 29). For instance, four zones in East Boston (6, 8, 11 and 12) were characterized by bad plumbing (in excess of 10% substandard). Renovation was simulated by fixing up all the plumbing and trimming the mean ages by nine to ten years. This latter move, which was done to varying extent in other zones, was used as a proxy for general improvement in the condition of the unit. Zones 50 (Fenway), 53 (Massachusetts and Columbia Avenues) and 58 (South Boston) were treated the same way. Finally, zones 45 and 49 (Back Bay-Fenway) and 54, 56, and 57 (southern part of South End) were more extensively renovated. Plumbing was improved, general renovation was simulated through age reductions, and both mean rooms and mean lot-size were increased. This last would simulate either the replacement of old buildings with new, somewhat roomier structures or merely the interior conversion of some existing structures. In total, the renewal and renovating project resulted in an increase of 406 units in the housing stock, about .05% of the base run stock.

When designing an experiment such as has been described, we no doubt have certain a priori expectations about the outcome, as must planners who implement such programs (with no allowance for error as we have in a model). A project is designed to achieve certain goals, and the designer should have an idea of how well he can meet these goals. Therefore, I shall present a short discussion of the results that we could expect from our experiment, given normal market response. We can then compare the results of the model to these expectations, and thus determine simultaneously how well the model reproduces these forces and how intuitive or counterintuitive the results are.

The easiest impact to determine is that of the high-rise development. From the base run, it can be observed that mean age is a variable with great leverage, that is, low values have a very high impact on rents. We can also see that the small room size (relative to the suburban tracts) will be insufficient to attract large families. We would therefore suspect, from these observations and from prior experience with such development, that these developments will attract the smaller, richer strata. Since 117 and 118 are not in the model, we would expect to find 127 and 128, or possibly 137 and 138, if we assume that these are young professionals just starting families, who will probably move to larger quarters as their families grow.

The other areas are not so clear, since the changes being made are not nearly so sweeping. For instance, despite the renovation in East Boston, there is still Logan Airport and the other noxious land uses to cope with. The units also remain rather small in terms of mean rooms. We would not expect the richest strata, then, to locate in these units, but rather middle-income, mid-size households, such as strata 125, 135, 136, 236, et cetera. In zones 32, 49, and 56-58, we have the same type renovation, and would expect generally the same results. Zones 50, 53, and 54 have very small rooms and lotsize, but also good plumbing (although not excellent), so we might expect some of the smaller, middle-income strata to locate there (i.e. 115, 124, 125). Finally, zones 45 and 49, being somewhat larger but also somewhat older than the ones just mentioned, would probably have somewhat larger households than the former, but the income spread would probably not be very great.

Determining the secondary impacts on location is much more difficult, and will not be attempted here. However, we can say something about the expected impact on the rent structure and the utility levels. The effect of the stock alterations will be to increase the supply of units at many higher "quality" levels (if we may for a moment think of housing as measurable unidimensionally in "quality"), and decrease the stock at the very lowest levels. Therefore, most strata inhabiting the higher levels will have to bid less to command the same level of quality. With everything else constant, those strata commanding the low quality units would have to pay more, and their utility should decline. However, by renovating the units that we have, we have removed from the stock those units that provided the numeraire; in fact, the rent structure would have to drop quite a bit just to resatisfy the requirement that the marginal unit rent for the reservation price. This drop in the rent structure would partially or totally offset the decrease in supply at the low quality levels, and thus we would expect the rents on all units (save for the converted or rebuilt ones) to be lower and the utilities of most strata to be higher or about the same.

The renovation run was started with the final utilities from the base run, and reached a fairly good equilibrium within 100 iterations. The results were good; in most cases predictable, in some cases very surprising. The most important are reported here.

As I said above, the effects of the high-rise development should be easiest to forecast, and the results bear out that statement. Typical of urban renewal, the rich moved in and the poor moved out. In all three high-rise zones, stratum 128 was the high bidder. Surprisingly, stratum 226 also appeared in the Fenway zone, but due to its small size (1274 households) and the fact that the model only located 672 of them, it is difficult to draw any general conclusions about the location pattern. It is also interesting to note that while rents in 31 and 33 were only separated by $374, due to the indifference in LANDUSE1, zone 44 was $1250 less than the lower of these two, which shows the tremendous importance of age and accessibility, the only two variables on which 31 and 45 significantly differ.

As predicted, the renovations were clearly dominated by the middle-income and mid-size households. First, let us look at East Boston. The renovated tracts were upgraded in both income and family size, but also showed a tendency to become black. For instance, tract 6, previously resided in by strata 125, 126 and 224, now is captured by 236 and 266. These same strata, with the addition of 246, pushed the former (plus 115) out of zone 8. Again, these same strata bid highest on zones 11 and 12, but here they merely superseded poorer blacks, namely 224 and 234. In zones 32, 49, and 56-58, which were mentioned earlier as being renovated in a similar fashion, we find the white, middle-income strata dominant, specifically 115, 124, 125, 135, 145, and 165. Surprisingly, though, we find 111 and 121 also making themselves at home in 56 and 57, possibly owing to the small rooms. Generally, the ousted strata were black (214, 215, 234) but 121 and 126 also declined the new quarters. Stratum 124 also staked a claim on the renewed units of zone 45, which had been vacant before. This is not surprising, since aside from the larger rooms, this tract does not differ much from the preceeding five, where we also find 124. Finally, we have zones 50, 53 and 54, those zones which were renovated but still allowed to remain primarily small units. Zones 53 and 54 went exclusively to stratum 121, while 50 was split between 121 and 215. Displaced from 53 and 54 were 111, 211, and 115. We can see that there is not much difference between those that moved out and those that moved in. In fact, the only important difference is in the household size, which increased due to the small increase in unit size.

Predictions about the rent structure and utility levels were also borne out, with somewhat surprising results. Rents throughout the SMSA dropped dramatically. Even in areas such as Weston, Wayland, and Sudbury, where the location pattern was completely unaffected, rents dropped by approximately $60. Closely substitutable tracts, that is, tracts at about the same "quality level" as the newly converted zones, seem to have the widest swings in the rents, as for example zone 95, which rented for $1393 before the renovation and only $1300 afterward. Zone 95 is fairly similar to the 56-58 group, so that the increase in supply in that area tended to drive prices down.

As predicted, utilities of all groups increased. However, the manner in which they increased far exceeded expectations. Evidently, the effect of the general lowering of rents far outweighed any added pressures on the market at the lower end. While utilities for most of the strata increased maybe 5% at the most, the utilities of strata 111, 121, 211, and 221 increased 23, 21, 12, and 72%. Strata 124 and 125, who were directly affected by the renovations, increased utilities only 3 and 8%, respectively. However, the structure of our model renders this result not quite so surprising. Working with the equations $U = M X_i^{a_i}$ and $M = Y-R$, we can easily show that the elasticity of utility with respect to rent, $(dU/U)/(dr/R)$, is equal to $R/(R-Y)$. Therefore, our low income groups, for whom rent is a large part of income, will have very high negative elasticities, and a small percent change in the rent structure will have a substantial effect on the utility levels. As further proof, we can look at stratum 128 which, while being one of the strata most involved in the changes, experiences only a .2% rise in utility.

It is difficult to generalize about the secondary locational effects in this run. For instance, stratum 128, which moved into the three new high-rise tracts, deserted three other tracts to keep the supply constant. However, the three tracts it deserted have very little in common. Tract 19 consists of mid-size houses of fairly considerable age in Charlestown, of mediocre condition but good accessibility. Zone 246 is similar in structural characteristics, but its lower tax rate and crime expenditures make up for the somewhat decreased accessibility of its Somerville location. The third zone is most unlike the other two, in that it is a rich, large-rooms large-lotsize tract in Newton. This zone (325) illustrates the heterogeneous nature of the housing good and the willingness of strata to pay more for more housing services. However, it also points up how difficult it is to predict which zones will be deserted by which strata, since the shape of the bid-rent curve is so complex. Therefore, I will attempt to point out the more interesting outcomes, but will not draw many general conclusions.

Some zones, not in the renovation plan, underwent extreme changes. For instance, zone 95, a Dorchester zone that was formerly the home of stratum 111, is now in addition the residence of middle-income strata 125, 126, 136, 145, and 146. However, since this was an area attractive to these groups in the base run (they located in 92, 93, 96, and 98; all are adjacent to 95) we should not be surprised that a small change in the rent structure would attract them into this additional zone. Most of the changes in the suburban communities were similar; strata entering or deserting zones very similar to ones they were already in. That is, most of these locational changes are more significant as adjustments within the model than indicative of changing preferences. We must note the behavior of stratum 211, though, which when evicted by urban renewal from zones 53 and 54, relocated in 23 and 26, which it shares with the illustrious company of strata 128. The reason for this is fairly clear. Since we did not restrict the strata in our model to those with "correct" signs from the estimations, wrong signs, such as those for plumbing for strata 211 and 128, were allowed to operate. Zones 23 and 26 have nearly 50% substandard plumbing, and are thus "attractive" to these strata. This points up the need for more disaggregate data, since it is difficult to believe that such a rich stratum as 128 would reveal a preference for bad plumbing.

What have we learned from this experiment? About 19000 housing units in the base run have been replaced or altered, comprising about 2% of the housing stock. The renovations clearly had a large impact on the concerned zones in terms of location, and significantly lowered the general rent profile. Perhaps most significantly, the secondary effects of the rent decrease raised the utility of those strata who need it most in many opinions, that is, the poorest strata. Secondary locational effects were not extensive, and those relocations that did occur were generally among similar strata. We could conclude then, that a moderate renewal plan combining both new construction and renovation, would have positive effects on the urban residential market.

LOW-INCOME HOUSING IN THE SUBURBS

Slum conditions are generally conceded to be fostered by the proximity of low-quality housing, and low-quality housing is often thought to be the exclusive province of the poor. Therefore, if we can disperse the poor, wouldn't we be able to eliminate all the social ills associated with slums? This is the reasoning, whether right or wrong, that led me to conduct this experiment. The basic premise here was to establish housing in the suburbs that would be attractive to low income groups. Hopefully, the poor would be attracted to the suburbs, the slum-dwelling groups would be dissipated and therefore slum conditions would not be able to develop.

The zones that were selected for low-income development had to be suffuciently far from the center of the city that there would be vacant land available for development, yet hopefully not so far that they would be totally inaccessible. Accordingly, twenty zones just outside Route 128 were selected, and were developed according to the amount of residential land already there. A full listing of the zones used is shown on the next page, but we should note here that the new zone numbers are merely the corresponding original zone number plus 500; that is, zone 875 had the same neighborhood as zone 375 in the base run. This is an application of the fract concept mentioned earlier; zones 375 and 875 have the same geographical location but different housing characteristics.

Expectations for the results are not as positive as for the previous policy. To reflect the fact that these were new units, we used an age variable of 20 years. As pointed out before, age has considerable leverage, especially among the richer strata. Since the units are relatively small, we would not expect the larger strata to be attracted, but it seems reasonable to suspect that the small, wealthier strata will outbid the poor for these units, in light of the fact that we have placed no restrictions on who may inhabit the units. However, the addition of some 21000 houses to the stock will send all rents plummeting, no matter who inhabits those new units. To reestablish equilibrium, then, many of the worst tracts (which are generally inner-city tracts) will be vacated, and it is this second-level effect which may help to eliminate the slum conditions.

LOW-INCOME HOUSING ZONES

NE NUMBER	LOCATION	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
672	Peabody	33.049	.053	.058	26.118	25.103	.039
674	Lynnfield	34.056	.033	.092	22.198	20.718	.035
789	Reading	33.549	.048	.026	24.777	29.049	.038
792	Woburn	32.201	.212	.012	24.863	19.886	.047
835	Lexington	32.105	.036	.048	20.455	23.999	.050
839	Burlington	34.918	.089	.004	22.504	23.266	.056
840	Bedford	40.326	.018	.006	21.567	20.989	.048
841	Lincoln	32.753	.005	.014	24.229	15.879	.026
843	Weston	29.518	.022	.07	20.142	34.501	.032
870	Wellesley	25.687	.013	.033	19.928	34.056	.038
875	Needham	26.843	.024	.028	22.214	37.338	.045
876	Needham	26.843	.044	.018	22.214	37.338	.045
878	Dedham	27.771	.012	.028	23.18	32.95	.044
880	Westwood	30.265	.041	.047	18.093	35.163	.040
885	Norwood	33.852	.182	.014	22.614	26.443	.038
888	Canton	33.049	.064	.145	23.285	25.79	.045
892	Milton	29.725	.013	.339	21.887	39.252	.034
910	Braintree	31.375	.085	.01	21.085	28.818	.048
921	Hingham	49.156	.049	.014	22.810	40.246	.049

L ZONES:

AGE: 20.0 LOTSIZE: .100

ROOMS: 5.5 PLUMBING: .100

Again, the run was started with the final utilities from the base run, and reached a reasonable equilibrium within three 50-iteration runs. Results were mostly predictable, but interesting nonetheless (much of the predictability seems to stem from an intense familiarity with the workings and limitations of the model, rather than any omniscient understanding of market forces).

The outcome of the run ran true to our forecast. Strata 125, 126 and 127 thoroughly dominated the new housing, accounting for all but 3273 of the 21004 new units (the others went to 136, 165, and 235). Clearly, the poor were not attracted to these units; in fact, only three of the new units rent for less than $1500, which does not make them prime targets for poor residents.

Secondary effects of this policy were significant, however. Many of the zones that were marginal in the base run went vacant due to the excess supply. Newly vacant zones were 11 and 12 (East Boston), 29, 30 and 31 (South End), and 58 (South Boston). Three zones in Lynn (187, 194 and 195) also went vacant, as did one in Chelsea (432). Generally, these zones were vacated by poor or black strata such as 111, 121, 211, 214, 224 and 234. We might say, then, that the upgrading in the suburbs caused a "filtering" downward of the good stock and thereby a general upgrading of housing.

Rents in this run fell considerably more than in the previous one, most drops being in the $100 to $140 range. This more than anything else helps to increase the utility of the various strata, such as 221, which experienced a 170% rise in its utility level. No matter what we feel about utility measures, this is certainly a significant rise.

There were some changes in the locational pattern, especially within Boston, that were very interesting. East Boston continued the trend towards black dominance, as the middle-income white strata took advantage of lower rent levels to vacate, allowing more blacks to move in (although 224 and 234 saw fit to leave zones 11 and 12 completely vacant). In fact, East Boston seems to be one of the least desirable areas in the model, with two of its twelve zones vacant and four others partially vacant. Contrast this with Roxbury, which, while the target of many social reformers, has only two zones out of twenty-one renting for less than $1400, and none for less than $1100. Of course, much of this is due no doubt to the lack of differentiation among the neighborhood variables, but it also points up the surprising quality of some "ghetto" housing.

Some of the filtering of the housing stock manifested itself in a flight to the suburbs of the middle class. For instance, many of those in strata 136, 146 and 156, who located in Roxbury, South Dorchester and Hyde Park wound up in the more suburban zones of Watertown, Waltham and Woburn. Richer strata, such as 137, 147 and 157, moved further out, going from Melrose and Belmont to Wellesley and Milton.

Both of the housing stock adjustment policies resulted in benefits for all strata in the model. This is not surprising, as relieving some of the demand pressure is bound to be beneficial. However, there are effects which the model does not foresee. First, it assumes that there is no response to demand other than that which we stipulate in our policies. That is, the equilibrium that the model assumes is a short-run equilibrium in which there is no supply response such as new construction or conversion, and the only adjustments that occur in the market occur on the demand side. Unfortunately, this may mask some of the aftereffects of our policies. For instance, we have seen that rents drop precipitously after the introduction of either of our policies, but especially the second. It is possible that such a change in the income stream could significantly alter investment decisions among home-builders, and thus have an unexpected long-run effect on the city. Similarly, implementation of the policies that we have proposed might, rather than augmenting the housing stock, merely replace private initiative with public programs. Private developers who might step in to fill the need for replacement housing (which is one way of reading a dynamic aspect into a static model) might be deterred by the public program, for fear of glutting the market, and thus we would be left no better off than before. These and other secondary effects, which could be better handled in a dynamic model, must be assumed to be of negligible importance here.

We can, however, make some important generalized conclusions from these two runs. First, due to the large marginal utility of income to those at the low end of the earnings scale, any relief on the housing market that results in a general decrease in rents, whether it starts at the high or low end, will be reflected in a significant increase in utility for those strata. Second, we can say that small-scale perturbations of the housing stock only have limited locational implications, especially as we become more and more removed from the source. Many of the moves that we noted may be due to the fact that the model did not reach a perfect equilibrium in either run, and that the final pattern is therefore a function of the particular iteration as well as the sum of all the market forces. We knew beforehand that increasing the supply would lead to lower prices, as in elementary economic theory; now let us move on and observe what happens when we alter the pressures on the demand side without affecting supply.

POLICY RUNS - TRANSFER PAYMENTS

An oft-espoused method for lifting the poor out of their housing miseries is the transfer payment, that is, give them a certain amount of money and let them spend it to purchase more in the way of housing services. This transfer payment can take one of two general forms: no strings attached, or strings attached. In the former, the recipient gets a lump sum that is in no way tied to his expenditures on housing; he can spend the money any way he wishes. The latter restricts the money in some way such that the amount of the transfer is dependent on the way it is spent.

The former method is clearly the most efficient, especially in terms of a utility-maximizing model, since it allows the recipient, by varying expenditures on all commodities as he sees fit, to purchase the optimum bundle (in his view) for a given amount of money. The latter method implicitly states that the policy maker's goals are necessarily different than and preferable to those of the recipient, and therefore the recipient's behavior pattern should be altered by use of incentives to fit those goals. Since neither approach has yet achieved a clear-cut victory in the sociological battle, I have attempted to simulate each type of policy in the model, namely an income-allowance policy and a rent-subsidy policy. Each will be discussed in turn.

RENT SUBSIDY

The rent-subsidy is a policy of the latter type, that is, there are strings attached. What it amounts to is that the amount of the subsidy is in direct proportion to the rent on the unit. For this run, I elected to use only the two poorest income groups as subjects. Unlike many studies of rent subsidies, the structure of this model requires a 100% participation rate below a certain income level and 0% above it. This does not introduce any great theoretical complications.

It was decided that the government would directly subsidize the rents offered by these low income groups in all housing units, regardless of quality or location. For this run, the amount of the subsidy was fixed at 30% of the total rent (or alternatively, the subsidy was calculated as 43% of the stratum's bid-rent for a unit: $R+S=R'$, $S=.3R'=.3(R+S)$, $.7S=.3R$, $S=.43R$). Since we used the two lowest income groups, strata eligible for the program were 111, 112, 121, 122, 211, 212, 221, 222 and 231, or about one-eighth of the total population. Final utilities from the base run were again used, and equilibrium required about 150 iterations.

Let us again try to investigate the theoretical consequences of the policy we have implemented. Perhaps the most obvious implication is that by introducing more spending power into the economy while not simultaneously increasing the amount of goods available, prices will experience a general rise. Due to this price increase, the forty-five strata not participating in the rent-subsidy program will have their utilities lowered. The nine poor strata will all have their utilities raised, since the only way they could be lowered would be if the rent structure rose by more than 30%, which is impossible since it would require raising all incomes (or rents) by 30%.

Questions of location are more difficult to resolve, however. We can make some predictions about the behavior of bids. Let us look at the strata participating in the program. If we denote the base run rent and utility levels as R^e_{ij} and U^e_i, and the new equilibrium levels as R^n_{ij} and U^n_i, then it can be easily shown that

$$R^n_{ij} - R^e_{ij} = C_{1i} - C_{2i} \Pi_{ij} \qquad \text{where } C_{1i} = .43\, Y_i$$

$$C_{2i} = ((1.43U^n_i + U^e_i)$$

We can see, then, that a rent-subsidy program will increase bid-rents differentially, and the absolute differential will be greatest where π_{ij} is the smallest, or where the original bid was the largest. We can see that the bid rent functions of those strata in the program will become steeper, and we can expect to find them more concentrated in their preferred areas. The only problem is that we cannot identify those tracts for which a stratum offers its highest bid; we can find only those tracts on which it is the highest bidder. It thus becomes very difficult to determine beforehand who will locate where. We can say something about where certain strata will definitely decline to locate, however. For instance, we know that stratum 121 will increase its bids on its favored zones much more than on the less favored zones. It can also be seen on inspection that its bids for zones 29, 30 and 31 ($945, $1046, and $1074) are relatively low compared to the other tracts in which it is located. We do not therefore expect this bid to increase much; in fact, it might very well decrease. Since bids of non-subsidized strata are increasing, we would not expect to find 121 in these zones in the new equilibrium (and, to eliminate the suspense, we don't).

Further locational implications are very difficult to work out. Since the very poor strata will be vacating certain low-attractiveness zones, and we have done nothing to the supply side, we would expect that the next-poorest strata such as 113 or 213 would move in. Beyond this, the workings of the market become too complex, and we can probably learn more by moving right on to the results.

As expected, the poorer strata deserted the less attractive zones in droves. Stratum 111 totally vacated former strongholds in East Boston, Charlestown and the South End, while the four poor white strata (111, 112, 121, 122) all moved out of the northern halves of Roxbury and Dorchester. Startum 211 left its enclave in the South End to move to somewhat better quarters in Charlestown, the North End, and South Boston. Strata 111 and 121, having deserted the northern halves, begin to take over the southern parts of Roxbury and Dorchester, in general getting larger units and a less noxious land use mix. Many of the other moves within these poorest strata were moves within towns, for instance an upgrading within Lynn, which are more difficult to grasp intuitively, but which often represent an improvement in land use mix and unit size (especially number of rooms).

Strata 112 and 122 are ones that made extensive and impressive moves. Large numbers of them left South Boston, Dorchester and Roxbury, as well as such non-Boston areas as

Malden, Medford, Everett and Somerville. With the rent subsidy, they were able to move into much better areas, specifically Belmont, Newton, Brookline, Needham and Milton. Two of the black strata showed significant improvement in their situations, moving from small-room, bad plumbing zones in Cambridge (stratum 222 moved from zone 218) and Malden (212 moved from zone 276) to two very nice Brookline zones. Stratum 231 upgraded its situation without moving far in space, simply shifting from zone 246 to 244 in Somerville. Stratum 221, the last stratum in the program, did not command any zones in the final near-equilibrium, so I am unable to report on any moves.

These improvements were not without any cost, however. Although the less attractive units could be vacated by the above strata, they have to be occupied, since we have added nothing to the housing stock. Actually, some stayed vacant; no one moved into zones 29, 30 and 31 after stratum 121 moved out. On the other hand, zone 45, which was vacant in the base run, is now occupied by stratum 224. The make-up of the city changed in more noticeable ways than just the shifting of vacancies, however. Many strata which had successfully outbid the poor strata in the suburbs now had to compete in the city as well. We note especially that strata 145, 146, 125, 135 and 136 seem to be close competitors of 111. This is obvious in such zones as 72, which was held exclusively by 111 in the base run and was shared by all six of the aforementioned strata in the policy run. We note all through the South End-Roxbury-Dorchester area that the exodus of 111 is followed by an influx of white strata in the fifth and sixth income brackets, although appearances of the large (5 and 6) family sizes are rare. These results are most surprising (at least to this writer) in light of my earlier prediction that the new residents in these zones would be in the third and fourth income classes. Some zones were captured by these strata, but there does not seem to be any general trend as there was in the higher income groups. It seems almost as if 111 and 121 exchanged places with the six or eight middle class strata without seriously disturbing the locations of too many other strata. Most of the other relocations seem to be of the type encountered earlier, that is, only minor shifts due to the imperfect nature of the equilibrium solution.

As expected, rents rose almost across the board. The pattern was not at all generalizable, however, with rents rising as much as $450 in the West End or falling as much as $173 in the South End. In those areas which were unaffected by the policy (in terms of population turnover), the rents increased by about $120.

What can this run tell us? What policy decisions are indicated by the new location pattern? The first thing that we see is that for dispersing the poor population and returning the middle-class to the city, the rent subsidy is much more effective than attempting to build low-cost and low-class housing in the suburbs. The composition of Boston proper changed radically, from poor to middle-income, while the poor distributed themselves throughout the suburbs. As mentioned before, this is not a costless policy. The government, be it federal or local, must absorb the cost of the rent subsidy, which is substantial for a program involving 100,000 people. Also, the general increase in rents leaves the other 700,000 worse off than in the base run. It is no longer a question, as it was in the previous policies, of taxing everyone to finance a policy that leaves everyone better off. Here, the majority is left worse off in terms of utility, and it is up to the policy-maker to determine the values of the various tradeoffs involved. If our aims are to disperse the people who are generally associated with slum conditions while simultaneously revitalizing the city, regradless of the expense to others, then the results of this run indicate that this policy is highly desirable. However, if we wish to consider all the people, then evaluation of the results must wait for the development of a measure of social costs and benefits, which is beyond the scope of this paper.

INCOME ALLOWANCE

This is the no-strings type of income transfer policy. The incomes of the low income group are supplemented by a fixed, predetermined amount which can be spent as the participant sees fit. We see that by imposing no restrictions on this income, we allow the individual to maximize his utility, and that this income transfer method is more efficient than the rent subsidy plan. However, this plan also contains the implicit assumption that the individual's preference function is generally the same as society's; that is, if society hopes to achieve better housing for the poor by implementing a straight income transfer, it must assume that the poor similarly value better housing. If it were not that this last point is open to debate, there would be very little reason for the rent-subsidy plan to be advocated.

In implementing this plan in the model, there is a large question that must be overcome immediately. When we estimated the utility functions initially, we were basing these estimates on certain income patterns. What had to be decided was whether supplementing the income of a low-income group caused that group to alter its preference pattern, or whether it merely followed its original pattern but with more resources at its disposal. Fortunately, there is a neat way around this problem. The difficulty stems from short-run versus long-run considerations, in that one would be much more inclined to suspect a change in preference given a permanent change in income rather than a transitory one, since the former implies a change in expectations rather than just resources for the moment. Since our model implies a time horizon in excess of a few years, we assumed that the income subsidization was a permanent one, and that there would be a distinct change in preference functions.

Having no idea of what any intermediate preference functions would look like, we moved the lowest income groups up into the higher groups; that is, we made stratum 111 part of 113, 122 part of 124,et cetera. We did this for 111, 112, 121, 122, 211, 212, 221, 222, and 231. The mechanics simply consisted of increasing the sizes of the 113, 114,et cetera strata and eliminating the nine listed above. Contained here is the assumption that a household shifted into a higher income group acquires the characteristics of that group, a contention which is certainly not attack-proof, but which must do in lieu of a more developed experiment.

What can we expect from such an experiment? Obviously, we are once again pumping money into the system without altering the supply any, so in general we would expect a rise in the rent structure. At the same time, though, we are seriously distorting the market, by simply removing nine bid-rent patterns from the market. We can not be sure, then, that all rents will rise, since upward pressure from the poorer strata will be removed from some zones. Utilities of the non-subsidized groups will most likely fall, since they now have to offer higher rents for the same locations. For the subsidized groups we can make no conclusions about utility, since changing the utility function eliminates any comparability.

The results of the rent-subsidy run might give some insight into possible results here. In the rent-subsidy, the poor strata, especially 111, moved out of many center city areas, and were succeeded in these areas by the smaller, middle-income strata. Although this is not a voluntary move, in that we are eliminating those strata, we still might expect these same middle-income strata to move into these areas. It will be somewhat difficult to follow the migrations of the poor strata, since they are now indistinguishable from their somewhat richer counterparts. Since these now enlarged strata have not changed in any dimension other than size, we might expect to find them spilling over into zones neighboring those in which they were already located, since these are usually very similar zones. Since generalizations are difficult to make here, let us move directly to the results.

The first thing that one notices on looking at the results is that the enlarged strata only rarely moved out of a zone; in fact, of all the zones, there were only ten places where a stratum (of this enlarged group) did not locate after having been there in the base run. We also note that there are two large areas, zones 192-249 and 297-446 that are hardly affected in a locational sense at all. This leaves only Boston and the northern suburbs as those which were affected, although the intuitive explanation for the latter is not so clear.

As predicted, the poor strata, violently ejected from all areas, were followed by the middle-income strata. Most of Boston proper, with the exception of Jamaica Plain, Roslindale, and parts of South Boston, Dorchester, South and East Boston, is now comprised of the fifth and sixth income groups, as it was in the rent-subsidy run.

There are many areas in which the poor stratum that was eliminated was replaced by its richer counterpart, possibly indicating that certain groups did not move. It happens often with strata 111 and 113, but there are even more areas where they locate independently, so it is not obvious that we can draw any implications from this.

Also as predicted, rents in general went up, about $90 to $100 in those areas which were generally unaffected by the location changes. However, in the central city area, rents were much more volatile. A new vacancy pattern developed as zones 29, 30 and 31 went vacant, while 45 was newly inhabited. Those 24 zones for which rents declined were, with one exception, exclusively inhabited in the base run by the poorest strata. When we eliminate the strata in this run, some of the pressure on these zones is relieved and, if the zones are not especially attractive to other strata, the rents will fall.

Again, it is difficult to draw any but generalized locational implications from the model due to the imprecise nature of the approximated solution. Many of the locational changes we see are adjustments of the near-equilibrium process. We do see, however, that this income-allowance program produces results that are very similar to the rent subsidy program in the previous section. The poor strata were enabled to buy better quality housing, and thus left the central-city area in droves. Increased pressure on the suburban housing markets caused some strata to move into these city zones, especially the smaller, middle income families. This last, however, hinges on the fact that we have allowed for no new development on the fringe of the city. Were we to operationalize the vacant land mechanism, it is highly possible that the center city would be vacated by <u>all</u> strata in this run. Effects of this unnatural restriction are difficult to determine.

Both of these plans involve considerable and important tradeoffs. Both the income allowance and the rent subsidy help the strata that participate directly in the program, but, unlike the housing stock programs, they increase the demand pressure on the market rather than decreasing it, thereby increasing rents for the non-participants. Both programs raise the rent structure; it seems as if the rent subsidy might raise it a little more, but the differences are so small and so dependent on arbitrarily chosen factors (e.g. the 30% subsidy and the amount of the income transfers) that any attempt to discuss the relative costs of the two programs would be fruitless.

Without further development, these runs cannot really determine which is the best policy. What they are useful for, however, is to point up the implications of the various policies and the questions that need to be addressed. In any case, the final evaluation of a policy must lie with the policymaker, not with the model. With this model, we can only asses the relative direction and magnitude of the impacts, and allow the policymaker to make more informed decisions.

CONCLUSIONS

Let us sit back and consider what has happened in these past pages. We have constructed a rather elaborate model of urban residential location, both in terms of the operational mechanisms and the data requirements. What can be learned from such an experiment; that is, what additional information do we have about the locational process (or the validity of the model) that makes this experiment worth the time and the money expended? This section is an attempt to answer this question both in light of the model as applied to this experiment and the model as a theoretical exercise. Also, some suggestions for further research (which presuppose the validity of the model) will be put forth in the latter part of this section.

First of all, the model has proved to be reasonably reliable in reproducing classical economic theory, in that an increase in supply results in a general decrease in prices, while an upward shift of the demand curve causes a general increase in prices. It has gone much further than this, however. The concepts of submarkets and partially-substitutable commodities, central to any theory of a housing market, are handled very well by this specification, as we saw in the income-allowance run, in which some zones were almost totally changed over while a large block remained almost perfectly stable. These submarkets have been at least nebulously defined without the use of a measure of housing "quality"; indeed, such a measure would be anathema to this model, which depends on ordering of preferences that are consistent only within a particular stratum. The results of the base run, given the restrictions imposed by the data, demonstrate the concept of the market being in a short-run equilibrium is not all that far-fetched and that, if we could model certain pervasive market imperfections such as racial discrimination, we could get a relatively good reproduction of the existing housing pattern.

Data limitations were, I believe, the major pitfall of this particular model application. The necessity for using aggregate data, both for the zones and strata, greatly compromised the richness of the model by eliminating much of the heterogeneity of the urban area. As pointed out in the various sections, this led to mislocations galore. Theoretically, I still have great faith in the model. In the more suburban areas, where the data on zonal characteristics is much more likely to be representative of the entire zone (due to the greater homogeneity of these zones, as well as the fact that the neighborhood data applies to smaller areas), the model did an exceptionally good job of prediction. This success might also seem to vindicate our use of the tie mechanism as a proxy for certain market frictions such as search and moving costs, although it does not necessarily validate the particular value of the tie range. In general, the short-term equilibrium seems to be a fair approximation to the urban pattern, although the effects of recent perturbations which have not fully worked themselves out would tend to bias the market away from this equilibrium. Given this, one can review the results of the model as the solution that would occur given the initial conditions, the specified perturbation (e.g. the rent-subsidy or the renovation program), and sufficient time, free of all other disturbances, for the effects to be felt throughout the system. We can see, then, that it would be dangerous to use the quantitative output of the model as a prediction of real-world occurrences, since the last assumption, that of a period of time free of other disturbances, will be constantly violated. I feel that the model's strength lies in predicting the direction of impact of a particular policy, and not the magnitude (or especially the revealed magnitude) of the effect.

Within this framework, I feel that there is much that could be done with the model, some of it extensions and some of it revisions in the original work. At the risk of harping overlong on the same point, let me say that the first order of business would be to obtain data on a more disaggregated scale. For example, simply getting different crime expenditure and pupil-teacher ratio data for the various Boston zones would have had a tremendous impact on the location pattern, and might even eliminate the phenomenon of whites in black zones and blacks in white zones. Different stratification than that provided by the census might have also been useful; the income classes are especially unreasonable. Using the fract concept explained earlier would have very positive effects; the more homogeneous the tract, the more accurate would be the model's predictions.

There are all sorts of projects that could be attacked with the model above and beyond refining the data. As we saw in the suburban poor-housing construction run, accessibility is a very important variable, and yet it hardly came up in the discussion. One of the most interesting questions in urban economics is how accessibility, that is, the transportation system, affects the location pattern. One could easily alter the accessibility measures to reflect proposed changes in the transportation system and observe the moves that took place; this could be very important in assessing the positive or negative impacts of such a system.

One factor that is already in the model, but was not used, is the vacant land mechanism. It is well known that one of the important responses to excess market demand is the supply response of new construction. In the runs here, however, we allowed only for price changes. Certainly, this was a very restrictive assumption and, while allowing for additional clarity in analyzing the results, serves at the same time to bias these results away from reality. More advanced experiments should certainly include developable land. In fact, it would not be too difficult (although it would be computationally expensive) to include a mechanism that compared the profitability of an existing unit to the profitability of razing that unit and constructing another in its place. This would in effect provide an endogenous urban renewal program, and would greatly enhance the dynamic applicability of the model.

As mentioned in the text, there is no explicit segregation in the model, and the methods which I proposed for inclusion are obviously unsatisfactory. One fruitful area of exploration would be the development of a segregation mechanism, other than that revealed in the utility parameters (given an explicit segregation mechanism, we might posit the effect of Fair Housing laws and their effect on the housing pattern, which we cannot do if the phenomenon of segregation is subsumed by the general utility estimation, such that preferences and segregation effects are indistinguishable). Unfortunately, my thought experiments in this direction have been less than satisfactory, and I cannot suggest directions that research might take.

Finally, and perhaps most ambitiously, one might wish to adapt the framework of this model to a location problem other than housing, for example, the commercial location problem. This might involve using a profitability function instead of a utility function, and types of business rather than strata, but it seems as if the formulation might be much the same; that is, that the difference in rent among locations is a function of the different profitabilities of these locations. This would be a Herculean task, in light of the fact that enterprises can vary size as well as location, and might not produce any better (or even as good) results than other location models, but at the same time it would be interesting to see just how well such a model, strongly based in theory, would do.

This exercise has certainly been useful, both as an exploration of one modelling technique and as an effort to answer certain questions facing urban policymakers. An equilibrium model, strongly grounded in a self-consistent model of the urban system, has been shown to give a reasonable reproduction of that system even when working with very aggregate and dirty data. It has been shown to respond with intuitive reasonableness to certain stimuli, both major and minor, while at the same time being sufficiently complex as to trace out some surprising second- and third-hand effects. It has conceptual weaknesses, but few that could not be rectified without disturbing the integrity of the model. On balance, I feel that this model, when properly refined and intelligently (and cautiously) analyzed, will be a valuable addition to the successful understanding and management of the urban residential location decision.

APPENDIX I

ZONE NUMBER CORRESPONDENCE

ZONES	TOWN	ZONES	TOWN
1-12	East Boston	145-152	Brighton
14-22	Charlestown	153-156	Hyde Park
23-28	North End	159	Wenham
29-32	South End	160-165	Beverly
33-36	West End	166	Danvers
37-42	South End	169-173	Peabody
43-45	Back Bay	174	Lynnfield
46-47	West End	175-177	Saugus
48-51	Back Bay	178-197	Lynn
52-57	South End	198-204	Salem
58-71	South Boston	205	Marblehead
72-77	Dorchester North	206	Swampscott
78-81	Roxbury	207	Nahant
82	Dorchester North	208	Topsfield
83-85	Roxbury	209-238	Cambridge
86	Back Bay	239-253	Somerville
87-91	Roxbury	254-260	Everett
92-106	Dorchester North	261-269	Medford
107-115	Roxbury	270-278	Malden
116-121	Jamaica Plain	279-282	Melrose
122	Roslindale	283-284	Stoneham
123	Jamaica Plain	285-288	Wakefield
124-128	Roslindale	289	Reading
129-132	West Roxbury	290	North Reading
133-144	Dorchester South	291	Wilmington

ZONES	TOWN	ZONES	TOWN
292-296	Woburn	383	Millis
297	Winchester	384	Walpole
298-304	Arlington	385-387	Norwood
305-312	Belmont	388	Canton
313-316	Watertown	389-392	Milton
317-326	Newton	393-404	Quincy
327-334	Waltham	405-409	Weymouth
335-337	Lexington	410-412	Braintree
339	Burlington	413	Holbrook
340	Bedford	414-415	Randolph
341	Lincoln	416	Sharon
342	Concord	417	Cohasset
343	Weston	418	Norfolk
344	Wayland	419	Hull
345	Sudbury	420-421	Hingham
346-350	Framingham	422	Rockland
351-353	Natick	423	Norwell
354	Sherborn	424	Scituate
356-367	Brookline	425	Marshfield
368-371	Wellesley	427	Pembroke
372-376	Needham	428	Hanover
377-379	Dedham	429-435	Chelsea
380	Westwood	436-441	Revere
381	Dover	442-446	Winthrop
382	Medfield		

APPENDIX II
STRATA COEFFICIENTS

STRATUM NUMBER	NUMB. OF PEOPLE	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
111	46336	0.31260	-2.11000	-0.83960	0.12100	2.88100	0.35460	-0.14610	3.85200	0.38030	0.56710
112	22452	-0.32030	-2.02700	-0.54250	0.08213	1.88600	0.16960	-0.07601	2.42800	0.16690	0.67580
113	29885	-0.32000	-0.52620	-0.35830	0.08122	1.01200	0.02012	-0.06420	1.50000	0.02959	0.40530
114	27605	-0.22300	-0.25340	-0.24910	0.05524	0.67690	0.01074	-0.05144	1.01100	0.04907	0.20200
115	24609	0.03755	-0.14570	-0.05944	0.00549	0.19830	0.00660	-0.01485	0.22200	0.02510	0.05280
116	14116	0.01774	-0.05964	-0.04224	0.00541	0.14890	0.00521	-0.00816	0.14570	0.02615	0.04789
121	16300	1.65500	-1.50810	-0.33510	0.23150	2.04000	0.35620	-0.14520	4.39900	0.39230	0.46690
122	13896	0.22730	-3.41100	-0.31600	0.04192	1.57800	0.19770	-0.04380	3.27000	0.08705	0.78100
123	28621	-0.12650	-0.77930	-0.39930	0.06774	1.24500	0.04746	-0.05146	1.84500	0.05827	0.39030
124	31542	-0.06047	-0.42430	-0.21190	0.05703	0.70710	0.03537	-0.02585	1.00500	0.06490	0.22820
125	48072	0.06253	-0.10630	-0.04317	0.00675	0.15220	0.01074	-0.00917	0.23960	0.02392	0.06316
126	60805	0.04415	-0.15730	-0.01967	0.00230	0.07000	0.00734	-0.00473	0.24010	0.01936	0.04788
127	35508	0.04196	-0.10960	-0.01055	-0.00348	0.05694	0.00505	-0.00293	0.14300	0.01413	0.02571
128	12502	0.04915	-0.27050	0.01451	-0.00833	0.04757	0.00412	0.00060	0.11740	0.01622	0.02408
134	12108	-0.14620	-0.70340	-0.22420	0.07015	0.83620	0.01772	-0.02492	1.06300	0.10690	0.28780
135	27745	0.05561	-0.17110	-0.04646	0.01232	0.14000	0.00873	-0.00791	0.24630	0.02159	0.06172
136	43046	0.03441	-0.12540	-0.02544	0.00000	0.05730	0.00545	-0.00423	0.17930	0.01253	0.04226
137	33115	0.03762	-0.14430	-0.00929	0.00133	0.03207	0.00533	-0.00315	0.15050	0.01153	0.03135
138	10090	0.05746	-0.24430	0.00795	-0.00534	0.03041	0.00444	-0.00234	0.13880	0.00819	0.02209
145	23466	0.05550	-0.21490	-0.03746	0.01310	0.11240	0.00741	-0.00782	0.24970	0.01977	0.06057
146	40672	0.02271	-0.12150	-0.02813	0.00855	0.05030	0.00567	-0.00466	0.17580	0.00895	0.03432
147	33423	0.03295	-0.15670	-0.00546	0.00301	0.02193	0.00549	-0.00314	0.15750	0.01154	0.03388
148	11620	0.03643	-0.10390	0.00023	0.00159	0.02906	0.00476	-0.00466	0.15030	0.01162	0.03072
155	14615	0.02653	-0.20100	-0.04353	0.01755	0.11270	0.00609	-0.00852	0.22300	0.01857	0.06110
156	28035	0.01630	-0.10320	-0.03350	0.00930	0.05630	0.00654	-0.00406	0.17110	0.00960	0.03393
157	24758	0.02625	-0.14490	-0.00928	0.00022	0.01601	0.00483	-0.00407	0.16020	0.00948	0.02829
158	9288	0.02794	-0.17450	-0.00277	0.00448	0.02200	0.00388	-0.00400	0.15490	0.01562	0.04157
165	14848	0.06287	-0.19250	-0.03630	0.01100	0.10420	0.00462	-0.00689	0.19960	0.01654	0.06544
166	30867	0.02835	-0.12520	-0.02428	0.00543	0.04654	0.00530	-0.00502	0.16100	0.00960	0.03897
167	27879	0.02320	-0.11410	-0.01153	0.00721	0.01492	0.00588	-0.00366	0.14910	0.00985	0.02980
168	10313	0.03850	-0.10630	-0.00250	0.00511	0.01926	0.00539	-0.00390	0.14370	0.00833	0.02841
211	2852	-0.76520	-3.55000	0.08599	-0.03265	1.22000	0.10670	-0.20500	3.68700	2.96800	-2.97100
212	1336	-0.21920	-0.70640	0.00031	-0.04927	0.02955	0.12490	-0.06054	-0.96400	-2.96100	2.88400
213	1969	-0.12210	-0.82370	0.09822	0.01101	0.14940	0.05406	-0.02876	1.22400	3.48100	-3.34700
214	1579	-0.01551	0.17290	-0.05794	0.00012	-0.05256	0.02784	-0.01516	0.40010	-0.58790	0.36310
215	880	0.03053	-0.14740	0.01754	0.02109	0.10280	0.01042	-0.01844	-0.09360	0.08950	-0.08813
221	1140	-1.32300	-5.87700	1.30900	0.30850	-0.08351	-0.05378	-0.16660	4.00000	12.00000	-11.00000
222	1087	-0.21520	-0.08120	0.23570	-0.07020	-0.62840	0.16800	-0.01627	7.12200	0.19680	1.83300
223	1845	-0.17330	-1.12900	0.16400	0.01919	-0.09125	0.02910	-0.02750	0.07671	-0.20370	0.14890
224	1582	-0.00025	-0.15830	0.01727	0.00251	-0.15110	0.01230	-0.00681	0.69760	0.10560	-0.35640
225	1407	0.08480	-0.22530	0.04333	0.00951	-0.00322	0.00794	-0.01503	0.17380	-0.04755	-0.00861
226	1274	0.04608	-0.19240	0.01489	0.02242	0.11450	0.00332	-0.00857	-0.06776	0.03914	-0.05482
231	766	-1.63000	-6.25800	1.20400	0.08143	-1.15800	-0.02724	-0.05026	-1.99000	0.11020	2.78100
233	1218	-0.34250	-1.32800	0.22720	0.05303	-0.10640	-0.01431	-0.01837	2.35300	7.13100	-6.88100
234	1020	-0.17370	-0.11430	0.04829	0.02777	-0.14060	-0.01425	-0.00198	0.14290	-0.16030	0.14970
235	1301	0.07053	-0.05112	0.02753	0.00370	-0.15160	0.01143	-0.00059	-0.49380	-0.55530	0.59860
236	1211	0.05598	-0.10430	-0.02238	0.02201	0.07069	-0.00275	-0.00741	0.00698	-0.04034	-0.00924
243	939	-0.10670	-0.10110	-0.34440	-0.02950	0.65030	-0.02872	-0.04977	2.47500	5.54300	-5.82000
244	706	-0.16360	0.32150	0.06403	0.01095	-0.59920	0.02962	0.01482	0.51610	0.27160	-0.46110
245	907	0.03818	-0.23570	0.02459	0.00944	-0.05753	0.00554	-0.01671	-0.23740	-0.27610	0.30240

STRATUM NUMBER	NUMB. OF PEOPLE	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
246	1096	0.03765	-0.19640	0.00423	0.03309	0.08295	-0.00613	-0.00719	0.01389	0.06157	-0.12310
264	1018	-0.23300	-0.06479	0.04641	0.02611	-0.23070	0.00945	0.00374	0.22210	-0.10320	-0.05364
265	1227	-0.05359	-0.04670	-0.00355	0.01919	-0.03552	0.00617	-0.00306	-0.12240	-0.00490	-0.05033
266	1096	-0.02256	-0.05716	-0.01070	0.01625	0.00590	-0.00625	-0.00155	0.29380	0.12600	-0.21200

APPENDIX III

ZONAL CHARACTERISTICS

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
1	2144	40.1050	6.3130	0.0560	0.0100	20.4300	0.0980	0.0650	21.6730	60.6430	0.0790
2	1613	38.3520	5.9570	0.0330	0.0140	20.4300	0.0960	0.1980	21.6730	60.6430	0.0790
3	1392	49.8920	5.9290	0.0250	0.0380	20.4300	0.2040	0.0960	21.6730	60.6430	0.0790
4	1248	49.8780	6.0830	0.0190	0.0250	20.4300	0.2760	0.0070	21.6730	60.6430	0.0790
5	1514	48.8300	5.7640	0.0180	0.0610	20.4300	0.2760	0.0070	21.6730	60.6430	0.0790
6	907	49.9090	5.3990	0.0210	0.1390	20.4300	0.2760	0.0070	21.6730	60.6430	0.0790
7	554	49.8610	5.4610	0.0170	0.0550	20.4300	0.2760	0.0070	21.6730	60.6430	0.0790
8	587	48.1220	5.3570	0.0190	0.1410	20.4300	0.3570	0.0210	21.6730	60.6430	0.0790
9	603	37.1460	5.3080	0.0230	0.0330	20.4300	0.3570	0.0210	21.6730	60.6430	0.0790
10	701	49.6680	5.4850	0.0120	0.1130	20.4300	0.3570	0.0210	21.6730	60.6430	0.0790
11	577	48.9160	5.1030	0.0150	0.3370	20.4300	0.3570	0.0210	21.6730	60.6430	0.0790
12	1026	48.1470	5.5070	0.0180	0.1520	20.4300	0.8420	0.0170	21.6730	60.6430	0.0790
14	532	33.0720	4.9280	0.0160	0.0500	18.2650	0.1130	0.0570	21.6730	60.6430	0.0790
15	600	42.5240	5.5300	0.0280	0.0050	18.2650	0.1130	0.0570	21.6730	60.6430	0.0790
16	1161	42.7040	6.1640	0.0160	0.0600	18.2650	0.1130	0.0570	21.6730	60.6430	0.0790
17	105	49.8590	6.5650	0.0210	0.0430	18.2650	0.8210	0.0290	21.6730	60.6430	0.0790
18	135	49.6600	6.5770	0.0490	0.0920	18.2650	0.6620	0.0090	21.6730	60.6430	0.0790
19	429	49.2670	6.5650	0.0240	0.0510	18.2650	0.1130	0.0570	21.6730	60.6430	0.0790
20	588	49.9090	6.2690	0.0230	0.0350	18.2650	0.1130	0.0570	21.6730	60.6430	0.0790
21	492	49.4680	6.1160	0.0310	0.1400	18.2650	0.1130	0.0570	21.6730	60.6430	0.0790
22	592	49.9750	6.5750	0.0220	0.0970	18.2650	0.1130	0.0570	21.6730	60.6430	0.0790
23	725	49.9410	4.7700	0.0020	0.4900	8.6450	0.2890	0.0860	21.6730	60.6430	0.0790
24	1194	48.5050	4.0480	0.0020	0.4040	8.6450	0.0380	0.0700	21.6730	60.6430	0.0790
25	53	39.6600	3.5410	0.0020	0.0020	8.6450	0.4770	0.0010	21.6730	60.6430	0.0790
26	1249	49.4950	4.7070	0.0020	0.4300	8.6450	0.1660	0.0490	21.6730	60.6430	0.0790
27	727	48.1210	4.7970	0.0020	0.3510	8.6450	0.0380	0.0700	21.6730	60.6430	0.0790
28	403	39.4630	3.6670	0.0020	0.0550	8.6450	0.0610	0.0010	21.6730	60.6430	0.0790
29	200	33.7450	2.8280	0.0020	0.1170	8.6450	0.0700	0.0010	21.6730	60.6430	0.0790
30	465	33.6570	2.9320	0.0030	0.1200	8.6450	0.0700	0.0010	21.6730	60.6430	0.0790
31	73	34.4980	2.6030	0.0020	0.1090	8.6450	0.0390	0.0010	21.6730	60.6430	0.0790
32	535	47.4350	3.5410	0.0010	0.1050	8.6450	0.0350	0.0010	21.6730	60.6430	0.0790
33	58	49.9090	3.5070	0.0150	0.3040	8.6450	0.0610	0.0010	21.6730	60.6430	0.0790
34	13	33.9060	3.5990	0.0380	0.1920	8.6450	0.0610	0.0010	21.6730	60.6430	0.0790
35	35	31.7060	3.5980	0.0240	0.1890	8.6450	0.0170	0.2130	21.6730	60.6430	0.0790
36	2255	29.8210	3.1720	0.0010	0.2020	7.9250	0.2500	0.0010	21.6730	60.6430	0.0790
37	520	7.3140	4.5330	0.0010	0.0170	14.2110	0.1520	0.0400	21.6730	60.6430	0.0790
38	69	6.8020	4.6520	0.0100	0.0130	14.2110	0.3150	0.0600	21.6730	60.6430	0.0790
39	1308	49.2030	4.2240	0.0130	0.2010	14.2110	0.0340	0.0300	21.6730	60.6430	0.0790
40	742	34.5670	4.0820	0.0140	0.0350	14.2110	0.3370	0.0290	21.6730	60.6430	0.0790
41	1482	49.6970	3.9130	0.0080	0.2750	11.8450	0.0510	0.0010	21.6730	60.6430	0.0790
42	270	49.9090	5.0520	0.0120	0.0040	14.2110	0.0320	0.0030	21.6730	60.6430	0.0790
43	1166	22.9880	3.9540	0.0020	0.0390	7.4190	0.0390	0.0010	21.6730	60.6430	0.0790
44	1145	48.0420	3.9090	0.0050	0.1920	14.9390	0.0400	0.0010	21.6730	60.6430	0.0790
45	4131	47.9110	3.4850	0.0050	0.0620	14.9390	0.0470	0.1490	21.6730	60.6430	0.0790
46	2331	49.4480	4.0040	0.0070	0.1330	7.4190	0.0160	0.0370	21.6730	60.6430	0.0790
47	3186	46.9040	4.5130	0.0070	0.0050	7.4190	0.0320	0.1190	21.6730	60.6430	0.0790
48	3152	47.1770	3.6530	0.0080	0.1350	7.4190	0.0630	0.2600	21.6730	60.6430	0.0790
49	2135	47.8340	3.0020	0.0030	0.1450	11.8220	0.0510	0.1070	21.6730	60.6430	0.0790
50	4527	48.0270	3.4550	0.0060	0.0480	14.9390	0.0600	0.1990	21.6730	60.6430	0.0790
51	3747	49.1920	3.5180	0.0050	0.1550	7.4190	0.0230	0.0740	21.6730	60.6430	0.0790

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
52	772	49.5730	5.5250	0.0120	0.2250	14.2110	0.0500	0.0260	21.6730	60.6430	0.0790
53	1124	49.5450	4.4830	0.0060	0.2320	14.2110	0.0720	0.0150	21.6730	60.6430	0.0790
54	946	49.1830	4.1390	0.0070	0.2910	14.2110	0.0930	0.0650	21.6730	60.6430	0.0790
55	497	49.3300	4.2320	0.0050	0.2800	14.2110	0.0580	0.0830	21.6730	60.6430	0.0790
56	461	49.3240	3.1940	0.0110	0.4590	14.2110	0.1040	0.0750	21.6730	60.6430	0.0790
57	306	49.9090	3.5490	0.0040	0.3750	14.2110	0.1330	0.0500	21.6730	60.6430	0.0790
58	371	49.9090	5.6360	0.0190	0.1370	18.4120	0.7520	0.0010	21.6730	60.6430	0.0790
59	556	38.8070	5.5130	0.0240	0.1020	18.4120	0.4450	0.0130	21.6730	60.6430	0.0790
60	1046	30.5570	5.3400	0.0150	0.0140	18.4120	0.4450	0.0130	21.6730	60.6430	0.0790
61	203	47.9260	5.7020	0.0230	0.1020	18.4120	0.4840	0.0010	21.6730	60.6430	0.0790
62	1586	48.8200	5.0650	0.0230	0.0920	18.4120	0.5240	0.1260	21.6730	60.6430	0.0790
63	1439	41.6930	5.9310	0.0250	0.0440	18.4120	0.0240	0.2880	21.6730	60.6430	0.0790
64	912	48.3950	6.0630	0.0290	0.0150	18.4120	0.0240	0.2880	21.6730	60.6430	0.0790
65	1372	48.9140	6.1830	0.0300	0.0540	18.4120	0.0570	0.2260	21.6730	60.6430	0.0790
66	2007	43.9510	5.9440	0.0240	0.0440	18.4120	0.0660	0.2080	21.6730	60.6430	0.0790
67	1319	38.6590	5.2580	0.0240	0.0430	18.4120	0.0660	0.2080	21.6730	60.6430	0.0790
68	852	49.9090	5.9030	0.0240	0.0850	18.4120	0.4450	0.0130	21.6730	60.6430	0.0790
69	298	46.8640	5.7680	0.0210	0.1130	18.4120	0.4450	0.0130	21.6730	60.6430	0.0790
70	565	44.7940	5.4650	0.0200	0.0430	18.4120	0.0660	0.2080	21.6730	60.6430	0.0790
71	867	43.9320	4.5580	0.0280	0.0030	18.4120	0.0660	0.2080	21.6730	60.6430	0.0790
72	478	48.3250	6.0890	0.0290	0.0150	23.7600	0.0570	0.0120	21.6730	60.6430	0.0790
73	1190	48.9800	6.2100	0.0340	0.0250	23.7600	0.3060	0.0200	21.6730	60.6430	0.0790
74	836	45.9540	6.2270	0.0320	0.0410	21.8890	0.0570	0.0110	21.6730	60.6430	0.0790
75	630	49.0750	5.9290	0.0480	0.0140	18.8590	0.0540	0.0090	21.6730	60.6430	0.0790
76	676	44.7390	6.3200	0.0430	0.0050	18.8590	0.0540	0.0090	21.6730	60.6430	0.0790
77	962	46.6450	5.8950	0.0360	0.0050	20.8430	0.0560	0.0100	21.6730	60.6430	0.0790
78	258	46.2700	5.3880	0.0300	0.0300	18.5340	0.4140	0.0240	21.6730	60.6430	0.0790
79	502	37.2550	5.5640	0.0360	0.0250	17.4270	0.2460	0.0160	21.6730	60.6430	0.0790
80	968	38.2990	5.7370	0.0300	0.0520	18.8590	0.1600	0.0330	21.6730	60.6430	0.0790
81	593	48.0480	6.0810	0.0280	0.0540	18.8590	0.0960	0.0280	21.6730	60.6430	0.0790
82	816	45.8290	6.1200	0.0450	0.0350	18.8590	0.0540	0.0090	21.6730	60.6430	0.0790
83	607	41.3530	5.0220	0.0210	0.0170	15.1500	0.1520	0.0380	21.6730	60.6430	0.0790
84	161	48.5440	5.5890	0.0490	0.0450	18.8590	0.1500	0.0410	21.6730	60.6430	0.0790
85	610	31.9640	5.5150	0.0110	0.0100	16.2150	0.1480	0.0480	21.6730	60.6430	0.0790
86	1324	32.6400	3.7990	0.0190	0.0450	16.4120	0.0220	0.0990	21.6730	60.6430	0.0790
87	1118	29.6270	5.1290	0.0200	0.0100	19.4340	0.0320	0.0820	21.6730	60.6430	0.0790
88	775	47.9850	5.7450	0.0390	0.0520	18.9730	0.1370	0.0370	21.6730	60.6430	0.0790
89	945	39.5770	5.2690	0.0220	0.0130	19.5310	0.0180	0.1080	21.6730	60.6430	0.0790
90	1918	42.0170	4.9440	0.0200	0.0250	18.3940	0.0170	0.0890	21.6730	60.6430	0.0790
91	1201	37.3190	5.5370	0.0230	0.0350	19.5310	0.0170	0.0890	21.6730	60.6430	0.0790
92	2353	29.4250	5.8770	0.0300	0.0250	23.7600	0.2690	0.1470	21.6730	60.6430	0.0790
93	1852	48.5200	6.5550	0.0310	0.0030	23.7600	0.0570	0.0120	21.6730	60.6430	0.0790
94	1301	44.5130	5.8830	0.0300	0.0490	23.7600	0.0570	0.0120	21.6730	60.6430	0.0790
95	1658	46.7540	6.2130	0.0270	0.0210	23.7600	0.0570	0.0120	21.6730	60.6430	0.0790
96	1005	47.7710	6.2420	0.0400	0.0150	23.7600	0.1840	0.0410	21.6730	60.6430	0.0790
97	1002	49.4140	6.5380	0.0490	0.0050	23.7600	0.0480	0.0570	21.6730	60.6430	0.0790
98	1203	48.4450	6.4730	0.0370	0.0250	23.7600	0.0880	0.0010	21.6730	60.6430	0.0790
99	1309	43.2240	6.5430	0.0350	0.0050	23.7600	0.0220	0.0040	21.6730	60.6430	0.0790
100	1018	47.8430	6.1810	0.0310	0.0110	18.8590	0.0190	0.0010	21.6730	60.6430	0.0790
101	757	42.9240	5.7790	0.0280	0.0050	18.8590	0.0540	0.0010	21.6730	60.6430	0.0790

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
102	1932	46.1460	6.2710	0.0440	0.0030	18.8590	0.0210	0.0030	21.6730	60.6430	0.0790
103	2105	41.2910	6.0830	0.0380	0.0090	21.8450	0.0280	0.0410	21.6730	60.6430	0.0790
104	1110	46.5530	6.5210	0.0540	0.0150	23.7600	0.0190	0.0520	21.6730	60.6430	0.0790
105	1805	47.8550	6.7280	0.0420	0.0140	23.7600	0.0530	0.0260	21.6730	60.6430	0.0790
106	1238	47.3520	7.1870	0.0430	0.0090	23.7600	0.0430	0.0540	21.6730	60.6430	0.0790
107	1218	47.2690	6.5990	0.0250	0.0510	18.8590	0.0220	0.0420	21.6730	60.6430	0.0790
108	1520	32.0360	5.0860	0.0260	0.0170	18.8590	0.0220	0.0420	21.6730	60.6430	0.0790
109	311	48.3410	6.5070	0.0520	0.0370	18.8590	0.0780	0.0390	21.6730	60.6430	0.0790
110	802	31.9580	6.0360	0.0460	0.0210	18.8590	0.0800	0.0170	21.6730	60.6430	0.0790
111	1362	41.0700	6.0360	0.0460	0.0140	18.8590	0.0870	0.0170	21.6730	60.6430	0.0790
112	1048	46.3790	6.4030	0.0380	0.0250	18.8590	0.0140	0.0070	21.6730	60.6430	0.0790
113	2016	37.6370	5.5060	0.0330	0.0050	18.8590	0.0130	0.0080	21.6730	60.6430	0.0790
114	1310	36.8630	5.0170	0.0360	0.0100	18.8590	0.1200	0.0150	21.6730	60.6430	0.0790
115	1423	29.3720	5.5800	0.0340	0.0220	19.5310	0.0230	0.1450	21.6730	60.6430	0.0790
116	785	48.6170	5.5670	0.0310	0.0430	19.5310	0.0210	0.1490	21.6730	60.6430	0.0790
117	970	30.3510	5.7220	0.0260	0.0120	19.5310	0.0210	0.1490	21.6730	60.6430	0.0790
118	1130	49.2630	6.1210	0.0270	0.0320	19.5310	0.0210	0.1490	21.6730	60.6430	0.0790
119	2401	45.8830	6.1150	0.0590	0.0420	19.5310	0.0230	0.2170	21.6730	60.6430	0.0790
120	1643	48.0520	6.2120	0.0410	0.0230	18.8970	0.1550	0.0150	21.6730	60.6430	0.0790
121	2370	38.5930	7.2060	0.0900	0.0170	19.5310	0.0890	0.3660	21.6730	60.6430	0.0790
122	2915	43.1600	7.0660	0.1050	0.0140	26.5560	0.0820	0.2910	21.6730	60.6430	0.0790
123	1464	42.4880	5.7910	0.0540	0.0550	19.5310	0.1170	0.1500	21.6730	60.6430	0.0790
124	2103	40.3490	6.0970	0.0590	0.0050	26.8970	0.0370	0.5520	21.6730	60.6430	0.0790
125	871	26.0280	5.5660	0.0560	0.0030	26.8970	0.0320	0.3600	21.6730	60.6430	0.0790
126	833	38.3560	6.3890	0.0720	0.0050	26.8970	0.0120	0.4580	21.6730	60.6430	0.0790
127	2449	42.8670	6.5540	0.0750	0.0110	26.8970	0.0460	0.0170	21.6730	60.6430	0.0790
128	2778	45.7430	6.6770	0.1030	0.0110	26.8970	0.0260	0.0650	21.6730	60.6430	0.0790
129	1937	36.5170	7.1450	0.1370	0.0030	26.1020	0.0800	0.2820	21.6730	60.6430	0.0790
130	1830	41.2080	7.5750	0.1110	0.0090	26.1020	0.0610	0.2140	21.6730	60.6430	0.0790
131	1760	42.1100	7.0230	0.1010	0.0100	26.1020	0.0270	0.0860	21.6730	60.6430	0.0790
132	4102	18.0570	5.9680	0.0870	0.0090	26.1020	0.0760	0.3130	21.6730	60.6430	0.0790
133	2256	47.6370	6.2980	0.0470	0.0190	23.7600	0.0820	0.0530	21.6730	60.6430	0.0790
134	2831	42.0800	6.4320	0.0600	0.0110	23.7600	0.1720	0.0410	21.6730	60.6430	0.0790
135	1554	44.9000	6.5710	0.0670	0.0010	23.7600	0.0520	0.0330	21.6730	60.6430	0.0790
136	2040	38.2690	6.4500	0.0590	0.0130	23.7600	0.0600	0.2390	21.6730	60.6430	0.0790
137	2244	44.4140	6.4320	0.0550	0.0250	23.7600	0.0190	0.0440	21.6730	60.6430	0.0790
138	1675	43.2140	6.5720	0.0710	0.0050	23.7600	0.0200	0.0190	21.6730	60.6430	0.0790
139	2276	36.9270	5.0460	0.0400	0.0150	23.7600	0.0410	0.1120	21.6730	60.6430	0.0790
140	1266	42.2120	6.2290	0.0330	0.0210	23.7600	0.0410	0.1120	21.6730	60.6430	0.0790
141	1379	43.6930	6.2410	0.0690	0.0030	23.7600	0.0380	0.0170	21.6730	60.6430	0.0790
142	2804	40.8040	6.3330	0.0350	0.0050	23.7600	0.0410	0.1120	21.6730	60.6430	0.0790
143	3815	25.1370	5.9310	0.0710	0.0030	23.7600	0.0290	0.0810	21.6730	60.6430	0.0790
144	1439	36.1440	6.5750	0.0810	0.0140	23.7600	0.0520	0.2540	21.6730	60.6430	0.0790
145	1274	46.7790	6.1450	0.0590	0.0300	20.3890	0.1880	0.1270	21.6730	60.6430	0.0790
146	2370	41.5120	5.3240	0.0360	0.0410	20.3890	0.2070	0.0630	21.6730	60.6430	0.0790
147	5232	43.2520	4.2370	0.0180	0.0110	20.3890	0.1130	0.0280	21.6730	60.6430	0.0790
148	3469	33.3440	5.6580	0.0250	0.0110	20.3890	0.0900	0.0290	21.6730	60.6430	0.0790
149	2958	39.7070	6.4350	0.0640	0.0130	20.3890	0.0790	0.0460	21.6730	60.6430	0.0790
150	2128	33.3900	6.4800	0.0550	0.0050	20.3890	0.0460	0.0720	21.6730	60.6430	0.0790
151	3134	41.9430	5.5000	0.0350	0.0110	20.3890	0.0290	0.1060	21.6730	60.6430	0.0790

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
152	3990	44.9010	4.5120	0.0300	0.0050	20.3890	0.0320	0.1120	21.6730	60.6430	0.0790
153	2382	29.6090	6.3380	0.0880	0.0140	30.2960	0.0470	0.0930	21.6730	60.6430	0.0790
154	1959	39.7320	6.3950	0.0810	0.0210	30.2950	0.1230	0.0330	21.6730	60.6430	0.0790
155	2781	30.7810	6.4850	0.1380	0.0130	30.2950	0.1210	0.0580	21.6730	60.6430	0.0790
156	4403	28.0650	6.3330	0.1010	0.0110	29.9340	0.0850	0.3900	21.6730	60.6430	0.0790
159	955	31.7350	8.3040	0.5850	0.0010	43.5850	0.0160	0.0310	21.1970	18.6160	0.0320
160	1715	29.1200	7.0260	0.6870	0.0070	37.0030	0.0120	0.0130	24.4920	33.7170	0.0480
161	1567	23.8150	6.7840	0.2300	0.0110	37.0030	0.1500	0.0100	24.4920	33.7170	0.0480
162	2304	33.2250	6.6860	0.1970	0.0070	37.0030	0.0610	0.1420	24.4920	33.7170	0.0480
163	1677	40.6080	6.1120	0.1330	0.0200	37.0030	0.0410	0.0520	24.4920	33.7170	0.0480
164	2475	44.3850	6.0310	0.0370	0.0300	37.0030	0.0860	0.0950	24.4920	33.7170	0.0480
165	1829	47.7200	5.0030	0.1030	0.0080	37.0030	0.0860	0.0950	24.4920	33.7170	0.0480
166	7130	28.4090	6.8970	0.2460	0.0140	31.3750	0.0390	0.0150	23.6540	19.8260	0.0420
169	1372	36.4880	6.1470	0.1130	0.0230	33.0490	0.1160	0.1260	26.1180	25.1030	0.0390
170	1231	39.7150	5.2790	0.1080	0.0830	33.0490	0.1160	0.1260	26.1180	25.1030	0.0390
171	1326	45.3030	5.9310	0.1140	0.0620	33.0490	0.1160	0.1260	26.1180	25.1030	0.0390
172	7311	16.1280	7.1620	0.2320	0.0030	33.0490	0.0530	0.0580	26.1180	25.1030	0.0390
173	2902	29.2310	6.5110	0.1830	0.0090	33.0490	0.0500	0.0940	26.1180	25.1030	0.0390
174	2969	24.0120	8.2840	0.4030	0.0130	34.0560	0.0330	0.0920	22.1980	20.7180	0.0350
175	2060	28.7050	6.7790	0.1960	0.0070	27.4400	0.0130	0.1820	24.5480	25.5850	0.0430
176	917	28.7900	7.0920	0.3050	0.0030	27.4400	0.0360	0.2800	24.5480	25.5850	0.0430
177	1684	37.2120	6.6150	0.1860	0.0150	27.4400	0.0170	0.0910	24.5480	25.5850	0.0430
178	1747	37.5780	6.8590	0.0860	0.0030	28.1910	0.0160	0.1730	25.6110	39.2910	0.0540
179	1904	40.7980	6.5760	0.0890	0.0100	28.1910	0.0170	0.2070	25.6110	39.2910	0.0540
180	1494	44.2700	6.2700	0.0580	0.0350	28.1910	0.0120	0.0770	25.6110	39.2910	0.0540
181	2887	33.7110	6.4690	0.1460	0.0050	28.1910	0.0120	0.2900	25.6110	39.2910	0.0540
182	2804	32.2770	6.1710	0.0950	0.0100	28.1910	0.0210	0.5990	25.6110	39.2910	0.0540
183	1327	46.5630	6.5510	0.0920	0.0070	28.1910	0.0400	0.0530	25.6110	39.2910	0.0540
184	1035	42.7680	5.6720	0.0830	0.0450	28.1910	0.0690	0.0270	25.6110	39.2910	0.0540
185	1098	48.4090	5.7170	0.0380	0.0480	28.1910	0.0420	0.0410	25.6110	39.2910	0.0540
186	1128	48.2090	5.3550	0.0470	0.0440	28.1910	0.0250	0.0650	25.6110	39.2910	0.0540
187	1380	45.2870	5.6640	0.0230	0.0550	28.1910	0.0770	0.0270	25.6110	39.2910	0.0540
188	1741	48.2900	5.5160	0.0530	0.0310	28.1910	0.0110	0.0310	25.6110	39.2910	0.0540
189	1313	45.0100	6.1200	0.0570	0.0110	28.1910	0.0110	0.0310.	25.6110	39.2910	0.0540
190	1693	48.0150	6.3940	0.0660	0.0050	28.1910	0.0110	0.0310	25.6110	39.2910	0.0540
191	1333	45.9190	5.9040	0.0620	0.0090	28.1910	0.0110	0.0310	25.6110	39.2910	0.0540
192	1727	47.4790	6.3010	0.0370	0.0010	28.1910	0.0650	0.0360	25.6110	39.2910	0.0540
193	1665	43.0190	5.2510	0.0380	0.0250	28.1910	0.0650	0.0360	25.6110	39.2910	0.0540
194	1360	45.5450	4.3740	0.0400	0.0320	28.1910	0.0660	0.0360	25.6110	39.2910	0.0540
195	1563	47.2820	3.8910	0.0240	0.3230	28.1910	0.0340	0.0280	25.6110	39.2910	0.0540
196	982	33.8650	5.1340	0.0580	0.0420	28.1910	0.2470	0.0230	25.6110	39.2910	0.0540
197	781	48.1720	6.0120	0.0540	0.0010	28.1910	0.3600	0.0150	25.6110	39.2910	0.0540
198	1617	46.0900	5.7210	0.0480	0.0680	35.1630	0.2430	0.1220	20.9330	38.2060	0.0580
199	1349	49.1550	5.2510	0.0650	0.1250	35.1630	0.1210	0.0700	20.9330	38.2060	0.0580
200	2181	44.8570	5.0870	0.0610	0.0490	35.1630	0.0930	0.0780	20.9330	38.2060	0.0580
201	1238	48.7770	5.7070	0.0490	0.1610	35.1630	0.1820	0.1060	20.9330	38.2060	0.0580
202	2077	43.8130	6.5160	0.0880	0.0100	35.1630	0.0520	0.4180	20.9330	38.2060	0.0580
203	2553	28.5330	6.2630	0.1060	0.0200	35.1630	0.0790	0.1580	20.9330	38.2060	0.0580
204	2616	40.1730	6.3530	0.0710	0.0070	35.1630	0.0700	0.0480	20.9330	38.2060	0.0580
205	7156	35.7700	7.4910	0.1980	0.0170	42.3510	0.0140	0.0530	22.7610	31.5000	0.0330

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
206	4304	40.5770	7.6530	0.1510	0.0050	35.9450	0.0600	0.1160	21.2690	43.4230	0.0450
207	1237	39.8050	7.1780	0.2440	0.0220	44.7910	0.0610	0.1480	20.1670	29.8150	0.0430
208	1355	25.0020	8.3170	0.5930	0.0010	43.2930	0.0130	0.0660	22.7560	12.9750	0.0450
209	660	46.9010	5.3850	0.0170	0.0740	17.4790	0.4560	0.0120	18.9350	42.9910	0.0520
210	535	49.1060	5.4720	0.0210	0.0340	17.4790	0.4560	0.0120	18.9350	42.9910	0.0520
211	774	48.9580	5.1770	0.0210	0.1430	17.4790	0.4560	0.0120	18.9350	42.9910	0.0520
212	952	43.5750	5.2780	0.0250	0.0190	17.4790	0.1890	0.0360	18.9350	42.9910	0.0520
213	1043	48.6000	5.7550	0.0140	0.0300	17.4790	0.0820	0.0240	18.9350	42.9910	0.0520
214	931	48.2220	5.5290	0.0250	0.0390	17.4790	0.1530	0.0390	18.9350	42.9910	0.0520
215	924	41.3550	5.4240	0.0210	0.1150	17.4790	0.1180	0.0320	18.9350	42.9910	0.0520
216	851	49.4420	5.3250	0.0260	0.0350	17.4790	0.0590	0.0190	18.9350	42.9910	0.0520
217	1384	44.5050	5.4830	0.0260	0.0590	17.4790	0.0270	0.0120	18.9350	42.9910	0.0520
218	2272	39.8930	3.5280	0.0130	0.3080	17.4790	0.0270	0.0120	18.9350	42.9910	0.0520
219	1302	27.4410	4.0580	0.0170	0.0270	17.4790	0.1790	0.0520	18.9350	42.9910	0.0520
220	1041	47.0050	5.5390	0.0260	0.0150	17.4790	0.3170	0.0620	18.9350	42.9910	0.0520
221	1723	47.6590	5.5540	0.0280	0.0540	17.4790	0.0740	0.1050	18.9350	42.9910	0.0520
222	657	45.7090	5.7860	0.0380	0.0240	17.4790	0.0740	0.1050	18.9350	42.9910	0.0520
223	1037	37.7400	5.3700	0.0310	0.0100	17.4790	0.0270	0.0120	18.9350	42.9910	0.0520
224	1645	44.5270	5.3340	0.0270	0.0400	17.4790	0.0130	0.0360	18.9350	42.9910	0.0520
225	1545	46.1400	4.5490	0.0260	0.0070	17.4790	0.0210	0.0340	18.9350	42.9910	0.0520
226	2309	43.9610	4.7660	0.0250	0.0420	17.4790	0.0270	0.0120	18.9350	42.9910	0.0520
227	906	32.3150	4.0640	0.0190	0.0550	17.4790	0.0180	0.0450	18.9350	42.9910	0.0520
228	1958	45.6460	4.0610	0.0320	0.0090	17.4790	0.0140	0.0390	18.9350	42.9910	0.0520
229	1715	40.6950	5.5120	0.0560	0.0430	17.4790	0.0050	0.1580	18.9350	42.9910	0.0520
230	1362	47.3460	8.5630	0.0920	0.0030	17.4790	0.0010	0.2060	18.9350	42.9910	0.0520
231	961	38.0610	6.3190	0.0660	0.0150	17.4790	0.0650	0.3890	18.9350	42.9910	0.0520
232	760	47.7740	6.5850	0.0490	0.0050	17.4790	0.2290	0.0460	18.9350	42.9910	0.0520
233	1380	45.1650	5.5090	0.0140	0.0250	17.4790	0.0010	0.0160	18.9350	42.9910	0.0520
234	1210	34.1800	6.3420	0.0430	0.0010	17.4790	0.3570	0.0740	18.9350	42.9910	0.0520
235	952	47.7340	6.1920	0.0400	0.0370	17.4790	0.1900	0.0700	18.9350	42.9910	0.0520
236	903	38.3310	5.5610	0.0120	0.0180	17.4790	0.1390	0.1010	18.9350	42.9910	0.0520
237	1713	33.1860	5.0330	0.0270	0.0110	17.4790	0.1390	0.1010	18.9350	42.9910	0.0520
238	921	43.4800	6.2050	0.0310	0.0150	17.4790	0.1390	0.1010	18.9350	42.9910	0.0520
239	2598	35.3630	5.0310	0.0370	0.0030	20.7390	0.0280	0.0570	23.6980	36.2340	0.0520
240	1137	49.4310	6.7250	0.0430	0.0010	20.7390	0.0190	0.0500	23.6980	36.2340	0.0520
241	680	49.0190	6.5020	0.0250	0.0200	20.7390	0.0190	0.0500	23.6980	36.2340	0.0520
242	749	49.1960	6.2170	0.0260	0.0010	20.7390	0.0190	0.0500	23.6980	36.2340	0.0520
243	1643	46.6540	5.5680	0.0140	0.0280	20.7390	0.0520	0.0430	23.6980	36.2340	0.0520
244	2209	49.0340	6.5460	0.0150	0.0180	20.7390	0.0350	0.0470	23.6980	36.2340	0.0520
245	905	48.3440	5.9980	0.0320	0.0180	20.7390	0.0800	0.0360	23.6980	36.2340	0.0520
246	2632	46.9050	6.3760	0.0140	0.0400	20.7390	0.0980	0.0320	23.6980	36.2340	0.0520
247	2409	47.3170	5.9220	0.0420	0.0300	20.7390	0.1430	0.0140	23.6980	36.2340	0.0520
248	2455	48.2540	6.2670	0.0300	0.0100	20.7390	0.0990	0.0220	23.6980	36.2340	0.0520
249	3167	43.0370	5.9370	0.0410	0.0020	20.7390	0.3390	0.0180	23.6980	36.2340	0.0520
250	2750	47.4410	5.9050	0.0370	0.0070	20.7390	0.0990	0.0220	23.6980	36.2340	0.0520
251	1687	44.5790	5.3860	0.0390	0.0490	20.7390	0.1450	0.0130	23.6980	36.2340	0.0520
252	3363	48.6140	5.5820	0.0350	0.0380	20.7390	0.1250	0.0260	23.6980	36.2340	0.0520
253	520	48.4510	5.3720	0.0490	0.0410	20.7390	0.6940	0.0030	23.6980	36.2340	0.0520
254	2929	36.8630	6.1410	0.0620	0.0050	23.9750	0.0030	0.3230	24.6670	49.1560	0.0640
255	3036	44.8290	6.2240	0.0580	0.0110	23.9750	0.0190	0.0190	24.6670	49.1560	0.0640

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
256	2278	48.4620	6.3420	0.0560	0.0210	23.9750	0.0470	0.0370	24.6670	49.1560	0.0640
257	1415	46.6640	5.9830	0.0430	0.0370	23.9750	0.2570	0.0170	24.6670	49.1560	0.0640
258	2246	47.5590	6.[illegible]430	0.0450	0.0220	23.9750	0.0340	0.0110	24.6670	49.1560	0.0640
259	1367	43.3680	6.[illegible]650	0.0630	0.0040	23.9750	0.0030	0.3230	24.6670	49.1560	0.0640
260	402	49.5090	6.[illegible]220	0.0380	0.0190	23.9750	0.7220	0.0030	24.6670	49.1560	0.0640
261	2141	37.1280	7.2850	0.1450	0.0030	24.7050	0.0080	0.2980	25.5700	33.6160	0.0470
262	4115	44.2910	7.5570	0.0940	0.0140	24.7050	0.0070	0.4800	25.5700	33.6160	0.0470
263	1089	46.8980	6.4740	0.1450	0.0070	24.7050	0.0040	0.4890	25.5700	33.6160	0.0470
264	2125	44.8640	6.7040	0.0560	0.0040	24.7050	0.0220	0.0360	25.5700	33.6160	0.0470
265	2137	36.2810	6.3360	0.0730	0.0150	24.7050	0.2150	0.0250	25.5700	33.6160	0.0470
266	1534	44.9150	6.2780	0.0530	0.0070	24.7050	0.0790	0.0280	25.5700	33.6160	0.0470
267	2135	48.0660	6.3270	0.0560	0.0110	24.7050	0.0790	0.0280	25.5700	33.6160	0.0470
268	1555	45.9570	6.7410	0.0700	0.0100	24.7050	0.0480	0.0720	25.5700	33.6160	0.0470
269	2617	39.9750	6.3810	0.0930	0.0110	24.7050	0.0310	0.2000	25.5700	33.6160	0.0470
270	1825	36.3460	6.4120	0.1410	0.0110	26.1540	0.0410	0.1890	24.9350	40.9770	0.0520
271	2116	30.9300	5.9180	0.0660	0.0110	26.1540	0.1140	0.0340	24.9350	40.9770	0.0520
272	1844	41.4620	6.5590	0.1020	0.0110	26.1540	0.0010	0.0070	24.9350	40.9770	0.0520
273	2292	39.0080	6.0440	0.0750	0.0130	26.1540	0.0240	0.2240	24.9350	40.9770	0.0520
274	1670	43.5450	6.[illegible]580	0.0560	0.0150	26.1540	0.1270	0.0360	24.9350	40.9770	0.0520
275	2092	45.2040	6.3070	0.0590	0.0220	26.1540	0.2170	0.0510	24.9350	40.9770	0.0520
276	1620	43.1880	4.9360	0.0450	0.1210	26.1540	0.0770	0.1270	24.9350	40.9770	0.0520
277	3062	42.5730	6.[illegible]560	0.0720	0.0300	26.1540	0.0200	0.1740	24.9350	40.9770	0.0520
278	2281	44.3640	6.1790	0.0560	0.0040	26.1540	0.0940	0.0510	24.9350	40.9770	0.0520
279	2191	39.8450	8.3130	0.1750	0.0030	31.5950	0.0140	0.1000	23.6020	28.3320	0.0420
280	2095	41.3350	7.4740	0.1410	0.0050	31.5950	0.0190	0.0680	23.6020	28.3320	0.0420
281	3417	30.6690	6.[illegible]850	0.0910	0.0120	31.5950	0.0310	0.3130	23.6020	28.3320	0.0420
282	2343	40.8310	7.3340	0.1830	0.0190	31.5950	0.0240	0.1990	23.6020	28.3320	0.0420
283	4638	21.8900	6.6480	0.1970	0.0150	30.4470	0.0240	0.3140	24.7780	28.7030	0.0400
284	1754	42.8920	6.5920	0.1370	0.0270	30.4470	0.0330	0.0330	24.7780	28.7030	0.0400
285	2146	33.0250	6.5700	0.2220	0.0170	32.2660	0.0450	0.0450	22.5510	25.5340	0.0390
286	2209	38.7830	6.7380	0.1710	0.0170	32.2660	0.0270	0.0350	22.5510	25.5340	0.0390
287	1783	35.2010	7.4050	0.2310	0.0130	32.2660	0.0320	0.1390	22.5510	25.5340	0.0390
288	1471	35.2780	7.[illegible]510	0.2070	0.0030	32.2660	0.0270	0.0490	22.5510	25.5340	0.0390
289	6386	31.5030	7.3370	0.2760	0.0070	33.5490	0.0480	0.0260	24.7770	29.0490	0.0380
290	2945	24.3790	7.[illegible]420	0.3690	0.0270	45.6040	0.0330	0.0500	23.6060	31.8810	0.0680
291	4340	25.9730	6.[illegible]210	0.3900	0.0130	36.3070	0.0510	0.0080	23.5170	25.1540	0.0540
292	1663	26.1610	6.4420	0.2760	0.0140	32.2010	0.2120	0.0120	24.8630	19.8860	0.0470
293	2557	36.6270	6.5760	0.1650	0.0420	32.2010	0.0740	0.0380	24.8630	19.8860	0.0470
294	1668	27.5350	6.5270	0.2140	0.0040	32.2010	0.1120	0.0390	24.8630	19.8860	0.0470
295	1555	39.2950	6.[illegible]930	0.1100	0.0430	32.2010	0.0880	0.0400	24.8630	19.8860	0.0470
296	3155	25.0530	7.[illegible]040	0.2560	0.0140	32.2010	0.0940	0.0500	24.8630	19.8860	0.0470
297	6431	34.4070	8.[illegible]650	0.2690	0.0090	30.9350	0.0170	0.2340	21.6750	33.8860	0.0440
298	2501	28.9030	7.7950	0.1850	0.0050	28.8470	0.0030	0.1330	21.6030	31.3120	0.0410
299	2363	29.2890	5.0870	0.0590	0.0070	28.8470	0.0130	0.0810	21.6030	31.3120	0.0410
300	2352	36.3720	6.1740	0.0620	0.0020	28.8470	0.0150	0.0800	21.6030	31.3120	0.0410
301	1403	45.9670	6.4380	0.0650	0.0010	28.8470	0.0110	0.0830	21.6030	31.3120	0.0410
302	3021	34.7400	7.[illegible]990	0.0950	0.0090	28.8470	0.0300	0.1280	21.6030	31.3120	0.0410
303	3517	32.2550	6.[illegible]860	0.1140	0.0110	28.8470	0.0360	0.0440	21.6030	31.3120	0.0410
304	2470	27.4970	6.0630	0.1180	0.0040	28.8470	0.0600	0.0580	21.6030	31.3120	0.0410
305	1420	31.3890	8.3620	0.2320	0.0030	27.6050	0.0200	0.1220	21.5430	35.1980	0.0330

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
306	1347	37.5810	9.2040	0.2550	0.0050	27.6050	0.0140	0.1320	21.5430	35.1980	0.0330
307	1337	42.4700	6.4030	0.1500	0.0030	27.6050	0.0130	0.1120	21.5430	35.1980	0.0330
308	980	45.8000	6.4890	0.0690	0.0140	27.6050	0.0190	0.0370	21.5430	35.1980	0.0330
309	915	44.8350	7.1400	0.0990	0.0080	27.6050	0.0590	0.0530	21.5430	35.1980	0.0330
310	909	47.8710	8.1260	0.1230	0.0050	27.5050	0.0120	0.1000	21.5430	35.1980	0.0330
311	1113	43.7040	9.1130	0.1170	0.0010	27.5050	0.0120	0.1000	21.5430	35.1980	0.0330
312	1307	42.9690	6.0200	0.1190	0.0120	27.6050	0.0120	0.1000	21.5430	35.1980	0.0330
313	2207	43.5470	6.2730	0.0570	0.0020	25.9450	0.1710	0.2410	22.3560	43.0340	0.0500
314	3302	43.0160	7.1770	0.1000	0.0090	25.9450	0.0490	0.2130	22.3560	43.0340	0.0500
315	2079	42.6040	6.4330	0.0730	0.0270	25.9450	0.0740	0.0580	22.3560	43.0340	0.0500
316	4973	30.9100	6.2170	0.0790	0.0040	25.9450	0.0820	0.0620	22.3560	43.0340	0.0500
317	2035	43.3390	6.5940	0.0910	0.0330	22.7600	0.0330	0.0380	19.9450	39.0170	0.0460
318	1661	43.8140	6.4250	0.0370	0.0470	22.7600	0.1000	0.0620	19.9450	39.0170	0.0460
319	2361	40.1180	6.7560	0.1290	0.0210	22.7600	0.0450	0.1110	19.9450	39.0170	0.0460
320	3404	39.2720	7.2050	0.1030	0.0020	22.7600	0.0170	0.0890	19.9450	39.0170	0.0460
321	2371	39.1150	7.7620	0.2420	0.0050	22.7600	0.0660	0.1980	19.9450	39.0170	0.0460
322	2898	39.7850	10.5040	0.3210	0.0070	22.7600	0.0140	0.1480	19.9450	39.0170	0.0460
323	2757	41.1210	7.5420	0.1900	0.0050	22.7600	0.0960	0.1100	19.9450	39.0170	0.0460
324	3427	40.2600	8.5890	0.1980	0.0070	22.7600	0.0070	0.1500	19.9450	39.0170	0.0460
325	2411	43.3430	9.7940	0.3080	0.0070	22.7600	0.0170	0.1560	19.9450	39.0170	0.0460
326	3631	18.5670	8.4050	0.2300	0.0020	22.7600	0.0080	0.1390	19.9450	39.0170	0.0460
327	252	13.2420	5.0590	0.1630	0.0010	28.4170	0.0110	0.2400	21.1400	36.0530	0.0530
328	6453	23.3990	6.5630	0.1030	0.0070	28.4170	0.0430	0.1130	21.1400	36.0530	0.0530
329	2869	35.4350	6.2700	0.1490	0.0070	28.4170	0.1090	0.2040	21.1400	36.0530	0.0530
330	2309	33.9630	6.5590	0.1000	0.0170	28.4170	0.0700	0.1340	21.1400	36.0530	0.0530
331	1215	46.1960	4.2580	0.0410	0.2290	28.4170	0.0900	0.1040	21.1400	36.0530	0.0530
332	2368	44.6800	5.5440	0.0570	0.0000	28.4170	0.0900	0.1040	21.1400	36.0530	0.0530
333	972	43.4800	5.5770	0.0540	0.0250	28.4170	0.0900	0.1040	21.1400	36.0530	0.0530
334	1772	34.1510	5.0900	0.0980	0.0400	28.4170	0.1260	0.0330	21.1400	36.0530	0.0530
335	4557	25.7530	8.2330	0.4040	0.0070	32.1050	0.0360	0.0480	20.4550	23.9990	0.0500
336	2516	27.3650	8.1610	0.3740	0.0210	32.1050	0.0160	0.0530	20.4550	23.9990	0.0500
337	1675	29.4440	7.0560	0.4040	0.0030	32.1050	0.0110	0.0500	20.4550	23.9990	0.0500
339	5237	16.6370	7.2800	0.3000	0.0030	34.9180	0.0890	0.0040	22.5040	23.2660	0.0560
340	3257	20.2320	7.8230	0.3780	0.0050	40.3260	0.0180	0.0060	21.5670	20.9890	0.0480
341	1890	23.7130	8.7280	0.6250	0.0050	32.7530	0.0550	0.0140	24.2290	15.8790	0.0260
342	4339	30.5320	8.3980	0.4510	0.0110	37.6000	0.0130	0.0350	20.4730	32.8840	0.0260
343	2752	27.6070	9.0600	0.9830	0.0080	29.5180	0.0220	0.0700	20.1420	34.5010	0.0320
344	3607	27.1210	8.5440	0.5670	0.0050	34.5700	0.0200	0.0320	19.7840	22.3990	0.0440
345	3274	21.3510	8.7490	0.5510	0.0050	41.0590	0.0190	0.0200	20.0660	23.5230	0.0460
346	3658	19.0950	7.3750	0.2910	0.0050	36.3790	0.0500	0.0150	22.7480	29.3410	0.0430
347	4389	13.4520	8.1400	0.2080	0.0030	36.3790	0.0230	0.0160	22.7480	29.3410	0.0430
348	4014	20.0190	6.5200	0.1920	0.0080	36.3790	0.0280	0.0210	22.7480	29.3410	0.0430
349	2909	37.7710	5.9840	0.1440	0.0400	36.3790	0.0460	0.0770	22.7480	29.3410	0.0430
350	3941	29.0800	5.5830	0.0880	0.0220	36.3790	0.1740	0.0130	22.7480	29.3410	0.0430
351	3324	31.1230	7.5700	0.3080	0.0090	29.4590	0.0350	0.0340	23.1850	37.6000	0.0490
352	4219	32.1510	6.8520	0.2000	0.0140	29.4590	0.0540	0.0650	23.1850	37.6000	0.0490
353	1308	39.7980	7.7200	0.6720	0.0050	29.4590	0.0220	0.0150	23.1850	37.6000	0.0490
354	881	29.6350	9.3500	0.7760	0.0070	42.5910	0.0410	0.0010	16.6400	23.1960	0.0390
356	2241	40.7900	6.5770	0.0440	0.0020	21.3920	0.0170	0.1210	18.5440	55.5900	0.0360
357	1900	37.4490	5.7880	0.0360	0.0050	21.3920	0.0110	0.0210	18.5440	55.5900	0.0360

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
358	1638	43.9150	6.0310	0.0330	0.0140	21.3920	0.0110	0.0210	18.5440	55.5900	0.0360
359	2257	35.4540	5.9640	0.0420	0.0030	21.3920	0.0160	0.0300	18.5440	55.5900	0.0360
360	2329	39.5180	5.9950	0.0480	0.0050	21.3920	0.0170	0.0330	18.5440	55.5900	0.0360
361	2323	42.6220	6.5740	0.0790	0.0040	21.3920	0.0420	0.0330	18.5440	55.5900	0.0360
362	1315	44.3030	6.5710	0.0550	0.0200	21.3920	0.0180	0.0700	18.5440	55.5900	0.0360
363	2297	30.0950	5.5330	0.0360	0.0240	21.3920	0.0130	0.0550	18.5440	55.5900	0.0360
364	1660	20.9500	5.1250	0.0300	0.0170	21.3920	0.0180	0.0700	18.5440	55.5900	0.0360
365	1189	43.2090	6.8920	0.0530	0.0300	21.3920	0.0180	0.0700	18.5440	55.5900	0.0360
366	1334	34.6530	9.0070	0.4510	0.0040	21.3920	0.0090	0.0600	18.5440	55.5900	0.0360
367	2018	30.2770	7.0900	0.2980	0.0020	21.3920	0.0110	0.2810	18.5440	55.5900	0.0360
368	405	30.4430	8.3310	0.3860	0.0100	25.6870	0.0320	0.0600	19.9280	34.0560	0.0380
369	2921	38.6000	9.5480	0.4520	0.0030	25.6870	0.0180	0.0710	19.9280	34.0560	0.0380
370	2911	30.0030	8.3760	0.2980	0.0030	25.6870	0.0130	0.0330	19.9280	34.0560	0.0380
371	1403	37.5850	7.9170	0.3420	0.0000	25.6870	0.0230	0.0910	19.9280	34.0560	0.0380
372	1672	30.6710	7.4130	0.3100	0.0130	26.8430	0.1220	0.0370	22.2140	37.3380	0.0450
373	1353	34.0780	8.1600	0.2580	0.0050	26.8430	0.0330	0.0090	22.2140	37.3380	0.0450
374	2253	28.1310	8.0950	0.3270	0.0040	26.8430	0.0340	0.0930	22.2140	37.3380	0.0450
375	1102	37.5350	8.6010	0.3650	0.0190	26.8430	0.0240	0.0280	22.2140	37.3380	0.0450
376	2130	25.1840	7.0510	0.3160	0.0020	26.8430	0.0440	0.0180	22.2140	37.3380	0.0450
377	2771	36.6820	6.5040	0.1960	0.0190	27.7710	0.0660	0.0910	23.1800	32.9500	0.0440
378	1105	36.1600	8.6870	0.5660	0.0220	27.7710	0.0120	0.0280	23.1800	32.9500	0.0440
379	4106	31.0790	7.3430	0.2070	0.0070	27.7710	0.1170	0.2010	23.1800	32.9500	0.0440
380	3528	28.0230	8.2450	0.4920	0.0170	30.2650	0.0410	0.0470	18.0930	35.1630	0.0400
381	1260	25.6090	9.7650	0.3680	0.0210	40.4880	0.0010	0.0010	16.7060	22.9660	0.0210
382	2330	26.7600	8.3830	0.3820	0.0160	39.7650	0.0230	0.0820	20.8340	12.3540	0.0320
383	1554	28.1780	7.1990	0.3920	0.0140	41.6370	0.0250	0.0240	21.4040	21.9110	0.0590
384	4770	29.4790	7.3440	0.3420	0.0100	38.5900	0.0480	0.0160	21.4010	16.1510	0.0450
385	3509	25.5410	6.0020	0.1930	0.0020	33.8520	0.1820	0.0140	22.6140	26.4430	0.0380
386	1219	39.9250	6.2080	0.2410	0.0450	33.8520	0.0540	0.0410	22.6140	26.4430	0.0380
387	4278	31.7960	6.7190	0.1450	0.0220	33.8520	0.0910	0.0340	22.6140	26.4430	0.0380
388	4566	29.1840	7.0490	0.4100	0.0150	33.0490	0.0640	0.1450	23.2850	25.7900	0.0450
389	2244	42.1650	8.1220	0.2760	0.0040	29.7250	0.0350	0.2230	21.8870	39.2520	0.0340
390	1554	41.1860	8.0920	0.2550	0.0100	29.7250	0.0150	0.0590	21.8870	39.2520	0.0340
391	2290	40.6310	7.7530	0.2460	0.0030	29.7250	0.0110	0.0420	21.8870	39.2520	0.0340
392	1859	32.3830	8.7650	0.6730	0.0010	29.7250	0.0130	0.3390	21.8870	39.2520	0.0340
393	1074	33.1460	6.9700	0.1420	0.0120	28.0780	0.0910	0.0140	22.7300	35.5520	0.0510
394	231	26.7820	6.0860	0.1010	0.0010	28.0780	0.1080	0.0270	22.7300	35.5520	0.0510
395	2427	42.2090	6.5140	0.1070	0.0050	28.0780	0.0480	0.0870	22.7300	35.5520	0.0510
396	3320	43.5220	6.3520	0.0820	0.0150	28.0780	0.0590	0.0410	22.7300	35.5520	0.0510
397	3237	42.8320	6.4740	0.0520	0.0120	28.0780	0.0390	0.1900	22.7300	35.5520	0.0510
398	1706	43.2160	6.9590	0.1200	0.0150	28.0780	0.0640	0.0880	22.7300	35.5520	0.0510
399	1853	30.2760	6.4920	0.1580	0.0050	28.0780	0.0590	0.4640	22.7300	35.5520	0.0510
400	2300	40.8600	6.4900	0.1250	0.0370	28.0780	0.1250	0.0640	22.7300	35.5520	0.0510
401	2638	36.0400	5.0680	0.0740	0.0410	28.0780	0.0530	0.1130	22.7300	35.5520	0.0510
402	3015	36.4010	6.2520	0.1220	0.0130	28.0780	0.0200	0.0330	22.7300	35.5520	0.0510
403	2871	37.9320	5.9630	0.0800	0.0100	28.0780	0.3700	0.0120	22.7300	35.5520	0.0510
404	3816	38.4180	5.9830	0.1070	0.0210	28.0780	0.0970	0.2800	22.7300	35.5520	0.0510
405	4823	34.3560	6.0580	0.1960	0.0240	35.5520	0.1400	0.0360	27.5940	27.7710	0.0500
406	2578	35.5150	6.4520	0.2290	0.0040	35.5520	0.1010	0.0190	27.5940	27.7710	0.0500
407	2418	28.6760	6.2210	0.1940	0.0180	35.5520	0.0550	0.0360	27.5940	27.7710	0.0500

ZONE NUMBER	NUMB. OF HOUSES	AGE	ROOMS	LOTSIZE	PLUMBING	TRAVEL COSTS	LANDUSE1	LANDUSE2	PUPIL RATIO	CRIMEEXP	PROPTAX
408	1966	29.6030	6.0600	0.2700	0.0120	35.5520	0.0300	0.0110	27.5940	27.7710	0.0500
409	3786	23.4900	6.5900	0.2220	0.0050	35.5520	0.0690	0.0110	27.5940	27.7710	0.0500
410	2700	38.7110	6.9770	0.2050	0.0150	31.3750	0.0850	0.0100	21.0850	28.8180	0.0480
411	3383	36.2150	7.3040	0.2240	0.0100	31.3750	0.0460	0.1300	21.0850	28.8180	0.0480
412	3464	28.6030	7.2680	0.2370	0.0130	31.3750	0.0640	0.0740	21.0850	28.8180	0.0480
413	3174	26.5510	6.7060	0.2940	0.0130	47.0870	0.0590	0.0070	22.9900	17.0470	0.0420
414	4109	21.3870	6.9540	0.2180	0.0040	35.0580	0.0240	0.2440	24.5930	18.7460	0.0410
415	3182	25.2260	6.6490	0.2340	0.0100	35.0580	0.0350	0.0420	24.5930	18.7460	0.0410
416	3404	26.3640	7.9050	0.3930	0.0110	40.9360	0.0170	0.0260	20.1840	18.4670	0.0420
417	2078	35.4550	8.4470	0.7110	0.0090	58.2070	0.0140	0.1740	22.8640	42.8200	0.0440
418	1041	29.0220	7.1260	0.5570	0.0590	49.9990	0.0190	0.0020	23.4510	14.0270	0.0350
419	2763	37.9700	7.4090	0.2500	0.0050	57.2260	0.0670	0.0770	23.4570	53.7320	0.0790
420	2950	38.0150	7.7750	0.3800	0.0030	49.1560	0.0370	0.0510	22.8100	40.2460	0.0490
421	2211	26.5040	8.3670	0.4070	0.0050	49.1550	0.0490	0.0140	22.8100	40.2460	0.0490
422	4174	35.4020	6.0860	0.2770	0.0230	40.7720	0.0440	0.0090	25.7020	18.9730	0.0510
423	2048	26.1120	7.6730	0.5820	0.0250	43.7720	0.0200	0.0030	19.9450	19.3950	0.0420
424	4553	29.1250	7.0740	0.5080	0.0120	60.2800	0.0150	0.0340	22.7220	36.3430	0.0520
425	4125	24.3010	7.3150	0.4740	0.0100	56.2050	0.0180	0.0170	22.2550	24.6800	0.0520
427	2902	23.5320	6.7260	0.4720	0.0250	51.2130	0.0170	0.0110	22.8760	14.7910	0.0380
428	2558	26.6840	7.7040	0.5500	0.0120	41.5950	0.0360	0.0060	21.9520	20.0000	0.0400
429	2728	46.5940	5.9350	0.0510	0.0210	19.8660	0.1460	0.0120	24.3770	47.4650	0.0630
430	2410	37.8420	6.1680	0.0620	0.0130	19.8660	0.0530	0.0330	24.3770	47.4650	0.0630
432	1554	48.2670	5.0950	0.0270	0.1130	19.8660	0.5460	0.0070	24.3770	47.4650	0.0630
434	1452	49.2430	5.5680	0.0190	0.0270	19.8650	0.2380	0.0240	24.3770	47.4650	0.0630
435	1750	45.2190	6.0560	0.0250	0.0150	19.8660	0.4150	0.0120	24.3770	47.4650	0.0630
436	1587	40.7830	6.0020	0.0700	0.0430	23.9030	0.0340	0.0630	24.9240	39.8850	0.0530
437	3190	32.7600	6.2980	0.0960	0.0270	23.9030	0.0990	0.0160	24.9240	39.8850	0.0530
438	3370	31.7620	6.2440	0.0920	0.0130	23.9030	0.0210	0.0100	24.9240	39.8850	0.0530
439	1583	41.7780	6.5900	0.0900	0.0050	23.9030	0.0210	0.0090	24.9240	39.8850	0.0530
440	2557	42.3230	5.7740	0.0500	0.0230	23.9030	0.2220	0.0300	24.9240	39.8850	0.0530
441	1739	42.5560	6.1260	0.0490	0.0140	23.9030	0.0350	0.0480	24.9240	39.8850	0.0530
442	1231	43.4510	6.4350	0.0580	0.0030	26.3640	0.0340	0.1140	23.1490	28.4740	0.0380
443	1607	39.9260	6.2860	0.0830	0.0090	26.3640	0.3350	0.0280	23.1490	28.4740	0.0380
444	1127	47.3320	7.7530	0.1020	0.0030	26.3640	0.3350	0.0280	23.1490	28.4740	0.0380
445	759	38.1600	6.5920	0.0860	0.0220	26.3640	0.2910	0.0390	23.1490	28.4740	0.0380
446	1668	45.0320	6.2580	0.0720	0.0300	26.3640	0.0340	0.1140	23.1490	28.4740	0.0380
999	0	1.0000	1.0000	1.0000	1.0000	47.0870	0.0590	0.0070	22.9900	17.0470	0.0420

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