

Essays on Economic Growth and Informational Frictions

by

Samuel Pienknagura

B.A. Economics, London School of Economics and Political Science (2003)

M.A. Economics, Centro de Estudios Monetarios y Financieros (2005)

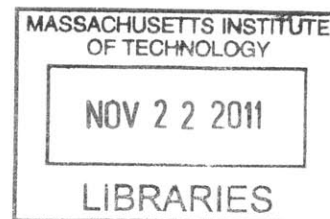
Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2011



ARCHIVES

© 2011 Samuel Pienknagura. All rights reserved.

The author hereby grants to Massachusetts Institute of Technology permission to
reproduce and
to distribute copies of this thesis document in whole or in part.

//

Signature of Author
Department of Economics
August 15, 2011

Certified by
Daron Acemoglu
Elizabeth and James Killian Professor of Economics
Thesis Supervisor

Certified by
Abhijit Banerjee
Ford International Professor of Economics
Thesis Supervisor

Accepted by
Michael Greenstone
3M Professor of Environmental Economics
Chairman, Departmental Committee on Graduate Studies

Essays on Economic Growth and Informational Frictions

by

Samuel Pienknagura

Submitted to the Department of Economics
on August 15, 2011, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Abstract

This thesis consists of three chapters on Economic Growth and Informational Frictions. Chapter 1 investigates the relation between financial development, R&D expenditure and aggregate growth. It provides empirical evidence that financial development has a large positive effect on both growth and R&D, and that the effect of financial development on growth is likely to be explained by its effect on R&D. I also study a general equilibrium model in with predictions which are consistent with the empirical regularities mentioned above. In particular, aggregate growth increases as financial development increases. The model also predicts that financial development produces large welfare gains, specially at low levels of financial development. Finally I show that the model studied suggests that R&D policy is welfare improving and that policy should be conditional on the level of financial development.

Chapter 2 gives an empirical assessment of the world income distribution. In particular, I take a CES production function implied by a Skill-Biased technical change model and fit this production function to the data. The calibration results give evidence of the importance of including different skills to account for the observed income differences over time. I also show that the calibration exercise is validated by the estimated values of the parameters of the model.

In Chapter 3 I study a model of entry under uncertainty. In particular, I analyze an economy where potential entrants make entry decisions after receiving noisy signals of the true demand levels for the different sectors of the economy. I show that equilibrium strategies depend on the precision of the signals received by agents. When precision is low the equilibrium of the game is a pure strategy equilibrium where agents enter the sector for which they receive a higher signal. On the other hand when precision is high the optimal strategy is to randomize over which sector to enter. The model also highlights the non-monotonic relations between the discrepancy between the equilibrium and efficient entry levels and both the precision of the signal and the true relative demand between sectors.

Thesis Supervisor: Daron Acemoglu
Title: Elizabeth and James Killian Professor of Economics

Thesis Supervisor: Abhijit Banerjee
Title: Ford International Professor of Economics

Contents

1	Financial Development, R&D and Growth	6
1.1	Introduction	6
1.2	Empirical Evidence	11
1.2.1	Data	12
1.2.2	Results	14
1.2.3	From R&D to Growth	22
1.3	Model	24
1.3.1	Preferences and Technology	25
1.3.2	Equilibrium	29
1.3.3	Steady-State	33
1.4	Numerical Solution	39
1.4.1	Computational Strategy	39
1.4.2	Value Function, R&D, and μ	41
1.4.3	Aggregate Results and Financial Development	41
1.5	Policy	42
1.5.1	Uniform R&D Subsidy	44
1.5.2	Size-Dependent Two-Level R&D Subsidy	45
1.6	Conclusion	46
1.7	References	49
1.8	Appendix 1: Proofs	52
1.9	Appendix 2: Figures and Tables	59
2	Skill-Biased Technical Change and the Evolution of the World Income Distribution: An Empirical Assesment	77

2.1	Introduction	77
2.2	Skill-Biased Technical Change and the World Income Distribution	80
2.2.1	Skill Biased-Technical Change: An Overview	80
2.2.2	A Simple Calibration Exercise	82
2.3	Comparison Across Models	86
2.3.1	Results	89
2.4	Estimation	90
2.5	Conclusion	92
2.6	References	93
2.7	Appendix 1: Figures and Tables	96
3	Demand Uncertainty, Information Quality and Sectoral Entry	122
3.1	Introduction	122
3.2	The Model	125
3.2.1	Preferences and Technology	125
3.2.2	Equilibrium	127
3.3	The Model with Dispersed Information	132
3.3.1	Equilibrium characterization	133
3.4	Equilibrium Characterization: Special Case	137
3.4.1	Dispersed Information and Welfare	139
3.5	Conclusion	141
3.6	References	143
3.7	Appendix 1: Proofs	145
3.7.1	Second Stage Equilibrium of the Game	145
3.7.2	Proof of Proposition 7	146
3.7.3	Proof of Proposition 8	151
3.7.4	Proof of Lemma 9	152
3.7.5	Proof of Proposition 10	154
3.7.6	Proof of Proposition 11	158
3.7.7	Proof of Proposition 12	159
3.8	Appendix 2: Figures and Tables	160

Acknowledgements

I am deeply indebted with my advisors Daron Acemoglu and Abhijit Banerjee, I appreciate their patience and guidance during my years at MIT. I would like to thank specially Daron Acemoglu for his neverending confidence in me. He has pushed when I needed a push and he has been supportive when I needed support. The privilege of working with him as a student, research assistant and advisee has made me a better economist.

I would also wish to thank all the outstanding faculty that I have come across in my years as an MIT PhD student for nurturing my curiosity and understanding of economics. Life at MIT would not have been any easier without the fantastic staff at the economics department. Special thanks to Gary King, Peter Hoagland and Lauren Fahey for their responsiveness and speediness in solving any problem.

In my years at MIT I had the pleasure to meet fantastic people and make some extraordinary friends. I wish to thank Leopoldo, Ximena, Mauro, Nico, Pablo Kurlat and Pam, Pablo Querubin and Sahar, Luis, Felipe, Arthur, Johanness, Dani, Will and Keka, Konrad, Monica, Laura, Jen-Jen and all the amazing people I met for all the fun times and laughs we shared during this years. Special thanks to Mauro and Leopoldo who have become a part of my extended family.

In my road to this thesis I had the support of many important people who believed in me. I wish to specially thank Juan Vilaseca, Omar Maluk and Silvia Maluk for their support, friendship and for giving me the push I needed to leave Ecuador and start this journey. Without you this thesis would not be possible.

The hardest part of the PhD experience was being away from my family. I owe a big part of this thesis to Susana, Isaac, Gaby, Irene, Kiersten and Melina. Despite the great distance separating us you were always by my side giving me the strength I needed. I also want to thank Joanna and Daniel for being my little family island in Boston. You are also an important part of this thesis.

Last but not least, all my gratitude and love to Sandra. Thank you for your courage, patience, understanding and love which were crucial at all times. Without you by my side this thesis wouldn't be possible.

Chapter 1

Financial Development, R&D and Growth

1.1 Introduction

A large literature emphasizes the positive effects of financial development on economic growth. Figure 1, which shows a plot of the aggregate growth rate in real GDP between 1980 to 2005 for 146 countries against private credit, provides evidence of a positive correlation between financial development and growth at the aggregate level. The same pattern emerges when one looks at the relation between financial development and firm level growth in sales (Figure 2). One of the possible channels by which financial development could affect growth is by giving firms financial slack and allowing them to invest in growth-enhancing activities. Figure 3, for example, shows that financial development and R&D expenditure are positively correlated. This suggests that understanding the relation between financial development, R&D and growth is very important when trying to understand the process of economic growth. The challenge when trying to interpret these results as causal comes from the fact that measures of financial development are likely to be correlated with other country characteristics affecting growth and R&D. Robinson (1952), for example, argues that entrepreneurial activity paves the way for financial development, suggesting a potential feedback from economic development to financial development. There are two main contributions of the present paper. First it provides empirical and theoretical evidence that financial development affects growth through the effect it has on R&D investment by firms. Second, it studies the effect that this mechanism has on the optimal design of policy.

This paper provides evidence of an economically important effect of financial development on both growth and R&D. To address the endogeneity concern raised above I use a difference-in-difference strategy which exploits the idea that financial development should affect differentially the growth and R&D investment of firms operating in sectors with different needs for external funds. This idea is based on the following intuition. Financial underdevelopment will make raising funds to finance growth-enhancing activities, such as R&D, harder. This problem will be specially relevant for firms operating in industries that, for technological reasons, require larger amounts of liquid funds to invest. For this reason, an increase in financial development should have a larger effect on growth and R&D expenditure for firms operating in sectors with higher dependence on external financing. This intuition was first introduced by Rajan and Zingales (1998) to analyze the relation between financial development and industry growth. To study the empirical relation between financial development, R&D expenditure and growth, I use firm level data from Compustat (Global and North America), and find that differences in growth rates and R&D expenditure between firms in sectors with high and low financial dependence are higher as financial development increases. This result is consistent with the results found using industry level data (Rajan and Zingales (1998)). The findings support the idea that both R&D and growth are affected by financial development. I show these reduced form findings are robust after controlling for a variety of firm level characteristics. I also show that the results are robust to alternative measures of financial dependence, alternative measures of financial development and to the inclusion of various country characteristics. Finally I point at the important relation of financial development and firm dynamics.

The results obtained when estimating these reduced form relations point at the importance of the effect of financial development on growth and R&D. In particular, the predicted difference in growth between a firm operating in the machinery industry and a firm operating in the paper industry (industries located at the 75th and 25th of financial dependence, respectively) is 0.9% higher in Australia than in India (countries located at the 75th and 25th of financial development, respectively). A similar exercise suggests a 5.6% difference in R&D expenditure. These numbers suggest the economic importance of the effect of financial development on growth and R&D.

The reduced form relations also highlight the possibility that the effect of financial development on firm growth is driven by the effect it has on R&D. To test this hypothesis, I run the baseline specification for the growth regression controlling for R&D expenditure. I find that once we control for R&D, the interaction between financial development and financial dependence has no

statistical significance in explaining differences in growth. One caveat to this finding is the potential endogeneity of R&D and growth. To take this into account, I follow two approaches. First, I follow a 2SLS strategy where I use past values of R&D as an instrument. Second, I use variations across countries and time of the tax component of the user cost of capital as an instrument for R&D expenditure. The results support the hypothesis that financial development affects growth through the effect it has on R&D.

The ideas behind the methodology described above raise questions which are important for our understanding of the relation between growth and financial development. Which industries and firms are those which rely more on external financing and what is driving these differences? To understand the channels through which financial dependence affects R&D and consequently, growth, I study an infinite-horizon general equilibrium model. Using an infinite-horizon will be useful to capture the dynamics of R&D decisions by firms and the interaction of these dynamics with the level of financial development, a feature which is present in the data. In the model firms are heterogeneous in two dimensions: they differ in their labor productivity and in a technological parameter which affects the R&D production function. Differences in labor productivity are standard in most Schumpeterian growth models. The novel ingredient is the heterogeneity in the R&D production function which is sector specific. There are differences between these two firm characteristics. On the one hand, firms can improve their labor productivity, and therefore reduce their marginal cost, by engaging in R&D investment. The R&D technological parameter, on the other hand, is constant over the entire life of the firm and can be interpreted as the R&D ability associated to the sector the firm operates in. In the model firms operating in high R&D ability sectors will be growing faster and investing more in R&D, which as the empirical results suggest is related to higher financial dependence. At the heart of the model is the assumption that firms have to borrow in order to finance their R&D expenditure and they can only borrow a proportion of their profits. I prove the existence of a steady-state general equilibrium in this economy and characterize the properties of R&D for different labor productivity-ability pairs. I find that firms with higher R&D ability invest more on R&D which implies these will be the more financially dependent firms in the economy.

In order to address if the model can match the empirical facts mentioned above, I use numerical methods to solve for the steady-state general equilibrium of the economy and find three patterns consistent with the data:

- 1) aggregate growth is increasing in financial development,*

ii) differences in expected growth between firms in the high R&D ability sector and firms in the low R&D ability sector are increasing in financial development, and

iii) differences in R&D expenditure between firms in the high R&D ability sector and firms in the low R&D ability sector are increasing in financial development.

The results also suggest that financial development increases welfare and the gains in welfare are larger for low levels of financial development.

The model used sheds light on the design of policy. In particular it shows there are three dimensions which the policy maker should consider when designing policy. On the one hand, even without financial constraints, one would expect policy to be targeted specially towards small firms operating in the high R&D ability sector. On the other hand, once I introduce financial constraints, this conclusion is reinforced as small firms in the high R&D ability sector, which are the ones who contribute more to growth, are also the ones hit harder by financial constraints. I restrict the analysis to uniform and size-dependent policies and find that the optimal uniform R&D subsidy is decreasing in financial development. This result highlights the role of subsidies in both increasing growth and in relaxing the liquidity constraints. I then show that a size-dependent R&D subsidy yields higher welfare than a uniform subsidy. A novel result is the shape of the optimal R&D subsidy. More specifically, I find that in the absence of financial frictions, the optimal R&D subsidy is non-decreasing in labor productivity. This result emerges as a consequence of the positive correlation between labor productivity and the firm's R&D ability. Once we introduce financial frictions this result does not necessarily hold. In particular, for low levels of financial development the optimal R&D subsidy will be decreasing in a firm's labor productivity. In this case the "liquidity" effect of subsidies dominates the "ability composition" effect of subsidies.

This paper contributes to a large body of literature which emphasizes the relation between financial development and growth¹. King and Levine (1993), following Goldsmith (1969), were among the first to study empirically the effect of financial development on aggregate growth finding a positive correlation between the two. Their study, however, does not take into account the potential feedback from aggregate growth to financial development. In order to study the causal effect of financial development on growth, Beck et al. (2000) use an instrumental variable dynamic panel approach to correct for the potential endogeneity of financial development. Consistent with King and Levine, they find a positive effect of financial development on aggregate growth. A second strategy used to identify the causal relation of financial development on growth was first

¹For a detailed review of the literature see Levine (2003).

proposed by Rajan and Zingales (1998). Using a difference-in-difference approach they find that financial development increases growth specially in industries with stronger dependence of external finance. This methodology provides further insights as it highlights a potential mechanism through which financial development affects growth. Since this influential work, several authors have used this empirical strategy to address similar questions. Raddatz (2006), for example, uses the same methodology to highlight the effect of financial constraints in sectoral volatility. Similarly, Aghion, Hemous, and Kharroubi (2009) use this methodology to study the effect of cyclical fiscal policy on sectoral growth and R&D. They find that growth and R&D respond in the same direction to cyclical policy which is consistent with what I find. In a similar vein, Aghion, et al. (forthcoming) study the effect of the interaction of financial development and volatility on aggregate growth and R&D. They find that both R&D expenditure and aggregate growth are more affected by volatility in countries where financial development is smaller. Both of these papers highlight the tight link between movements in R&D expenditure and movements in aggregate growth. The main contribution of the present paper is to take the extra step of addressing whether the effect of financial development on growth is caused by the effect it has on R&D expenditure.

The present paper also contributes to the endogenous growth literature by highlighting the importance of the interaction between size, R&D technology and financial development in the process of economic development. On the theoretical side, this paper is closest to Akcigit (2009). The main difference between the model presented there and this work is the introduction of heterogeneity in R&D production and financial constraints. This paper is also related to a large body of theoretical literature which highlights the positive effect of financial development on growth. Most of these studies stress the importance that the financial sector has on reducing investment risk and investment volatility (Greenwood and Jovanovic (1990), Saint-Paul (1992), Aghion et al. (2005), Laeven (2009)). Like Aghion et al. (forthcoming), this paper highlights the role of the financial sector in providing liquidity for productive activities.

Finally, this paper contributes to the literature on R&D policy and state-dependent policy. This literature typically argues that size-dependent policies are detrimental for the economy as they misallocate resources (Guner et al. (2008), Restuccia and Rogerson (forthcoming)). On the other hand, recent work on endogenous growth has highlighted the potential role of size-dependent policies in improving competition and innovation (Acemoglu and Akcigit (2009), Akcigit (2009)). This paper shares some of the insights pointed in these two papers. The main contribution here is to highlight how the presence of financial development shapes the optimal size-dependent policies,

showing that policies which are increasing in size are optimal for high levels of financial development while policies which are decreasing in size are optimal for low levels of financial development.

The rest of the paper is organized as follows: Section 2 describes the econometric analysis and presents the main empirical findings. Section 3 presents the model and presents the main theoretical results. Section 4 presents the numerical results and analyzes the general equilibrium effects of financial constraints on firm level and aggregate growth. Section 5 analyzes the policy implications of financial constraints on the model. Finally, Section 6 concludes.

1.2 Empirical Evidence

The objective of this section is to analyze empirically the effect a country's financial development has on firm level growth and R&D investment. The difficulty of assessing this question is that measures of financial development are likely to be correlated with other country characteristics which affect firm growth and R&D decisions. One way to deal with this problem was suggested in a seminal paper by Rajan and Zingales (1998) (henceforth RZ). Their methodology uses a difference-in-difference approach which isolates the effect of financial development from other country characteristics. Using this approach I show three results which are the main findings of this section. First I show that the interaction between financial development and the financial dependence of a sector has a positive effect on firm level growth. Second, I show that the same pattern emerges when I analyze the relation between the interaction between financial development and financial dependence and firm level R&D expenditure. Finally I show evidence that the effect of a country's financial development on growth is potentially caused by the effect that financial development has on R&D and that this effect is significant even after addressing potential endogeneity concerns.

At the heart of this empirical section lies the hypothesis that firms which are in sectors with high financial dependence will be more favored by financial development. This methodology, first studied empirically by RZ, has become popular in recent years (see Raddatz (2006), Aghion, Hemous and Kharrobi (2009)). The empirical literature, however, has given little convincing evidence of the channel through which financial development affects growth. The main hypothesis I test in this section is that one potential channel through which financial development, financial dependence, and growth interact is through R&D investment by firms.

The rest of this section is organized as follows. First I describe the data I use in detail. Then

for each of the three regressions of interest I present the empirical strategy and the main results, followed by robustness checks.

1.2.1 Data

Firm level data

The two main data sources I use are Standard and Poor's Compustat North America Annual and Standard and Poor's Compustat Global Annual, both of which are comparable between each other. Compustat North America includes information on a large number of publicly held companies in the US and Canada since 1950, while Compustat Global focuses on publicly held firms in non-US and non-Canadian marketplace starting in 1987. It includes firms from 98 different countries in all continents. I focus on the period 1987-2006 which is the period of time for which both data sets match. Since data from Compustat global comes in the domestic currency, I have deflated and converted all monetary items to US dollars. Data for the exchange rates have been taken from the IMF's International Financial Statistics².

The literature using the RZ methodology has focused on industry level data. Instead, I use firm level data which will allow me to control for country, industry and firm characteristics. The other important advantage of using firm level data is that it will allow me to treat certain country level variables as exogenous to the firm. This will prove useful specially when analyzing the effect of R&D on growth. On the downside, using Compustat has the disadvantage of having only publicly traded firms which biases the sample towards large firms. As shown on Figure 4, this becomes less of an issue over time as the average size in the sample becomes smaller. This, plus the increase in the sample size over time of both data sets, suggests that Compustat has become more representative of the population of firms. I follow the common practice of focusing only on industrial firms, where I define industries using 4-digit International Standard Industry Classification codes (ISIC REV.2). This definition of industry is consistent with the one used in RZ and Raddatz (2006).

Since my goal is to analyze the long run effects of financial development, I will use variables aggregated over 5 year periods. Other options would be to use yearly data or to aggregate over the whole sample period. The former could be capturing the effect of short-run variation which I want to isolate from. On the other hand, the latter could be capturing other long run trends of the economy which could contaminate the effect of financial development. For this reason, I

²For countries who have changed the exchange rate during the period of analysis, I have obtained the exchange rate from various different sources.

construct aggregate measures of the variables of interest for a given interval. I define Sales and R&D expenditure for interval T as the average of the variables over the interval. Age is defined as the number of years from the initial public offering date of firm i and the initial year of the interval, t . A firm's growth rate for interval T will be calculated as the annualized growth rate in sales for the period. I also construct a proxy for the book value of a firm. This is defined as the assets held by the firm at the beginning of interval T minus the liabilities of the firm at the beginning of interval T .

Financial Dependence

Compustat North America is also used to construct the measure of financial dependence of a sector. In particular I follow the methodology in RZ in constructing an industry specific measure of financial dependence as the median ratio of capital expenditure over sales for firms in the US³. I construct the measure for the 1987-2006 period which is the period of time used in the regressions below. I also construct the same measure for the period 1970-1987 both as a robustness check of the stability of the measure and for use in the 2SLS estimation⁴.

As mentioned in Raddatz (2006) the use of US firms can be justified by the observation that the US financial markets are among the most developed in the world. The assumption I make is that a firm's financial dependence affects not only how much a firm can borrow for working capital and capital expenditure, but also how much a firm can invest in R&D. One concern one might have is that the measure of financial dependence used in RZ is not capturing the right technological difference in R&D investment across sectors. To test this I construct an equivalent R&D financial dependence measure. The two measures are positively correlated with a correlation coefficient of 0.4.

Financial Development and Other Country Measures

Measures of financial development come from different sources. The main measure of financial development I use is private credit as a fraction of GDP, which can be interpreted as the access to bank loans that firms in a particular country have. The value for this variable is taken directly from the World Bank's World Development Indicators (WDI) and is the country's average value for

³See Rajan and Zingales (1998) for the specifics of how the measure is constructed.

⁴As highlighted in RZ, the validity of this methodology rests on two assumptions. First, there is a technological reason for which certain sectors in the economy rely more on external funds to finance R&D projects. Second, this industry differences are common across countries and across time.

private credit over GDP during the period 1980-1995. We can think of this measure as the initial value of financial institutions for each country in the 1987-2006 period.

When doing robustness checks I use two other measures of financial development used in the literature. The first is a country's stock market capitalization and the second is a country's accounting standards. Stock Market Capitalization, taken from Beck et al. (2001), is a measure of the size of a country's equity markets with respect to GDP. The second alternative measure used is quality of accounting standards. This measure is obtained from La Porta et al. (1998) and, as mentioned in RZ, it captures how easy it will be for firms to raise funds from a wider circle of investors.

Measures of financial development are likely to contain measurement error and to be correlated with other country characteristics. In order to control for this potential endogeneity, I use dummy variables representing a country's legal origin (British, french, German, and Scandinavian) This instrument, which has been very popular recently in the Political Economy and Law and Finance literature, is taken from Laporta et al. (1998).

To test the effect of other institutions on growth and R&D, I use PPP adjusted GDP per capita in 1987 obtained from the World Bank's World Development Indicators. As in Acemoglu and Johnson (2005) I will instrument GDP per capita by using settler mortality.

Finally, in order to control for the fact that Compustat firms are potentially bigger than a average firm, I use average employment in each ISIC industry per country. This is obtained from the United Nations Industrial Development Organization (2001), Industrial Statistics Database (UNIDO).

Summary statistics of the variables used are presented in Table I.

1.2.2 Results

Financial Dependence and Financial Development: Basic Specification

I begin the empirical analysis by establishing the effect of financial dependence on growth and R&D. In particular I estimate the following two relations:

$$g_{ijTk} = \alpha_1 + \omega_{1k} + \delta_{1T} + \beta_1 * FDEP_j + \\ + \rho_1 * age_{ijTk} + \mu_1 * \ln(Sale_{ijTk}) + \varepsilon_{itk}$$

$$\begin{aligned}\ln(RD/Sale)_{ijTk} &= \alpha_2 + \omega_{2k} + \delta_{2T} + \beta_2 * FDEP_j + \\ &+ \rho_1 * age_{ijTk} + \mu_1 * \ln(Sale_{ijTk}) + e_{itk}\end{aligned}$$

where g_{ijTk} is the growth rate of firm i , at interval T , in industry j from country k , $\ln(RD/Sale)_{ijTk}$ is the log of R&D intensity, $FDEP_j$ is sector j 's measure of financial dependence, age_{ijTk} is the age of firm i at the beginning of interval T , ω_k, δ_T are country and time fixed effects respectively, and α is a constant. The results are shown in columns (1) and (5) of Table 2. The point estimates indicate that industries in which financial dependence is higher grow faster and invest more in R&D than firms with lower financial dependence. If the identification strategy is valid, this would suggest an important channel through which financial development affects economic growth: it will benefit more those firms which invest more in R&D and grow faster.

I will now study the effect of financial development on growth and R&D expenditure. To investigate these effects the following two equations will be estimated:

$$\begin{aligned}g_{ijTk} &= \alpha_1 + \theta_{1j} + \omega_{1k} + \delta_{1T} + \beta_1 * FDEV_k * FDEP_j + \\ &+ \rho_1 * age_{ijTk} + \mu_1 * \ln(Sale_{ijTk}) + \varepsilon_{itk}\end{aligned}$$

$$\begin{aligned}\ln(RD/Sale)_{ijTk} &= \alpha_2 + \theta_{2j} + \omega_{2k} + \delta_{2T} + \beta_2 * FDEV_k * FDEP_j + \\ &+ \rho_1 * age_{ijTk} + \mu_1 * \ln(Sale_{ijTk}) + e_{itk}\end{aligned}$$

where now θ_{1j} and θ_{2j} are industry fixed effects and the term $FDEV_k * FDEP_k$ is the interaction between country k 's measure of financial development and sector j 's measure of financial dependence. The interaction term will be the object of interest and will be capturing the effect of financial development on R&D investment and growth. As will be clear in what follows, the patterns that emerge from these two estimations are very similar. Furthermore, the concerns and consistency checks of the two reduced form estimations are almost identical. For this reason, in

what follows I will analyze the two equations simultaneously making clear when one deserves a different discussion than the other.

Growth, R&D Intensity and Financial Development

Column (2) of Table 2 shows the basic OLS regression of the growth regression. Consistent with the finding in RZ we have that the interaction term is positive and significant, with a point estimate of 0.027. The interpretation of this coefficient is that of a cross-partial derivative. In particular, it indicates that the difference in growth rates between firms which operate in a sector with high financial dependence compared to ones which operate in a sector with low financial dependence should be increasing as the country's financial development increases. In order to get a sense of the magnitude of this coefficient take a firm with average sales ($\log Sales = 0.0511$) in the country at the 75th percentile of financial development, Austria, and one in the country at the 25th percentile of financial development, India. The coefficient tells us that the predicted differential growth for a firm located in the 75th percentile of financial dependence (high dependence) compared to a firm operating in a sector at the 25th percentile of financial dependence (low dependence) in Austria is 0.9 percentage points per year higher than in India. This magnitude is large and economically important. Column (6) of Table 2 suggests a similar pattern for R&D intensity. To see the practical importance of the effect of the interaction term on R&D intensity I will perform a similar example as the one above. Take a firm with average size ($\log Sales = 0.0511$) with average R&D intensity ($\log(RD/Sale) = -3.31$) and assume sales grow at 10% annual rate (61.05% in five years). The estimated coefficient of R&D tells us that the predicted differential R&D expenditure for a firm with average sales located in the 75th percentile of financial dependence (high dependence) compared to a firm operating in a sector at the 25th percentile of financial dependence (low dependence) in Austria should be 5.58 percentage points per year higher than in India. This again sheds light on the economic importance of this channel.

Measurement Error and Endogeneity

One concern one might have about the previous results for growth and R&D intensity is that we are using proxies for financial development and financial dependence which could suffer from measurement error and bias the estimates. Other concern is that financial development might be endogenous to unobserved factors that affect growth. I deal with this concern in Columns (3), (4), (7) and (8) of Table 2. In columns (3) and (7) of Table 2 I use a 2SLS approach where I

instrument financial development with a country's legal origin. The validity of this instrument lies in the assumption that the country's legal origin affects growth only through financial development. For this reason one should take this result with caution as one can argue that a country's legal origin may affect firm-level growth and R&D through channels different than the level of financial development⁵. Columns (4) and (8) of Table 2 use the same instruments as before for financial development and also instruments financial dependence with the lagged value of the measure. The estimates for both growth and R&D intensity are still positive and significant and the estimated values are higher than the OLS counterparts, supporting the attenuation bias conjecture.

Reporting Bias

Another concern in the estimation of the R&D intensity regression is that R&D expenditure is not reported by many firms in Compustat. To correct for this selection concern, I apply Heckman's two-step procedure to deal with this issue. The instrument I use in this case is a propensity-to-report index similar to the one constructed in Akcigit (2009)⁶. The index is constructed in the following way. I choose 12 Compustat items common to all manufacturing firms across countries. The index is the share of these items that a firm reports in a given year and one would expect R&D reporting to be positively correlated with this index. Columns (9) and (10) in Table 2 report the estimates of the selection equation and the outcome equation respectively. Column (9) shows that the expected sign for the propensity to report index is robust to the assumption that firms in Compustat misreport items consistently. Column (10) shows that the inverse mills ratio is significant supporting the hypothesis of selection bias. As for the point estimate of the interaction term, it increases with respect to the OLS estimate suggesting that not taking into account the non-reporters dampens the effect of the interaction between financial development and financial dependence.

Other Measures of Financial Dependence and Financial Development

First I start by investigating how the results are affected by the fact that we are using a particular measure of financial dependence. We can think of an alternative measure of financial dependence where we use R&D expenditure as opposed to capital expenditure in the numerator. There are

⁵See Glaeser and Shleifer (2002) and Acemoglu and Robinson (2005) for a discussion.

⁶The items used to construct the index in Akcigit (2009) are not always reported in Compustat Global. For this reason the selected variables I use to construct the index do not match Akcigit (2009).

two problems with this measure of financial dependence. First, this measure is more likely to be correlated with the error term even when using firm level data. Second, as was discussed above, misreporting of R&D expenditure will bias this measure towards low financial dependence. For this reason I will take the results with caution. To try to deal with the first concern I restrict the analysis in the R&D regressions to non-US firms. The results are presented in column (1) of Table 3 and column (1) of Table 4. The estimated coefficients suggest that the effect of the interaction term becomes much bigger and is still significant when we use the alternative measure of financial dependence.

So far I have assumed that technological differences across sectors make some firms rely more on external financing for all types of expenditures. One possibility is that firms have different external dependence for different types of expenditure. To study the importance of differences in external dependence on growth and R&D I run the baseline specifications including two interaction terms, one using a measure of financial dependence using R&D expenditure and another using a measure of financial dependence using capital expenditures. Column (2) of Table 3 and Column (2) of Table 4 show that the two interaction terms are positive and significant suggesting that both measures of external dependence are important in explaining differences in growth and R&D expenditure. The magnitudes of the estimated coefficients are smaller than the estimated values of the regressions where each interaction term is considered independently which was expected as the two measures are positively correlated⁷.

The measure of financial development I use is a proxy for the true variable of interest which is access to private credit. In what follows I will check for the robustness of the measure used. In particular I use two alternative measures of financial development. The first alternative measure is Stock Market Capitalization. This variable is a measure of how developed the stock market of a country is. The second alternative measure used is a country's measure of accounting standards. Columns (3) and (4) of Table 3 and Columns (3) and (4) of Table 4 report the estimates of the interaction coefficient using the two alternative measures. The estimates for the interaction term using the alternative measures gives significant coefficients for both growth and R&D intensity, making us confident that the results are not driven by the choice of an arbitrary proxy.

⁷The coefficient for the R&D regression when using Non-US firms is 0.0299 and significant at 1% significance level.

Alternative Institutions

One explanation for the findings in the previous subsections is that financial development is capturing the effect of other institutions and country characteristics. For example, Acemoglu, Johnson, and Robinson (2001) find evidence that property rights institutions affect long run growth. In a similar vein Acemoglu and Johnson (2005) argue that measures of contractual institutions affect private credit as a fraction of GDP. Column (5) of Table 3 and Column (5) of Table 4 show the OLS estimations of the basic reduced form regression including the interaction of log GDP in 1987 with Financial Dependence, where log GDP is used as a proxy for other institutions. For the growth regression the added interaction term is positive and significant and the original interaction term remains positive and significant but the estimated value is smaller than in the baseline regression. For R&D intensity a similar picture emerges. In this case the added interaction term is positive but not significant while the estimated coefficient for the interaction between financial dependence and financial dependence is positive and significant but smaller in magnitude. Both of this results suggest that the baseline estimated coefficients could have been capturing the effect of other institutions which also affect growth and R&D investment. However, once we control for these other institutions, the effect of financial development on growth and R&D investment is still present.

Sample Selection

An additional concern is the use of Compustat. As mentioned above, Compustat firms are expected to be larger than the average firm in a country. This implies that Compustat firms could potentially be less constrained than the average firm⁸. If our only concern was that firms in Compustat are bigger but share the same characteristics as an average firm in the economy, this would make our estimates a lower bound for the true effect of financial development in a firm's growth and R&D responses. But there could be other unobservables intrinsic to Compustat firms. Using data from the United Nation's Industrial Statistics Database, I construct a measure of average employment in industry j for country k and then I construct a measure of a firm's employment relative to the average firm⁹. Column (6) of Tables 3 and 4 show the results when this extra control is added. Notice the coefficients for this extra control are positive and significant. This suggests that there

⁸Compustat firms are publicly held firms, which by definition means they are less constrained in raising funds than a small firm.

⁹Average employment is the ratio of total employment and number of establishments.

are firm characteristics in the Compustat sample which makes firms grow faster and invest more in R&D relative to the average firm. Also notice that in both growth and R&D regressions the coefficient for the interaction term increases but is still positive and significant and the difference with respect to the baseline estimate is small. The fact that the coefficients are similar suggests that the baseline results were not driven by the fact that Compustat firms are different than the average firm.

Financial Constraints and Size

The previous discussion raises one related question. Many models of financial constraints, like the seminal paper of Kiyotaki and Moore (1997), simplify financial frictions by imposing a borrowing limit. This borrowing limit will usually depend on a firm's current assets and market value. Precisely for this reason the effect of financial development on both R&D and growth could be affected by firm size. To analyze this possibility I study how the interaction term of financial dependence and financial development is affected by the firm's valuation. As discussed above, one could expect that the differential effect of financial development on firms with different levels of financial dependence drops as a firm becomes bigger since the financial constraints the firms are facing are relaxed with higher firm value. Ideally I would like to measure the firm's market valuation using observed stock prices and outstanding shares as in Baker and Wurgler (2002) and Fama and French (2002). Unfortunately, Compustat Global does not have entries for stock prices and for this reason I use a proxy for a firm's Book Value.

Table 5 shows the results of these regressions. First of all variables are deviations from the mean which makes the interpretation of the coefficient equivalent as in previous regressions. The main findings of these regressions are summarized in columns (1) and (4) of Table 5. The results support the significance of the interaction between Financial Dependence and Financial development for both R&D intensity and growth. The second thing to notice from these regressions is that the coefficient of the triple interaction term has the predicted sign but is not significant. One possible explanation for this pattern is that the relation between the interaction of financial dependence and financial development, although important, is non monotonic in a firm's size. This would explain the zero coefficient for the linear approximation. This particular explanation will be supported by the model presented in the next section.

As discussed above the use of proxies for a firm's value as well as for financial institutions could

be biasing the effects found in columns (1) and (4) of Table 5. To take this into account I run a 2SLS regression where first I instrument financial development with the country's legal origin (Columns (2) and (5) of Table 5) and then I also instrument financial dependence with lagged values (Columns (3) and (6) of Table 5). As can be seen the predictions from columns (1) and (4) follow through when we instrument financial development and financial dependence. As in the baseline regressions the coefficient for the interaction term between financial dependence and financial development increases when using a 2SLS approach.

Financial Constraints, Industry Heterogeneity and Firm Heterogeneity

So far I have exploited sectoral differences in financial dependence in order to identify the effect of financial development on growth and R&D expenditure. One might also suspect that there is a differential effect of financial development on firms with different growth profiles. The idea is that firms which for some technological reason have steeper growth profiles should gain more from financial development. To analyze this hypothesis I run the following two regressions

$$g_{ijTk} = \alpha_1 + \omega_{1k} + \delta_{1T} + \beta_1 * FDEV_k * high_i + \tau_1 * high_i + \rho_1 * age_{ijTk} + \mu_1 * \ln(Sale_{ijTk}) + \varepsilon_{itk} \quad (1.1)$$

$$\ln(RD/Sale)_{ijTk} = \alpha_2 + \omega_{2k} + \delta_{2T} + \beta_2 * FDEV_k * high_i + \tau_1 * high_i + \rho_1 * age_{ijTk} + \mu_1 * \ln(Sale_{ijTk}) + e_{itk} \quad (1.2)$$

where $high_i$ is a dummy variable which takes a value of 1 if firm i 's average growth is above the average growth rate in country k . Columns (1) and (4) of Table 6 shows the results for the OLS estimations. The results indicate that firms with above average growth grow faster and invest more in R&D than firms with below average growth. Furthermore, the interaction term is positive and significant which indicates that firms with above average growth benefit more from financial development. The proposed empirical strategy relies on the assumption that the differential effect of financial development on firms with different growth profiles works through financial constraints. To check whether the assumption is plausible I estimate the above regressions for firms with high and low sales. One expects that the effect interaction term between financial development and the

growth dummy will be stronger for firms with low levels of sales as these will be the more financially constrained firms in the economy. Columns (2) and (3) compares the estimation results for the growth regression for firms with sales above the average sales level and below the average sales level respectively. Similarly columns (6) and (7) do the same exercise for the R&D regression. Columns (2) and (6) show that when we analyze firms with high levels of sales the interaction term is lower than the original estimates and non significant. This implies that the differential effect of financial development on firms with above average growth vanishes when we analyze big firms. Columns (3) and (7) on the other hand analyze the effect of the interaction term when one looks at firms with low levels of sales. The results suggest that higher financial development has an important effect on growth and R&D specially for firms with high growth profiles. Taking these two observations together confirm the hypothesis that financial development affects growth and R&D by relaxing the financial constraint of firms.

Columns (4) and (8) analyze the estimation of equations (1.1) and (1.2) when we include the interaction term for financial development and financial dependence, $FDEV_k * FDEP_j$. The estimates indicate an important effect of financial development by favoring both firms with higher financial dependence as well as firms with above average growth. Furthermore, the results presented an important relation between financial development and firm dynamics.

This concludes the analysis of the reduced form relation between the interaction of financial dependence and financial development with growth and R&D. The main results are summarized below:

i) The difference in growth of firms in sectors with high financial dependence compared to those in sectors with low financial dependence is increasing in the financial development of the country.

ii) The difference in R&D intensity of firms in sectors with high financial dependence compared to those in sectors with low financial dependence is increasing in the financial development of the country.

iii) The effect of financial development on firm growth and R&D intensity will affect differentially firms with growth rates above average, suggesting an important relation between financial development and firm dynamics.

1.2.3 From R&D to Growth

Any theoretical model of endogenous growth would emphasize the relation from R&D to growth both at the aggregate level and at the firm level. That is why a natural reading from the predictions

above is that the effect of the interaction between financial development and financial institutions on growth could be caused by the impact this interaction has on R&D. One could proceed to estimate a regression of the form

$$g_{ijTk} = \alpha_1 + \theta_{1j} + \omega_{1k} + \delta_{1T} + \phi \log(RD/Sale_{ijTk}) + \beta_1 * FDEV_k * FDEP_j + \\ + \rho_1 * age_{ijTk} + \mu_1 * \log(Sale_{ijTk}) + \varepsilon_{itk}$$

and test the hypothesis that $\beta_1 = 0$. This subsection tries to study precisely this hypothesis. In particular column (1) of Table 7 shows that once we control for R&D intensity the coefficient of the interaction term becomes negative but not significant.

The problem with estimating the above equation is that it is very likely to suffer of endogeneity, *i.e.* $Cov(\varepsilon_{itk} | \log(RD/Sale)) \neq 0$. To address the potential endogeneity of $\log(RD/Sale)$ I follow two approaches.

The first approach I follow is to instrument $\log(RD/Sale)$ with lagged values of the variable. This approach is usually used when treating for classical measurement error. Column (2) of Table 7 shows the results for this 2SLS regression. In this case the coefficient for R&D expenditure is positive but smaller than in column (1) and highly significant while the coefficient for the interaction term is not significant.

As mentioned before, our measure of financial development is likely to be correlated with variables affecting a firm's growth and R&D decisions. For this reason in column (3) I instrument both $\log(RD/Sale)$ with lagged values of R&D and financial institutions with the legal origin dummy. The results show that the coefficient for R&D intensity is almost unchanged and remains highly significant. The coefficient for the interaction term increases but remains non-significant.

The results above are subject to the following critique. One can argue that past R&D decision by firms are forward looking and can be correlated with contemporaneous firm level shocks. If this is the case, the estimated coefficients on the growth regression using lagged R&D intensity as an instrument would be biased. To overcome this concern ideally one would like to instrument R&D with a variable which is not affecting growth directly. One plausible source of exogenous variation in R&D are unexpected changes in R&D tax and credits. Hall and van Reenen (2000) provide a summary of some papers which review the tax treatment of R&D for several countries. The problem with such literature is that most of it does not have any time variation in the tax structure. One study which provides a unified overview of changes in R&D tax and credits is

Bloom, Griffith and van Reenan (1999). In particular they construct a panel of countries over 15 years for which they calculate the tax component of R&D user cost. I use this measure as an instrument for R&D intensity. The basic assumption for the validity of the instrument is that changes in R&D tax structure affect growth only through R&D intensity. The use of firm level data makes this assumption more likely to hold. The use of this instrument, however, comes at the cost of a much smaller sample as observations for the tax component of user cost of R&D are available for only half the period in the sample and nine countries. With this caveat in mind I show the results for the estimation of the growth regression where R&D intensity is instrumented with changes in the tax component of the user cost of R&D. Column (4) of Table 7 shows the results of the 2SLS regression where only R&D is instrumented while column (5) shows the case when financial development is also instrumented. We can see from both columns (4) and (5) of Table 7 that the coefficients for R&D intensity increase dramatically when we use tax changes as an instrument and although the sample size is largely reduced this does not affect the significance of the effect of R&D. In both cases the coefficient for the interaction term becomes negative and non significant. One concern is that there is little variation in the financial development measure and that even when excluding R&D intensity the interaction term is non-significant for the subsample of countries. Column (6) in Table 7 examines this concern. The first thing to notice is that the coefficient for the interaction term is positive as in the full sample case. The estimated coefficient is larger than in the full sample case but the variance is larger as was expected. Despite this, the coefficient is still significant at the 10% significance level.

The main result of this subsection is summarized below:

ii) The main channel through which the interaction between financial dependence and financial development affects a firm's growth is through the effect it has on R&D expenditure.

1.3 Model

In this section I present a modified version of the model used in Akcigit (2009). I will start by describing the basics of the model. Then I continue to define and characterize a Markov Perfect Equilibrium (MPE) of the model. Next I turn my attention to Steady-State equilibria of the model, that is, equilibria where aggregate variables grow at a constant rate. I conclude this section by showing that the model's predictions are consistent with the empirical regularities found in the previous section.

1.3.1 Preferences and Technology

Households and Final Good Producer

I will start by describing the baseline dynamic model. Consider a discrete time economy where the representative household maximizes the expectation of an infinite sum of discounted utility, with intertemporal preferences of the following form,

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau} \log C_{\tau} \quad (1.3)$$

where C_t denotes consumption at time t , $\beta \in (0, 1)$ is the discount factor and E_t is the expectation operator conditional on the information at time t . The choice of the logarithmic per-period utility is both a widely used and convenient assumption as it implies a simple relation between the interest rate, the growth rate and the discount factor.

Let Y_t be the total production of the final good at time t . The final good is produced by a perfectly competitive firm using inputs from two sectors, sector H and sector L . Specifically the production function for the final good takes the following form:

$$Y_t = 2Y_H^{1/2}Y_L^{1/2} \quad (1.4)$$

Throughout the exposition of the model, I take the price of the final good as the numeraire and denote the price of sector $j \in \{H, L\}$ at time t by P_{jt} .

The maximization problem of the final good producer implies that the demands for input from sector j satisfies

$$Y_{jt} = \frac{Y_t}{P_{jt}} \quad (1.5)$$

Producers in sectors H and L are perfectly competitive and each uses a continuum 1 of intermediates indexed by i and use the following Cobb-Douglas production function¹⁰

$$\ln Y_{jt} = \int_0^1 \ln \left(\frac{y_{ijt}}{2} \right) di \quad (1.6)$$

From sector j producer's maximization problem and denoting the price of intermediate i in

¹⁰I could allow the set of active firms to be determined by free entry as in Akcigit (2009). This would create an extra difficulty when trying to solve numerically the model and would introduce an extra effect of financial frictions which, although an interesting one, I shut down.

sector j at time t by p_{ijt} , we have that the demand for intermediate i in sector j will take the following form

$$y_{ijt} = \frac{P_{jt}Y_{jt}}{p_{ijt}} \quad (1.7)$$

Taking (1.5) and (1.7) together we have that the demand for intermediate i in sector j follows satisfy the following equation:

$$y_{ijt} = \frac{Y_t}{p_{ijt}} \quad (1.8)$$

Households are endowed with 1 unit of labor which will be used for production by intermediates and for R&D. A representative household holds a balanced portfolio of all the firms in the economy which implies the following budget constraint

$$C_t + A_{t+1} \leq \int_0^1 \Pi_{iHt} di + \int_0^1 \Pi_{iLt} di + w_t + (1 + r_t)A_t \quad (1.9)$$

where Π_{it} are the profits of firm i at time t , w_t is the wage rate in the economy at time t , r_t is the interest rate, and A_t are the savings of the representative consumer. Assets held by the consumer will be used to finance R&D expenditure by firms.

Intermediate Goods Sector

Each intermediate $i \in I$ is produced by an infinitely-lived monopolist which takes decisions about production and R&D. All these decisions will be described in detail below.

Production. Production of intermediates satisfies the following linear technology:

$$y_{ijt} = q_{ijt}l_{ijt} \quad (1.10)$$

where l_{ijt} is the labor hired by intermediate producer i in sector j at time t and $q_{ijt} \in [\underline{q}_t, \infty)$ is firm-specific labor productivity with an economy-wide distribution function $\Sigma_t(Q_t)$. As in Akgigit (2009) the lower bound on the set of possible qualities, \underline{q}_t , reflects the fact that in each period there is a threshold below which the technology is outdated and has no productive value. For mathematical convenience I will assume $\underline{q}_t > w_t$ ¹¹. Furthermore, as in Aghion and Griffith (2005), I assume there is a competitive fringe of imitators who can produce variety i in sector j at a marginal cost of $\chi w_t / \bar{q}_t$, where $\bar{q}_t = \int q_{it} d\Sigma_t$ is an aggregate labor productivity index. We can

¹¹This assumption will guarantee that all firms earn positive profits over the labor productivity space.

interpret the term χ as a measure of intellectual property rights in the economy. This assumption implies the monopolist faces a limit price it can charge equal to $\chi w_t / \bar{q}_t$.

The production function for intermediate i in sector j , (1.10), implies that the marginal cost of producing intermediate i is

$$MC_{ijt} = \frac{w_t}{q_{ijt}} \quad (1.11)$$

Putting (1.8) and (1.11) together we get that "operational" profits (profits exclusive of R&D expenditure) for an intermediate producer are

$$\pi_{ijt} = Y_t \left(1 - (p_{ijt} \hat{q}_{ijt})^{-1} \right) \quad (1.12)$$

where $\hat{q}_{ijt} \equiv q_{ijt} / w_t$ is firm i 's relative labor productivity and will prove to be important in the analysis that follows.

Labor Productivity and R&D Technology. So far I have treated labor productivity as a constant. The key ingredient of the model will be investment in R&D which improves a firm's labor productivity. In particular, each intermediate producer has a quality ladder along which she can improve her current labor productivity, q_{ijt} , through additive step-by-step innovation. Innovation is stochastic, such that the intermediate producer cannot choose directly future productivity, but only the probability of success. This implies that the actual R&D decision undertaken by the intermediate producer is the choice of a success probability which I will define as $\phi_{ijt} \in [0, \bar{\phi}]$, $\bar{\phi} < 1$. Putting these two assumptions together, we have that next period's labor productivity for intermediate producer i will be given by

$$q_{ijt+1} = \begin{cases} q_{ijt} + \lambda_t & \text{with probability } \phi_{ijt} \\ q_{ijt} & \text{with probability } 1 - \phi_{ijt} \end{cases}$$

where λ_t is the additive step size which is common across firms. I will assume that the step size is proportional to the wage rate, $\lambda_t = \hat{\lambda} w_t$. This convenient normalization will turn useful when I analyze the steady-state equilibrium of the model.

In order to generate this success probability the intermediate producer must hire labor. In particular, I will assume that in order to have a success probability of ϕ , $h_j(\phi, \kappa_j)$ workers must be employed. There is heterogeneity across sectors in the R&D cost function. In particular, I assume that sectors differ in their ability to perform R&D. This ability is captured by a parameter κ_j with

$\kappa_H > \kappa_L$. This implies that in the economy there is fraction 1/2 of firms which have ability κ_H and a fraction 1/2 which have ability κ_L . We can now define the distribution of labor productivity-type pairs as

$$\Gamma_t : Q_t \times \{\kappa_L, \kappa_H\} \rightarrow [0, 1]$$

where this distribution is consistent with the distribution of intermediates within sectors and with Σ_t .

I assume that for a given success probability the R&D cost function satisfies the following inequality, $h(\phi_{ijt}, \kappa_H) < h(\phi_{ijt}, \kappa_L)$. This implies that sector H will be the R&D productive sector in the economy. I will assume that firms have an infinite cost of switching sectors which implies that they operate in the same sector as long as they live. This means that, as opposed to labor productivity, a firm's ability will be constant over time. Going back to the empirical section this assumption captures the observation that there is great variation in growth rates and R&D expenditure across sectors. Furthermore, consistent with the empirical section, the described model will imply that more financially dependent sectors in the economy will be growing faster and investing more in R&D.

Following Kortum (1993) and Acemoglu and Akgigit (2009), I will use the following production function for an R&D:

$$h(\phi_{ijt}, \kappa) = \frac{\gamma}{\theta \kappa_j} \phi_{ijt}^\theta \quad (1.13)$$

with $\theta > 1$. This specification implies that the total R&D cost for an intermediate firm is

$$w_t h(\phi_{ijt}, \kappa) = \frac{w_t \gamma}{\theta \kappa_j} \phi_{ijt}^\theta \quad (1.14)$$

The choice of R&D by firms is not unconstrained. In particular, firms have to borrow from households in order to invest in R&D. I assume, for example because of commitment issues, that firms can only invest a constant $\mu \in (0, \infty]$ of their operational profit, π_{ijt} . This implies the following borrowing constraint

$$\frac{w_t \gamma}{\theta \kappa_j} \phi_{ijt}^\theta \leq \mu \pi_{ijt} \quad (1.15)$$

This will be the key ingredient of the model in generating the relation between financial dependence, financial development, growth and R&D which we observe in the data.

Exit Firms will exit whenever their labor productivity is outdated, that is when labor produc-

tivity falls below the threshold \underline{q} . For simplicity I assume that an exiting firm's outside option is the current value of the production unit.

Labor Market The intermediate producer will hire labor both for production and for R&D. This implies that the following labor market clearing condition has to be satisfied

$$1 = \int_{i \in I_H} (l_{iHt} + h_{iHt}) di + \int_{i \in I_L} (l_{iLt} + h_{iLt}) di \quad (1.16)$$

To close the description of the model we can summarize the timeline as follows:

- Beginning of period t , firms invest in R&D using the amount borrowed in $t - 1$. Current labor productivity is q_{ijt} .
- Production of the intermediate goods, the final good, and wages paid. Profits distributed among consumers.
- R&D outcome realized, q_{ijt+1} determined.
- Intermediate firms borrow from households. End of period t .

1.3.2 Equilibrium

Throughout I will use Markov perfect equilibria (MPE) as the equilibrium concept, where strategies are only functions of the payoff-relevant variables¹². For the intermediate producer the payoff-relevant variables are the current labor productivity q_{ijt} , the firm's ability, κ , as well as the distribution of labor productivities, $\Gamma_t(Q_t)$, the final good's production, Y_t , the wage rate, w_t and the interest rate.

Before characterizing the equilibrium of the model, I define an allocation as follows:

Definition 1 (Allocation) *An allocation in this economy is*

- (i) *a sequence of consumptions and Assets holdings by the households $\{A_t, C_t\}_{t=0}^{\infty}$,*
- (ii) *a sequence of productions for the final good producer $\{Y_t\}_{t=0}^{\infty}$, and a sequence of productions of inputs, $\{Y_{jt}\}_{t=0}^{\infty}$ $j \in \{H, L\}$*

¹²Using MPE allows us to ignore more elaborate interactions between economic agents. Given the continuum assumption it makes sense to ignore such interactions. Furthermore, the use of this equilibrium concept is generally used for this class of models.

(iii) a sequence of productions and R&D decisions by the intermediate producers of each sector $\{\mathbf{y}_{jt}, \phi_{jt}, \mathbf{l}_{jt}, \mathbf{h}_{jt}\}_{t=0}^{\infty}$,

where $\mathbf{y}_{jt} \equiv (y_{ijt})_{i \in [0,1]}$, $\phi_{jt} \equiv (\phi_{ijt})_{i \in [0,1]}$, $\mathbf{l}_{jt} \equiv (l_{ijt})_{i \in [0,1]}$, $\mathbf{h}_{jt} \equiv (h_{ijt})_{i \in [0,1]}$

(iv) a sequence of prices $\{w_t, r_t, \mathbf{p}_{Ht}, \mathbf{p}_{Lt}, \mathbf{P}_{Ht}, \mathbf{P}_{Lt}\}$ where $\mathbf{p}_{jt} \equiv (p_{ijt})_{i \in [0,1], j \in \{L, H\}}$,

(v) and a sequence of labor productivity Distributions $\{\Gamma_t\}_{t=0}^{\infty}$

Next, I define an equilibrium in this economy. As I mentioned above, we are going to restrict to markovian strategies. For the intermediate producers this implies that R&D decisions, prices and production can be represented by the following mappings:

$$\begin{aligned} \phi_{ijt} &: Q_t \times \kappa \times \Gamma \times \mathbb{R}_+^3 \rightarrow [0, \bar{\phi}] \\ y_{ijt} &: Q_t \times \kappa \times \Gamma \times \mathbb{R}_+^3 \rightarrow \mathbb{R}_+ \\ p_{ijt} &: Q_t \times \kappa \times \Gamma \times \mathbb{R}_+^3 \rightarrow \mathbb{R}_+ \end{aligned}$$

where Q_t is the space of possible labor productivities at time t , Γ is the space of distribution functions and \mathbb{R}_+^3 stands for aggregate output, Y_t , the wage rate, w_t , and the interest rate, r_t . Taking these elements into account I have the following equilibrium definition:

Definition 2 (Equilibrium) A Markov Perfect Equilibrium is given by an allocation

- $\{Y_t^*, Y_{Ht}^*, Y_{Lt}^*, C_t^*, A_t^*, w_t^*, r_t^*, \mathbf{y}_{Lt}^*, \mathbf{y}_{Ht}^*, \phi_{Lt}^*, \phi_{Ht}^*, \mathbf{l}_t, \mathbf{h}_t, \mathbf{p}_{Lt}^*, \mathbf{p}_{Ht}^*, \mathbf{P}_{Lt}^*, \mathbf{P}_{Ht}^*, \Gamma_t^*\}_{t=0}^{\infty}$ such that
- i) $\{\mathbf{y}_{jt}^*, \mathbf{p}_{jt}^*\}_{t=0}^{\infty}$ solves the intermediate producer's maximization problem conditional on $\{\mathbf{q}_t, Y_t^*, w_t^*, r_t^*, \Gamma_t^*\}_{t=0}^{\infty}$
 - ii) $\{\phi_t^*\}_{t=0}^{\infty}$ solves the R&D maximization problem conditional on $\{\mathbf{q}_t, Y_t^*, w_t^*, r_t^*, \Gamma_t^*\}_{t=0}^{\infty}$,
 - iii) $\{\mathbf{l}_t, \mathbf{h}_t\}_{t=0}^{\infty}$ satisfy (1.10) and (1.13) respectively,
 - iv) $\{C_t^*, A_{t+1}^*\}_{t=0}^{\infty}$ solves the household's maximization of (1.3) subject to (1.9),
 - v) $\{Y_t^*\}_{t=0}^{\infty}$ is consistent with (2.1),
 - vi) $\{w_t^*\}_{t=0}^{\infty}$ clears the labor market, $\{r_t^*\}_{t=0}^{\infty}$ clears the savings market,
- and
- viii) $\{\Gamma_t^*\}_{t=0}^{\infty}$ is consistent with $\{\phi_t^*\}_{t=0}^{\infty}$.
 - ix) $\{Y_{jt}^*\}_{t=0}^{\infty}$ is consistent with (1.6),

Having defined an equilibrium in this economy I proceed to characterize the equilibrium.

Intermediate Producers I start by pointing out that the intermediate producers' problem can be split into two: an intratemporal problem and the R&D (intertemporal) problem. The intratemporal problem of intermediate producer i is the static profit maximization of (1.12). We can see from (1.12) that the profits of a monopolist are increasing in p_{ijt} which implies that the optimal price the monopolist will charge is the limit price $\chi w_t/\bar{q}_t$. Using the previous observation we have the following optimal quantities, prices and labor demands

$$y_{ijt}^*(q_{ijt}; w_t^*, Y_t^*) = \frac{Y_t^* \bar{q}_t}{\chi} \quad (1.17)$$

$$p_{ijt}^*(q_{ijt}; w_t^*, Y_t^*) = \frac{w_t \chi}{\bar{q}_t} \quad (1.18)$$

$$l_{ijt}^*(q_{ijt}; w_t^*, Y_t^*) = \frac{Y_t^* \bar{q}_t}{w_t \chi \hat{q}_{ijt}} \quad (1.19)$$

where, as defined above, $\hat{q}_{ijt} = q_{ijt}/w_t^*$ and $\bar{q}_t = \bar{q}_t/w_t^*$. Putting (1.17) and (1.18) together with (1.12) we have that firm i 's "operational" profits are

$$\pi_{ijt}^*(q_{ijt}; w_t^*, Y_t^*) = Y_t^* \left(1 - \frac{\bar{q}_t}{\chi \hat{q}_{ijt}} \right) \quad (1.20)$$

Notice that equilibrium operational profits of firm i are concave in the relative labor productivity.

Next we turn to the R&D problem faced by firm i . We can write this intertemporal problem in recursive form and define the value of firm i , in sector j , at time t as follows:

$$V_{ijt}(q_{ijt}, \kappa_j; K_t^*) = \max_{\phi} \left\{ \Pi_{ijt} + \left(\frac{1}{1+r_{t+1}} \right) \left(\begin{array}{l} \phi V_{ijt+1}(q_{ijt} + \lambda, K_{t+1}^*) + \\ +(1-\phi)V_{ijt+1}(q_{ijt}; K_{t+1}^*) \end{array} \right) \right\} \quad (1.21)$$

subject to (1.15)

where

$$\Pi_{ijt} = \pi_{ijt}^*(q_{ijt}; w_t, Y_t) - (1+r_t^*) w_t^* h(\phi, \kappa_j)$$

are the profits obtained by firm i and K_t^* is the set of state variables at time t . Per-period

profits have two components. First we have the equilibrium operational profits. The second term is the R&D cost of the firm which is paid back to lenders. The value function presented in (1.21) captures the basic R&D problem of firm i . On the one hand the firm incurs in a cost in the current period and on the other hand this R&D investment allows the firm to have a higher labor productivity in the future. The optimal R&D choice by firm i satisfies (1.21).

Having described the problem of the intermediate producer I turn to the equilibrium in the final good's market.

Final Good First, using the intermediate demands we can check that the zero profit condition of the final good producer is immediately satisfied.

Also, by combining the production function of the final good producer (2.1) and (1.17) I can pin down the average relative labor productivity in the economy as

$$\bar{q}_t = \chi \tag{1.22}$$

The equilibrium average relative labor productivity implies that the equilibrium wage rate of the economy will be $w_t = \bar{q}_t/\chi$ which points at the importance of average labor productivity as the main force of growth in this economy.

Households Households maximize (1.3) subject to (1.9), which gives the standard Euler equation

$$\frac{C_{t+1}^*}{C_t^*} = \beta(1 + r_t^*) \tag{1.23}$$

for the law of motion of consumption. We also have that the budget constraint of the representative household will be binding in equilibrium. Combining the binding budget constraint of consumers together with the final good producer's zero profit condition and the market clearing condition for the asset market we get the following resource constraint:

$$C_t^* + A_{t+1}^* - w_t^* H_t^* = Y_t^*$$

where $H_t^* = \int_{i \in I_H} h_{iHt}^* di + \int_{i \in I_L} h_{iLt}^* di$ is aggregate employment in R&D. This condition implies that output and last period's assets can be used for two purposes, consumption and saving.

Labor Market Combining the labor market equilibrium condition, (1.16), and (1.22) we can

pin down equilibrium output in the economy as

$$Y_t^* = \frac{(1-H)}{\left(\frac{1}{\bar{q}}\right)} w_t$$

For future reference I will define relative wage, the ratio of the wage rate with respect to output, as

$$\tilde{w}_t = \frac{1}{(1-H_t^*)} \overline{\left(\frac{1}{\bar{q}}\right)} \quad (1.24)$$

My next goal is to solve for the steady state equilibrium of the economy. The focus on the steady state equilibrium is both for convenience and it's importance. The importance of understanding the steady-state behavior of the model is that it allows me to predict and understand the long run behavior of the economy.

1.3.3 Steady-State

I define a steady-state equilibrium as an equilibrium in which output, consumption, average labor productivity and the wage rate grow at a constant rate g . Before going to the formal definition of a steady state equilibrium, I will define a convenient normalization. In particular the normalized value of some variable x_t with respect to Y_t will be denoted by \tilde{x} .

Definition 3 (Steady-State Equilibrium) A steady state equilibrium is a tuple $(V^*, \phi^*, \hat{\Gamma}^*, \tilde{w}^*, g^*)$ such that

i) V^* satisfies (1.21)

ii) ϕ^* solves (1.21)

iii) $\hat{\Gamma}^*$ forms an invariant distribution over the state space $\hat{Q} \times \{\kappa_H, \kappa_L\}$ and this invariant distribution is generated by ϕ^*

iv) the relative wage rate, \tilde{w}^* , clears the labor market

and

v) $C_t^*, Y_t^*, w_t^*, \bar{q}^*$, all grow at the constant growth rate g^* which is consistent with the equilibrium R&D decision ϕ^* .

Value Functions and Policy Functions

In what follows I will characterize the steady state equilibrium and show that the predictions of the model are consistent with the empirical predictions from Section 2. The first thing to notice is that in the steady-state equilibrium the only relevant state variables from an intermediate's perspective are the relative wage rate, the growth rate, the firm's relative labor productivity and the firm's R&D ability, κ . This reduction in the space of state variables occurs because conditional on these three variables firms can fully predict current and future prices along the equilibrium path. Having made this observation I can write the R&D problem of the firm as

$$\tilde{V}(\hat{q}, \kappa; \tilde{w}, g) = \max_{\phi \in \Xi(\hat{q})} \left\{ \begin{aligned} & \left(1 - \frac{1}{\hat{q}}\right) - \frac{(1+g)}{\beta} \tilde{w}h(\phi, \kappa) + \\ & + \beta \left(\phi \tilde{V}\left(\frac{\hat{q} + \lambda}{1+g}, \kappa; \tilde{w}, g\right) + (1 - \phi) \tilde{V}\left(\frac{\hat{q}, \kappa; \tilde{w}, g}{1+g}\right) \right) \end{aligned} \right\} \quad (1.25)$$

where $\Xi(\hat{q}) = \left\{ \phi \in [0, \bar{\phi}] : \tilde{w}h(\phi, \kappa) \leq \mu \left(1 - \frac{1}{\hat{q}}\right) \right\}$ is the relevant constraint set and \tilde{V} is the normalized value function. Given the stationarity of the problem, I have dropped the time indices. The above equation uses the Euler equation to pin down the equilibrium interest rate, $r^* = \frac{(1+g)}{\beta}$. One interesting observation which can be made from (1.25) are the externalities that aggregate R&D will create on individual firms. In particular, higher aggregate R&D investment will have opposing forces on individual R&D. On the one hand, it will increase the cost of R&D by increasing both the interest rate and the wage rate. On the other hand, the growth of the wage rate depreciates future relative labor productivity, giving an incentive to firms to overcome this through R&D.

The next proposition describes the general properties of (1.25). The proof of this proposition and the proofs for other results in the rest of the paper can be found in the appendix at the end.

Proposition 4 *For any given tuple $(\hat{\Gamma}^*, \tilde{w}^*, g^*)$, the relative value function (1.25) exists, is unique, continuous, strictly increasing in the relative labor productivity, differentiable and strictly concave. This implies that optimal policy functions ϕ^* exist and they are continuous functions.*

Next I proceed to characterize in more detail the R&D decision by firms. Understanding a firm's R&D choices is crucial for understanding the forces which affect individual and aggregate growth. I'll start by focusing on the unrestricted choice of a firm with labor productivity \hat{q} and ability κ . Using the results from Proposition 4 (concavity and differentiability of the value function) we can characterize the R&D decision of an unconstrained firm by using the first order conditions.

This implies that the unconstrained optimal choice of R&D will be:

$$\phi^u(\hat{q}, \kappa) = \left(\kappa \beta^2 \frac{\tilde{V}\left(\frac{\hat{q}+\hat{\lambda}}{1+g}, \kappa; \tilde{w}, g\right) - \tilde{V}\left(\frac{\hat{q}}{1+g}; \kappa, \tilde{w}, g\right)}{\gamma \tilde{w}(1+g)} \right)^{1/(\theta-1)} \quad (1.26)$$

First, as will be proven later, ϕ^u is increasing in κ_H . We can also see that as idiosyncratic labor productivity increases firms will choose a lower R&D probability. This comes from the concavity of the value function: as a firm's labor productivity increases the differential gain of increasing labor productivity by $\hat{\lambda}$ will be smaller.

The effect of the growth rate on the unconstrained choice is not an obvious one. First notice that an increase in the growth rate increases the cost of R&D through the interest rate. On the other hand the growth rate will shrink relative labor productivity tomorrow making the differential gain of successful R&D greater.

The unconstrained R&D choice will not be attainable for constrained firms. In particular we will have a threshold value such that firms with \hat{q} high enough will not be restricted. We summarize this in the following Lemma.

Lemma 5 *For each κ there will be a threshold $\bar{q}(\kappa)$ such that for $\hat{q} \geq \bar{q}(\kappa)$ we have $\phi^*(\hat{q}, \kappa) = \phi^u(\hat{q}, \kappa)$ and for $\hat{q} < \bar{q}(\kappa)$ we have $\phi^*(\hat{q}, \kappa) = \phi^r(\hat{q}, \kappa)$, where*

$$\phi^r(\hat{q}, \kappa) = \left(\mu \frac{\theta \kappa}{\gamma \tilde{w}} \left(1 - \frac{1}{\hat{q}} \right) \right)^{1/\theta} \quad (1.27)$$

Furthermore, $\bar{q}(\kappa_L) < \bar{q}(\kappa_H)$

Lemma 5 tells us that R&D choices of a firm will be hump-shaped in \hat{q} . In particular, for $\hat{q} < \bar{q}(\kappa)$ optimal R&D will be increasing and concave in \hat{q} while for $\hat{q} \geq \bar{q}(\kappa)$ it will be decreasing. It also points at one important feature of the model, the set of \hat{q} 's for which firms in the high ability sector are constrained is larger than that of the low R&D ability sector. This follows from the fact that the high ability firm is spending more on R&D than the low ability firm for a given value of \hat{q} .

Existence of a Steady-State Equilibrium

In what follows I will show that a Steady State equilibrium exists. First I start by characterizing the equilibrium growth rate for a given Γ^* . In equilibrium we have that

$$\bar{q}^* = \int_0^1 \hat{q}_i d\hat{\Gamma}^*$$

is constant and equal to χ . Using this observation we have that the aggregate growth rate of the economy satisfies the following equation

$$g^* = \frac{\hat{\lambda} \int \phi^*(\hat{q}, \kappa) d\hat{\Gamma}^*}{\chi} \quad (1.28)$$

Next I show the existence of a steady state distribution. One useful observation is that for a given κ , there exists a $\check{q} = \lambda/g^*$ such that for a given $g^* > 0$, if $\hat{q}_t > \check{q}$ then $\hat{q}_{t+\tau} \leq \check{q}$ for some finite $\tau > 0$. This implies that all states $\hat{q}_t > \check{q}$ are transient.

Proposition 6 *Assume $g^* > 0$. Then for a equilibrium policy function $\phi^*(\hat{q}, \kappa)$ there exists a unique Steady-State Distribution $\hat{\Gamma}^* : \hat{Q} \times \{\kappa_L, \kappa_H\} \rightarrow [0, 1]$*

To finish the characterization of a Steady-State Equilibrium the following proposition shows that a steady-state equilibrium with a positive growth rate g^* exists.

Proposition 7 *Consider the economy above. A Steady-State Equilibrium $m^* = (V^*, \phi^*, \hat{\Gamma}^*, \tilde{w}^*, g^*)$ exists. Furthermore, $g^* > 0$.*

Financial Dependence and Financial Development

Next I will analyze what is the effect of sectoral differences on R&D ability, κ , and financial development, μ , on a firms's decisions. One question that comes naturally from the analysis above is whether higher ability firms, firms in sector H , are choosing higher R&D levels conditional on their size. The next lemma shows that this is the case.

Lemma 8 *The policy function, ϕ^* , is increasing in κ for a given value of \hat{q} .*

Having characterized the policy function we turn to the analysis of three variables of interest at the firm level: expected growth, R&D intensity and financial dependence. The study of these

variables will be important when trying to map the model to the observed patterns in the data. As the reader can expect, the results I will show for these three variables of interest will be closely related to the results for R&D probabilities, ϕ_{ijt} .

I will start by defining a firm's growth in labor productivity as

$$\varsigma(\hat{q}, \kappa) = \frac{\hat{\lambda}\phi^*(\hat{q}, \kappa) - \hat{q}g^*}{\hat{q}(1 + g^*)} \quad (1.29)$$

From this equation we can see that a firm's growth is determined by three factors. On the one hand a firm's growth is affected by her current labor productivity. As pointed above, the effect of labor productivity on growth will be hump-shaped. At low levels of labor productivity the firm will be financially constrained and increases in labor productivity will relax the constraint. This channel will allow firms to invest more in R&D and grow faster. On the other hand, as the firm's constraint is slack, R&D is decreasing in labor productivity, which then causes growth to slow down. The second channel is aggregate growth. On the one hand growth has a direct negative effect on individual growth as it depreciates future sales. On the other hand there is an indirect effect through R&D decisions which is ambiguous. The third channel is the firm's R&D ability. Finally Lemma 6 implies that for a given labor productivity level, \hat{q} , individual growth will be weakly increasing in κ .

Next we define R&D intensity (R&D expenditure divided by operational profits) as

$$\mathfrak{R}(\hat{q}, \kappa) = \frac{\hat{q}(1 + g)\tilde{w}\gamma\phi^*(\hat{q}, \kappa)^\theta}{\theta\kappa\beta(\hat{q} - 1)} \quad (1.30)$$

Similarly we will define a firm's financial dependence (borrowing over profits) as

$$\mathcal{F}(\hat{q}, \kappa) = \frac{\hat{q}\tilde{w}\gamma\phi^*(\hat{q}, \kappa)^\theta}{\theta\kappa(\hat{q} - 1)}$$

It is obvious from these definitions that in this model financial dependence and R&D intensity are proportional.

One key question in the analysis is how financial dependence is related to a R&D ability. To answer this I will characterize how R&D intensity varies as we vary the firm's R&D ability.

Lemma 9 *For a given relative labor productivity \hat{q} , R&D intensity and Financial Dependence, $\mathfrak{R}(\hat{q}, \kappa)$ and $\mathcal{F}(\hat{q}, \kappa)$ respectively, are weakly increasing in κ .*

This result implies that R&D ability and financial dependence are tied together. In particular, we have that firms operating in sector H will have higher financial dependence. So far, and consistent with the data, the model suggests that sectors which have higher κ are more financial dependent, grow faster and invest more in R&D. But the empirical section also suggests a differential effect of financial development on the growth rate and R&D investment of firms in sector H compared to those in sector L .

Partial Equilibrium Analysis So far I have shown that firms with higher R&D ability have higher growth, higher R&D choices and higher financial dependence. But one natural question one can ask is how are these variables affected by the measure of financial development of the economy, μ . Answering this question is difficult due to the general equilibrium effect that μ has on prices and the firm's value function. Because of this I will split the analysis into two. First I will characterize the effects of μ on the variables of interest assuming that the Value functions, the wage rate and the growth rate are constant while the general equilibrium effect will be analyzed in detail in the next section. Studying this partial equilibrium case will be useful in highlighting the main channel through which financial development and a firm's ability interact. Two important results emerge from this partial equilibrium analysis. The first result shows how financial development shapes the range of firms which are financially constrained. Second, as a consequence of the effect of financial development in the set of constrained firms and holding the g^*, \tilde{w}^* and \tilde{V}^* fixed, an increase in μ will have a bigger impact in the R&D investment of those firms with higher R&D ability, sector H .

The next Lemma shows the main results of the partial equilibrium analysis. I show first that for given g^* and \tilde{w}^* higher financial development increases both R&D intensity and growth of a firm, and second that this higher financial development increases also the differential growth and R&D intensity between a high ability and a low ability firm.

Lemma 10 *For given relative labor productivity \hat{q} , and constant \tilde{V}^* , g^* and \tilde{w}^* ,*

- 1) *firm growth and R&D intensity, $\mathfrak{R}(\hat{q}, \kappa)$, are weakly increasing in μ .*
- 2) *Furthermore we will have that for $\mu_H > \mu_L$ the following conditions must hold*

$$\begin{aligned} \frac{\mathfrak{R}(\hat{q}, \kappa_H; \mu_H)}{\mathfrak{R}(\hat{q}, \kappa_L; \mu_H)} &\geq \frac{\mathfrak{R}(\hat{q}, \kappa_H; \mu_L)}{\mathfrak{R}(\hat{q}, \kappa_L; \mu_L)} \\ \frac{\varsigma(\hat{q}, \kappa_H; \mu_H)}{\varsigma(\hat{q}, \kappa_L; \mu_H)} &\geq \frac{\varsigma(\hat{q}, \kappa_H; \mu_L)}{\varsigma(\hat{q}, \kappa_L; \mu_L)} \end{aligned}$$

3) *The inequalities are non monotonic in \hat{q} .*

The result follows from the fact that high types will be more severely affected by low levels of financial development as they are the ones who are spending more on R&D. This result is consistent with the findings in Section 2 and points to a particular channel as a possible explanation of how financial development affects firm growth and aggregate growth: low levels of financial development affect disproportionately firms which have a higher R&D ability as opposed to those that don't.

One should beware as the results I find stem from one important modeling assumption. In particular I have assumed that the borrowing limit is subject only to current profits which are independent of ability. A more general framework would allow this borrowing constraint to depend on a firm's value. This would end up favoring high ability types who will have higher value functions. However, the total effect is ambiguous as high types are still investing more on R&D which makes the financial constraint more likely to be binding. The choice of this simpler constraint is a matter of tractability of the model¹³.

So far I have showed that the predictions of the model are consistent with the empirical prediction above. However, this predictions assume that the Value functions of the firm, the relative wage and the growth rate are constant as μ changes. Once I take this general equilibrium effect into account, one could expect the predicted patterns to change. The next section shows that even when we take this general equilibrium effects into account the predictions of the model remain consistent with the predictions in the data.

1.4 Numerical Solution

1.4.1 Computational Strategy

The purpose of this section is to solve the model numerically in order to assess and quantify the general equilibrium effects of financial development. First I start by describing the computational strategy to be followed.

The computational solution of the model consists of the following routine. First there is an outer layer which consists of two variables, namely the aggregate growth rate g^* and the labor share, in this outer layer I use a bracketing procedure in order to find the equilibrium prices. Next, taking

¹³I solved the model numerically with the different R&D restriction. For high levels of μ convergence was achieved and the main predictions remain unchanged. The computational challenge arose for low values of μ in which case I didn't get convergence of the value functions.

these values as given inside the inner nest, a firm's value function is solved using a value function iteration routine. For this, I calculate the unconstrained policy and evaluate if the constraint is slack. If it is, then this is the policy function $\phi^* = \phi^u$, otherwise we calculate the restricted value using (1.27), $\phi^* = \phi^R$. Since the state space \widehat{Q} is continuous I use a cubic splines collocation method to approximate the value function at exactly $n = 150$ points.

Once the value function iteration converges I use the policy function ϕ^* to calculate a transition function and with this a Steady-State Distribution over the n points. With this distribution I calculate the growth rate and the relative wage rate using (1.28) and (1.24) respectively. These values are compared to the original guess and we increase or decrease the new guesses depending on the sign of the difference between the old guess and the predicted values.

I have 7 parameters in the model and their values are taken from several studies. First, the value for θ is set to two which is within the range of values suggested in Kortum (1993). The value of the innovation step size, $\widehat{\lambda}$, is taken from Akcigit and set to 0.25. I use a value for the discount factor, β , of 0.975 which is consistent with values used in the literature. The value for χ is chosen to be 2 which implies a mark-up of a 100%. This mark-up value is consistent with the estimates by Hall (1988) and Broda and Weinstein (2004)¹⁴. This leaves me with 2 parameters to set: the low ability level κ_L and the constant γ which are calibrated such that the model with no financial frictions matches the moments from the US Compustat firms¹⁵. To do this, I use the 1980-2005 period and divide firms into two groups, low financial dependence and high financial dependence. In particular I calculate the 33rd percentile and 66th percentile of the distribution of the financial dependence measure calculated in Section 2. Using these thresholds I define a high financial dependence sector as a sector with financial dependence above the 66th percentile and equivalently a low financial sector a sector with financial dependence below the 33rd percentile. Using this classification of sectors, I calculate three statistics from the sample:

- the difference in average growth between the high financial dependence sector and the low financial dependence sector,
- the difference in average R&D intensity between the high financial dependence sector and the low financial dependence sector,

¹⁴In particular, Hall (1988) estimates mark-ups for US industries which are bigger than the one I use. On the other hand, Broda and Weinstein (2004) estimate elasticities of substitution using trade data and the implied mark-ups from their study are smaller than the one I use. I have chosen a value consistent with both these studies.

¹⁵The parameter κ_H is normalized to 1.

- the difference in average labor productivity between the high financial dependence sector and the low financial dependence sector

The next table shows the results obtained from the calibration exercise

	$g_H - g_L$	$RD_H - RD_L$	$q_H - q_L$
Data	0.012	0.08	0.06
Model	0.011	0.06	0.08

1.4.2 Value Function, R&D, and μ

Before analyzing the effect of μ on aggregate growth, the equilibrium wage rate and welfare, I start by analyzing the effect that changes in μ has on the Value functions and the policy functions. Section 3 showed that if we treated the growth rate, the wage rate and the value functions as constant and for a given labor productivity level, differences in R&D intensity and growth rates between firms of high and low R&D ability are increasing in μ . Next, I will present the calibration results for the economy with $\mu = \infty$ and one with $\mu = 0.6$. Figures 5 and 6 show that the partial effect of μ might not hold once we allow for this general equilibrium interactions. First notice that policy functions satisfy the properties highlighted above. For the economy with financial constraints, R&D is always decreasing in labor productivity and the high ability invests more for every labor productivity value. Similarly, for the constrained case we have a hump-shaped best response as predicted in the theory. One important observation to be made is that when we have financial frictions the value function becomes very steep for firms which are financially constrained. This could create a region in which differences in R&D choices by high and low ability firms might be larger in the constrained case than in the unconstrained. Despite this possibility, the analysis that follows shows that on average the "partial effect" result from Section 3 still holds

1.4.3 Aggregate Results and Financial Development

To analyze the effect of differences in financial dependence I have solved the general equilibrium model for different values of μ . I will start analyzing the results by showing the effect of financial development on growth. Figure 7 shows the plot of the equilibrium growth rate for various values of μ . The dashed line is the growth rate of an economy with no financial friction ($\mu = \infty$). The first thing to notice is that growth and financial dependence are positively correlated and the gains in growth from increases in financial development are particularly big for low levels of financial

development. This result is consistent with the evidence presented in Figure 2, where the same pattern emerges. Figure 8 presents a similar pattern for the equilibrium wage rate in the economy. The co-movement of the wage rate and the measure of financial development comes from the fact that when μ is higher firms are investing more in R&D which requires them to hire more labor, pushing up the labor demand.

Figures 9 and 10 present the second successful prediction of the model. Both the expected difference in firm growth and R&D intensity between firms in sector H and sector L (high financial dependence and low financial dependence, respectively) are increasing in financial development. This goes in line with the empirical section which showed evidence of a positive effect of the interaction between financial dependence and financial development on growth and R&D intensity.

The next goal is to assess the welfare gains of higher financial development. From (1.3) and the final good market clearing condition, we can approximate aggregate welfare for an economy with financial development μ as

$$W(\mu) \simeq -\frac{\ln(\tilde{w}^*(\mu))}{1-\beta} + \frac{\beta g^*(\mu)}{(1-\beta)^2} \quad (1.31)$$

Figure 11 shows this relation. The pattern that emerges is exactly the one we observe for growth and the wage rate. As with growth, this implies that there are big gains to be made from financial development for economies with low levels of μ .

The discussion above suggests that there is room for policy interventions to increase the welfare of the economy. In particular there are three dimensions for which policy might be important in contributing to welfare. First, as in Akcigit (2009), under no financial constraint, small firms are doing more R&D conditional on ability which calls for subsidies for small firms. In addition to that, firms with higher R&D are ability are contributing more to growth which suggests that they should be targeted by policy. Finally, financial constraints are affecting R&D decisions precisely of those firms which had a higher contribution to growth in the first place. All this suggests that optimal R&D policy should take into account all three channels. The next Section deals with the discussion of policy design in a world of financial frictions and heterogeneity among firms.

1.5 Policy

In the model presented in Section 3 there are three reasons for which the decentralized economy is inefficient: First, monopoly power of intermediate producers generates a distortion in their

production. Second, because of this distortion in production the value to an intermediate producer of improvements in labor productivity is smaller than the social value of such improvement. Third, the presence of financial frictions implies that firms are investing in R&D less than what is socially optimal. If we would give the policy maker the ability to use production and R&D subsidies which are conditional on labor productivity, R&D ability and the level of financial development, then first best allocations could be attained. In what follows I will analyze the optimal policies restricting the analysis to a particular subset of policies, two-level size-dependent R&D subsidies. This subset of policies will be important given the two inefficiencies mentioned above regarding R&D investment.

There are three important caveats to this analysis. First, I am not conditioning policies on R&D ability, that is sector specific subsidies. Not considering this dimension will be crucial in the results obtained below as firm size will also be informative of R&D ability. This restriction would be plausible if, for example, the policy maker can observe the production level of the firm but can't observe in which sector it operates. This would be a realistic assumption if the set of intermediates used by the two sectors is similar. Second, I will restrict to a subset of all size-dependent subsidies. I impose this restriction to simplify the numerical analysis which follows. Finally, as will become clear below, subsidies will relax the financial constraint of firms. An alternative way to relax the financial constraint of a firm would be to offer size-dependent lump-sum transfers which I am not considering in the analysis. One reason for assuming this restriction is lack of commitment by firms to use the transfer for R&D. This lack of commitment by firms, which may be precisely the underlying reason for the presence of financial constraints, makes the assumed restriction plausible.

The practical importance of analyzing this set of policies arises from the growing debate over the suboptimality of private R&D and the role for policy to align the social and private returns of R&D investment. As a result of this debate policymakers around the world have engaged in R&D subsidy programs to stimulate R&D investment. One important observation is that such programs vary greatly in their intensity as well as in their shape¹⁶. In particular, certain countries tend to have special treatment for a Small and Medium Enterprises (SME) and for startups. One goal of this section is to rationalize in the context of the model presented in Section 3 the variety of R&D and production subsidy regimes observed in the world. One important question I will ask is how these regimes change as the financial development of the economy varies and how the optimal mix between R&D subsidies and production subsidies.

I study a setup in which the government provides R&D subsidies to finance a fraction τ_i of

¹⁶See Hall and Van Reenen (2000) for a survey of different R&D subsidy programs around the world.

firm i 's R&D investment. Given the widespread use of size-dependent subsidies, I will allow the subsidies to be size-dependent which in the context of the model is equivalent to having labor productivity dependent subsidies

$$\tau_i^{RD} = \tau^{RD}(\hat{q})$$

The proposed R&D subsidy regime implies that firm i will only pay $(1 - \tau_i^{RD})(1 + r_t)w_t h(\phi_i)$ instead of the full R&D cost. The government finances R&D subsidies through lump sum taxes T to consumers. The government follows a balanced budget which implies

$$T = \int \tau^{RD}(\hat{q}) w_t h(\phi(\hat{q})) d\Sigma$$

In what follows I will focus on Welfare-maximizing subsidy schedules, that is, schedules which maximize (1.31).

Before analyzing different R&D subsidy regimes I start by presenting the results when no R&D subsidies are in place. The following table reports the average R&D probability $(\bar{\phi})$, the equilibrium relative wage rate \tilde{w}^* , the equilibrium growth rate g^* and the resulting welfare for the frictionless economy and an economy with a value of $\mu = 0.6$.

Table 7. No Subsidy

	\tilde{w}^*	g^*	$\bar{\phi}$	τ	T	Welfare
$\mu = \infty$	0.6106	0.0338	0.1577	0	0	13.89
$\mu = 0.6$	0.6080	0.0319	0.1503	0	0	13.19
$\mu = 0.05$	0.5810	0.0175	0.0793	0	0	7.18

As was discussed in Section 4 a higher level of μ will imply a higher growth rate and relative wage rate as firms are investing more in R&D. This positive relation between financial development and R&D spending can be seen more clearly when we compare the average success probability in the economy, $\bar{\phi}$, which as can be seen almost doubles when we move from an economy with $\mu = 0.05$ to a frictionless economy ($\mu = \infty$). Next we turn to the analysis of R&D subsidy.

1.5.1 Uniform R&D Subsidy

Next we turn to the analysis of a uniform R&D subsidy, that is, a subsidy which is independent of any firm characteristic. Formally this corresponds to the case in which $\tau^{RD}(\hat{q}) = \tau^{RD} \forall \hat{q}$. Under

this policy regime the model generates the following results:

Table 8. Uniform Subsidy

	\tilde{w}^*	g^*	$\bar{\phi}$	τ^{RD}	T	Welfare
$\mu = \infty$	0.8156	0.0526	0.3720	0.5860	0.1766	20.19
$\mu = 0.6$	0.8126	0.0524	0.3680	0.6150	0.1839	20.14
$\mu = 0.05$	0.7598	0.0469	0.2751	0.8925	0.1885	18.12

The first thing to observe is that for any financial development level there are welfare gains to be made by a uniform R&D subsidy. As we can see such a policy increases growth compared to the no subsidy regime at the cost of a decrease in initial consumption through a higher wage rate. The overall welfare effects of a uniform R&D subsidy are large specially for low levels of financial development. The second thing to notice is that the optimal uniform R&D subsidy is decreasing in financial development. As was discussed earlier R&D subsidies will be a useful tool to correct the two frictions present in this model: financial frictions and monopoly power. For this reason, as the level of financial development in the economy is smaller the optimal R&D subsidy becomes more aggressive.

1.5.2 Size-Dependent Two-Level R&D Subsidy

As was mentioned at the beginning of this Section, many countries have differential R&D subsidies for firms of different sizes. In the context of the model presented in this paper there are three forces for why a policy maker would want to target subsidies based on size. First, there is a size effect: in the absence of financial frictions and for a given R&D ability, firms with smaller labor productivity will engage in higher R&D investment. This implies that targeting R&D subsidies to firms of low labor productivity is a more efficient way to boost growth which is the effect present in Akcigit (2009). Second, there is a financial constraint effect: in the presence of financial frictions size-dependent policies have the extra benefit of targeting those firms which are financially constrained. The third side to size-dependent policies is the composition effect: high ability firms have in equilibrium higher labor productivity. This implies that targeting firms with high labor productivity can be beneficial as this subsidizes indirectly high ability firms.

To analyze this channels I will consider a two-level size-dependent subsidy of the following shape

$$\tau^{RD}(\hat{q}) = \begin{cases} \tau_s^{RD} & \text{if } \hat{q} < \bar{q} \\ \tau_b^{RD} & \text{if } \hat{q} \geq \bar{q} \end{cases}$$

This type of policy resembles what we observe in countries such as the UK. The next table presents the results from such a subsidy regime

Table 9. Two-Level Subsidy

	\tilde{w}^*	g^*	$\bar{\phi}$	τ_s^{RD}	τ_b^{RD}	T	Welfare
$\mu = \infty$	0.8052	0.0528	0.3851	0.4567	0.6538	0.2308	23.08
$\mu = 0.6$	0.8029	0.0525	0.3691	0.5671	0.6235	0.1885	22.12
$\mu = 0.05$	0.7890	0.0495	0.3154	0.9115	0.7532	0.1823	19.25

Two important results emerge from the size-dependent two-level subsidy analysis. First we can see that for all levels of financial development a size-dependent subsidy always yields higher welfare compared to a uniform subsidy case. This result should not be surprising as the latter is a special case of the former. This result goes in line with Acemoglu and Akcigit (2009) and Akcigit (2009) which find that state-dependent policy is optimal in the context of endogenous growth models.

The second and most novel result is the shape of the optimal R&D subsidy. In particular we can see that for the frictionless economy ($\mu = \infty$) the optimal size-dependent subsidy is decreasing in size. This implies that the composition effect discussed above dominates the fact that smaller firms spend more in R&D. As financial development decreases the financial constraint effect becomes more important and the optimal R&D subsidy is decreasing in size.

1.6 Conclusion

Motivated by a large literature emphasizing the role of financial development on growth, this paper investigates in detail the potential channels that cause this relation. The channel I emphasize is the effect of R&D on growth and I argue that financial constraints are likely to affect long run growth precisely through the impact they have on R&D. Although this channel seems obvious,

little empirical work has been done to explore it. The first contribution of this paper is to show empirically that financial development affects disproportionately firms which have high financial needs compared to those that have a low financial need. The results found are consistent with the seminal work of Rajan and Zingales (1998). I also show that there is a similar pattern when considering the effect of financial development on R&D intensity by firms, which is consistent with other work which has found a co-movement of growth and R&D with respect to other variables using a similar methodology as the one I use (see for example Aghion, Hemous and Kharroubi (2009)). These relations are shown to be robust to a number of checks. But the most important empirical contribution is the analysis of the hypothesis that financial constraints affects firm growth through the effect they have on R&D. In particular I show that once one controls for R&D in the growth regression, the effect of the interaction between financial dependence and financial development on growth disappears. This result is shown to be robust to the possible endogeneity of R&D in the growth regression.

Second I rationalize this finding by studying a theoretical infinite horizon general equilibrium model where firms are financially constrained in their R&D investment decisions. I assume firms are heterogeneous in two dimensions. The first dimension of heterogeneity is that firms are different in their labor productivity. At the heart of the model is the fact that labor productivity can be increased through R&D. This dimension of heterogeneity is widely used in models of Schumpeterian growth. The second dimension of heterogeneity comes from sectoral differences in the R&D production function. The model assumes that firms operating in one sector of the economy are more able in doing R&D and that firms can't switch sectors. The model predicts that for perfect capital markets the firm's R&D and expected growth are decreasing in the firm's size. When financial constraints are introduced there is a hump-shaped relation between R&D decisions and a firm's size. Second the model predicts that, irrespective of the level of financial development of the economy, firms with higher ability will invest more on R&D conditional on their labor productivity. Two results that emerge from these predictions is that financial constraints will hit harder firms with lower levels of labor productivity and higher ability. This result will be crucial when confronting the model to the empirical prediction. The model also predicts that conditional on size, the difference between R&D expenditure and growth between a high ability firm and a low ability firm is increasing in financial development. I interpret this result to be consistent with the observed empirical findings.

Then by using a numerical approach, I show that the aggregate growth rate of the economy

and the wage rate of the economy are monotonically increasing concave in financial development. The second observation is consistent with the empirical pattern observed in Figure 1. The results also find big welfare increases for low levels of financial development.

The model presented sheds light on the design of R&D policy. In particular it shows that there are three dimensions which R&D policy affect. First, for a given size and financial development level, policies which target high ability firms will attain higher welfare as these policies boost growth at a minimum consumption cost. Second, in the absence of financial frictions and for a given R&D ability, firms with lower labor productivity spend more on R&D. Third, there is a financial constraint effect: in the presence of financial frictions R&D subsidies targeted to small firms relax the financial constraint of small firms. I compare two policy regimes: a uniform subsidy (size-independent) and size-dependent subsidy. The results of this analysis suggest large gains of implementing R&D subsidies, in particular, size-dependent subsidies. An important result found is the shape of such subsidies. I find that for high levels of financial development the optimal size-dependent subsidy is increasing in size since firms with higher R&D ability have in equilibrium higher labor productivity values. As financial development decreases the financial constraint effect dominates and the optimal size-dependent R&D subsidy becomes decreasing in size.

1.7 References

- **Acemoglu, Daron and Akcigit, Ufuk (2009)** "State Dependent Intellectual Property Rights Policy" MIT mimeo
- **Acemoglu, Daron and Simon Robinson (2005)** "Unbundling Institutions" *Journal of Political Economy*, Vol 113, No.5, pp. 949-995.
- **Aghion, Philippe, George-Marios Angeletos, Abhijit Banerjee and Kalina Manova (forthcoming)** "Volatility and Growth: Credit Constraints and the Composition of Investment" *Journal of Monetary Economics*
- **Aghion, Phillippe, David Hemous, and Enisse Kharroubi (2009)** "Credit Constraints, Cyclical Fiscal Policy and Industry Growth" Harvard mimeo
- **Aghion, Phillippe, Peter Howitt and David Mayer-Foulkes (2005)** "The Effect of Financial Development on Convergence: Theory and Evidence" *Quarterly Journal of Economics*, vol. 120, pp. 173-222.
- **Akcigit, Ufuk (2009)** "Firm Size, Innovation Dynamic and Growth" University of Pennsylvania manuscript
- **Baker, Malcolm and Jeffrey Wurgler (2002)** "Market Timing and Capital Structure" *The Journal of Finance*, Vol. LVII, No.1
- **Beck, Thorsten, Asli Demirguc-Kunt, and Ross Levine (2001)** "The Financial Structure Database." In *Financial Structure and Economic Growth: A Cross-Country Comparison of Banks, Markets, and Development*. Cambridge, MA: MIT Press.
- **Beck, Torsten, Ross Levine and Norman Loayza (2000)** "Financial Intermediation and Growth: Causality and Causes" *Journal of Monetary Economics*, Vol. 46, pp. 31-77
- **Bencivenga, Valerie and Bruce Smith (1991)** "Financial Intermediation and Endogenous Growth" *The Review of Economic Studies*, Vol. 58, pp. 195-209
- **Bloom, N., Griffith, R. and Van Reenen, J. (1998)** "Do R&D tax credits work? Evidence from an International Panel of countries 1979-1994" Institute of Fiscal Studies Working Paper W99/8

- **Broda, Christian and David Weinstein (2004)** "Variety Growth and World Welfare" The American Economic Review, Vol.94, No.2, pp 139-144
- **De Gregorio, Jose and Pablo Guidotti (1995)** "Financial Development and Economic Growth" World Development, Vol. 23, pp. 433-448
- **Demirguc-Kunt, A. and V. Maksimovic (1998)** "Law, Finance and Firm Growth", Journal of Finance, 53, pp. 2107-2137
- **Fama, Eugene and Kenneth French (2002)** "Testing Tradeoff and Pecking Order Predictions about Dividends and Debt" Review of Financial Studies 15 pp. 1-37
- **Glaeser, Edward L., and Andrei Shleifer (2002)** "Legal Origins" Quarterly journal of Economics vol. 117, pp. 1193-1229
- **Greenwood, Jeremy and Boyan Jovanovic (1990)** "Financial Development, Growth and the Distribution of Income", Journal of Political Economy, Vol. 98 pp. 1076-1107
- **Guner, Nezih, Gustavo Ventura and Yi Xu (2008)** "Macroeconomic Implications of Size-Dependent Policies" Review of Economic Dynamics, Vol. 11, pp. 721-744
- **Hall, Bronwyn and John Van Reenen(2000)** "How effective are fiscal incentives for R&D? A review of the evidence." Research Policy, 29 (4-5) pp. 449-469. ISSN 0048-7333
- **Hall, Robert (1988)** "The Relation between Price and Marginal Cost in U.S. Industry" The Journal of Political Economy, Vol. 96, pp. 921-947
- **King, Robert and Ross Levine (1993)** "Finance and Growth: Schumpeter Might be Right" The Quarterly Journal of Economics, Vol. 108, pp. 717-737
- **La Porta, Rafael, Florencio Lopez de Silanes, Andrei Shleifer, and R. Vishny (1998)** "Law and Finance" Journal of Political Economy, 106(6), pp. 113-155.
- **Laeven, Luc, Ross Levine and Stelios Michalopoulos (2009)** "Financial Innovation and Endogenous Growth" mimeo.
- **Levine, Ross (2003)** "Finance and Growth: Theory, Evidence and Mechanisms" Handbook of Economic Growth edited by Philippe Aghion and Steven Durlauf.

- **Kortum, Samuel (1993)** "Equilibrium R&D and the Patent-R&D Ratio: US Evidence," American Economic Review, 83, pp. 450-457
- **Raddatz, Claudio (2006)** "Liquidity Needs and Vulnerability to Financial Underdevelopment" Journal of Financial Economics, 80, pp. 667-772
- **Rajan, Raghuram and Luigi Zingales (1998)** "Financial Dependence and Growth" The American Economic Review, Vol. 88, No.3, pp-559-586
- **Restuccia, Diego and Richard Rogerson (2007)** "Policy Distortions and Aggregate Productivity with Heterogeneous Plants" NBER Working Paper No. 13018
- **Saint-Paul, Gilles (1992)** "Technological Choice, Financial Markets and Economic Development" European Economic Review, Vol. 36, pp. 763-781
- **Stokey, Nancy and Robert Lucas (1989)** "Recursive Methods in Economic Dynamics" Cambridge, Harvard University Press.
- **United Nations, Statistical Division.** Industrial Statistical Yearbook, Vol.1. New York: United Nations, 2001.

1.8 Appendix 1: Proofs

Proof of Proposition (4)

To prove this Proposition we will check the assumptions in Stokey-Lucas (1989). Through this proof I will assume that κ is a continuous variables with $\kappa \in [\kappa_{\min}, \kappa_{\max}]$.

Assumption 9.4 S-L First notice that the relevant state space is $\widehat{Q}^* = [\underline{q}, \widehat{q}_{\max}]$, where $\widehat{q}_{\max} = \widehat{\lambda}/g < \infty$. The logic for this upper bound on relative labor productivities comes from the fact that values above this upper bound will be depreciated by the growth rate even in the case of a successful shock. Therefore labor productivities will always lie in \widehat{Q} , which is clearly a Borel set in \mathbb{R} .

Assumption 9.5 S-L As shown in Akcigit (2009) the set $Z = [0, \bar{\phi}]$ is a compact Borel set in \mathbb{R} and the transition function

$$T(\widehat{q}, \widehat{q}') = \begin{cases} \phi & \text{if } \widehat{q}' = \frac{\widehat{q} + \widehat{\lambda}}{1+g} \\ 1 - \phi & \text{if } \widehat{q}' = \frac{\widehat{q}}{1+g} \\ 0 & \text{otherwise} \end{cases}$$

satisfies the Feller property.

Assumption 9.6 S-L Take the set

$$\Xi(\widehat{q}, \kappa) = \left\{ \phi \in [0, \bar{\phi}] : \phi \leq \left(\mu \frac{\theta \kappa}{\gamma \bar{w}} \left(1 - \frac{1}{\widehat{q}} \right) \right)^{1/\theta} \right\}$$

It is clear that for any (\widehat{q}, κ) the set is non-empty. Compactness comes from the fact that for every (\widehat{q}, κ) the set is closed and is bounded. Continuity follows from the fact that $\left(\mu \frac{\theta \kappa}{\gamma \bar{w}} \left(1 - \frac{1}{\widehat{q}} \right) \right)^{1/\theta}$ is continuous in \widehat{q}, κ .

Assumption 9.7 S-L We now turn to the question of compactness of the profit function

$$\Pi(\widehat{q}, \kappa, \phi) = \left(1 - \frac{1}{\widehat{q}} \right) - \frac{(1+g)}{\beta} \bar{w} \frac{\gamma}{\theta \kappa} \phi^\theta$$

This function is clearly bounded below in $\Xi \times \widehat{Q}$ by $-\frac{(1+g)}{\beta} \bar{w} \frac{\gamma}{\theta \kappa} \phi^\theta$ and is bounded above by 1.

Given this, the existence and uniqueness of the value function follows from theorem 9.6 in Stokey-Lucas (1989).

Assumption 9.8 S-L The profit function $\Pi = \left(1 - \frac{1}{\widehat{q}} \right) - \frac{(1+g)}{\beta} \bar{w} \frac{\gamma}{\theta \kappa} \phi^\theta$ is increasing in \widehat{q} and κ for all values of ϕ .

Assumption 9.9 S-L Take $\hat{q}' > \hat{q}, \kappa' > \kappa$. Then we have

$$\begin{aligned} \left(\mu \frac{\theta \kappa}{\gamma \bar{w}} \left(1 - \frac{1}{\hat{q}'} \right) \right)^{1/\theta} &\geq \left(\mu \frac{\theta \kappa}{\gamma \bar{w}} \left(1 - \frac{1}{\hat{q}} \right) \right)^{1/\theta} \\ \left(\mu \frac{\theta \kappa'}{\gamma \bar{w}} \left(1 - \frac{1}{\hat{q}} \right) \right)^{1/\theta} &\geq \left(\mu \frac{\theta \kappa}{\gamma \bar{w}} \left(1 - \frac{1}{\hat{q}} \right) \right)^{1/\theta} \end{aligned}$$

this implies that $\Xi(\hat{q}, \kappa) \subseteq \Xi(\hat{q}', \kappa)$ and $\Xi(\hat{q}, \kappa) \subseteq \Xi(\hat{q}, \kappa')$.

This proves that the value function is increasing in both relative labor productivity, \hat{q} , and R&D ability, κ .

Assumption 9.10 S-L The profit function is concave in both \hat{q}, ϕ . Furthermore, if $\theta \geq 2$ the profit function is jointly concave in \hat{q}, ϕ, κ .

Assumption 9.11 S-L The function $\left(\mu \frac{\theta \kappa}{\gamma \bar{w}} \left(1 - \frac{1}{\hat{q}} \right) \right)^{1/\theta}$ is concave in \hat{q}, κ . This implies that the set Ξ is convex in \hat{q}, κ .

Assumption 9.12 S-L The profit function is continuously differentiable on the interior of $\hat{Q} \times \Xi$.

Putting this last observations, and by virtue of Theorems 9.9 and 9.10 in Stokey-Lucas (1989) we have that the Value function is strictly concave and differentiable in \hat{q} .

Proof of Lemma (5)

Define

$$\begin{aligned} \varrho(\hat{q}, \kappa) &= \mu \pi(\hat{q}) - \bar{w} h(\phi^u(\hat{q}, \kappa), \kappa) \\ &= \mu \left(1 - \frac{1}{\hat{q}} \right) - \frac{1}{\theta} \left(\frac{\kappa}{\gamma \bar{w}} \right)^{1/(\theta-1)} \left(\beta^2 \frac{\Delta \tilde{V}(\hat{q}, \kappa)}{(1+g)} \right)^{\theta/(\theta-1)} \end{aligned}$$

where $\Delta \tilde{V}(\hat{q}, \kappa) = \tilde{V}\left(\frac{\hat{q}+\hat{\lambda}}{1+g}, \kappa\right) - \tilde{V}\left(\frac{\hat{q}}{1+g}, \kappa\right)$.

First using Proposition 4 we know that \tilde{V} is continuous in \hat{q} which implies that $\varrho(\hat{q}, \kappa)$ is continuous. Second, because of the concavity of \tilde{V} we have that $\Delta \tilde{V}(\hat{q}, \kappa)$ is decreasing in \hat{q} . This implies that $\varrho(\hat{q}, \kappa)$ is increasing in \hat{q} . Finally, and using the fact that \tilde{V} is strictly increasing, we have that $\varrho(1, \kappa) < 0$ and $\lim_{\hat{q} \rightarrow \infty} \varrho(\hat{q}, \kappa) > 0$. All this implies that there exists a unique $\bar{q}(\kappa) \in \mathbb{R}_+$ which satisfies

$$\mu \left(1 - \frac{1}{\bar{q}(\kappa)} \right) = \frac{1}{\theta} \left(\frac{\kappa}{\gamma \bar{w}} \right)^{1/(\theta-1)} \left(\beta^2 \frac{\Delta \tilde{V}(\bar{q}(\kappa), \kappa)}{(1+g)} \right)^{\theta/(\theta-1)} \quad (1.32)$$

Now we want to analyze how $\kappa \Delta \tilde{V}(\bar{q}(\kappa), \kappa)^\theta$ changes as κ changes. Notice that if $\Delta \tilde{V}(\bar{q}, \kappa)$ is increasing in κ we will have $\bar{q}(\kappa_H) > \bar{q}(\kappa_L)$. Now suppose $\Delta \tilde{V}(\bar{q}, \kappa)$ is decreasing in κ . In this case we need to determine if

$$\kappa_H \left(\Delta \tilde{V}(\bar{q}, \kappa_H) \right)^\theta \geq \kappa_L \left(\Delta \tilde{V}(\bar{q}, \kappa_L) \right)^\theta \quad (1.33)$$

Using the fact that $\tilde{V}(\hat{q}, \kappa)$ is increasing in κ implies that condition (1.33) is satisfied and $\bar{q}(\kappa_H) > \bar{q}(\kappa_L)$.

Proof of Proposition (6)

First of all, for a given κ , ϕ^* is continuous and bounded on \hat{Q} . Furthermore I assumed that $\phi < \bar{\phi} < 1$. This implies that $\exists n^* < \infty$ such that for every \hat{q} we have

$$\frac{\hat{q}}{(1 + g^*)^{n^*}} = 0$$

Then conditional on κ , for any $\hat{q} \in \hat{Q}$

$$P^{n^*}(\hat{q}, 0) \geq (1 - \bar{\phi})^{n^*} > \epsilon > 0$$

Since for all $A \subset 2^{\hat{Q}}$, either $0 \in A$ or $0 \notin A$, we have

$$P^{n^*}(\hat{q}, A) \geq P^{n^*}(\hat{q}, 0) \geq \epsilon > 0$$

or

$$P^{n^*}(\hat{q}, A^c) \geq P^{n^*}(\hat{q}, 0) \geq \epsilon > 0$$

This is condition M in chapter 11 of Stokey-Lucas (1989).

This proves the existence and uniqueness of a conditional steady-state distribution $\Psi^*(\hat{q}|\kappa)$.

This implies the Steady-State Distribution $\hat{\Gamma}^*(\hat{q}, \kappa)$ exists, is unique and satisfies:

$$\hat{\Gamma}^*(\hat{q}, \kappa) = \Pr(\kappa) \Psi^*(\hat{q}|\kappa)$$

where

$$\Pr(\kappa) = \begin{cases} p & \text{if } \kappa_H \\ 1 - p & \text{otherwise} \end{cases}$$

Proof of Proposition (7)

The proof follows directly from Akcigit (2009).

Proof of Lemma (8)

As shown in Lemma 5 we have $\bar{q}(\kappa_H) > \bar{q}(\kappa_L)$. This implies that we can divide the analysis in three region

i) for $\hat{q} \leq \bar{q}(\kappa_L)$ we have

$$\phi^*(\hat{q}, \kappa) = \left(\mu \frac{\theta \kappa}{\gamma \bar{w}} \left(1 - \frac{1}{\hat{q}} \right) \right)^{1/\theta}$$

which is increasing in κ .

ii) For $\hat{q} \geq \bar{q}(\kappa_H)$ we have

$$\phi^*(\hat{q}, \kappa) = \left(\kappa \beta^2 \frac{\Delta \tilde{V}(\hat{q}, \kappa_H)}{\gamma \bar{w} (1 + g)} \right)^{1/(\theta-1)}$$

which as shown in Lemma 6 is increasing in κ .

iii) For $\bar{q}(\kappa_L) < \hat{q} < \bar{q}(\kappa_H)$ we have a firm that a firm with κ_H is restricted and a firm with κ_L is unrestricted. This implies that R&D intensity of firm κ_H will be

$$\phi^*(\hat{q}, \kappa_H) = \phi^r(\hat{q}, \kappa_H) > \phi^r(\hat{q}, \kappa_L)$$

Since $\bar{q}(\kappa_L) < \hat{q}$ we have that $\phi^r(\hat{q}, \kappa_L) > \phi^u(\hat{q}, \kappa_L) = \phi^*(\hat{q}, \kappa_L)$. Putting the two inequalities together we have

$$\phi^*(\hat{q}, \kappa_L) < \phi^*(\hat{q}, \kappa_H)$$

This completes the proof.

Proof of Lemma (9)

As shown in Lemma 5 we have $\bar{q}(\kappa_H) > \bar{q}(\kappa_L)$. This implies that we can divide the analysis in three region

i) for $\hat{q} \leq \bar{q}(\kappa_L)$ we have that R&D intensity is

$$\Re(\hat{q}, \kappa) = \frac{\mu(1+g)}{\beta}$$

which is independent of κ .

ii) For $\widehat{q} \geq \bar{q}(\kappa_H)$ we have

$$\mathfrak{R}(\widehat{q}, \kappa) = \frac{\beta^{(\theta+1)/(\theta-1)} \widehat{q}}{\theta(\widehat{q}-1)} \left(\frac{\kappa}{\gamma \widetilde{w}(1+g)} \right)^{1/(\theta-1)} \left(\Delta \widetilde{V}(\widehat{q}, \kappa_H) \right)^{\theta/(\theta-1)}$$

which as shown in Lemma 6 is increasing in κ .

iii) For $\bar{q}(\kappa_L) < \widehat{q} < \bar{q}(\kappa_H)$ we have a firm that a firm with κ_H is restricted and a firm with κ_L is unrestricted. This implies that R&D intensity of firm κ_H will be constant. Since $\bar{q}(\kappa_L) < \widehat{q}$ and for a given κ RD intensity is decreasing in \widehat{q} , we will have

$$\mathfrak{R}(\widehat{q}, \kappa_L) < \mathfrak{R}(\widehat{q}, \kappa_H)$$

This completes the proof.

Proof of Lemma (10)

I start by proving part (1) of Lemma (10). Take two financial development levels, μ , such that $\mu_H > \mu_L$. Using condition (1.32) in Lemma 5 we have that $\bar{q}(\kappa; \mu_H) < \bar{q}(\kappa; \mu_L)$. Then, holding \widehat{q} , \widetilde{w}^* , κ and \widehat{q} , there are three relative regions to analyze:

i) For $\widehat{q} < \bar{q}(\kappa; \mu_H)$ we have

$$\phi^*(\widehat{q}, \kappa; \mu) = \phi^r(\widehat{q}, \kappa; \mu)$$

Using condition (1.27) we have that $\phi^r(\widehat{q}, \kappa; \mu_H) > \phi^r(\widehat{q}, \kappa; \mu_L)$. This immediately implies that $\phi^*(\widehat{q}, \kappa; \mu_H) > \phi^*(\widehat{q}, \kappa; \mu_L)$; $\mathfrak{R}(\widehat{q}, \kappa; \mu_H) > \mathfrak{R}(\widehat{q}, \kappa; \mu_L)$

ii) For $\bar{q}(\kappa; \mu_H) < \widehat{q} < \bar{q}(\kappa; \mu_L)$ the optimal R&D choices are

$$\phi^*(\widehat{q}, \kappa; \mu_H) = \phi^u(\widehat{q}, \kappa)$$

$$\phi^*(\widehat{q}, \kappa; \mu_L) = \phi^r(\widehat{q}, \kappa; \mu_L)$$

We know that $\phi^u(\widehat{q}, \kappa)$ is the unrestricted optimal choice for \widehat{q}, κ which implies $\phi^u(\widehat{q}, \kappa) > \phi^r(\widehat{q}, \kappa; \mu_L)$. This means $\phi^*(\widehat{q}, \kappa; \mu_H) > \phi^*(\widehat{q}, \kappa; \mu_L)$, $\mathfrak{R}(\widehat{q}, \kappa; \mu_H) > \mathfrak{R}(\widehat{q}, \kappa; \mu_L)$.

iii) For $\widehat{q} > \bar{q}(\kappa; \mu_L)$ firms are unrestricted and their R&D decisions are independent of μ , or $\phi^*(\widehat{q}, \kappa; \mu_H) = \phi^*(\widehat{q}, \kappa; \mu_L)$; $\mathfrak{R}(\widehat{q}, \kappa; \mu_H) = \mathfrak{R}(\widehat{q}, \kappa; \mu_L)$.

This proves part (1) of the Lemma

For the second part we will have the following cases:

i) $\widehat{q} < \bar{q}(\kappa_L; \mu_H)$

In this case bot types are constrained for both financial development levels. This implies

$$\frac{\phi^r(\widehat{q}, \kappa_H; \mu)}{\phi^r(\widehat{q}, \kappa_L; \mu)} = \left(\frac{\kappa_H}{\kappa_L}\right)^{1/\theta}$$

which is independent of μ .

ii) $\bar{q}(\kappa_L; \mu_H) < \widehat{q} < \min\{\bar{q}(\kappa_L; \mu_L), \bar{q}(\kappa_H; \mu_H)\}$

In this case when $\mu = \mu_L$ both firms are constrained which implies

$$\frac{\phi^*(\widehat{q}, \kappa_H; \mu_L)}{\phi^*(\widehat{q}, \kappa_L; \mu_L)} = \frac{\phi^r(\widehat{q}, \kappa_H; \mu_L)}{\phi^r(\widehat{q}, \kappa_L; \mu_L)}$$

For μ_H only high ability types are constrained which implies

$$\frac{\phi^*(\widehat{q}, \kappa_H; \mu_H)}{\phi^*(\widehat{q}, \kappa_L; \mu_H)} = \frac{\phi^r(\widehat{q}, \kappa_H; \mu_H)}{\phi^u(\widehat{q}, \kappa_L)}$$

Notice that since $\widehat{q} > \bar{q}(\kappa_L; \mu_H)$, we have the following inequality $\phi^u(\widehat{q}, \kappa_L) < \phi^r(\widehat{q}, \kappa_L; \mu_H)$

This implies

$$\frac{\phi^*(\widehat{q}, \kappa_H; \mu_H)}{\phi^*(\widehat{q}, \kappa_L; \mu_H)} > \left(\frac{\kappa_H}{\kappa_L}\right)^{1/\theta} = \frac{\phi^r(\widehat{q}, \kappa_H; \mu_L)}{\phi^r(\widehat{q}, \kappa_L; \mu_L)}$$

which shows that

$$\frac{\phi^*(\widehat{q}, \kappa_H; \mu_H)}{\phi^*(\widehat{q}, \kappa_L; \mu_H)} > \frac{\phi^*(\widehat{q}, \kappa_H; \mu_L)}{\phi^*(\widehat{q}, \kappa_L; \mu_L)}$$

This result immediately shows that firm growth follows the same pattern. As for \mathfrak{R} we have

$$\frac{\mathfrak{R}(\widehat{q}, \kappa_H; \mu_H)}{\mathfrak{R}(\widehat{q}, \kappa_L; \mu_H)} > \frac{\mathfrak{R}(\widehat{q}, \kappa_H; \mu_L)}{\mathfrak{R}(\widehat{q}, \kappa_L; \mu_L)}$$

which follows from the fact that

$$\begin{aligned} \frac{\mathfrak{R}(\widehat{q}, \kappa_H; \mu_H)}{\mathfrak{R}(\widehat{q}, \kappa_L; \mu_H)} &= \frac{\mathfrak{R}^r(\widehat{q}, \kappa_H; \mu_H)}{\mathfrak{R}^u(\widehat{q}, \kappa_L; \mu_H)} > \\ &> \frac{\mathfrak{R}^r(\widehat{q}, \kappa_H; \mu_H)}{\mathfrak{R}^r(\widehat{q}, \kappa_L; \mu_H)} = \frac{\mathfrak{R}(\widehat{q}, \kappa_H; \mu_L)}{\mathfrak{R}(\widehat{q}, \kappa_L; \mu_L)} \end{aligned}$$

iii) $\bar{q}(\kappa_H; \mu_L) > \widehat{q} > \max\{\bar{q}(\kappa_L; \mu_L), \bar{q}(\kappa_H; \mu_H)\}$

In this case case the only restricted firms are high ability facing low financial development. This implies

$$\begin{aligned}\frac{\phi^*(\widehat{q}, \kappa_H; \mu_L)}{\phi^*(\widehat{q}, \kappa_L; \mu_L)} &= \frac{\phi^r(\widehat{q}, \kappa_H; \mu_L)}{\phi^u(\widehat{q}, \kappa_L)} \\ \frac{\phi^*(\widehat{q}, \kappa_H; \mu_H)}{\phi^*(\widehat{q}, \kappa_L; \mu_H)} &= \frac{\phi^u(\widehat{q}, \kappa_H)}{\phi^u(\widehat{q}, \kappa_L)}\end{aligned}$$

Using the fact that $\phi^u(\widehat{q}, \kappa_H) > \phi^r(\widehat{q}, \kappa_H; \mu_L)$ we get

$$\frac{\phi^*(\widehat{q}, \kappa_H; \mu_H)}{\phi^*(\widehat{q}, \kappa_L; \mu_H)} > \frac{\phi^*(\widehat{q}, \kappa_H; \mu_L)}{\phi^*(\widehat{q}, \kappa_L; \mu_L)}$$

which implies the same relation growth. The pattern for RD intensity follows from the fact that $\mathfrak{R}(\widehat{q}, \kappa_H; \mu_H) = \mathfrak{R}^u(\widehat{q}, \kappa_H) > \mathfrak{R}^r(\widehat{q}, \kappa_H; \mu_L) = \mathfrak{R}(\widehat{q}, \kappa_H; \mu_L)$.

$$\text{iv) } \min\{\bar{q}(\kappa_L; \mu_L), \bar{q}(\kappa_H; \mu_H)\} < \widehat{q} < \max\{\bar{q}(\kappa_L; \mu_L), \bar{q}(\kappa_H; \mu_H)\}$$

Suppose first $\bar{q}(\kappa_L; \mu_L) > \bar{q}(\kappa_H; \mu_H)$. This implies that firms are constrained only for low levels of financial development. From Lemma 6 we have that $\left(\frac{\kappa_H}{\kappa_L}\right)^{1/\theta} \frac{\Delta\tilde{V}(\widehat{q}, \kappa_H)}{\Delta\tilde{V}(\widehat{q}, \kappa_L)} > 1$ which implies

$$\frac{\phi^*(\widehat{q}, \kappa_H; \mu_H)}{\phi^*(\widehat{q}, \kappa_L; \mu_H)} > \frac{\phi^*(\widehat{q}, \kappa_H; \mu_L)}{\phi^*(\widehat{q}, \kappa_L; \mu_L)}$$

This condition implies the same relation for growth and R&D intensity.

Now suppose $\bar{q}(\kappa_L; \mu_L) < \bar{q}(\kappa_H; \mu_H)$. This implies that high ability firms are constrained for both high and low levels of financial development, while low ability firms are never constrained. In this case we have

$$\frac{\phi^*(\widehat{q}, \kappa_H; \mu_H)}{\phi^*(\widehat{q}, \kappa_L; \mu_H)} > \frac{\phi^*(\widehat{q}, \kappa_H; \mu_L)}{\phi^*(\widehat{q}, \kappa_L; \mu_L)}$$

which follows from the fact that $\phi^*(\widehat{q}, \kappa_L; \mu_H) = \phi^*(\widehat{q}, \kappa_L; \mu_L) = \phi^u(\widehat{q}, \kappa_L)$ and that $\phi^*(\widehat{q}, \kappa_H; \mu_H) = \phi^r(\widehat{q}, \kappa_H; \mu_H) > \phi^r(\widehat{q}, \kappa_H; \mu_L) = \phi^*(\widehat{q}, \kappa_H; \mu_L)$.

This condition implies the same relation for growth and R&D intensity.

$$\text{v) } \widehat{q} > \bar{q}(\kappa_L; \mu_H)$$

In this case bot firms are unconstrained which implies that differences in R&D are independent of μ .

The non monotonicity follows immediately from the analysis above.

This finishes the proof.

1.9 Appendix 2: Figures and Tables

Figure 1. Financial Development and Aggregate Growth

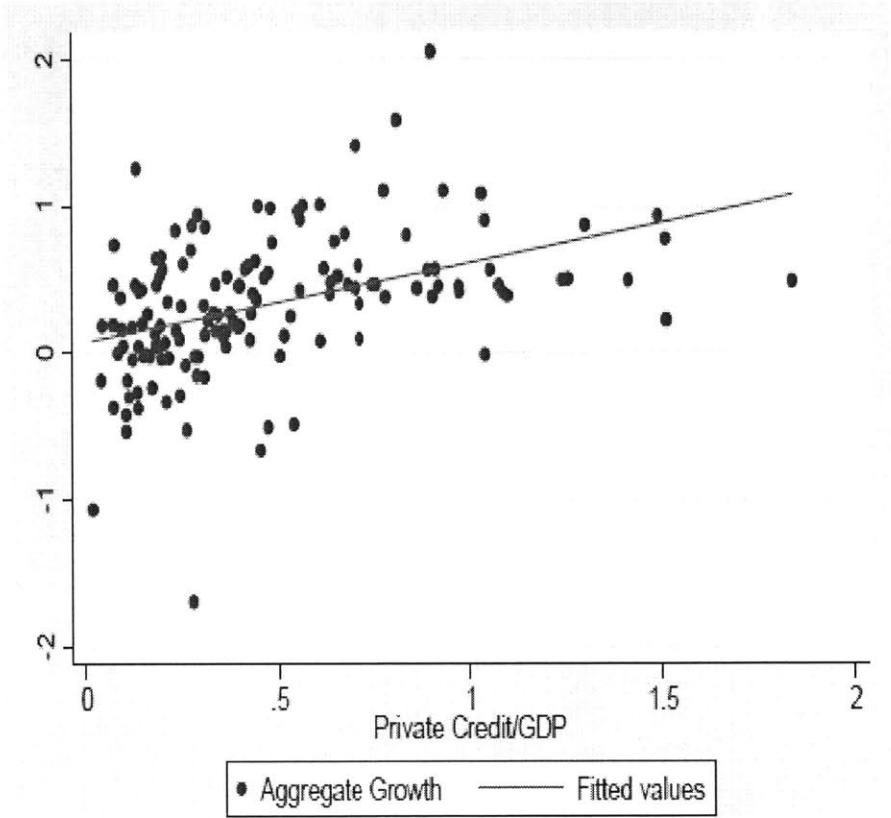


Figure 2. Financial Development and Firm Level Growth

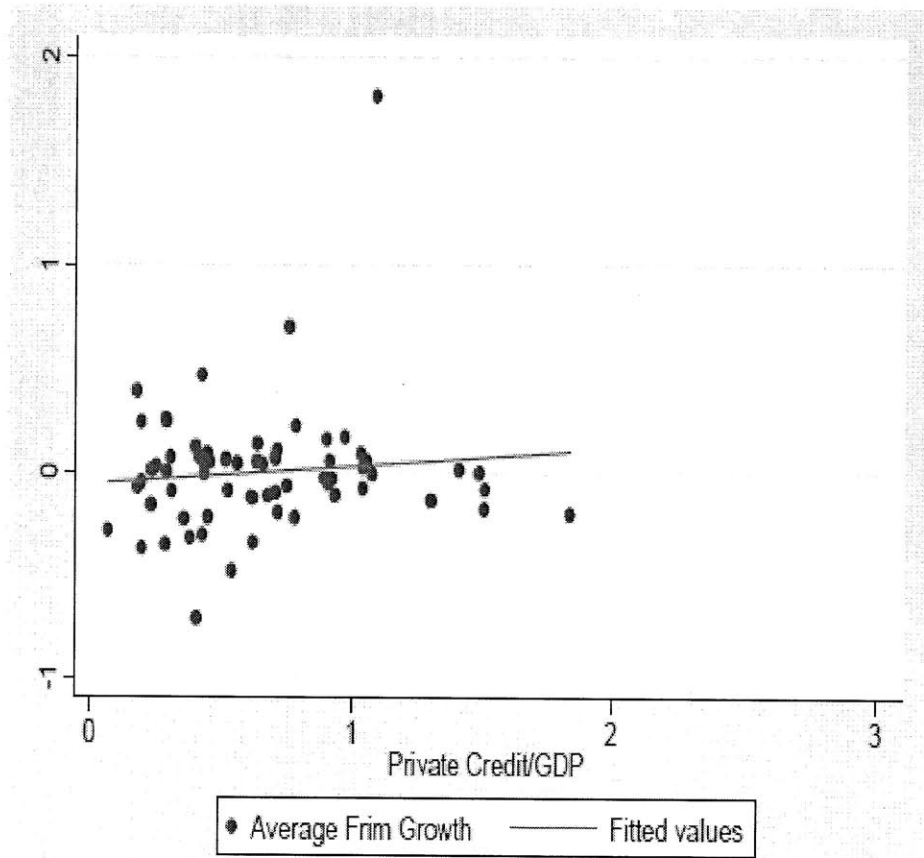


Figure 3. Financial Development and Firm Level R&D

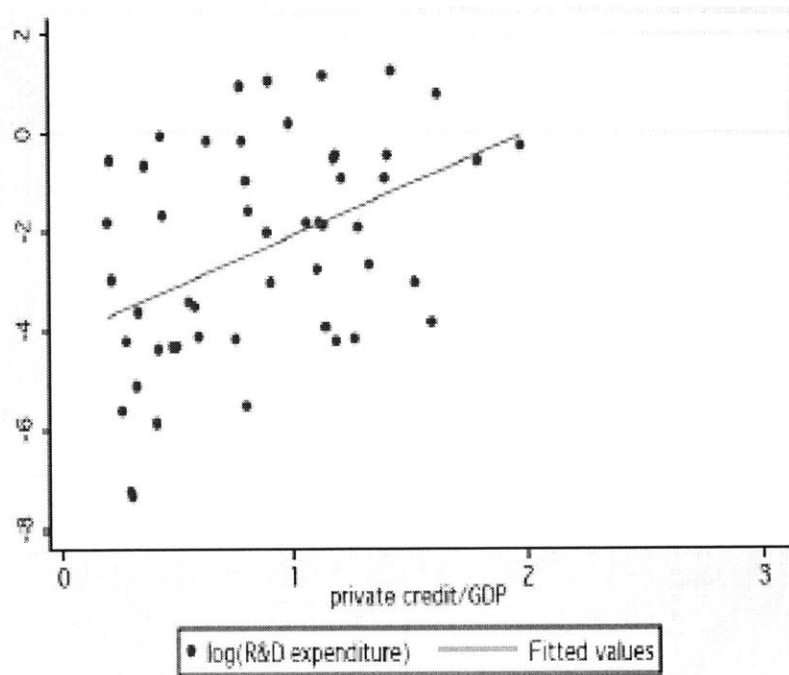


Figure 4. Average Sale vs. Year

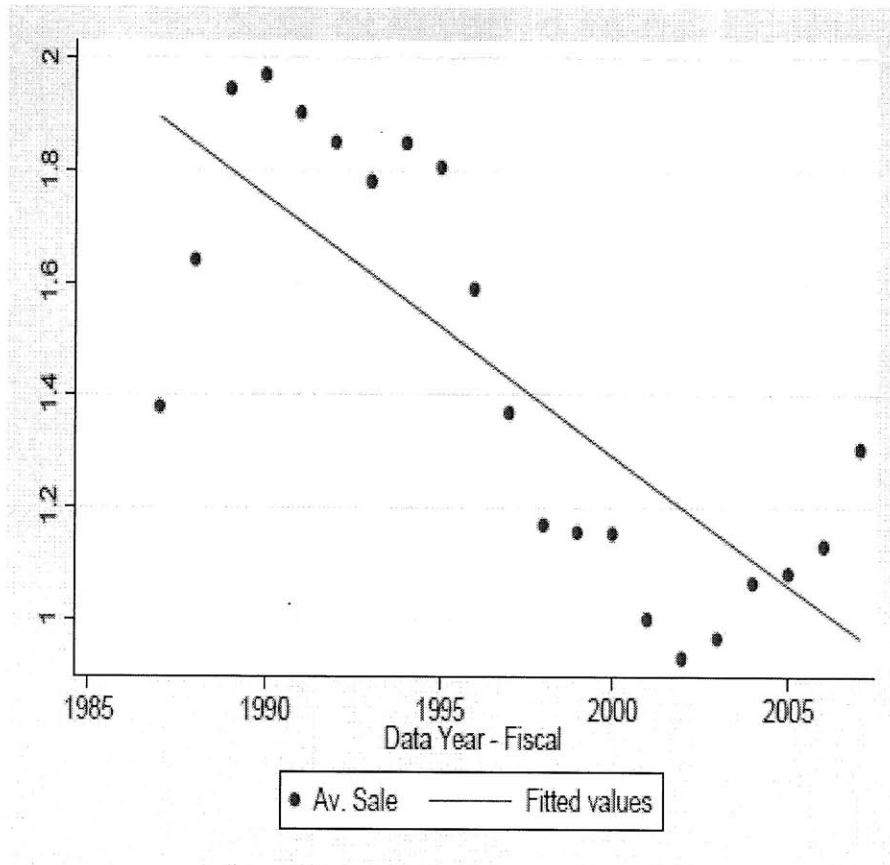


Figure 5

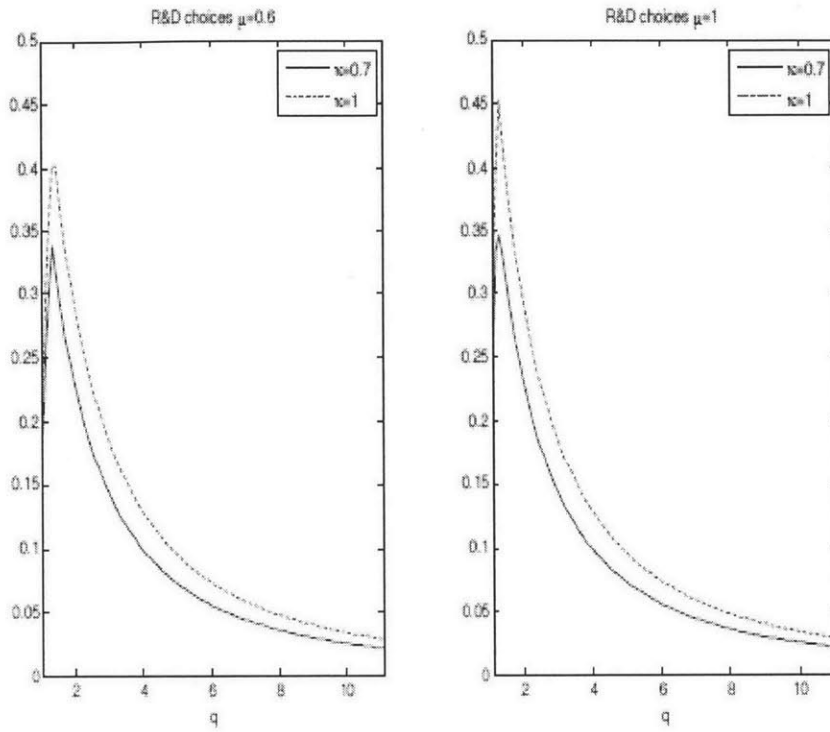


Figure 6

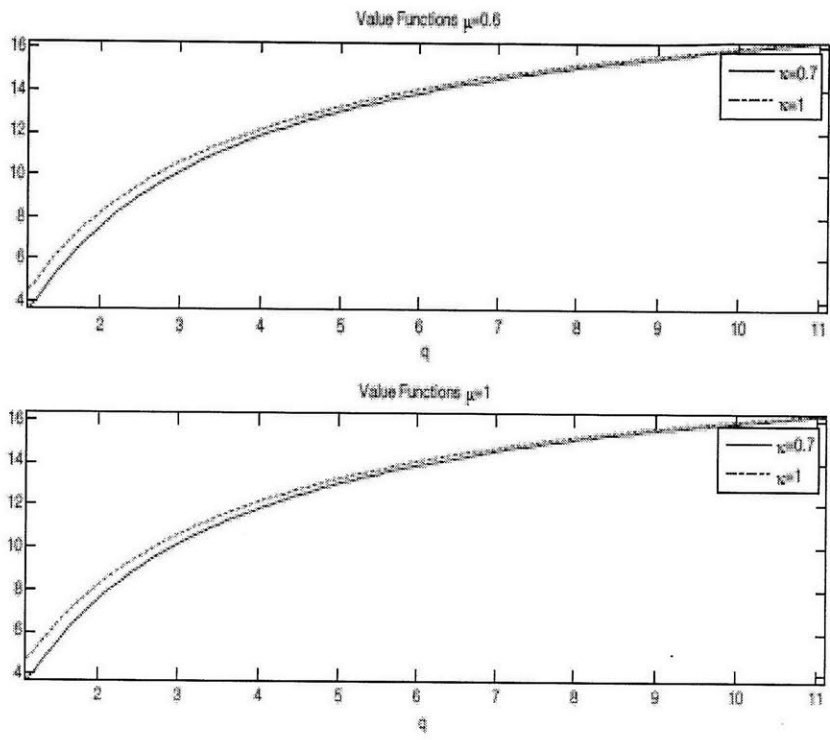


Figure 7. Equilibrium Growth Rate and Financial Development

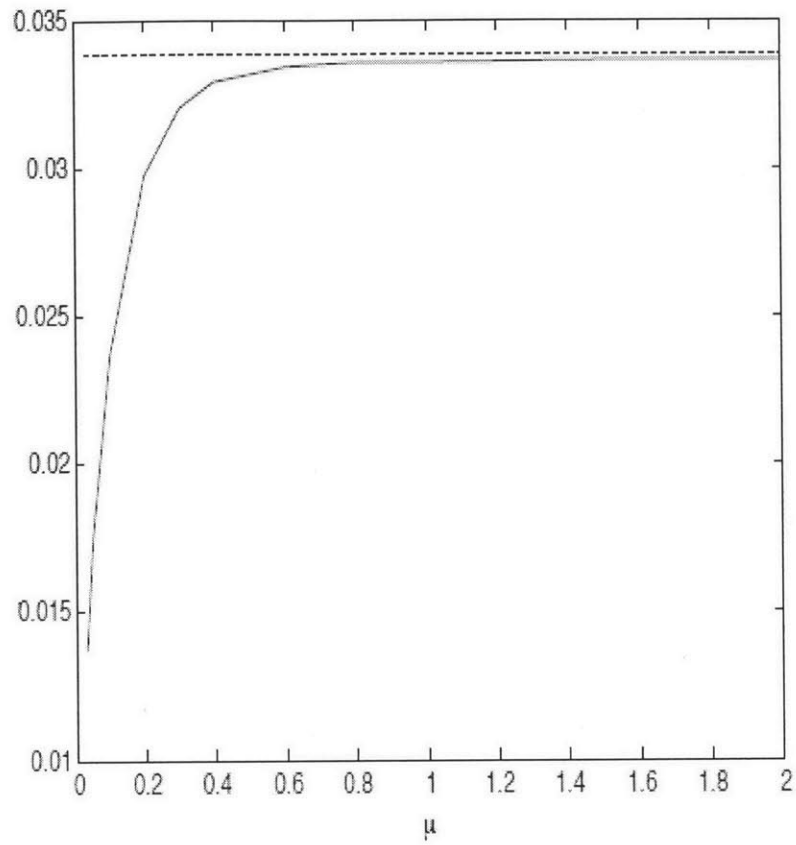


Figure 8. Equilibrium Relative Wage Rate and Financial Development

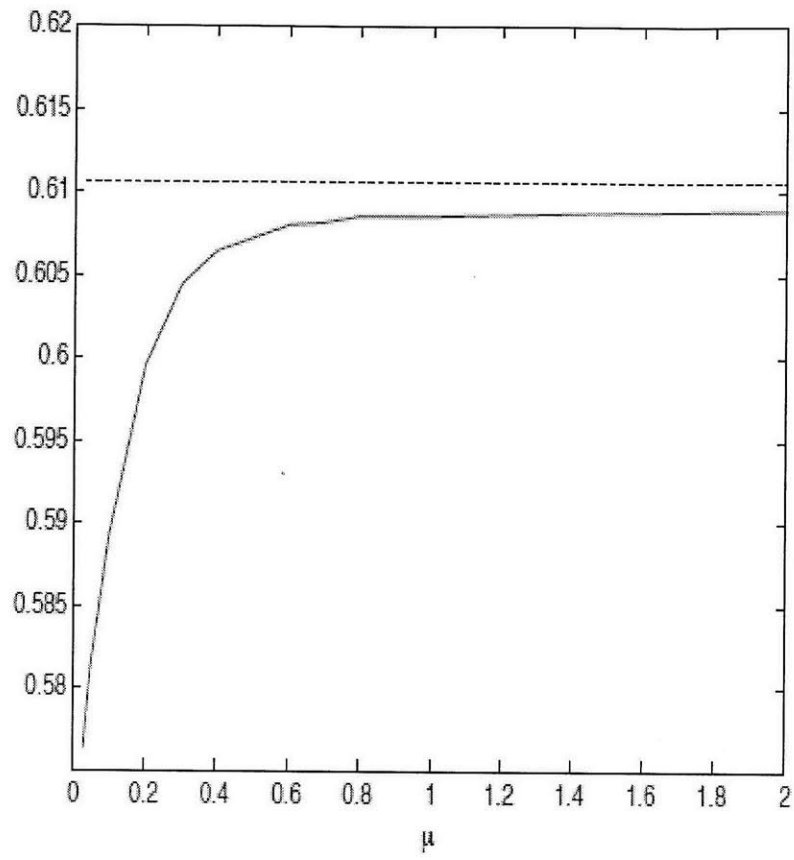


Figure 9. Expected Diff. in Growth between Sector *H* and Sector *L*

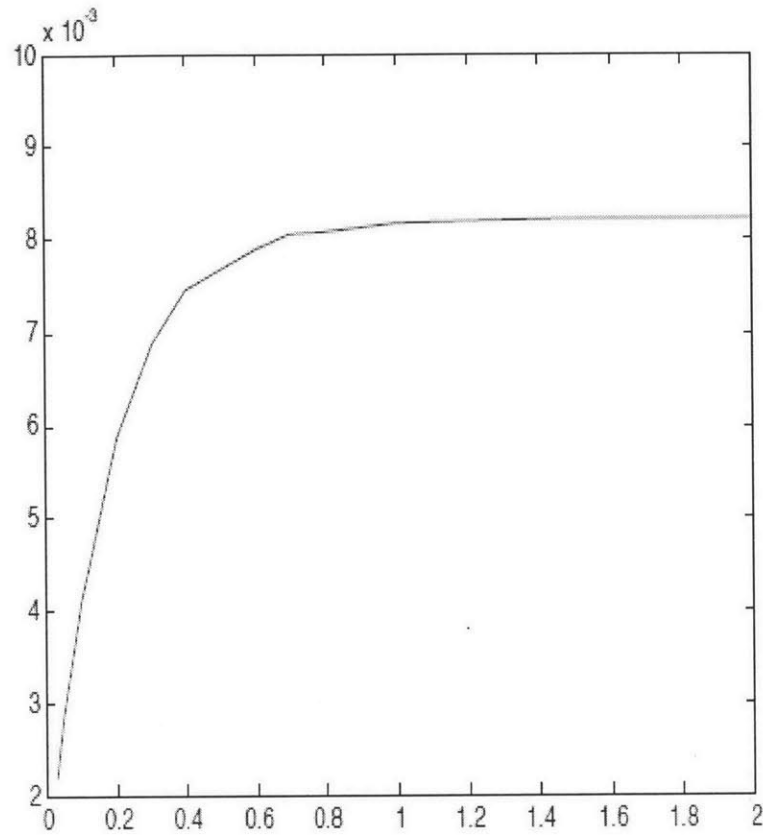


Figure 10. Expected Diff. in R&D Expenditure between Sector H and Sector L

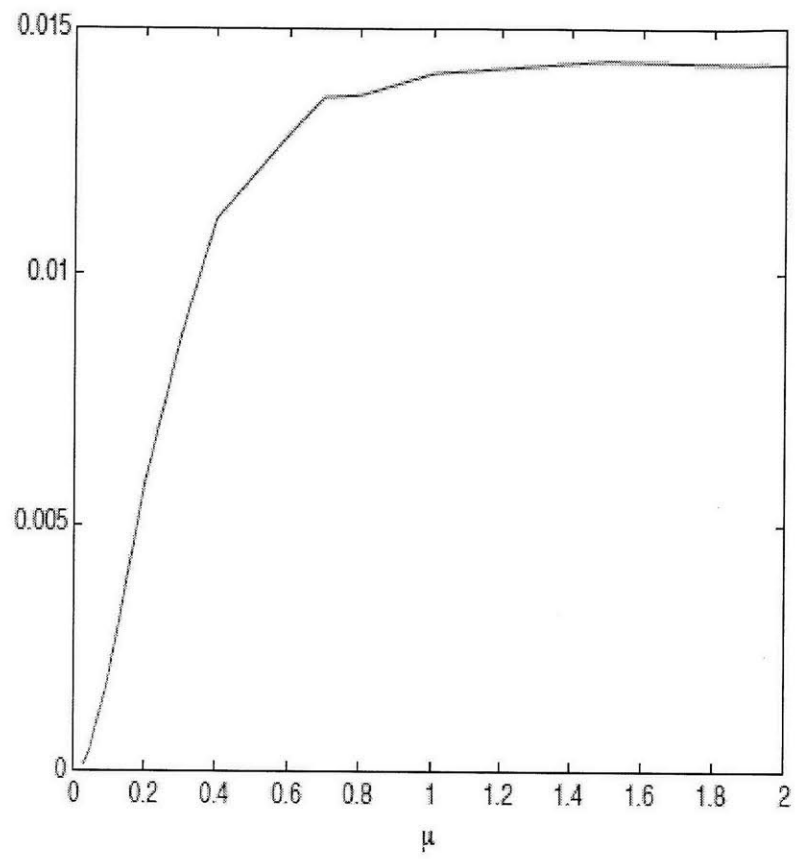


Figure 11. Welfare and μ

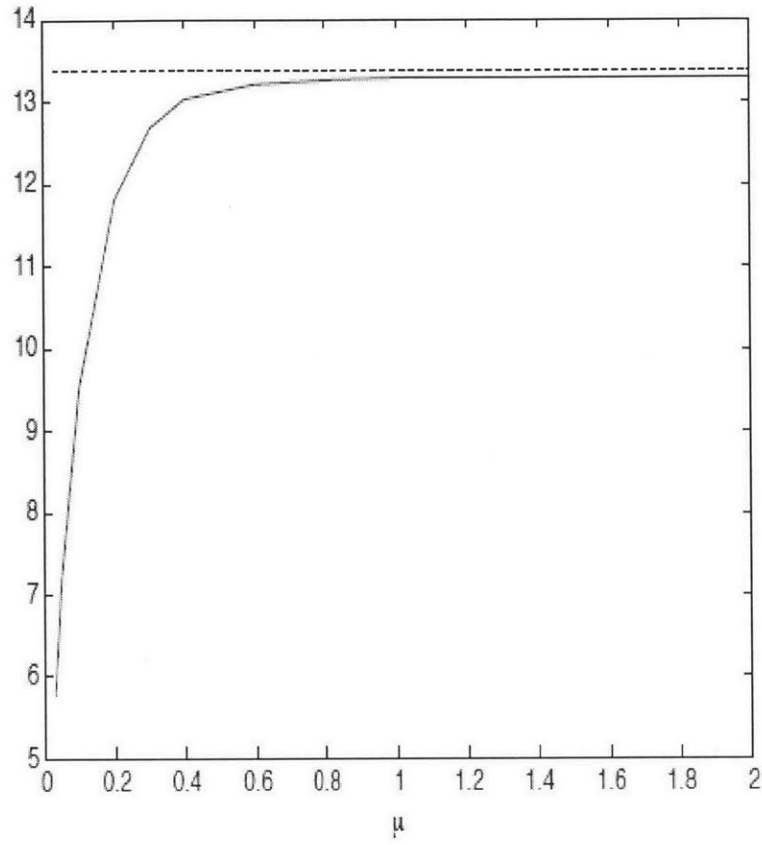


Table 1. Summary Statistics

	Mean	St. Dev.	Observations	Source
Firm Level Data				
Growth of Sales	0.2096	1.981	37431	Compustat
log (Sales)	0.0511	2.2536	39074	Compustat
log (R&D Expenditure/Sales)	-3.3105	2.2317	19579	Compustat
Age	11.231	13.11	39074	Compustat
log (Market Value)	1.1401	2.7428	37830	Compustat
Industry Level Data				
Financial Dependence	0.1977	0.4148	68	Compustat
Employment	14444	59866	7090	UNIDO
Av. Employment	164.38	448.75	5828	UNIDO
Country Level Data				
PPP log GDP 1987	24.25	2.67	168	World Bank WDI
Private Credit/GDP	0.5381	0.3421	47	Raddatz (2008)
Stock Market Capitalization	0.2521	0.3034	46	Beck et. al. (2001)
Accounting Standards	61.0278	13.98	36	Laporta et al. (1998)
log (Settler Mortality)				Acemoglu, Johnson and Robinson (2001)
Legal Origins				Laporta et al. (1998)

Note: All firm level data is normalized by using the GDP deflator and converted to US dollars

Table 2: Growth and R&D Regressions

	Dependent Variable: Growth			Dependent Variable: R&D Intensity				
	OLS	2SLS	2SLS	OLS	2SLS	2SLS	Heckman 2 Step	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Interaction (External Dependence x Private Credit)	0.027** [0.011]	0.038*** [0.012]	0.041*** [0.015]	0.059*** [0.012]	0.072*** [0.025]	0.090*** [0.029]	0.068*** [0.008]	0.060*** [0.010]
log sale	-0.063*** [0.010]	-0.062*** [0.010]	-0.062*** [0.010]	-0.314*** [0.007]	-0.314*** [0.007]	-0.314*** [0.007]	0.076*** [0.004]	-0.309*** [0.006]
age	-0.001** [0.001]	-0.001* [0.001]	-0.001* [0.001]	0.005*** [0.001]	0.005*** [0.001]	0.005*** [0.001]	0.003*** [0.001]	0.005*** [0.001]
Inverse mills ratio								0.209*** [0.065]
propensity to report							5.015*** [0.134]	
Country, Time and Industry Dummies	YES	YES	YES	YES	YES	YES	YES	YES
Observations	35367	35349	35349	19599	19589	19589	38044 (19599 censored)	
R-squared	0.118	0.117	0.117	0.605	0.605	0.605		

Note: Dependent Variable is the annualized growth rate of real sales between period t and t+5

Note: Dependent Variable is log (RD/Sale) in period t

All standard errors are clustered at the country level and are shown in brackets

***p<0.01, **p<0.05, *p<0.1

Table 3. Growth Regression

	OLS	OLS	OLS	OLS	OLS	OLS
	[1]	[2]	[3]	[4]	[5]	[6]
Interaction (External Dependence x Private Credit)		0.025* [0.013]			0.013*** [0.005]	0.029*** [0.010]
Interaction (R&D External Dependence x Private Credit)	0.316* [0.179]	0.198* [0.115]				
Interaction (External Dependence x Stock Market Capitalization)			0.027** [0.012]			
Interaction (External Dependence x Accounting Standards)				0.002*** [0.000]		
log sale	-0.072*** [0.010]	-0.071*** [0.010]	-0.070*** [0.011]	-0.064*** [0.010]	-0.063*** [0.010]	-0.248*** [0.034]
age	-0.001 [0.001]	-0.001 [0.001]	-0.002*** [0.001]	-0.001* [0.001]	-0.001* [0.001]	-0.003*** [0.001]
Interaction (log Real GDP per cap. 1987 x External Dependence)					0.007* [0.004]	
log (employment/average employment)						0.245*** [0.031]
Country, Time and Industry Dummies	YES	YES	YES	YES	YES	YES
Observations	35914	35914	26469	32321	36057	38344
R-squared	0.122	0.117	0.098	0.11	0.122	0.119

Note: Dependent Variable is the annualized growth rate of real sales between period t and t+5

All standard errors are clustered at the country level and are shown in brackets

**p<0.01, *p<0.05, †p<0.1

Table 4. R&D Intensity Regression

	OLS	OLS	OLS	OLS	OLS	OLS
	[1]	[2]	[3]	[4]	[5]	[6]
Interaction (External Dependence x Private Credit)		0.0267* [0.016]			0.022*** [0.005]	0.058*** [0.013]
Interaction (R&D External Dependence x Private Credit)	0.322* [0.189]	0.234* [0.137]				
Interaction (External Dependence x Stock Market Capitalization)			0.048*** [0.054]			
Interaction (External Dependence x Accounting Standards)				0.005*** [0.001]		
log sale	-0.263*** [0.011]	-0.263*** [0.011]	-0.340*** [0.007]	-0.311*** [0.007]	-0.314*** [0.007]	-0.653*** [0.015]
age	0.013*** [0.001]	0.011*** [0.001]	-0.002 [0.001]	0.005*** [0.001]	0.005*** [0.001]	0.003*** [0.001]
Interaction (log Real GDP per cap. 1987 x External Dependence)					0.012 [0.018]	
log (employment/average employment)						0.455*** [0.017]
Country, Time and Industry Dummies	YES	YES	YES	YES	YES	YES
Observations	9326	9326	15193	19085	19559	17621
R-squared	0.541	0.547	0.618	0.599	0.604	0.628

Note: Dependent Variable is log (RD/Sale) in period t

All standard errors are clustered at the country level and are shown in brackets

***p<0.01, **p<0.05, *p<0.1

Table 5: Growth and R&D Regressions

	Dependent Variable: Growth			Dependent Variable: R&D Intensity		
	OLS [1]	2SLS [2]	2SLS [3]	OLS [4]	2SLS [5]	2SLS [6]
log Value	-0.012** [0.005]	-0.012** [0.005]	-0.011** [0.005]	-0.081*** [0.008]	-0.076*** [0.009]	-0.076*** [0.009]
Interaction (log Value x Private Credit)	0.044*** [0.012]	0.011 [0.010]	0.009 [0.010]	0.084*** [0.017]	0.053* [0.027]	0.047* [0.027]
Interaction (log Value x External Dependence)	-0.007** [0.003]	-0.006** [0.003]	-0.006** [0.003]	-0.002 [0.003]	-0.009*** [0.003]	-0.008** [0.004]
Interaction (Private Credit x External Dependence)	0.059*** [0.018]	0.054*** [0.013]	0.063*** [0.017]	0.072*** [0.016]	0.121*** [0.028]	0.129*** [0.033]
Interaction (Private Credit x log Value x External Dependence)	-0.001 [0.006]	-0.017 [0.018]	-0.012 [0.011]	-0.014 [0.016]	-0.023 [0.014]	-0.04 [0.036]
age	-0.005*** [0.001]	-0.005*** [0.001]	-0.005*** [0.001]	-0.015*** [0.001]	-0.014*** [0.001]	-0.014*** [0.001]
Country, Time and Industry Dummies	YES	YES	YES	YES	YES	YES
Observations	35367	35349	35349	19019	19009	19009
R-squared	0.118	0.117	0.117	0.542	0.541	0.539

Note: Dependent Variable is the annualized growth rate of real sales between period t and t+5

Note: Dependent Variable is log (RD/Sale) in period t

All standard errors are clustered at the country level and are shown in brackets

***p<0.01, **p<0.05, *p<0.1

Table 6: Growth and R&D Regressions

	Dependent Variable: Growth				Dependent Variable: R&D Intensity			
	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Interaction (External Dependence x Private Credit)				0.018* [0.011]				0.055*** [0.012]
Interaction (high growth dummy x Private Credit)	0.188*** [0.025]	0.03 [0.022]	0.192*** [0.029]	0.185*** [0.025]	0.127** [0.080]	-0.112 [0.173]	0.176** [0.086]	0.117** [0.059]
high growth dummy	0.162*** [0.030]	0.150*** [0.037]	0.156*** [0.034]	0.165*** [0.030]	0.061 [0.102]	0.275 [0.296]	0.101 [0.152]	0.074 [0.102]
log sale	-0.074*** [0.010]	0.006 [0.004]	-0.103*** [0.014]	-0.074*** [0.010]	-0.320*** [0.007]	0.172*** [0.020]	-0.446*** [0.021]	-0.320*** [0.007]
age	-0.004*** [0.001]	-0.001*** [0.000]	-0.007*** [0.001]	-0.004*** [0.001]	0.003*** [0.001]	0.004*** [0.001]	-0.007 [0.008]	0.003*** [0.001]
Country, Industry and Time Dummies	YES	YES	YES	YES	YES	YES	YES	YES
Observations	38317	10808	25509	38317	19509	6189	10430	19509
R-squared	0.141	0.765	0.135	0.141	0.608	0.584	0.643	0.608

Note: Dependent Variable is the annualized growth rate of real sales between period t and t+5

Note: Dependent Variable is log (RD/Sale) in period t

All standard errors are clustered at the country level and are shown in brackets

***p<0.01, **p<0.05, *p<0.1

Table 7. Growth Regression

	OLS	2SLS	2SLS	2SLS	2SLS	OLS
	[1]	[2]	[3]	[4]	[5]	[6]
Interaction (Private Credit x External Dependence)	0.018 [0.012]	0.01 [0.010]	0.008 [0.010]	-0.029 [0.365]	-0.05 [0.034]	0.057* [0.029]
log (RD/Sale)	0.099*** [0.017]	0.025*** [0.005]	0.026*** [0.005]	2.212* [1.128]	1.085*** [0.379]	
log sale	-0.042*** [0.010]	0.003 [0.010]	0.003 [0.010]	0.569 [0.365]	0.212** [0.098]	-0.094*** [0.011]
age	-0.001* [0.001]	0.001 [0.000]	0.001 [0.000]	-0.018* [0.008]	-0.009*** [0.003]	-0.003 [0.002]
Country, time and industry dummies	YES	YES	YES	YES	YES	YES
Observations	18793	9555	9555	6296	6296	11034
R-squared	0.105	0.299	0.299	0.287	0.295	0.023

Note: Dependent Variable is the annualized growth rate of real sales between period t and t+5

All standard errors are clustered at the country level and are shown in brackets

**p<0.01, *p<0.05, †p<0.1

Chapter 2

Skill-Biased Technical Change and the Evolution of the World Income Distribution: An Empirical Assessment

2.1 Introduction

Output per worker varies enormously across countries. To highlight this, a simple calculation shows that income per worker in the richest country in 1980 was approximately fifty times bigger than that of the poorest. Furthermore, disparities in output have been amplified over the course of the last decades, with the ratio of the richest to the poorest country almost doubling in 2000 compared to the 1980 value. This point can be illustrated more clearly with Figure 1 where we present the kernel estimates for the distribution of output per worker relative to the US in 1980 and in 2000. Figure 1 shows a bimodal distribution for 1980 and 2000, a point made by Quah (1996) and Jones (1997), with a shift to the left in the two modes of the 2000 distribution. This suggests that the previously mentioned increase in the relative income gap is a phenomenon that not only occurs in the left tail of the distribution. Numerous theories have emerged to explain this observed patterns. Some economists have given empirical support for the neoclassical model pointing at the importance of physical and human capital accumulation in explaining income differences (e.g. Mankiw, Romer and Weil (1992), Barro and Sala-i-Marti (1995) or Young (1995)). Other authors have proposed theories of endogenous technological change as a way to rationalize the observed income differences, a view which has been supported by the large TFP differences across countries (see Romer (1990), Islam

(1995), Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999) among others). Moreover, in recent years some economists have highlighted the importance of differences in the relative supply of skills in explaining the observed TFP differences (e.g. Acemoglu and Zilibotti (2001) and Caselli and Coleman (2006)). Figure 2 emphasizes the previous argument. In this figure we plot the distribution for the supply of skills relative to the supply of skills in the United States for both 1980 and 2000. This graph shows a similar pattern as the one observed for income, with the modal value shifting to the left and an increase in the number of countries with low skill supply relative to the United States. An influential argument connecting the evidence presented in Figures 1 and 2 was laid by Acemoglu and Zilibotti (2001). More specifically, they propose a theory of Skilled-Biased technical change with two underlying assumptions, technological advances originate in developed countries and technology flows freely across countries¹. One important implication of this model is that technological advances are appropriate for the skill supply from developed countries, suggesting that differences in skill supplies across countries can account for income differences.

In this paper we argue the mechanism presented in Acemoglu and Zilibotti (2001) is an empirically relevant one. We start by presenting evidence that the model can fit the evolution of the income distribution in the period 1980-2000 as well as predicting the observed growth rates in income per capita for the same period. More precisely, we calibrate a CES production function with Skill-Biased technical change and predict the income levels for 1990 and 2000. Following the set-up presented in Acemoglu and Zilibotti (2001), we make two assumptions: technology is produced in the U.S. and is used in the production process across countries. Using this assumption we calibrate the skill-augmenting technological parameter with the observed evolution of the skill premia in the United States. The calibration exercise shows that the predicted growth rates for 1980-2000 are positively correlated with the observed 1980-2000 growth rates and the hypothesis that the fitted line lies on the 45 degree line going through the origin cannot be rejected for most parameter values and definitions of skill supply. All this shows evidence that the model can successfully capture the 1980-2000 growth experience. Furthermore the model is also successful in predicting the observed income distribution for 1990 and 2000, specially for the lower tail of the distribution.

In order to contrast the importance of the model presented in Acemoglu and Zilibotti (2001) we compare it to two alternative growth hypotheses. The first is the neoclassical benchmark where difference in output are explained by factor differences. This comparison will allow us to contrast

¹The first assumption is supported by the disproportionate percentage of innovation originating from OECD countries observed in the data.

the role of productivity differences generated by skill mismatches to technology with the role of capital accumulation. The second model we calibrate is the model presented in Caselli and Coleman (2006). As Acemoglu and Zilibotti (2001), Caselli and Coleman (2006) also point at the importance of skill supply in explaining productivity differences. However, contrary to Acemoglu and Zilibotti, they assume each country chooses a different technology level to be used in the production of goods. By comparing the Acemoglu and Zilibotti model to the Caselli and Coleman model we are able to compare two different channels through which the supply of skills might affect productivity: the choice of technology and the appropriateness of technology. One challenge that arises in these comparisons is the data availability for the skill premia. To be precise, in the calibration exercise we use the cross country data presented in Caselli and Coleman (2006) which only estimates the skill premia for 1980. To tackle this issue we extrapolate the 1990 and 2000 values using alternative assumptions for the evolution of the skill premia. For this reason the results presented are subject to the validity of this assumptions. This exercise suggests that the model presented in Acemoglu and Zilibotti outperforms the neoclassical benchmark and the Caselli and Coleman model in explaining the data. More precisely, we show that for all parameter values used and all definitions of skills, the Acemoglu and Zilibotti model matches better average income, variance and interquartile range observed in the data. Furthermore this model has a better goodness of fit than the two alternative models. One important point from this exercise is that predicted average income and goodness of fit of the neoclassical model is very close to those of the Acemoglu and Zilibotti model, but there are significance difference between the models in explaining volatility. This suggests that skill mismatches are particularly important to explain the dispersion observed in the data.

The calibration exercise discussed above relies on the chosen parameter values. For this reason we estimate a loglinear approximation of the CES production function used for the calibration of the Acemoglu and Zilibotti model. We find that for all the alternative definitions of skills used the estimated values for the elasticity of substitution between skilled and unskilled workers lies in the range of values used in the calibration exercise. Furthermore we show that the 95 percent confidence interval from the estimation is very close to those estimated in the labor literature (Katz and Murphy (1992)).

This paper contributes to the literature explaining cross-country income differences. A number of papers have empirically addressed the observed income differences and growth experience in the post-war decades. Mankiw, Romer and Weil (1992) and Young (1995) try to explain cross-country income differences through differences in factors of production, while Islam (1995), Klenow and

Rodriguez-Clare (1997) and Hall and Jones (1999) point at the importance of TFP differences in explaining income differences. Our paper differs from the above mentioned literature in pointing at the empirical relevance of differences in skill supplies in explaining TFP differences. In this sense the closest papers to ours are Acemoglu and Zilibotti (2001) and Caselli and Coleman (2006). The latter differs to this paper in the channel through which skill supply affects productivity. Furthermore Caselli and Coleman (2006) do not contrast their mechanism to alternative ones. In this respect the approach presented here is closely related to Acemoglu and Zilibotti (2001). There are three important differences between the present paper and theirs. First we extend the comparison across models to include the mechanism presented in Caselli and Coleman which is an important alternative explanation for why skill supply differences are important for income differences. Second we check the robustness of the calibration exercise by using a range of values fro the parameters of the model and we estimate these to validate the exercise. Finally we focus on the relevance of the model presented in Acemoglu and Zilibotti (2001) for the evolution of the income distribution.

This paper also highlights the role of "appropriateness" of technology in explaining cross-country income differences. Other papers like Atkinson and Stiglitz (1969) and Basu and Weil (1998) have also arise attention to this point. One important difference is that they focus on "appropriateness" relative to capital per worker while we investigate the the relevance of "appropriateness" with respect to relative skill supplies.

The plan of the paper is as follows. Section 2 describes the methodology used in the calibration exercise and compares the predictions of the Acemoglu and Zilibotti model to the observed data. Section 3 compares the predictions of the Acemoglu and Zilibotti model to the predictions of the alternative models described above. Section 4 presents the estimation results and finally Section 5 concludes. All figures and tables are shown in the appendix.

2.2 Skill-Biased Technical Change and the World Income Distribution

2.2.1 Skill Biased-Technical Change: An Overview

There is large evidence which shows a large increase in inequality between skill groups in the United States. Acemoglu (2002), for example, documents the sharp increase in the college wage premium in the U.S. in the last century, with an increase of more than a hundred percent in the period between

1950 and 1996. Surprisingly, this massive increase in the college wage premium has occurred in a time in which the relative supply of college graduates has more than doubled. The recent consensus is that technical change in advanced economies favors more skilled workers compared to unskilled workers, which exacerbates inequality between these two groups. Furthermore, as Acemoglu (1998) shows in the context of a growth model, technical change could be driven by the relative supply of skills in the economy. To highlight this point, suppose the economy has perfectly competitive markets and has the following production function:

$$Y_t = K_t^\alpha [\gamma_h (A_{Ht} H_t)^\rho + (1 - \gamma_h) (A_{Lt} L_t)^\rho]^{(1-\alpha)/\rho} \quad (2.1)$$

where Y_t is output at time t , K_t is capital at time t , H_t is the number of skilled workers in the economy at time t , L_t is the number of unskilled workers in the economy at time t , $\alpha \in (0, 1)$, ρ, γ_h are constants and A_{Ht}, A_{Lt} are technological parameters changing over time. This implies that the relative wage of a skilled worker with respect to an unskilled worker is

$$\frac{w_{Ht}}{w_{Lt}} = \frac{\gamma_h}{1 - \gamma_h} \left(\frac{A_{Ht}}{A_{Lt}} \right)^\rho \left(\frac{H_t}{L_t} \right)^{\rho-1} \quad (2.2)$$

The argument made in Acemoglu (1998) emphasizes that if $0 < \rho < 1$ and both $\frac{H_t}{L_t}$ and $\frac{w_{Ht}}{w_{Lt}}$ were increasing over time, it had to be that $\frac{A_{Ht}}{A_{Lt}}$ was increasing fast enough to compensate for the increase in $\frac{H_t}{L_t}$. Table 3 shows the sharp increase in $\frac{A_{Ht}}{A_{Lt}}$ for values of ρ which are consistent with the empirical estimates of the elasticity of substitution between skilled and unskilled labor in Katz and Murphy (1992) and Autor et. al. (1998). In particular we estimate the skill premia for different educational categories in the U.S. using the CPS files and use this estimated skill premia to solve for the value of $\frac{A_{Ht}}{A_{Lt}}$ from (2.2) for different levels of ρ and different definitions of skilled and unskilled workers². This exercise highlights the dramatic increase in $\frac{A_{Ht}}{A_{Lt}}$ from 1962 to 2000.

One important implication of Skill-Biased technical change is the effect it has in the cross-country income patterns. Acemoglu and Zilibotti (2001) (hereafter AZ) argue that most of the R&D investment in the world is undertaken in the OECD and in particular in the United States. In the context of the discussion above, this suggests that technological advances in the world are mainly driven by the skills supply in the U.S. and the developed countries. If this is the case, differences in skill supplies across countries will be crucial to understand the observed income

²The details of the construction of Table 3 are presented in the next subsection.

disparity in the world. This point is highlighted in the model presented by AZ who assume that skill-biased technical emerges from developed countries and is adopted by the rest of the world. They show that this model successfully predicts both cross-industry TFP differences and cross-country income differences. In what follows we will add to this evidence by investigating if the AZ model can account for two important empirical characteristics: growth rate differences across countries and the observed evolution of the income distribution discussed in the introduction.

2.2.2 A Simple Calibration Exercise

The objective of this section is to compare the predictions of the AZ model to the data. For this purpose we present the results of a calibration exercise using an extended version of the production function in (2.1). In particular we calibrate the following production function

$$\hat{y}_{it} = B_i \tilde{A}_t k_{it}^\alpha \left[\gamma_h \left(\left(\frac{A_{Ht}}{A_{Lt}} \right)^{US} h_{it} \right)^\rho + (1 - \gamma_h) l_{it}^\rho \right]^{(1-\alpha)/\rho} \quad (2.3)$$

where B_i is a country specific effect, \tilde{A}_t is a skill neutral technological parameter, \tilde{y}_{it} is the predicted output per worker at time t in country i , k_{it} is capital per worker at time t in country i , h_{it} is the share of skilled workers in country i at time t , l_{it} is the share of unskilled workers in country i at time t , $\alpha \in (0, 1)$, $\rho = (\sigma - 1)/\sigma$ where σ is the elasticity of substitution between skilled and unskilled workers, γ_h is a constant and $(A_{Ht}/A_{Lt})^{US}$ is a skill-augmenting technological parameters in the U.S. at time t .

Data and Methodology

We use data for y_{it} , k_{it} , h_{it} and l_{it} . The measure for y , which we will compare against the calibrated value \tilde{y} , is output per worker in international dollars (i.e. PPP adjusted) and is obtained directly from Summers, Heston and Aten (2002). Real per-worker capital stock, k , is constructed using the method of perpetual inventory described in Hall and Jones (1999). For this we use the series of investment per worker in international dollars from Summers, Heston and Aten (2002). Crucial to our analysis is the construction of relative skills for each country. We use the data set constructed in Barro and Lee (2001) to construct our measure of h_{it} and l_{it} . In this data set we have information on the percentage of the population with age 25 or more who fall in one of seven categories. Following Acemoglu (2002) I use three different measures of skills. The first one (Some College) defines a skilled worker as all high school graduates with some college education. This implies

that h_{it} will be constructed summing the last two bins in the Barro and Lee (2001) dataset. The second classification (College Graduates) defines a skilled worker as all those who have college education. This implies that h_{it} will be constructed summing the last bin in the Barro and Lee (2001) dataset. Finally, the third classification (College Equivalents) defines a skilled worker as $h_{it} = \text{college graduates} + (0.5 * \text{some college})$.

The values for α and ρ are taken from the literature. In particular the value of α is set to $1/3$, which roughly matches the historical average of the capital share in the U.S. economy. In the exercise we use three different values for ρ : $0.1667, 0.2856$ and 0.4117^3 . These values lie in the interval estimated by Katz and Murphy (1992) and Autor et.al. (1998) who argue that plausible values for ρ satisfy $\rho \in (0, 1/2)$.

The parameters γ_h and $\left(\frac{A_{Ht}}{A_{Lt}}\right)^{US}$ are constructed as follows. We compute the three categories of skills described above using the March CPS for 1962, 1970, 1980, 1990 and 2000. With the CPS data we estimate the following equation:

$$\ln w_{jt} = \beta'_t X_{jt} + u_{jt}$$

where X_{jt} includes experience up to the quartic power, a female dummy, a non-white dummy, a dummy for having some college, a college dummy and a dummy for having a post-graduate degree. We allow the coefficient on the covariates to change through time to capture changes in the labor market across decades. More specifically, we perform a year by year regression and use the dummy variables for the different educational attainment groups to estimate the skill premia⁴. We set $\left(\frac{A_{H1962}}{A_{L1962}}\right)^{US} = 1$ such that

$$\frac{\gamma_h}{1 - \gamma_h} = \frac{w_{H,1962}^{US}}{w_{L,1962}^{US}} \left(\frac{H_{US,1962}}{L_{US,1962}}\right)^{1-\rho}$$

and the values $\left(\frac{A_{Ht}}{A_{Lt}}\right)^{US}$ are recovered using the relative wage equation (2.2)⁵.

Finally the two last parameters to be obtained are B_i and \tilde{A}_t . The value of B_i for the U.S. is set one and the parameter \tilde{A}_t is set such that $\tilde{y}_{US,t} = y_{US,t}$ for every t . For countries other than the U.S. I set the value of B_i such that the predicted value output per worker matches the observed

³If we define $\rho = (\sigma - 1)/\sigma$, where σ captures the elasticity of substitution between skills, these values correspond to $\sigma \in \{1.2, 1.4, 1.7\}$

⁴The estimation results are presented in Table 1.

⁵The values of $\left(\frac{A_{Ht}}{A_{Lt}}\right)^{US}$ are presented in Table 3. The calibrated changes in $\left(\frac{A_{Ht}}{A_{Lt}}\right)^{US}$ are consistent with those found in Acemoglu (2002).

value in 1980, *i.e.* $\tilde{y}_{i,1980} = y_{i,1980}$.

Results

We begin by presenting the relation between the observed growth rates and the growth rates predicted by the AZ model. This relation can be seen graphically in Figure 3 which highlights the positive correlation between predicted growth rates and the observed growth rates⁶. In particular, we present the scatter plot and the fitted line for two samples. The first sample is the full sample of countries for which there is information on the capital stock, output per worker and skills for both 1980 and 2000. The second sample is the subset of the first sample for which Caselli and Coleman (2006) report relative wages which will be used in the next section when we compare alternative models. The positive correlation presented in Figure 1 can be explored further by running the following growth regression:

$$\ln y_{i,2000} - \ln y_{i,1980} = \alpha + \beta(\ln \tilde{y}_{i,2000} - \ln \tilde{y}_{i,1980}) + \varepsilon_i$$

where \tilde{y} is predicted output per worker and y is observed output per worker. We present the results of this regression in Table 3, where Panel A presents the full sample, Panel B presents the Caselli and Coleman (2006) subsample and each column represents a pair of skill definition and value for the elasticity of substitution (σ). Table 3 shows that the positive correlation between observed and predicted growth observed in Figure 1 is significant for all samples and for the all combinations of elasticity of substitution and skill definition. Furthermore, the estimated coefficient for β is close to 1 and in for all nine cases in the two subsamples a coefficient of 1 lies in the 95 percent confidence interval. An extra point to be made is that if the model were to fully describe the observed growth we would expect to find the fitted line to lie on the 45 degree line. We test this hypothesis and show the in the bottom part of the two panels in Table 3. We can see that for the full sample this hypothesis cannot be rejected in 8 of the 9 cases. This implies we can't reject a one to one relation between the two growth rates for 8 of the 9 skill definition-elasticity of substitution pairs. For the restricted sample this hypothesis is rejected when we use the College Graduates skill definition and for one additional case. Table 3 also shows that for the parameter values and skill classifications used the model explains between 26% and 61% of the observed cross-country

⁶For this figure we use $\sigma = 1.4$ and the College Equivalents definition of skills. I also constructed this figure for other definitions but I leave the extended discussion for the regression analysis.

variation in growth rates. Furthermore, for a given skill classification the R-squared of the growth regression is increasing in σ , and for a given value of σ the R-squared increases as we narrow the definition of a skilled worker. Finally we can see that the fit of the model is higher in the restricted sample used in Caselli and Coleman (2006) than in the full sample.

So far we have analyzed the ability of the AZ model to capture the observed differences in growth rates but the AZ model also has implications for the income distribution. To study this we present the estimated income distribution for both 1990 and 2000 using the income levels relative to the U.S. predicted by the AZ model and compare these to the estimated income distributions using the observed income values relative to the United States. This normalization will not affect the comparison between distributions since we normalized the calibrated values to fit exactly the income level in the U.S. for all years. The results for this exercise are presented in Figures 4 and 5⁷⁸. The first thing to notice is that, independently of the value of σ , the predicted distributions capture a salient fact, the bimodal shape of the income distributions in 1990 and 2000. This fact has been documented by Quah (1996) and Jones (1997) for the 1980's and is still present in the two years we study. Secondly, the model successfully captures the lower tail of the distribution and upper mid range of the distribution for both years studied. On the other hand, the model is not as successful in predicting the right tail of the distribution as well as the upper mid range of the distribution. More precisely, the model predicts a higher number of countries with middle-to-high income (the range between -0.5 and -1.7 in the graph) relative to the observed distribution and a smaller number of high and middle-to-low income countries relative to the observed distribution. This result stems from the following fact: if we calculate the ratio $H/L/H_{US}/L_{US}$ by income levels we find that the difference in this ratio between rich, middle-to-high income countries and middle-to-low income countries is very small. For this reason the model overpredicts the number of middle-to-high income countries. One last point to notice is that, as with the growth rates, the model is more successful in capturing the observed distribution for higher σ .

One point made in the introduction is the shift observed in the income distribution from 1980 to 2000. Figure 6 shows that the model can partly predict this shift for the sample used in the calibration exercise. More specifically, Figure 6 shows that the model can match the decrease in the

⁷For simplicity I present the results only for the College graduates definition of Skills.

⁸Differences in the estimated distribution for observed output in Figure 1 and Figure 5 arise as a consequence of sample used. On the one hand, in Figure 1 we estimate the 2000 density using all countries with output reported in 2000. In Figure 5 on the other hand we estimate the density using countries for which there is information on the capital stock, output per worker and skills for both 1980 and 2000.

density around the two modes, specially for the lower modal value. Furthermore, like the observed distribution, the model predicts a decrease in the mass of countries with high and middle-to-low income and an increase in the mass of countries with middle-to-high and low levels.

So far we have found that the AZ model successfully predict some salient facts of the income distribution and we have shown the positive correlation between the growth rates predicted by the model and the observed growth rates. The analysis so far, however, hasn't confronted the data to alternative hypotheses for the process of growth. The next section will address this and show that the model outperforms two popular alternative growth models.

2.3 Comparison Across Models

Since the late 1980's the literature on economic growth has been revived and many explanations have emerged in order to explain the observed differences in income levels across countries. On the one hand, since Romer (1986, 1990), a big emphasis has been given to endogenous growth theories in explaining income disparities. On the other hand, as mentioned in Klenow and Rodriguez-Clare (1997), a revival of neoclassical growth theory has taken place (see Mankiw, Romer and Weil (1992), Barro and Sala-i-Marti (1995) and Young (1995)). In this spirit we confront the AZ/Skilled-Biased technical change model of economic growth with two competing models. In doing this exercise we are not taking any one model as the real data generating process, the objective is to isolate potential channels for the observed output disparities. The first alternative model considered is the neoclassical Solow growth model. This model emphasizes the role of factor accumulation and exogenous technological progress as the key drivers of economic force. Comparing this model with the neoclassical benchmark will be useful in examining the importance of cross-country relative skills mismatches in the observed output differences. The second alternative model we consider is the model presented by Caselli and Coleman (2006) (hereafter CC). This model takes into account the role of relative skills and skill-biased technical change in an economy but differs from AZ by arguing that not all countries share the same $\frac{A_{Ht}}{A_{Lt}}$. Hence, the CC model will be an important benchmark to highlight the role of technology transfer and appropriate technology relative to the skills of an economy in explaining income cross-country income differences. The objective of the following exercise will be to compare and quantify the predictive power of these three growth models.

For this purpose we perform a calibration exercise similar to the one described in the last section

and extending it to predict the neoclassical model and the CC model. For the neoclassical model we use a Cobb-Douglass production function:

$$\tilde{y}_{i,t}^{NC} = \tilde{A}_t^{NC} B_i^{NC} k_{i,t}^\alpha N_{i,t}^{1-\alpha}$$

where we construct labor as $N_{i,t} = L_{i,t} + \left(\frac{w_{Hi,t}}{w_{Li,t}}\right) H_{i,t}$ to adjust for the different productivities between the two skill groups. Values for $\frac{w_{Hi}}{w_{Li}}$ are obtained from CC.

For the CC model on the other hand we use a CES production function similar to (2.3):

$$\tilde{y}_{i,t}^{CC} = B_i^{CC} \tilde{A}_t^{CC} k_{it}^\alpha \left[\gamma_h \left(\left(\frac{A_{Hit}}{A_{Lit}} \right) h_{it} \right)^\rho + (1 - \gamma_h) l_{it}^\rho \right]^{(1-\alpha)/\rho}$$

where $\frac{A_{Hit}}{A_{Lit}}$ is constructed using cross-country skill premium data from CC. As in Section 2 we set $B_i^{NC} = B_i^{CC} = 1$ and calculate \tilde{A}_t^{NC} and \tilde{A}_t^{CC} such that $\tilde{y}_{US,t}^{NC} = \tilde{y}_{US,t}^{CC} = y_{US,t}$. This implies that

$$\begin{aligned} \tilde{A}_t^{NC} &= \frac{y_{US,t}}{k_{US,t}^\alpha N_{US,t}^{1-\alpha}} \\ \tilde{A}_t^{CC} &= \frac{y_{US,t}}{k_{it}^\alpha \left[\gamma_h \left(\left(\frac{A_{Hit}}{A_{Lit}} \right) h_{i,t} \right)^\rho + (1 - \gamma_h) l_{i,t}^\rho \right]^{(1-\alpha)/\rho}} \end{aligned}$$

Finally the values for B_i^{NC} and B_i^{CC} are calculated such that the predicted income for both models match exactly the 1980 output for all countries in the sample, $\tilde{y}_{i,1980}^{NC} = \tilde{y}_{i,1980}^{CC} = y_{i,1980}$, which implies:

$$\begin{aligned} B_i^{NC} &= \frac{y_{i,1980}}{\tilde{A}_{1980}^{NC} k_{i,1980}^\alpha N_{i,1980}^{1-\alpha}} \\ B_i^{CC} &= \frac{y_{i,1980}}{\tilde{A}_{1980}^{CC} k_{i,1980}^\alpha \left[\gamma_h \left(\left(\frac{A_{Hit}}{A_{Lit}} \right) h_{i,1980} \right)^\rho + (1 - \gamma_h) l_{i,1980}^\rho \right]^{(1-\alpha)/\rho}} \end{aligned}$$

One challenge faced by the calibration exercise of the alternative models is that data for skill premia in certain countries is available only for the 1980's. For this reason we have adopted five different approaches when constructing the predicted values. In these five approaches we make assumptions on the evolution of either $\frac{w_{Hi,t}}{w_{Li,t}}$ or $\frac{A_{Hit}}{A_{Lit}}$ and predict the other value with the relative wage equation.

The first approach assumes that $\frac{w_{Hi,t}}{w_{Li,t}}$ has remained constant in the period between 1980 to

2000 and use the 1980 values for entire exercise. If we extrapolate the U.S. experience to the rest of our sample this seems to be a very bad approximation. Acemoglu (2003) however shows that the sharp increase in the skill premium experienced in the U.S. from 1980 to 2000 is the exception among a selected set of developed economies. All this suggests that this first approach is not as bad as one might first think. To allow for the possibility of secular variation in the wage ratio we adopt two approaches. First, we assume that the relative wage $\frac{w_{Hi,t}}{w_{Li,t}}$ has increased at the same rate as in the U.S. (i.e. $\frac{w_{Hi,t}/w_{Li,t}}{w_{Hi,t-10}/w_{Li,t-10}} = \frac{w_{HUS,t}/w_{LUS,t}}{w_{HUS,t-10}/w_{LUS,t-10}}$) and we also perform the calibration assuming that the wage ratio in the world has responded to changes in the skill ratio as in the U.S., that is

$$\left(\frac{H_{US,t-10}}{H_{US,t-10}} \frac{w_{HUS,t}}{w_{LUS,t}}\right) * \left(\frac{w_{HUS,t-10}}{w_{LUS,t-10}} \frac{H_{US,t}}{H_{US,t}}\right)^{-1} = \left(\frac{H_{i,t-10}}{H_{i,t-10}} \frac{w_{Hi,t}}{w_{Li,t}}\right) * \left(\frac{w_{Hi,t-10}}{w_{Li,t-10}} \frac{H_{i,t}}{H_{i,t}}\right)^{-1}$$

As was discussed earlier, the increase in relative wages in the U.S. has been higher than in other countries. Furthermore the relative supply of skills in the U.S. is extremely high compared to the rest of the world. For this reason the above assumptions might be overestimating the increase in the wage ratio. For this reason we calibrate the model using a fourth approach. More specifically, we run the regression of the logarithm of relative wages in 1980 on the logarithm of the relative skills in the economy allowing the coefficients to vary with skill levels, i.e.

$$\ln\left(\frac{w_{Hi,1980}}{w_{Li,1980}}\right) = \alpha + \alpha_R * I(H/L_i > h') + \beta \ln(H_{i,1980}/L_{i,1980}) + \quad (2.4)$$

$$+ \beta_R \ln(H_{i,1980}/L_{i,1980}) * I(H/L_i > h') + \varepsilon_i, \quad (2.5)$$

Finally we calibrate the CC model and the

where $I(\cdot)$ is the indicator function and h' is chosen to be median skill supply in the sample. We predict the relative wage relation using the estimated coefficients $\hat{\alpha}, \hat{\beta}, \hat{\alpha}_R, \hat{\beta}_R$.

The final approach assumes that the frontier of technology for all countries has grown at the same rate as the U.S. technology, which implies $\frac{A_{Hi,t}}{A_{Li,t}} / \frac{A_{Hi,t-10}}{A_{Li,t-10}} = \frac{A_{HUS,t}}{A_{LUS,t}} / \frac{A_{HUS,t-10}}{A_{LUS,t-10}}$

In what follows, to avoid excessive repetitiveness, we present results only for approach one, as this is the one that has the best fit compared to the observed data for both the neoclassical benchmark as well as the CC model⁹.

⁹In particular this approach gave the highest \mathfrak{R}^2 out of the five approaches for both the CC model and the neoclassical benchmark. The definition of \mathfrak{R}^2 will be explained in detail below.

2.3.1 Results

I start by comparing three statistics across models for the sample of countries for which CC report skill premia data. The first statistic we analyze is the average predicted output relative to the observed average output. The second statistic we compare is the predicted variance relative to the observed variance. Finally we analyze a measure of goodness of fit proposed in AZ, \mathfrak{R}^2 , where \mathfrak{R}^2 is the "constrained R^2 ". In particular, let y denote output per worker in the data and let \tilde{y}_s denote predicted output per worker by model s , then we define $\mathfrak{R}_s^2 = 1 - \sum(y - \tilde{y}_s)^2 / \sum(y)^2$. This is the " R^2 " from a regression of output per worker in the data on predicted values when we constrain the slope to be equal to 1 and the constant to be 0. In general \mathfrak{R}_s^2 would be equal to 1 if model s can fully predict observed output and could potentially be negative if the fit of the model is particularly bad.

The results are presented in Tables 4 through 6. Each table presents the results for a different value of ρ , or equivalently σ , in the range of values proposed by Katz and Murphy (1992). In each Table we present three panels corresponding to the three definitions of skills we have used. Within each panel (i.e. for each σ and skill definition) I present the three statistics for the 1990 subsample, for the 2000 subsample, and for the full sample.

The first thing to notice is that the CC model consistently underperforms compared to the neoclassical benchmark and the AZ model for the three statistics presented. Moreover, it performs particularly worse than the other two models for lower values of σ and for broader definitions of skills. When comparing the Solow model and the AZ model we can see that these two models predict very well the average income in the sample of countries we study. To be precise, both models predict average incomes which are 0.94 to 1.04 times the observed average income. Furthermore there is no clear pattern over which model is better at predicted average income. Turning to the variance the three models underpredict the observed standard deviation in the sample for all values of σ , but this underprediction is smaller for the AZ model. More specifically, the standard deviation predicted by the AZ model is at least 84 percent of the observed variance while the Solow model predicts at most 84 percent of the observed variance. The analysis for the R-squared shows a similar picture to the one presented for the variance. In particular we see that the AZ model has a higher restricted R-squared than the other two models for all values of σ and definition of skills. The tables also suggest two important points. The first is that the Solow model performs particularly well compared to the CC model and performs almost as well as the AZ model when analyzing

the relative average and the constrained R-squared. As is pointed out by AZ, this highlights the importance of differences in human capital and physical capital in explaining income differences. The second point to notice is that both the fit of the AZ model and the CC seem to be increasing in σ . Furthermore the sensitivity of the CC model to changes in σ seems to be much higher than those of the AZ model. In Section 4 of the paper we will estimate the models and show that the estimated σ lies in the range of values that we have chosen for the calibration exercise which reassures the above results.

Section 2 showed that the AZ model captured some important features of the income distribution for the full sample of countries. So far we have shown, for the restricted sample in CC, that the AZ model performs better than the neoclassical and CC models in capturing the observed average income and the variance. In order to see if the models can capture other characteristics of the income distribution, we compare the ability of the three models to explain the interquartile range of the income distribution. In particular we plot the ratio of the 75th percentile to the 25th percentile (75/25) of income for the observed and predicted levels. Figures 4 to 13 show the plot for the four series. The figures highlight the increasing pattern from 1980 to 2000 in the observed 75/25 ratio for the sample countries considered. This pattern is successfully captured by the AZ and the Solow models while the CC model predicts a decrease from 1990 to 2000. One point to highlight is that both the Solow model and the AZ model predict better the 75/25 ratio in 2000 than in 1990, but in both cases the latter model is closer to the observed ratio.

2.4 Estimation

The results obtained in Sections 2 and 3 rely on the set of values chosen for the parameters α and ρ . The aim of this section is to validate the calibration exercise by estimating the production function in (2.3) and comparing the estimated values to those used in the previous exercise. To do this I will use a log linearized version of the production function and estimate the resulting equation by OLS. In particular let the production function be

$$y_{it} = U_{it} B_i \tilde{A}_t k_{it}^\alpha \left[\gamma_h \left(\left(\frac{A_{Ht}}{A_{Lt}} \right) h_{it} \right)^\rho + (1 - \gamma_h) l_{it}^\rho \right] \quad (2.6)$$

where y_{it} is output per worker in country i at time t , k_{it} is capital per worker in country i at time t , h_{it} and l_{it} are the the share of workers in country i at time t with high skills and low skill respectively, \tilde{A}_t is a time varying technological parameter, B_i is a country specific term and U_{it} .

The term U_{it} captures the effect on output of unobservable factors.

Taking the log of (2.6) we obtain the following expression:

$$\ln y_{it} = b_i + \tilde{a}_t + \alpha \ln k_{it} + \ln M_{it} + u_{it} \quad (2.7)$$

where lower b_i, \tilde{a}_t, u_{it} stand for $\ln B_i, \ln \tilde{A}_t$ and $\ln U_{it}$ respectively, and $M_{it} = \left[\gamma_h \left(\left(\frac{A_{Ht}}{A_{Lt}} \right) h_{it} \right)^\rho + (1 - \gamma_h) l_{it}^\rho \right]$

Now, doing a quadratic approximation of $\ln M$ around $\rho = 0$ we have

$$\begin{aligned} \ln M_{it} \simeq & (1 - \alpha)\gamma_H \ln h_{it} + (1 - \alpha)(1 - \gamma_H) \ln l_{it} + (1 - \alpha)\gamma_H \ln \frac{A_{Ht}}{A_{Lt}} + \delta_t \\ & + \beta \left((\ln h_{it})^2 + (\ln l_{it})^2 - 2 \ln h_{it} \ln l_{it} \right) + \beta \ln \frac{A_{Ht}}{A_{Lt}} (\ln h_{it} - \ln l_{it}) \end{aligned} \quad (2.8)$$

where $\delta_t = (1 - \alpha)\gamma_H \ln \frac{A_{Ht}}{A_{Lt}} + \frac{\gamma_H(1-\gamma_H)(1-\alpha)\rho}{2} \left(\ln \frac{A_{Ht}}{A_{Lt}} \right)^2$ and $\beta = \frac{\gamma_H(1-\gamma_H)(1-\alpha)\rho}{2}$.

Putting together (2.8) and (2.7) and applying the $\Delta(\Delta x_t = x_t - x_{t-10})$ operator we obtain

$$\begin{aligned} \Delta(\ln y_{it} - \ln l_{it}) = & \theta_{1t} + \alpha \Delta(\ln k_{it} - \ln l_{it}) + (1 - \alpha)\gamma_H \Delta(\ln h_{it} - \ln l_{it}) \\ & + \beta \Delta(\ln h_{it} - \ln l_{it})^2 + \pi_{1t}(\ln h_{it} - \ln l_{it}) + \\ & + \pi_{2t}(\ln h_{it-10} - \ln l_{it-10}) + \Delta u_{it} \end{aligned} \quad (2.9)$$

where $\theta_{1t} = \Delta \tilde{a}_t + \Delta \delta_t$.

We estimate equation (2.9) with OLS. When performing the OLS estimation we include a time dummy to estimate θ_{1t} and include time dummies interacted with the $(\ln h_{it} - \ln l_{it})$ and $(\ln h_{it-10} - \ln l_{it-10})$. We use robust standard errors in order to take into account for the possibility of having non-spherical errors. The crucial assumption for the consistency of our estimated coefficients is to have $E(\Delta u_{it} | X_{it}) = 0$ where X_{it} is our set of controls.

Table 7 presents the estimation results for the three definitions of skills used in Sections 2 and 3. Panel A shows the estimated coefficients from equation (2.9) and Panel B shows the estimates for α, ρ and γ_H implied by these coefficients¹⁰. Table 7 shows the estimated model can account for approximately 60 percent of the observed variation of $\Delta(\ln y_{it} - \ln l_{it})$. Panel B shows that independently of the measure of skills used the estimated coefficient for ρ lies in the $(0, 1/2)$ proposed by Katz and Murphy (1992). Furthermore we can see that, for the exception of the

¹⁰The standard deviation for α, ρ and γ_H is obtained using the delta method.

column for Some College, the 95% confidence interval for the ρ is a subset of the Katz and Murphy set. We can also see that the point estimate for α is close to the value used in our calibration (1/3) and that this values lies in the 95% confidence interval.

One caveat for the results found is the assumption of exogeneity of the controls included in the regression. More specifically, physical capital and skills are likely to be correlated with unobserved characteristics determining output. Furthermore this variables are likely to be measured with error aggraviating the potential biases in our estimates. Unfortunately finding good instruments for this variables is very difficult and goes beyond the scope of this paper. With this caveat in mind, the above results suggest that the parameters used in the calibration exercise are consistent with the observed data.

2.5 Conclusion

In the past twenty years several theories have emerged to explain the observed income differences across countries. One popular theory proposed by Acemoglu and Zilibotti (2001) highlights the effect of Skill-Biased technical change in developing countries as a source of productivity differences. In particular, in line with the large literature on skill-biased technical change, the authors assume that technological change depends crucially on the supply of skills of the country where technology is produced. The aim of this paper was to assess the empirical relevance of this source of productivity differences. First, we find that the AZ model can account for the observed differences in growth rates from 1980 to 2000. In particular we show that the growth rates predicted by the model are positively correlated with the observed growth rates for all the definitions of skilled workers used in the exercise. Moreover, the model captures some important features of the world income distribution. Specifically the model predicts a bimodal distribution close to the one observed in the data.

In order to to test the mechanism hypothesis porposed in Acemoglu and Zilibotti (2001), we compare the predictions of their model to two alternative models. In particular we calibrate the three models and compare the predicted values to the ones observed in the data. First, we compare the model to a Cobb-Douglass production function with factor-neutral technological progress. The neoclassical benchmark will allow us to contrast the role of physical capital per-worker in explaining income differences to the one porposed by AZ. We also contrast the AZ model to the model presented in Caselli and Coleman (2006). This model assumes that each country adopts a different

technology depending on their own skill supply as opposed to adopting the technology produced by developed countries. We find that the AZ model performs better than the two alternative model when explaining the observed income distributions. In particular the AZ can account better for the observed mean, variance and interquartile range than both alternative models and has a better goodness of fit, captured by a constrained R-squared measure. The neoclassical benchmark performs almost as well in terms of predicting the observed averages and has a high R-squared but underperforms in terms of capturing the volatility of output. Finally, I estimate the parameters of the model and show that the estimates are close to the ones used in the calibration exercise which reassures the results described earlier.

I conclude by pointing at two important limitations of the present paper. The first point we want to raise is the issue of the sample used in Section 3. In particular we only used countries for which we had estimated values for relative wages. This sample is not necessarily a representative sample of the object of interest which is the entire sample¹¹. A related issue comes from the fact that we only have values for relative wages around 1980 and we only observe one value per country. Our results relied on extrapolations of the relative wage which might be far from the true evolution of relative wages. It is our opinion that extending the data for relative wages is a promising direction which would allow us to address the questions raised in this paper in a more accurate way.

2.6 References

- **Acemoglu, Daron** (1998). "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." *Quarterly Journal of Economics*, vol. 113, pp. 1055-108
- **Acemoglu, Daron** (2002). "Technical Change, Inequality and the Labor Market." *Journal of Economic Literature*, vol. 40, pp. 7-72
- **Acemoglu, Daron** (2002). "Directed Technical Change." *Review of Economic Studies*, vol. 69, pp. 781-810.
- **Acemoglu, Daron** (2003). "Cross-Country Inequality Trends." *Economic Journal*, vol. 113, pp. 121-149.
- **Acemoglu, Daron and Zilibotti, Fabrizio** (2001). "Productivity Differences." *Review of Economic Studies*, vol. 69, pp. 781-810.

¹¹To be precise, this sample underrepresents very poor countries.

- **Autor, David, Katz, Lawrence F. and Kearney, Melissa** (2005). "Trends in U.S. Wage inequality: Re-Assesing the Revisionists." Unpublished Manuscript.
- **Autor, David, Katz, Lawrence and Krueger, Alan** (1998). "Computing Inequality: Have Computers Changed the Labor Market:?" Quarterly Journal of Economics, 1998, vol. 113, pp. 1169-1213.
- **Barro, Robert J. and Lee, Jong-Wha** (2000). "International Data.on Educational Attainment: Updates and Implications." CID Working Paper No. 42.
- **Barro, Robert J. and Sala-i-Marti, Xavier** (1995). "Economic Growth." New York:McGraw-Hill.
- **Basu, Susanto and Weil, David** (1998). "Appropriate Technology and Growth." Quarterly Journal of Economics, vol. 113, pp. 1025-1054.
- **Caselli, Francesco and Coleman, Wilbur J.** (2006). "The World Technological Frontier." American Economic Review, vol 96, pp. 503-522.
- **Hall, Robert E. and Jones, Charles I.** (1999). "Why Do Some Countries Produce so Mouch More Output per Worker Than Others." Quarterly Journal of Economics, vol. 114, pp. 83-116.
- **Summers, Robert, Heston, Alan, Aten, Bettina.** (2002). "Penn World Table Version 6.1" Center of International Comparison at the University of Pennsylvania (CICUP), October 2002.
- **Jones, Charles I.** (1997). "On the Evolution of the World Income Distribution." The Journal of Economic Perspectioves, vol. 11, pp. 19-36.
- **Katz, Lawrence F. and Murphy, Kevin M.** (1992). "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." Quarterly Journal of Economics, vol. 107, pp. 35-78.
- **Krusell, Per, Ohanian, Lee, Rios-Rull, Jose-Victor and Violante, Giovanni** (2000). "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." Econometrica, vol. 68, pp. 1029-1053.
- **Lucas, Robert. E, Jr.** (1988). "On the Mechanics of Economic Development" Journal of Monetary Economics, vol.22, pp. 3-42.

- **Mankiw, N. Gregory, Romer, David and Weil, David** (1992). "A Contribution to the Empirics of Economic Growth." *Quarterly Journal of Economics*, vol.107, pp.407-438.
- **Quah, Danny.** (1996). "Twin Peaks: Growth and Convergence in Models of Distribution Dynamocs" *Economic Journal*, vol. 106, pp. 1045-1055.
- **Romer, Paul M.** (1986). "Increasing Returns and Long-Run Growth." *Journal of Political Economy*, vol. 94, pp. 1002-1037.
- **Romer, Paul M.** (1990). "Endogenous Technological Change." *Journal of Political Economy*, vol. 98, S71-S102.
- **Wooldridge, Jeffrey M.** (2002). "Econometric Analysis of Cross Section and Panel Data." The MIT Press.
- **Young, Alwyn.** (1995). "The tyranny of numbers: Confronting the statistical realities of the east Asian growth experience." *Quarterly Journal of Economics*, vol.110, pp. 641-680.

2.7 Appendix 1: Figures and Tables

Figure 1. Distribution of Income Relative to the US 1980-2000

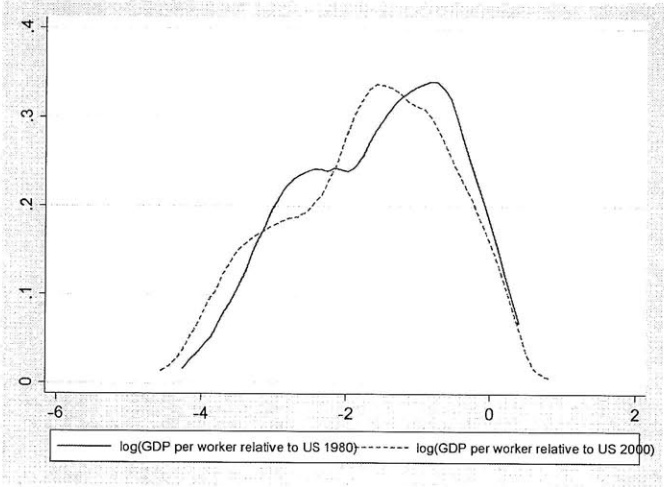


Figure 2. Distribution of Skill Supply Relative to the US 1980-2000

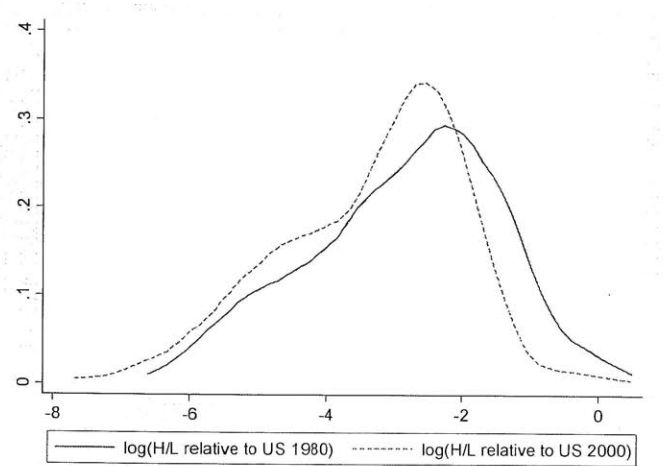
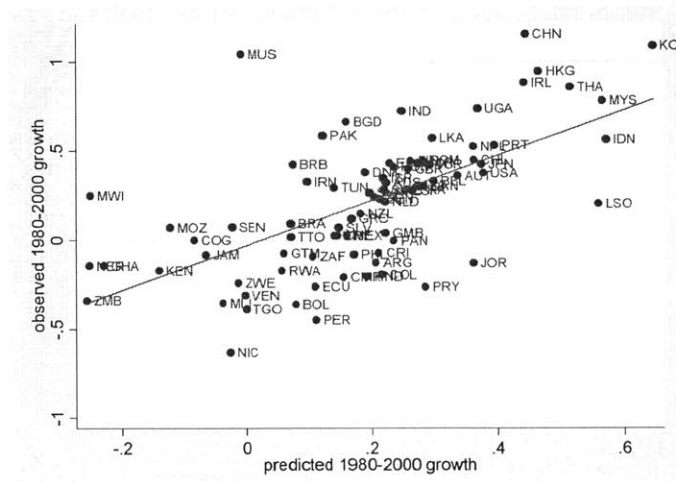


Figure 3. Observed and Predicted Growth Rates, $\sigma = 1.4$, College Equivalents

Full Sample



Caselli-Coleman Sample

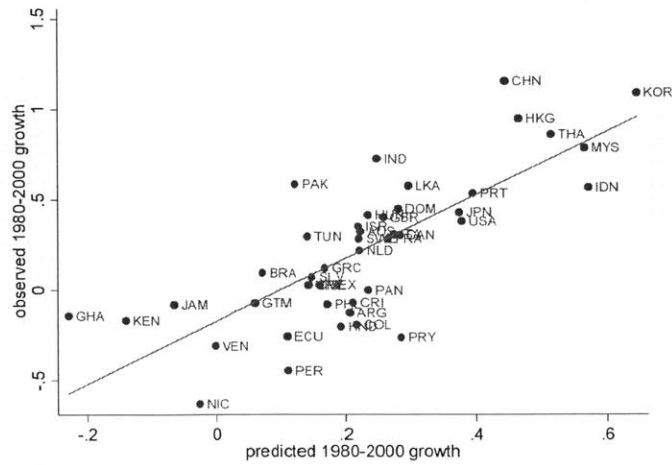


Figure 4. 1990 Observed Income Dist and Predicted Income Dist., College Graduates

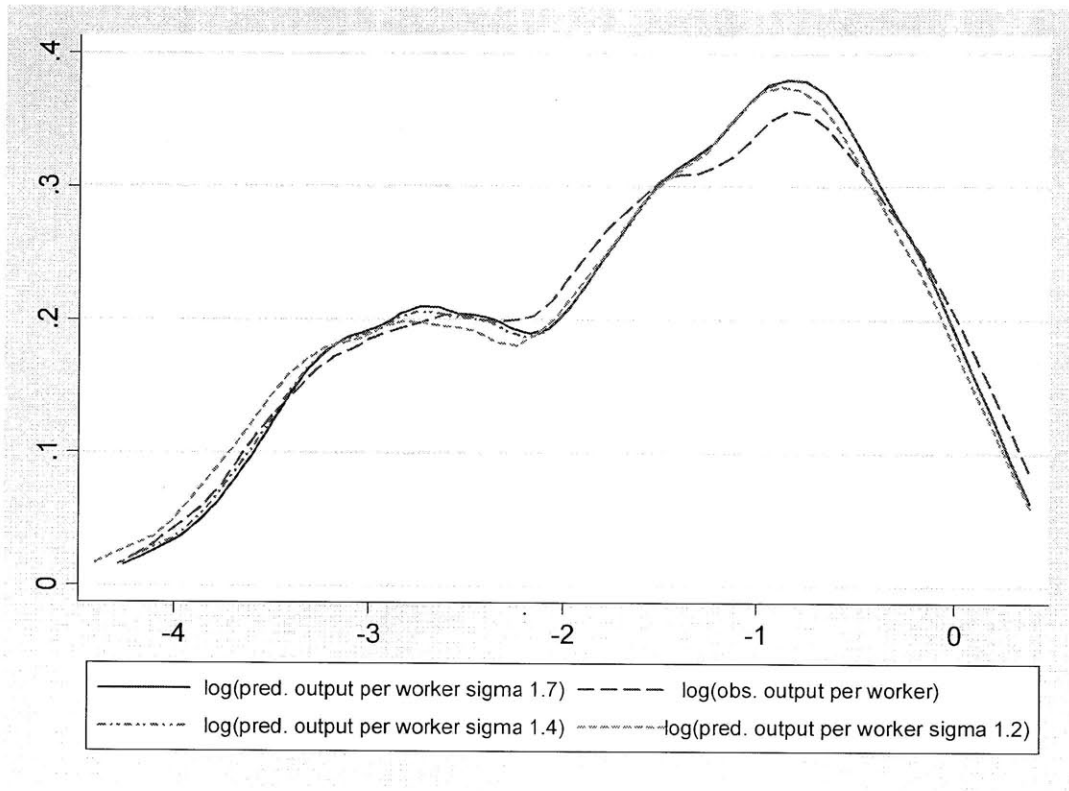


Figure 5. 2000 Observed Income Dist and Predicted Income Dist., College Graduates

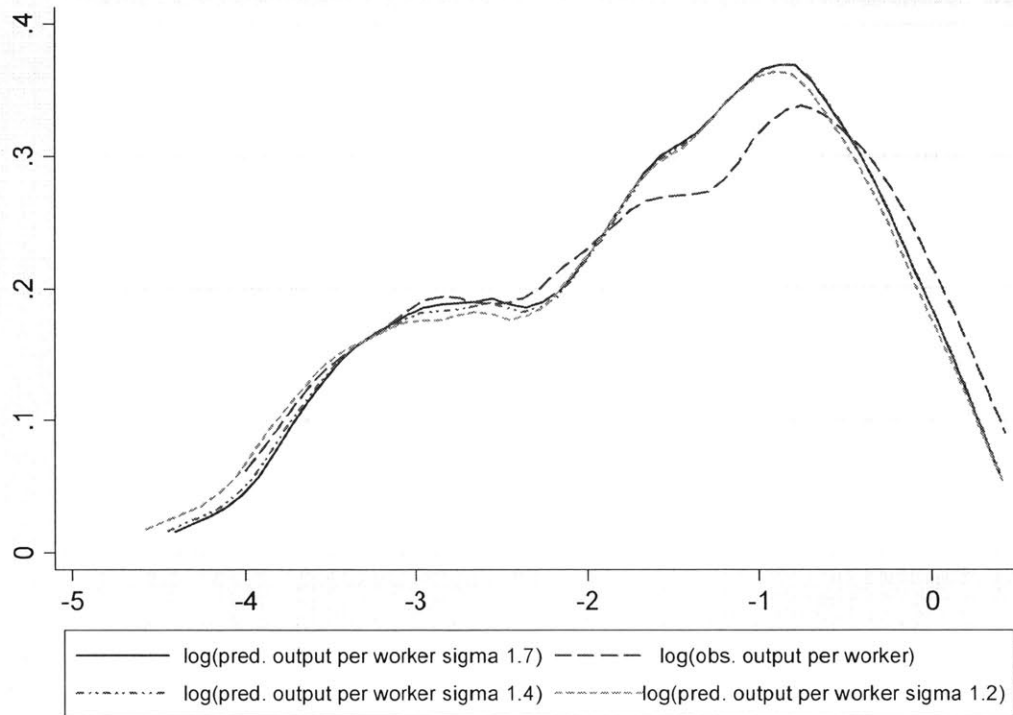


Figure 6. Evolution of the Income Distribution, 1980-2000

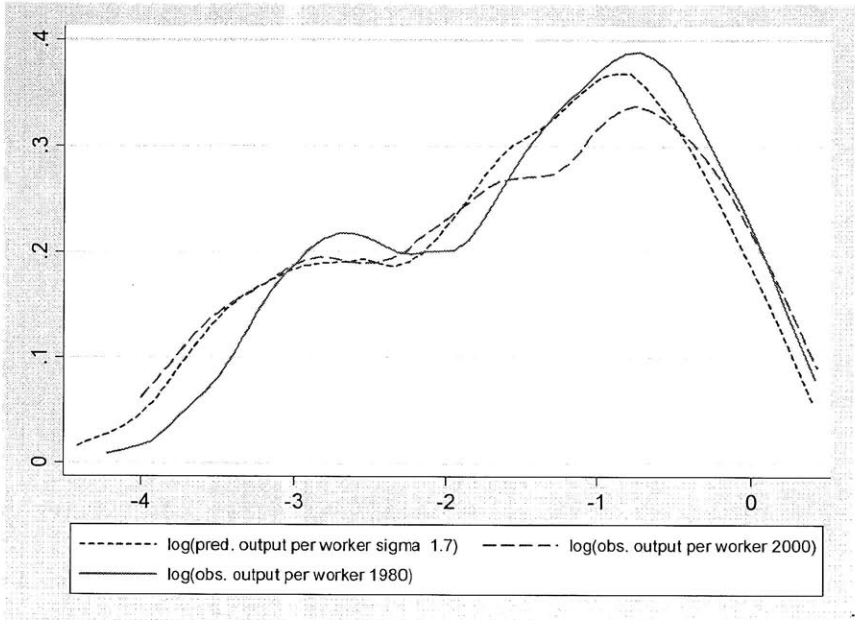


Figure 7. 75/25 Ratio, Constant Wage, $\sigma = 1.2$, College Equivalents

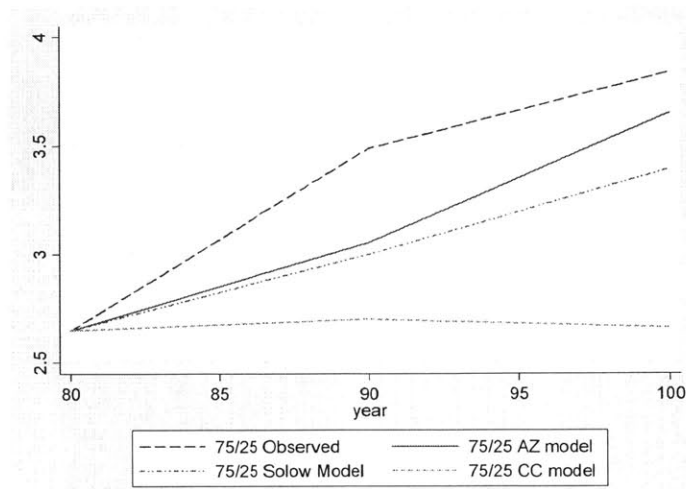


Figure 8. 75/25 Ratio, Constant Wage, $\sigma = 1.2$, Some College

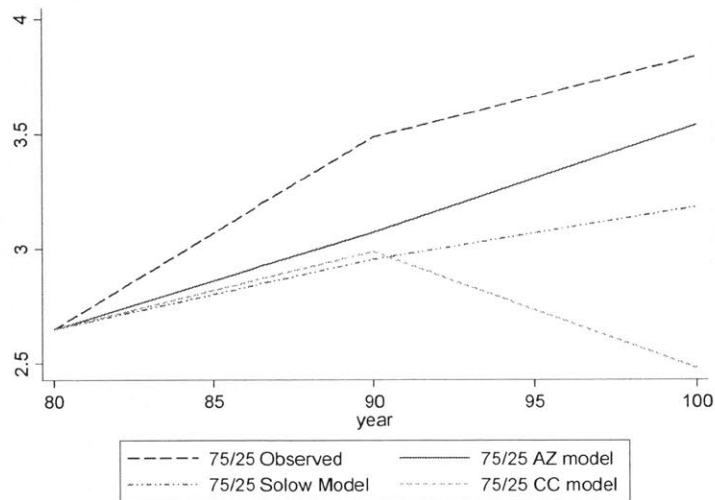


Figure 9. 75/25 Ratio, Constant Wage, $\sigma = 1.2$, College Graduates

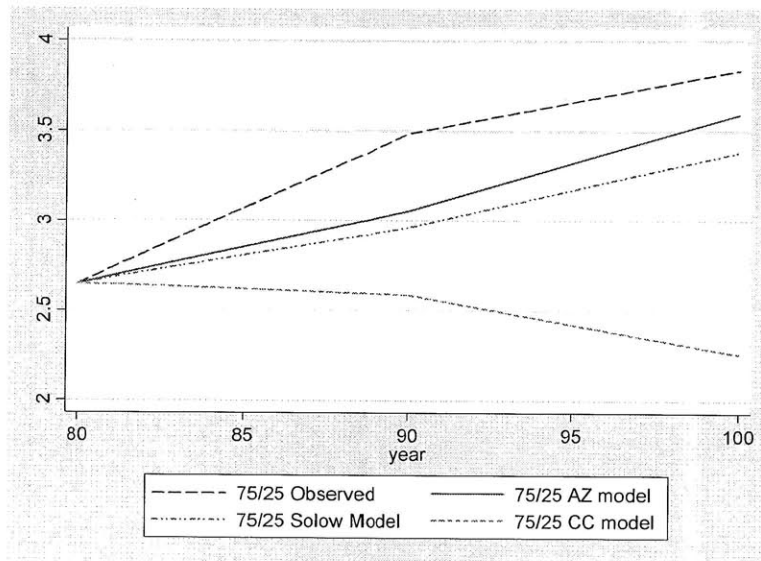


Figure 10. 75/25 Ratio, Constant Wage, $\sigma = 1.4$, College Equivalents

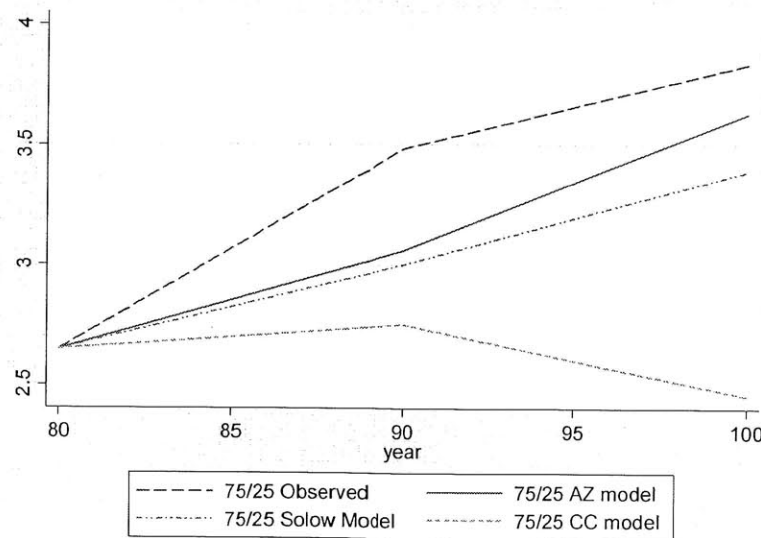


Figure 11. 75/25 Ratio, Constant Wage, $\sigma = 1.4$, Some College

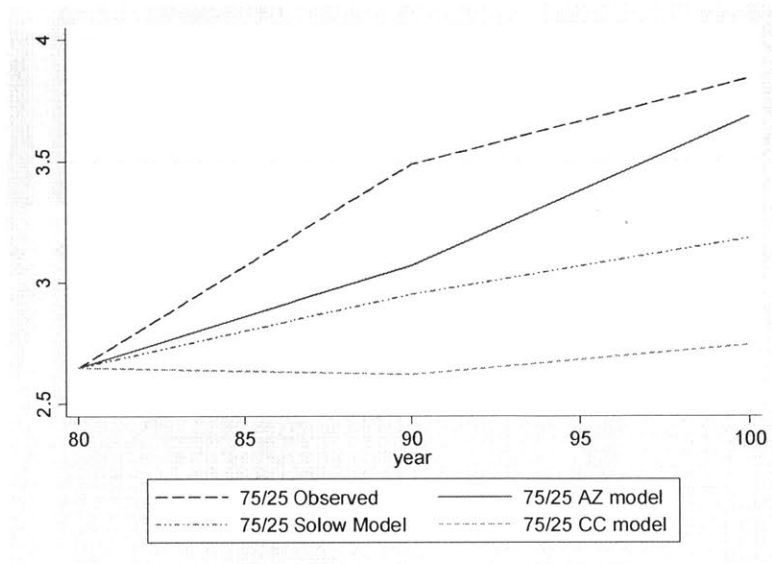


Figure 12. 75/25 Ratio, Constant Wage, $\sigma = 1.4$, College Graduates

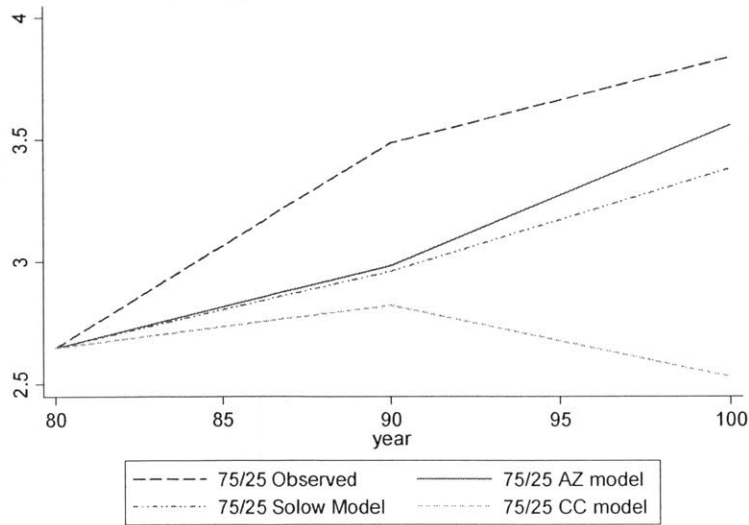


Figure 13. 75/25 Ratio, Constant Wage, $\sigma = 1.7$, College Equivalents

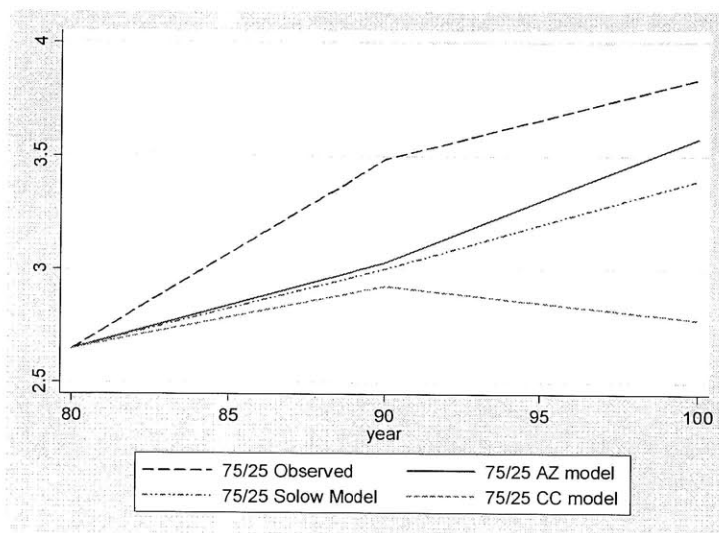


Figure 14. 75/25 Ratio, Constant Wage, $\sigma = 1.7$, Some College

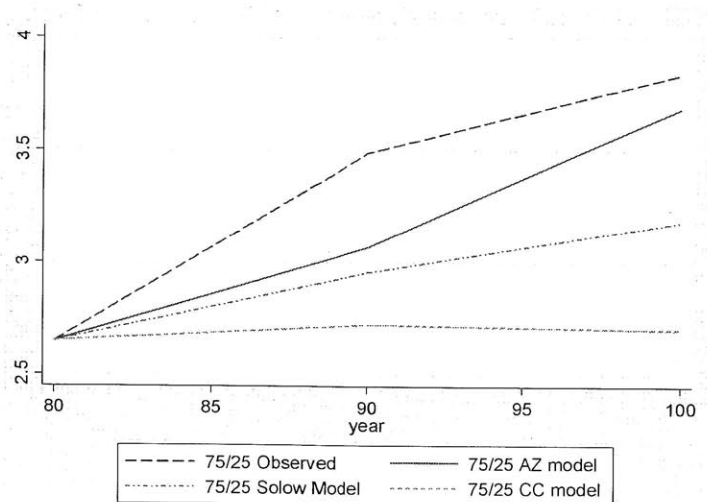


Figure 15. 75/25 Ratio, Constant Wage, $\sigma = 1.7$, College Graduates

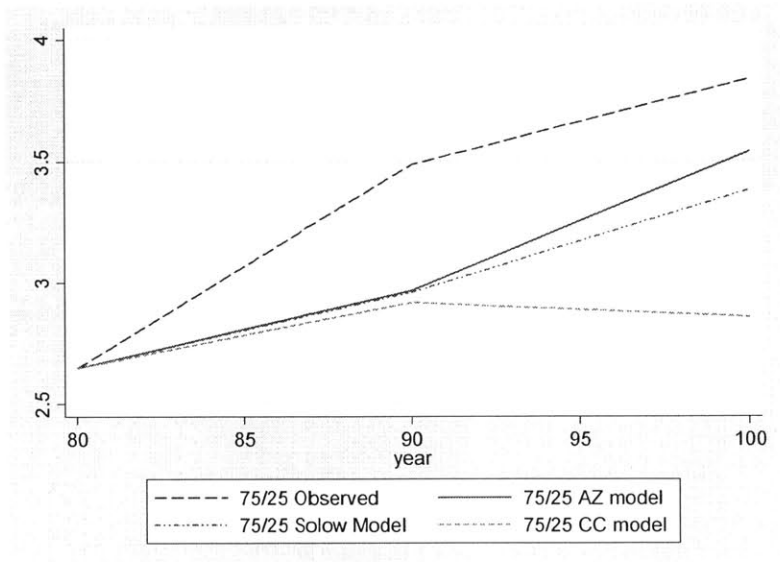


Figure 16. 75/25 Ratio, Constant A, $\sigma = 1.2$, College Equivalents

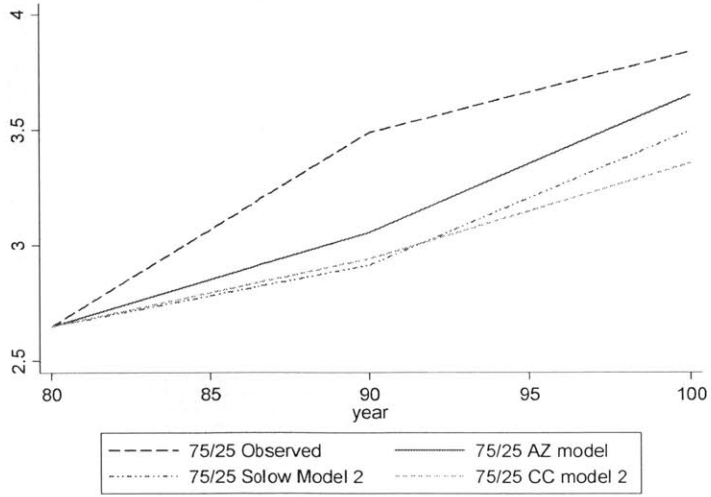


Figure 17. 75/25 Ratio, Constant A, $\sigma = 1.2$, Some College

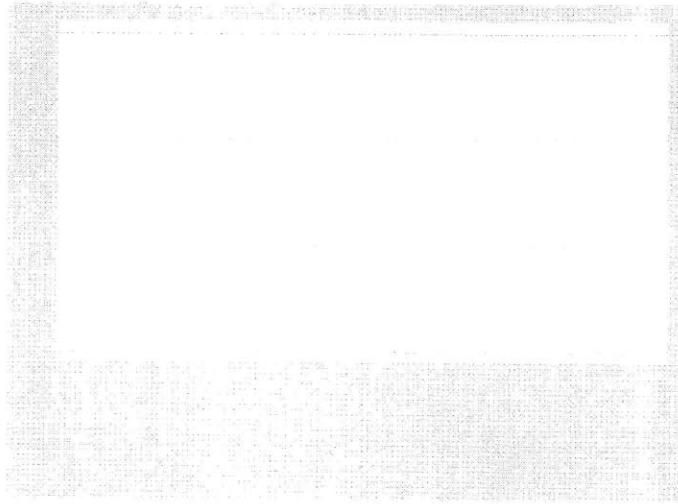


Figure 18. 75/25 Ratio, Constant A, $\sigma = 1.2$, College Graduates

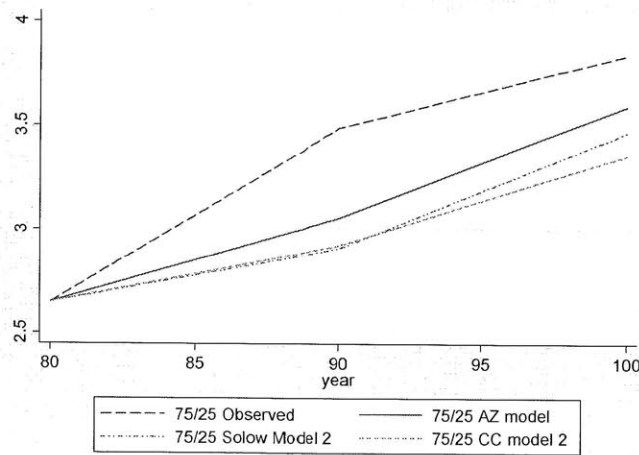


Figure 19. 75/25 Ratio, Constant A, $\sigma = 1.4$, College Equivalents

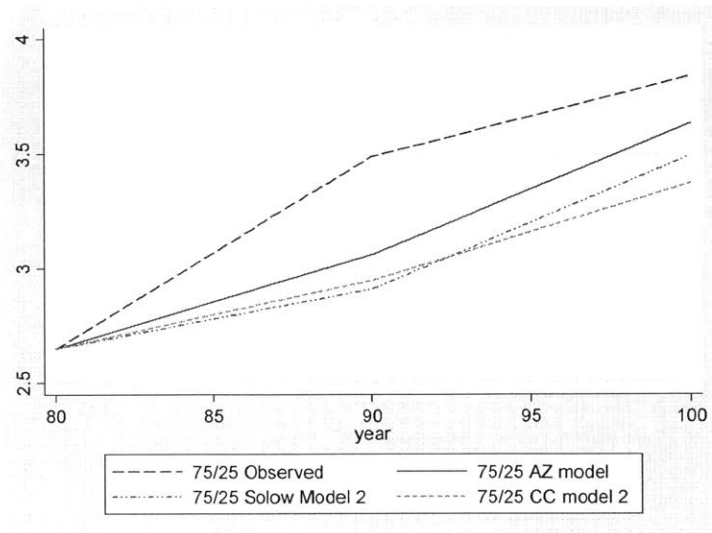


Figure 20. 75/25 Ratio, Constant A, $\sigma = 1.4$, Some College

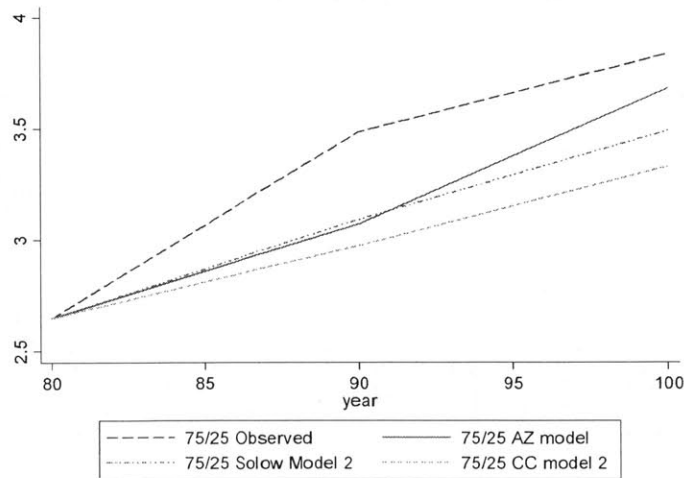


Figure 21. 75/25 Ratio, Constant A, $\sigma = 1.4$, College Graduates

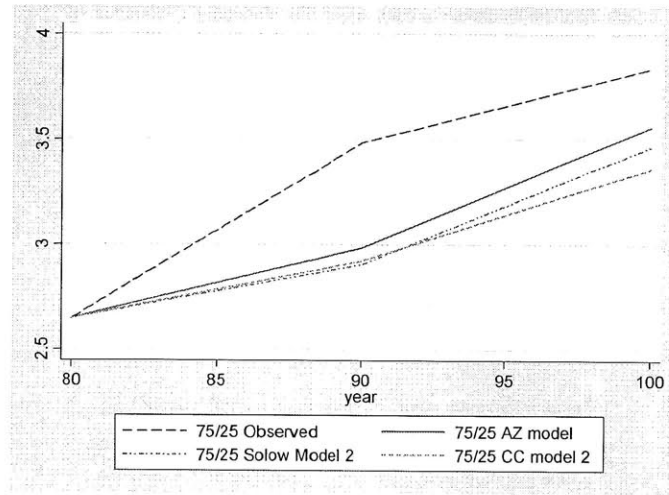


Figure 22. 75/25 Ratio, Constant A, $\sigma = 1.7$, College Equivalents

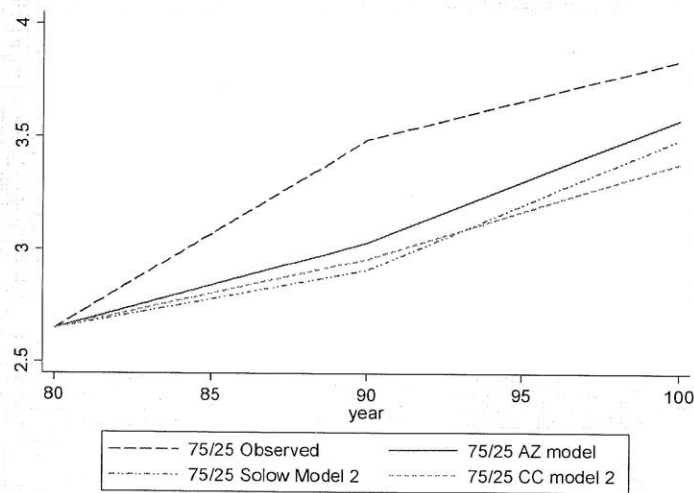


Figure 23. 75/25 Ratio, Constant A, $\sigma = 1.7$, Some College

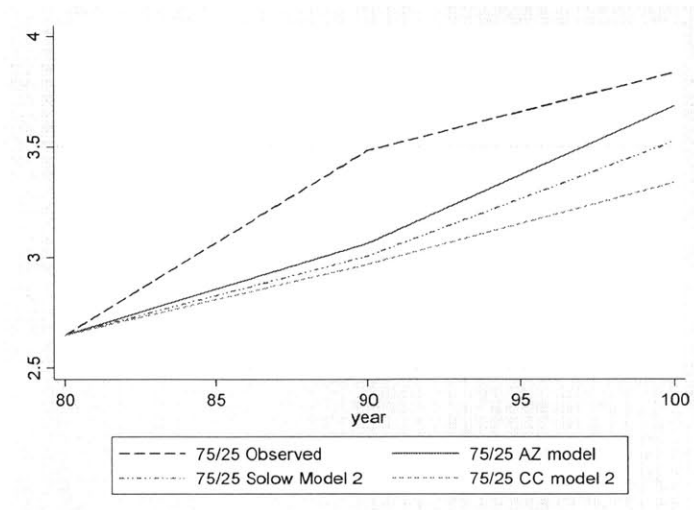


Figure 24. 75/25 Ratio, Constant A, $\sigma = 1.7$, College Graduates

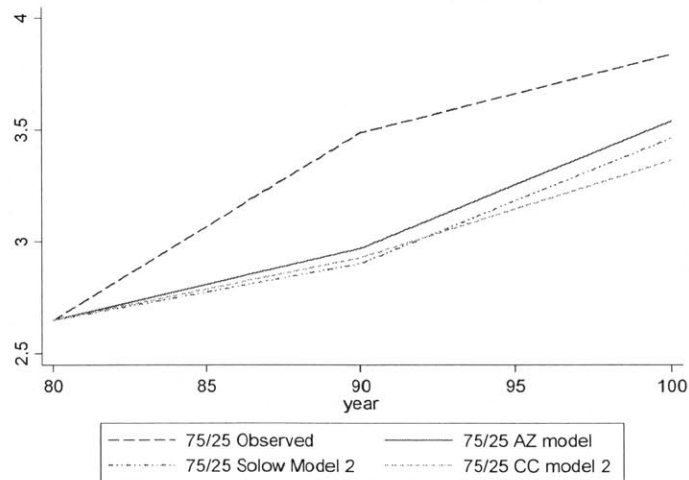


TABLE 1. Estimation of wage premium

	1962	1970	1980	1990	2000
college premium	0.5211 [0.0165]	5.021 [0.0098]	4.848 [0.0071]	5.928 [0.0074]	6.387 [0.0076]
some college	0.2141 [0.0144]	0.1617 [0.0080]	0.1323 [0.0057]	0.2150 [0.0063]	0.2455 [0.0064]
post graduate	0.0805 [0.0313]	0.1021 [0.0186]	0.1753 [0.0121]	0.1875 [0.0116]	0.2662 [0.0112]
female (fem)	-0.1615 [0.0325]	-0.1701 [0.0173]	-0.2581 [0.0132]	-0.1452 [0.0165]	-0.1933 [0.0192]
non white (nw)	-0.5125 [0.0190]	-0.3511 [0.0123]	-0.2306 [0.0101]	-0.1904 [0.0109]	-0.1263 [0.00111]
experience (exp)	0.1368 [0.00601]	0.1571 [0.00381]	0.1347 [0.00291]	0.1512 [0.00361]	0.1526 [0.00451]
exp ²	-0.0075 [0.0060]	-0.0091 [0.0003]	-0.0072 [0.0003]	-0.0081 [0.0003]	-0.0091 [0.0004]
exp ³	0.0002 [1.47E-5]	0.0002 [8.14E-6]	0.0002 [8.32E-6]	0.0002 [1.04E-5]	0.0002 [1.48E-5]
exp ⁴	-1.49E-06 [1.47E-07]	-1.91E-06 [8.99E-08]	-1.57E-06 [8.52E-08]	-1.78E-06 [1.08E-07]	-2.48E-06 [1.67E-07]
fem ² nw	-0.0293 [0.0286]	0.1241 [0.0177]	0.2345 [0.0143]	0.2132 [0.0151]	0.1431 [0.0152]
fem ² exp	-0.0881 [0.0094]	-0.1124 [0.0054]	-0.0472 [0.0043]	-0.0431 [0.0051]	-0.0235 [0.0063]
fem ² exp ²	0.0043 [0.0007]	0.0057 [0.0004]	0.0006 [0.0004]	0.0011 [0.0004]	-5.28E-06 [0.0008]
fem ² exp ³	-0.0001 [2.32E-05]	-0.0001 [1.47E-05]	2.61E-05 [1.27E-05]	-8.68E-06 [1.50E-05]	2.62E-05 [2.10E-05]
fem ² exp ⁴	6.25E-07 [2.25E-07]	7.60E-07 [1.47E-07]	-4.60E-07 [1.33E-07]	-3.18E-07 [1.58E-07]	-3.91E-07 [0.0144]

Standard errors in brackets

TABLE 2. Implied Technology Levels and Shares

	$\sigma=1.2$			$\sigma=1.4$			$\sigma=1.7$		
	Y_w/Y_L	$A_w/A_{L(t)}$	$A_w/A_{L(t)}/A_w/A_{L(t-10)}$	Y_w/Y_L	$A_w/A_{L(t)}$	$A_w/A_{L(t)}/A_w/A_{L(t-10)}$	Y_w/Y_L	$A_w/A_{L(t)}$	$A_w/A_{L(t)}/A_w/A_{L(t-10)}$
Panel A - Some College									
1962	0.8208	1.0000	-	0.7157	1.0000	-	0.8323	1.0000	-
1970	0.8208	3.9613	3.9613	0.7157	1.9717	1.9717	0.8323	1.4620	1.4620
1980	0.8208	33.4759	8.4507	0.7157	5.4595	2.7890	0.8323	2.5095	1.7185
1990	0.8208	491.5564	14.6839	0.7157	22.9829	4.2097	0.8323	6.1844	2.4644
2000	0.8208	2405.8357	4.8943	0.7157	52.0272	2.2837	0.8323	10.0600	1.6267
Panel B - College Graduates									
1962	0.3074	1.0000	-	0.3919	1.0000	-	0.5069	1.0000	-
1970	0.3074	1.7032	1.7032	0.3919	1.2918	1.2918	0.5069	1.1482	1.1482
1980	0.3074	9.2443	5.4275	0.3919	2.8999	2.2218	0.5069	1.7378	1.5135
1990	0.3074	125.5835	13.5849	0.3919	11.6246	4.0505	0.5069	4.1893	2.4107
2000	0.3074	352.9001	2.8101	0.3919	19.99297	1.7144	0.5069	5.8139	1.3878
Panel C - College Equivalents									
1962	0.4582	1.0000	-	0.5519	1.0000	-	0.6720	1.0000	-
1970	0.4582	2.8237	2.8237	0.5519	1.6640	1.6640	0.6720	1.3269	1.3269
1980	0.4582	17.9456	6.3553	0.5519	3.9919	2.3990	0.6720	2.0984	1.5814
1990	0.4582	231.7364	12.9132	0.5519	15.7616	3.9484	0.6720	4.9854	2.3758
2000	0.4582	822.6966	3.5501	0.5519	30.3822	1.9276	0.6720	7.3979	1.4839

Technology levels and shares coefficients obtained from the wage premiums regressions in Table 1 and the relative wage equation in Section 3.

Table 3. Growth Regressions: Observed 1980-2000 Growth and Predicted 1980-2000

Panel A. Full Sample									
	$\sigma=1.2, SC$	$\sigma=1.4, SC$	$\sigma=1.7, SC$	$\sigma=1.2, CE$	$\sigma=1.4, CE$	$\sigma=1.7, CE$	$\sigma=1.2, CG$	$\sigma=1.4, CG$	$\sigma=1.7, CG$
constant	0.044 [0.048]	0.05 [0.046]	0.047 [0.044]	0.08 [0.043]	0.048 [0.043]	0.103*** [0.037]	0.057 [0.041]	-0.028 [0.047]	-0.051 [0.047]
predicted 1980-2000 growth	0.971*** [0.148]	1.083*** [0.151]	1.168*** [0.158]	1.202*** [0.157]	1.049*** [0.164]	1.106*** [0.168]	1.084*** [0.167]	1.282*** [0.178]	1.381*** [0.180]
Observations	81	81	81	81	81	81	81	81	81
R-squared	0.261	0.287	0.318	0.31	0.342	0.38	0.349	0.393	0.419
Null Hypothesis: $\alpha=0, \beta=1$									
F	1.21	0.87	0.71	1.08	1.27	5.78	2.15	1.31	2.15
Prob>F	0.307	0.516	0.501	0.348	0.288	0.004	0.123	0.278	0.123
Panel B. Caselli-Coleman Sample									
	$\sigma=1.2, SC$	$\sigma=1.4, SC$	$\sigma=1.7, SC$	$\sigma=1.2, CE$	$\sigma=1.4, CE$	$\sigma=1.7, CE$	$\sigma=1.2, CG$	$\sigma=1.4, CG$	$\sigma=1.7, CG$
constant	-0.085 [0.073]	-0.088 [0.089]	-0.084 [0.088]	-0.063 [0.084]	-0.078 [0.082]	-0.002 [0.055]	-0.075 [0.080]	-0.171** [0.084]	-0.198*** [0.084]
predicted 1980-2000 growth	1.180*** [0.208]	1.284*** [0.217]	1.377*** [0.224]	1.357*** [0.218]	1.377*** [0.224]	1.438*** [0.233]	1.488*** [0.328]	1.542*** [0.327]	1.558*** [0.327]
Observations	48	48	48	48	48	48	48	48	48
R-squared	0.431	0.447	0.474	0.479	0.508	0.508	0.527	0.584	0.614
Null Hypothesis: $\alpha=0, \beta=1$									
F	0.89	0.86	1.42	1.38	2.34	4.12	3.34	5.35	7.11
Prob>F	0.419	0.304	0.251	0.287	0.109	0.023	0.045	0.008	0.002

Standard errors in brackets, *** p<0.01, ** p<0.05, * p<0.1

SC stands for some college, CE stands for College Equivalents and CG stands for College Graduates.

Table 4. Comparison Across Models, $\sigma=1.2$

Panel A. Some College

	Acemoglu Zilibotti Model			Augmented Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	1.01168	0.93277	0.97905	0.94910	0.78290	0.97205	0.14443	0.28118	0.22013
2000	1.04620	0.96576	0.96642	0.97120	0.76573	0.95550	0.11359	0.29876	0.17959
full sample	1.02948	0.95801	0.97200	0.96048	0.77722	0.96280	0.12848	0.29180	0.19783

Panel B. College Equivalents

	Acemoglu Zilibotti Model			Augmented Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.96974	0.87560	0.97860	0.96500	0.82599	0.97883	0.43509	0.32327	0.61169
2000	0.96868	0.88118	0.96831	0.97320	0.78973	0.96033	0.44273	0.35925	0.59891
full sample	0.97845	0.88165	0.97285	0.96920	0.80707	0.96761	0.43902	0.34305	0.60473

Panel C. College Graduates

	Acemoglu Zilibotti Model			Augmented Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.95330	0.86270	0.97805	0.97508	0.84301	0.97838	0.52886	0.35402	0.70716
2000	0.96524	0.84983	0.96729	0.97855	0.79703	0.96190	0.53420	0.37861	0.69109
full sample	0.95942	0.85718	0.97204	0.97735	0.81799	0.96917	0.53063	0.36787	0.69272

Mean and Variance are the ratio of the prediction of the statistic by the model compared to the observed statistic.

All the values are calculated using the sample from Casselli and Coleman (2008).

Table 5. Comparison Across Models, $\sigma=1.4$

Panel A. Some College

	Acemoglu Zilibotti Model			Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.99703	0.91814	0.97944	0.94910	0.78290	0.97205	0.33092	0.29071	0.49698
2000	1.02293	0.94032	0.96856	0.97120	0.76573	0.95550	0.26932	0.30164	0.41472
full sample	1.01037	0.93540	0.97324	0.96048	0.77722	0.96280	0.29905	0.29515	0.45121

Panel B. College Equivalents

	Acemoglu Zilibotti Model			Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.96820	0.93694	0.97907	0.96500	0.82599	0.97683	0.63815	0.43311	0.78742
2000	0.97957	0.93610	0.96824	0.97320	0.78973	0.96033	0.64101	0.44334	0.76571
full sample	0.97403	0.93749	0.97302	0.96920	0.80707	0.96781	0.63980	0.43543	0.80858

Panel C. College Graduates

	Acemoglu Zilibotti Model			Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.98316	0.94500	0.98065	0.97508	0.84301	0.97838	0.70984	0.48443	0.86983
2000	0.99136	0.93498	0.96850	0.97955	0.79703	0.96190	0.71123	0.47104	0.84570
full sample	0.98735	0.94010	0.97386	0.97735	0.81799	0.96917	0.71053	0.47780	0.85640

Mean and Variance are the ratio of the prediction of the statistic by the model compared to the observed statistic.

All the values are calculated using the sample from Casselli and Coleman (2006).

Table 6. Comparison Across Models, $\sigma=1.7$

Panel A. Some College									
	Acemoglu Zilibotti Model			Augmented Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.983313	0.903327	0.979730	0.949095	0.782903	0.972050	0.509197	0.353949	0.70840181
2000	1.001983	0.915963	0.988813	0.971199	0.765725	0.955498	0.448375	0.345282	0.83724133
full sample	0.992920	0.913723	0.973833	0.960479	0.777225	0.962803	0.477723	0.348284	0.86873813
Panel B. College Equivalents									
	Acemoglu Zilibotti Model			Augmented Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.988416	0.880829	0.979572	0.965005	0.825991	0.976829	0.759360	0.545911	0.90394652
2000	0.974885	0.872081	0.988033	0.973197	0.789728	0.960327	0.762381	0.538111	0.88548988
full sample	0.971715	0.876954	0.973126	0.969195	0.807069	0.967608	0.760885	0.541174	0.89386882
Panel C. College Graduates									
	Acemoglu Zilibotti Model			Augmented Solow Model 1			Caselli-Coleman 1		
	Mean	Variance	R-squared	Mean	Variance	R-squared	Mean	Variance	R-squared
1990	0.984917	0.897182	0.981033	0.975080	0.843011	0.978383	0.811535	0.597120	0.92981808
2000	0.989275	0.872650	0.988374	0.979554	0.797030	0.961900	0.812552	0.567144	0.90908143
full sample	0.987123	0.884097	0.973960	0.977347	0.817987	0.969172	0.812022	0.584564	0.91825193

Mean and Variance are the ratio of the average prediction of a model compared to the observed statistic.

All the values are calculated using the sample from Casselli and Coleman (2008).

Table 7. Comparison Across Models, $\sigma=1.2$

Panel A. Some College

	Acemoglu Zilibotti Model			Augmented Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	1.01168	0.93277	0.97905	0.299180	0.320351	0.488009	0.970182	0.827777	0.97648
2000	1.04820	0.98578	0.98642	0.258994	0.337284	0.428076	0.995215	0.808799	0.959123
full sample	1.02948	0.95801	0.97200	0.277342	0.328681	0.454523	0.977909	0.818768	0.968768

Panel B. College Equivalents

	Acemoglu Zilibotti Model			Augmented Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.98974	0.87580	0.97880	0.774060	0.623891	0.928818	0.976223	0.854050	0.979187
2000	0.98888	0.88118	0.98831	0.703532	0.543681	0.878726	0.975288	0.811901	0.962323
full sample	0.97845	0.88165	0.97285	0.737554	0.572323	0.899822	0.975682	0.830345	0.968689

Panel C. College Graduates

	Acemoglu Zilibotti Model			Augmented Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.95330	0.86270	0.97805	0.774074	0.620444	0.927359	0.985081	0.869044	0.980047
2000	0.98524	0.84983	0.98729	0.712817	0.537324	0.884233	0.982613	0.821671	0.963682
full sample	0.95942	0.85718	0.97204	0.742362	0.587582	0.903231	0.983749	0.842148	0.970891

Mean and St.Dev. are the ratio of the average prediction of a model compared to the observed statistic.

All the values are calculated using the sample from Casselli and Coleman (2008).

Table 8. Comparison Across Models, $\sigma=1.4$

Panel A. Some College

	Acemoglu Zilibotti Model			Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.99703	0.91814	0.97944	0.365976	0.339589	0.576284	0.986120	0.819881	0.975852
2000	1.02293	0.94032	0.98858	0.314889	0.347080	0.508135	0.980088	0.797022	0.958542
full sample	1.01037	0.93540	0.97324	0.339532	0.340615	0.538190	0.973288	0.809807	0.986167

Panel B. College Equivalents

	Acemoglu Zilibotti Model			Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.98820	0.93694	0.97907	0.834563	0.692829	0.951183	0.974398	0.849855	0.978731
2000	0.97957	0.93810	0.98824	0.788853	0.602488	0.913036	0.974073	0.808011	0.962061
full sample	0.97403	0.93748	0.97302	0.800443	0.635261	0.929839	0.974184	0.826423	0.989404

Panel C. College Graduates

	Acemoglu Zilibotti Model			Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.98318	0.94500	0.98065	0.843510	0.703700	0.955005	0.983499	0.885284	0.979853
2000	0.99138	0.93498	0.98850	0.787175	0.611315	0.921580	0.981502	0.817754	0.963471
full sample	0.98735	0.94010	0.97386	0.814340	0.645412	0.936303	0.982419	0.838368	0.970887

Mean and St.Dev. are the ratio of the average prediction of a model compared to the observed statistic.

All the values are calculated using the sample from Casselli and Coleman (2006).

Table 9. Comparison Across Models, $\sigma=1.7$

Panel A. Some College

	Acemoglu Zilibotti Model			Augmented Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.983313	0.903327	0.979730	0.458548	0.377097	0.681880	0.962233	0.812098	0.975183
2000	1.001983	0.915983	0.968813	0.398422	0.368350	0.609782	0.975862	0.788270	0.967939
full sample	0.992920	0.913723	0.973633	0.425436	0.368452	0.641564	0.969235	0.801224	0.965535

Panel B. College Equivalents

	Acemoglu Zilibotti Model			Augmented Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.969416	0.880829	0.979572	0.900844	0.775343	0.969887	0.972544	0.845487	0.978438
2000	0.974885	0.872081	0.968033	0.841843	0.676217	0.941793	0.973096	0.804124	0.961787
full sample	0.971715	0.876954	0.973126	0.870189	0.712764	0.954167	0.972783	0.822468	0.969121

Panel C. College Graduates

	Acemoglu Zilibotti Model			Augmented Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.984917	0.897182	0.981033	0.902020	0.780417	0.970930	0.981889	0.861321	0.979838
2000	0.989275	0.872650	0.968374	0.852969	0.684404	0.945872	0.980524	0.813716	0.963244
full sample	0.987123	0.884097	0.973960	0.878613	0.720288	0.956797	0.981137	0.834463	0.970465

Mean and St.Dev. are the ratio of the average prediction of a model compared to the observed statistic.

All the values are calculated using the sample from Casselli and Coleman (2006).

Table 10. Comparison Across Models, Full Sample, Constant Wage
 Panel A. College Equivalents $\sigma=1.2$

	Acemoglu Zilibotti Model			Solow Model 1			Caselli-Coleman 1		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.96147	0.89786	0.97954	0.990238	0.889091	0.980501	0.522755	0.316135	0.707391
2000	0.92850	0.83803	0.95520	0.947325	0.806368	0.954033	0.507109	0.337352	0.665072
full sample	0.94318	0.85932	0.96576	0.967722	0.837121	0.965501	0.514607	0.326718	0.684481

Panel B. College Equivalents, $\sigma=1.4$									
	Acemoglu Zilibotti Model			Solow Model 1			Caselli-Coleman 1		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.99553	0.93507	0.98225	0.990238	0.889091	0.980501	0.714079	0.492541	0.873147
2000	0.95454	0.86666	0.95860	0.947325	0.806368	0.954033	0.683543	0.474028	0.831472
full sample	0.97404	0.89107	0.96885	0.967722	0.837121	0.965501	0.698061	0.479274	0.849923

Panel C. College Equivalents, $\sigma=1.7$									
	Acemoglu Zilibotti Model			Solow Model 1			Caselli-Coleman 1		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.99764	0.93634	0.98262	0.990238	0.889091	0.980501	0.820089	0.624341	0.933143
2000	0.95224	0.86062	0.95866	0.947325	0.806368	0.954033	0.783497	0.581218	0.896136
full sample	0.97380	0.88762	0.96904	0.967722	0.837121	0.965501	0.800881	0.596247	0.912333

Mean and St.Dev. are the ratio of the average prediction of a model compared to the observed statistic.

All the values are calculated using the sample of countries with information on Capital, output and Education.

Table 11. Comparison Across Models, Full Sample, Constant $\Delta \ln Y / \Delta t$
 Panel A. College Equivalents $\sigma=1.2$

	Acemoglu Zilibotti Model			Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.96147	0.89786	0.97954	0.782333	0.579638	0.926585	0.994264	0.894311	0.972179
2000	0.92650	0.83803	0.95520	0.679589	0.449508	0.854522	0.941449	0.787717	0.948310
full sample	0.94318	0.85932	0.96576	0.728293	0.497478	0.885564	0.967105	0.828439	0.957430

Panel B. College Equivalents, $\sigma=1.4$

	Acemoglu Zilibotti Model			Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.99553	0.93507	0.98225	0.854378	0.674583	0.955248	0.992751	0.890994	0.972011
2000	0.95454	0.88868	0.95880	0.753154	0.530353	0.897323	0.940622	0.785668	0.946120
full sample	0.97404	0.89107	0.96885	0.801226	0.583654	0.922235	0.965956	0.825946	0.957249

Panel C. College Equivalents, $\sigma=1.7$

	Acemoglu Zilibotti Model			Solow Model 2			Caselli-Coleman 2		
	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared	Mean	St.Dev.	R-squared
1990	0.99764	0.93834	0.98262	0.914909	0.761946	0.971680	0.991213	0.887533	0.971825
2000	0.95224	0.86062	0.95866	0.818145	0.610058	0.926316	0.939996	0.783786	0.945929
full sample	0.97380	0.86762	0.96904	0.864201	0.666324	0.945808	0.964904	0.823517	0.957061

Mean and St.Dev. are the ratio of the average prediction of a model compared to the observed statistic.

All the values are calculated using the sample of countries with information on Capital, output and Education.

Table 12. Estimation of CES Model

Panel A			
dependent variable: $\Delta \ln(y_{it}-l_{it})$	Some College	College Equivalents	College Graduates
$\Delta \ln(k_{it}-l_{it})$	0.304*** [0.033]	0.304*** [0.050]	0.302*** [0.033]
$\Delta \ln(h_{it}-l_{it})$	0.225*** [0.069]	0.203** [0.081]	0.173* [0.102]
$\Delta \ln(h_{it}-l_{it})$	0.064*** [0.017]	0.052*** [0.016]	0.042** [0.021]
Observations	326	326	323
R-squared	0.63	0.661	0.69

Panel B: Implied Parameters			
α	0.304*** [0.033]	0.304*** [0.05]	0.302*** [0.033]
γh	0.371*** [0.113]	0.335*** [0.1362]	0.284* [0.1681]
ρ	0.452*** [0.071]	0.3852*** [0.0574]	0.339*** [0.071]

Standard errors in brackets, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Chapter 3

Demand Uncertainty, Information Quality and Sectoral Entry

3.1 Introduction

Entry decisions are at the heart of many economic and strategic problems. Agents constantly have to make entry decisions into different activities and these decisions have consequences on the allocation of resources, competition and the efficiency of a market economy. Furthermore, in many circumstances these decisions are accompanied by uncertainty of the payoffs for these activities and the actions taken by other players. Examples of such situations vary from the researcher who chooses to write about field A over field B, the political candidate deciding whether to appeal to a conservative constituency or to a liberal constituency or the firm who has to decide in which sector of the economy to operate in. All this suggests that information and information quality have an important effect on agents' decision and competition. One important implication of this is the potential inefficiency of the equilibrium outcomes that arise in these models. Banerjee (1992, 1993) and Bala and Goyal (1994), for example, show how sequential decision making by agents who have incomplete information of the profitability of their investment opportunities can lead to inefficient choices. Rob (1987) also points at the inefficiencies of the market equilibrium in a static model of entry under uncertainty. Moreover, these inefficiencies can have important macroeconomic implications for the growth rate of the economy, specially when there are spillovers and learning from other agents in one's sector, and for the amplification of sectoral shocks (see for example Caplin and Leahy (1993)).

In this vein the present paper has two objectives. The first is to explore the relation between entry decisions by agents and the quality of the information they have prior to entry. For this purpose I study a simple two stage model of entry. In the first stage a continuum of agents decide the sector of the economy they will operate in. Following the entry stage agents engage in monopolistic competition where they make positive profits which depend negatively on the level of entry to the sector they operate in and positively on the demand of the sector. At the time of entry agents are uncertain of the true state of demand for each sector. This uncertainty is partly revealed by a private signal of the relative demand of sector one with respect to sector two which is observed before the entry decision is made. The signal has a direct and an indirect informational effect. On the one hand, a higher signal suggests a higher demand for sector one which, other things equal, would make entry to sector one more attractive. On the other hand, signals are correlated across entrants, which implies that a high signal also predicts higher potential entry into sector one. These two conflicting effects behind a high signal will be essential in determining the equilibrium of the game. I show that the quality of the information received by agents, measured by the preciseness of the signal they receive, will be crucial in determining which of the two forces dominates. In particular, when the precision of the signal is low, the direct effect of a high signal dominates the indirect effect and the equilibrium of the game is a pure strategy threshold equilibrium in which agents enter sector one whenever they receive a positive signal. Once the signal received by agents becomes more precise, the indirect effect becomes stronger and the equilibrium of the game is a mixed strategy equilibrium. Furthermore, I show that the equilibrium probability of entry to sector one is increasing in the value of the signal received and decreasing in the level of precision of the signals.

The second objective of the paper is to analyze the effect of both information quality and fundamentals on competition, generated by entry decisions by agents, and welfare. In general there are two potential sources of inefficiencies in the model discussed: the first one comes from the monopolistic structure of the post-entry game and the second one comes from the uncertainty faced by investors at the moment of entry. When analyzing welfare I will use a constrained efficient notion where I take the competitive structure of the two sectors as given and focus on the second source of potential inefficiency. I will also assume the social planner has full information of the demand for the two sectors. I start by showing that without uncertainty the equilibrium entry level into the two sectors coincides with the efficient entry levels. This implies that the level of competition in the two sectors is socially optimal. Once I introduce uncertainty the discrepancy

between the equilibrium entry level and the efficient entry level will depend crucially on both the precision of the signal received by agents and the value of the relative demand of sector one with respect to sector two. I start by showing that the relation between the precision of the signal received by agents and the discrepancy between the equilibrium and the efficient level of entry is non-monotonic. In particular, when the equilibrium of the game is a pure strategy equilibrium (a low precision level) there is a U-shaped relation between the precision of the signal and the discrepancy between the equilibrium and the efficient entry levels. Moreover, for each relative demand level there is a unique level of precision such that the equilibrium entry level is efficient. The last point suggests an important issue: small increases in information quality, captured by an increase in the precision of the signals, might generate excessive competition in one sector and too little competition in the other market. Once the precision increases, and the equilibrium of the game is a mixed strategy equilibrium, the discrepancy between the socially efficient entry level and the equilibrium entry level is decreasing in the precision of the signal. In particular, for low levels of precision there will be excessive competition in the sector with high relative demand and too little competition in the sector with low relative demand. As information quality increases, equilibrium competition levels become closer to the socially efficient level. The discussion above points at an important result of the paper, increases in information quality can be socially detrimental. More specifically, small improvements in information can be socially costly while drastic improvements in information quality always lead to welfare improvements. Next I show that the relation between relative demand and the discrepancy between the efficient and the equilibrium entry levels is non-monotonic. I show that, independently of the quality of information, when sectors are equally profitable (relative demand equal to one) the efficient entry level and the equilibrium entry level coincide. This result implies that uncertainty affects efficiency as long as consumers have a stronger taste for one sector over the other. When the two demand levels are not equal, the relation between relative demand and the discrepancy in entry levels is non-monotonic and will depend on the quality of information. On the one hand, when precision is low, there is a twin-peaked relation between relative demand and the discrepancy in entry levels while there is an inverted U-shaped relation when the precision is high. All this points at the intricate relation between information quality, relative demand and efficiency. It also highlights an important point of the paper: small increases in the level of precision of the signals might increase or decrease welfare depending on the value of relative demand and the quality of information.

This paper contributes to the literature of entry and efficiency. Dixit and Stiglitz (1977) and

Mankiw and Whinston (1985) were among the first to study the efficiency of entry into markets when there is full information. Both papers point at the importance of product substitutability in determining the efficiency of equilibrium entry. Two papers which analyze a similar entry decision but assuming demand uncertainty are Jovanovic (1981) and Rob (1987). In both models the equilibrium is one where agents are indifferent between entering or not entering and they randomize over this decision. Furthermore, Rob (1987) highlights that the random nature of equilibrium entry makes it inefficient, a result that comes from the assumption of a discrete number of entrants. There are two important differences between these papers and mine. The first difference is that they study the entry decision of a discrete number of entrants into one industry while I consider the case of a continuum of investors deciding between two sectors. This will imply that in my model entry is constant and not random. The second difference is that they solve their respective models assuming a particular precision level for the signals that agents receive prior to entry. This paper on the other side, allows the precision of the signals to vary which enriches the equilibrium and welfare analysis and, as I pointed above, highlights the interaction between information quality, fundamentals and welfare. In this sense, my work is related to Angeletos and Pavan (2007) and Morris and Shin (2003) who analyze the welfare implications in games where agents receive noisy signals of the fundamentals. Contrary to what Angeletos and Pavan (2007) find I show that the relation between welfare and precision is non-monotonic.

The paper is organized as follows. Section 2 will introduce the model and characterize the equilibrium of the game with full information. On section 3 I examine the model with incomplete information and characterizes the equilibria of the game in this case. Section 4 analyzes a tractable example which highlights the main results of the paper. The summary of the results and the conclusion are left to Section 5. All proofs and figures are left to the appendix.

3.2 The Model

3.2.1 Preferences and Technology

Household and Consumption Good Producers

I start by describing the baseline model. Consider the following two period economy where the representative household maximizes a utility function of the following form

$$U = \sum_{i=1}^2 A_i \ln C_i \quad (3.1)$$

subject to the budget constraint

$$\sum_{i=1}^2 P_i C_i \leq I$$

where C_i is a consumption good produced in sector i and I is the income level of the representative consumer. The choice of the utility function will give a simple relation between the expenditure in consumption good i and the income level of the household.

Let Y_i be total production of good i . Good i is produced by a perfectly competitive producer using a continuum M_i of intermediate goods indexed by j . The mass of intermediate producers M_i will be determined endogenously and will be of crucial interest in what follows. Specifically, the production function for consumption good i takes the following form:

$$Y_i = \left(\int_0^{M_i} y_{ij}^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)}, \sigma > 1 \quad (3.2)$$

Each household is endowed with one unit of labor, which is used for production in the two sector of the economy, and a balanced portfolio of all the firms in the economy. Taking this into account the budget constraint of the household can be rewritten as

$$\sum_{i=1}^2 P_i C_i \leq w + \int_0^{M_1} \Pi_{1j} dj + \int_0^{M_2} \Pi_{2j} dj$$

where w is the wage rate of the economy and Π_{ij} are the profits of firm j in sector i .

Intermediate Good Producers

Production Each intermediate good i is produced by a monopolist. Each monopolist has an identical linear production technology satisfying the following equation

$$y_{ij} = q * l_{ij} \quad (3.3)$$

where l_{ij} is the labor employed by firm j in sector i and q is a labor productivity parameter. The production function for the intermediate producers (3.3) implies that the marginal cost of

producing intermediate j in sector i is

$$MC_{ij} = \frac{w}{q}$$

Entry Before any production or consumption takes place a mass 1 of potential intermediate producers have to decide which sector to operate in. I assume that investors make this decision simultaneously and that their objective is to maximize future profits.

Labor Market The intermediate producers' production function implies the following labor market clearing condition:

$$1 = \int_0^{M_1} l_{1j} dj + \int_0^{M_2} l_{2j} dj$$

To close the description of the model I summarize the timing of events:

1. Investors decide which industry to operate in.
2. Given the entry decision, firms set prices and agents decide how much to consume.

3.2.2 Equilibrium

Throughout this section I will use Markov Perfect Equilibrium (MPE) as the equilibrium concept. In particular I will be able to solve the game using backwards induction. Before proceeding to the characterization of the equilibrium I will start by defining an allocation in this economy.

Definition 11 (Allocation) *An allocation in this economy is:*

- i) a pair of numbers (M_1, M_2)*
- ii) a pair of vectors of intermediate productions $(\mathbf{y}_{1j})_{j=0}^{M_1}, (\mathbf{y}_{2j})_{j=0}^{M_2}$*
- iii) a pair of vectors of labor demands $(\mathbf{l}_{1j})_{j=0}^{M_1}, (\mathbf{l}_{2j})_{j=0}^{M_2}$*
- iv) a pair of consumption levels (C_1, C_2)*
- v) a pair of consumption good productions (Y_1, Y_2)*
- vi) and prices $w, P_1, P_2, (P_{1j})_{j=0}^{M_1}, (P_{2j})_{j=0}^{M_2}$*

Next I will define an equilibrium in this economy. At the time of production and consumption all the agents in the economy have full information with respect to the fundamentals A_1, A_2 and the number of producers in each sector M_1, M_2 . This implies that the second stage equilibrium can be described as:

Definition 12 (Second Stage Equilibrium) *An equilibrium in the production and consumption stage is an allocation such that*

- i) (C_1^*, C_2^*) solve the utility maximization problem in (3.1)*
- ii) $(y_{ij}^*)_{j=0}^{M_i}, (P_{ij}^*)_{j=0}^{M_i}$ solves the profit maximization problem of intermediate firm j in sector i .*
- iii) (Y_i^*, P_i^*) solves the profit maximization problem of consumption good producer i and the zero profit condition.*
- iv) there is labor market clearing*
- v) all the goods market clear*

In the Appendix I show that there is a unique second stage equilibrium of the economy and this equilibrium is characterized by the following allocation:

$$\begin{aligned}
 y_{ij}^* &= \frac{\alpha_i q}{M_i}, \forall j & (3.4) \\
 l_{ij}^* &= \frac{\alpha_i}{M_i}, \forall j \\
 Y_i^* &= \alpha_i q M_i^{1/(\sigma-1)} \\
 w^* &= 1 \\
 P_{ij}^* &= \frac{\sigma}{(\sigma-1)q} \\
 P_i^* &= \frac{\sigma M_i^{1/(1-\sigma)}}{(\sigma-1)q}
 \end{aligned}$$

The second stage equilibrium implies that the profits of firm j in industry i are:

$$\Pi_{ij} = \frac{\alpha_i}{M_i (\sigma - 1)} \tag{3.5}$$

where $\alpha_i = A_i/(A_1 + A_2)$. Equation (3.5) will be crucial in the discussion which follows. In particular, it captures the problem that firms face in the first period. On the one hand firms will have an incentive to enter into the sector which yields the highest fundamental profitability of the two, *i.e.* the sector with the highest α_i . On the other hand, higher α_i will attract more entrants into sector i which will decrease the profits of a firm in that sector. For this reason higher fundamental profitability will have a direct positive effect on a firm's profit and an indirect negative effect through the equilibrium number of entrants. The interaction between the fundamental profitability and the number of entrants will be crucial when I analyze entry decisions in the next subsection and specially

in the game with incomplete information. In general the profit function in (3.5) highlights a tension which is important in many economic problems where good fundamentals attracts more economic agents into an economic activity.

This tension between sectoral profitability (or demand elasticity) and entry has been highlighted in contexts similar to the one presented here by Dixit and Stiglitz (1977), Jovanovic (1981), Mankiw and Whinston (1985) and Rob (1987). But the tension implied by equation (3.5) goes beyond this specific economic model and can be framed in many other contexts. To mention some examples of situations where this tension is present we have political candidates choosing platforms based on voter's preferences, firms investing in new technologies based on future demand or an academic choosing a research topic based on the current hot topic. For this reason the theoretical contribution of this paper goes beyond the scope of the specific model presented and highlights a more general economic force.

Equilibrium with Full Information

I start by characterizing the first stage equilibrium of the game when agents have full information of future profitability A_1 and A_2 . First stage strategies (s_i) will be mappings from the pair of sectoral profitabilities $\{A_1, A_2\}$ to the binary decision $\{\text{Enter Sector 1, entering Sector 2}\}$

$$s_j : \{A_1, A_2\} \rightarrow \{\text{Enter Sector 1, Enter Sector 2}\}$$

I assume the entry cost to all industries is zero. If this was not the case we would have three possible actions for each investors, no entry, entry into industry 1, and entry into industry 2. Setting the entry cost to zero will allow me to focus on the main goal of this paper which is to study the entry decision across industries¹. I start by defining an equilibrium of this game.

Definition 13 *An equilibrium of the entry game with full information is a set of strategies $\{s_j^*\}_{j=0}^1$, a set of industry participants, $\{M_i^*\}_{i=1}^2$, and an equilibrium number of opened industries, N^* , such*

¹Jovanovic (1981), Mankiw (1985), Rob (1987) introduce a fixed cost of entry and focus on the binary decision of whether to enter or not. As has been highlighted by Caplin and Leahy (1993), the sectoral entry decision adds important macroeconomic implications.

that the following conditions hold

$$\begin{aligned}\sum_{i=1}^2 M_i^* &= 1 \\ \frac{A_i}{M_i^*} &\geq \frac{A_j}{M_j^*}, \forall i, j \\ N^* &= 2\end{aligned}$$

These conditions are easy to interpret. The third condition tells us that with a continuum of players all industries must be opened. This follows from the fact that if this was not the case, opening a new industry would yield infinite profits to the deviator. The first condition is just the condition that all investors participate in some industry, which by assumption is costless. This will be the case since not entering yields zero profits while entering to one of the two industries will always result in positive profits. The second condition is a condition that no investor wants to move to another industry which is currently in place (No bunching). All these conditions together imply that the equilibrium number of entrants to each sector must satisfy:

$$\frac{A_1}{M_1^*} = \frac{A_2}{M_2^*}$$

Which means that any full information equilibrium of this game must have $M_i^* = \alpha_i = \frac{A_i}{A_1 + A_2}$ $\forall i \in \{1, 2\}$. Clearly there are multiple equilibria to this game since any strategy vector $\{s_j^*\}_{j=0}^1$ which yields $M_i^* = \alpha_i$ is an equilibrium. However, the equilibrium number of entrants to each sector, M_i^* , and the number of opened sectors, N^* , are unique in any equilibrium. All this is summarized in a proposition below.

Proposition 14 (Full Information Equilibrium) *There are multiple equilibria to the full information entry game presented above. In all of these equilibria the equilibrium number of entrants to each sector is uniquely determined by the following expression*

$$M_i^* = \frac{A_i}{A_1 + A_2} = \alpha_i$$

The optimal number of entrants into each industry reflects the underlying tension of the model: higher relative fundamental profitability of the sector implies higher profits and more entry into the sector. In equilibrium these two opposing forces balance out across sectors making entrants indifferent across sectors.

Socially Optimal Entry

So far we have focused on the profit maximizing behavior of entrants. This individually optimal behavior might not necessarily lead to an efficient allocation of resources in the economy. I will try to address this question in what follows.

In the economy described above there are two potential sources of inefficiencies. The first comes from the monopolistic competition which takes place in the second stage and the second potential source of inefficiency comes from the entry decision by investors. The goal of this paper is to understand the second potential source of inefficiency for which reason I will focus on the constrained efficient problem below. The objective function of the social planner is

$$W^S = \max_{M_i} \left\{ \left[\alpha_1 \ln \alpha_1 q M_1^{1/(\sigma-1)} + \alpha_2 \ln \alpha_2 q (1 - M_1)^{1/(\sigma-1)} \right] \right\}$$

where I use the second stage equilibrium consumption, $C_i^* = \alpha_i q M_i^{1/(\sigma-1)}$.

Solving this problem yields the optimal social number of firms:

$$M_1^S = \alpha_1, M_2^S = \alpha_2$$

We can see that the socially optimal number of firms is equivalent to the equilibrium number of firms which implies that equilibrium entry in the game with full information over the fundamental profitability of each sector is constrained efficient. This is summarized in the following proposition.

Proposition 15 (Socially Optimal Entry Level) *The full information equilibrium number of entrants to each sector, $\{M_1^*, M_2^*\}$, is constrained efficient. That is, taken the second stage competition as given, $\{M_1^*, M_2^*\}$ maximize the utility of the representative consumer.*

One question we might ask now is how robust is this result to changes in the information that investors have at the moment of entry. In the next subsection we change the model such that agents have imperfect signals of α_1 and α_2 at the time of entry. Introducing this change in the information structure of the game will yields a set of interesting predictions compared to the full information case. In particular I will show that the equilibria of the game are not necessarily socially efficient and that the level of inefficiency varies with the true value of α_1 and with the precision of the signal that agents get prior to entry.

3.3 The Model with Dispersed Information

In this section I will assume that the information available to investors is imperfect. In particular, I assume that investor's j entry decision is taken before observing the true realizations of A_1, A_2 but after receiving a private signal, x_j , of the relative future profitability of the two sectors which takes the following form:

$$x_j = \widehat{\alpha} + \varepsilon_j$$

where $\widehat{\alpha} = \ln A_1 - \ln A_2$ is the log of the relative profitability of sector 1 with respect to sector 2 and ε_j is a shock which is independent and identically distributed across individuals. For simplicity I assume $\widehat{\alpha}$ to be randomly drawn from the real line (improper prior) and ε_j to be identically and independently distributed across agents with a distribution F , symmetric, with mean zero and variance σ^2 . The improper prior assumption implies that agents will only use information on the signal and the distribution of ε when making forecasts about other agents' signals². Furthermore I assume the following condition holds

$$\begin{aligned} f'(z) &< 0, \forall z > 0 \\ f'(z) &> 0, \forall z < 0 \end{aligned} \tag{3.6}$$

where $f(z) = F'(z)$. Notice that before investor j receives the signals both industries are identical. In this sense we say that we have ex-ante homogeneity across industries.

However, before deciding which industry to enter investors receive a signal about the state of the fundamental profitability of the two sectors. The above discussion implies that the expected profitability difference conditional on the signal is:

$$\widehat{\alpha}|x_j = x_j - \varepsilon_j|x_j$$

To finish the description of the primitives of the model, notice that when forecasting player h 's signal, player j calculates the following distribution

$$x_h|x_j = x_j + \varepsilon_h - \varepsilon_j$$

²A milder assumption would be to have a proper distribution for $\widehat{\alpha}$ which is much more dispersed than the distribution of ε .

I will call the distribution of the difference $\varepsilon_h - \varepsilon_j$ G , which from the properties of F is symmetric and has zero mean. Furthermore, given that (3.6) holds, we have the following condition

$$\begin{aligned} g'(z) &< 0, \forall z > 0 \\ g'(z) &> 0, \forall z < 0 \end{aligned} \tag{3.7}$$

where $g(z) = G'(z)$. Distribution G will determine how informative the individual signals are about other player's actions. This will prove to be essential when characterizing the equilibrium of the game in the next section.

3.3.1 Equilibrium characterization

Before characterizing the equilibrium of the game let's define strategies in the game with dispersed information. A (symmetric) strategy here is a mapping from the signals to the simplex, the probability of entering industry 1 in detriment of industry 2. Hence we have

$$S_j : x_j \rightarrow [0, 1]$$

where x_j is the signal that agent j receives. Notice I am restricting the analysis to symmetric equilibria in which agents with the same signal value play the same strategy, or in other words the identity of the investors is irrelevant, only the signal value matters. Let's define $\widehat{q}_j(x_j)$ to be the probability that player i enters industry 1 under the strategy profile \widehat{S}_i and conversely let $1 - \widehat{q}_j(x_j)$ be the probability that agent i enters industry 2. Define a strategy profile $\widehat{S} = \left(\widehat{S}_i \right)_{i \in [0,1]}$ to be the collection of all strategies from investors. To finish with the description of the game we have to define the payoffs of each investor. We know that once the entry decision has been made the profits on investor j are

$$\Pi_i = \frac{\alpha_i}{(\sigma - 1)M_i}$$

Given the logarithmic preferences of agents, we can focus on the maximization of the expected log differential profits. This differential profit for an agent with signal value x_j given a strategy profile \widehat{S} is:

$$E \left[\Delta\pi | x_j, \widehat{S} \right] = x_j + \ln \left(\frac{1 - M_1(x_j, \widehat{S})}{M_1(x_j, \widehat{S})} \right)$$

where $M_1(x_j, \widehat{S})$ is the expected number of entrants to industry 1 given strategy profile \widehat{S} and a signal value x_j , and $\Delta\pi = \ln \Pi_1 - \ln \Pi_2$. The formulation highlights two important features of the model. The first is that the number of entrants to each industry is an endogenous object which depends on the strategies. This implies that the signal value has two effects in the expected differential profits. The first is the direct effect of signals on profits, a higher signal value makes the expected profits of industry 1 higher. The second effect is the indirect effect which works through the forecast of the mass investors in each industry. In general these two effects will go in different directions as higher entry into a sector yields lower profits for each entrant in that sector.

Next I define an equilibrium of the entry game with imperfect information.

Definition 16 *An equilibrium of the entry game with private signals is a set of strategies $S^* = (q_i^*)_{i \in [0,1]}$ such that for every $i \in [0,1]$*

$$\begin{aligned} q_i^* &= 1 \Leftrightarrow E[\Delta \ln \pi | x_j, S^*] \geq 0 \\ q_i^* &\in (0, 1) \Leftrightarrow E[\Delta \ln \pi | x_j, S^*] = 0 \\ q_i^* &= 0 \Leftrightarrow E[\Delta \ln \pi | x_j, S^*] \leq 0 \end{aligned}$$

and

$$M_1(x_i, S^*) = \int_{-\infty}^{\infty} q^*(y) g(y - x_i) dy$$

Notice that the last line requires investors to make rational forecasts of the number of investors in each industry. Also, the last condition uses the fact that

$$\begin{aligned} \Pr(x_h < y | x_i) &= \Pr(x_i + \varepsilon_h - \varepsilon_j < y | x_i) \\ &= G(y - x_i) \end{aligned}$$

Pure Strategy Equilibrium

I will start by characterizing the pure strategy equilibrium of the game with dispersed information. I choose this starting point for two reasons. The first reason is that it will highlight the properties that $G(\cdot)$ must have such that investors make positive expected profits. The second reason is that for a similar class of games with dispersed information but with strategic complementarity of actions, a pure strategy threshold equilibrium is the unique equilibrium of the game³. This class

³See Morris and Shin (2003) and Angeletos and Pavan (2007).

of games are dominant solvable for which reason the finding the unique equilibrium of the game is straightforward. The entry game presented here is not dominant solvable precisely because of the strategic substitutability in players' actions, making the characterization of the pure strategy equilibrium of the game less straightforward. However, as Proposition 7 shows, if there exists a pure strategy equilibrium of the game it must be a threshold strategy where everyone enters the sector with the highest signal.

Proposition 17 (Pure Strategy Equilibrium) *If there is a pure strategy equilibrium of the game it must be a threshold strategy where everyone enters the market with the highest signal.*

That is

$$q_i^* = \begin{cases} 1 & \text{if } x_i > 0 \\ [0, 1] & \text{if } x_i = 0 \\ 0 & \text{if } x_i < 0 \end{cases}$$

This is an equilibrium of the game if and only if the following condition (condition P)

$$G(z) < \frac{\exp(z)}{1+\exp(z)} \text{ if } z > 0$$

$$G(z) > \frac{\exp(z)}{1+\exp(z)} \text{ if } z < 0$$

holds.

Proof. In the Appendix ■

Condition P gives us the requirement for a pure strategy equilibrium to exist. The economics behind this condition will be important to understand. The existence of a pure strategy equilibrium requires that the direct effect of the signal dominates the indirect effect for every signal level. Condition P suggests that this will occur if the distribution of the difference in shocks is sufficiently dispersed, that is, if signals give entrants little information of the signals of other investors.

So far I have showed that under condition P there exists a pure strategy equilibrium of the game. In what follows I will analyze what will happen when condition P is violated and whether there are other types of equilibria of the game. In particular I will show that when the signals received by investors become more precise the direct and the indirect effect of the signal balance out giving room for mixed strategy equilibria.

Mixed strategies

Now we turn to equilibria where a positive mass of investors randomize over which industry to enter. It will be useful to write the expected number of entrants to industry 1 conditional on a

signal x and strategy profiles \widehat{S} as:

$$\widehat{M}_1(x, \widehat{S}) = \int_{-\infty}^{\infty} \widehat{q}(y)g(y-x)dy.$$

where $\widehat{q}(x)$ is the probability that a player with a signal x enters sector 1 under strategy profile \widehat{S} . From the pure strategy case one can hint that any mixed strategy equilibrium will have a threshold structure, that is there will be an interval of indifference and outside of it investors play a pure strategy entering the sector with the highest signal⁴. For this reason I restrict the analysis to strategies of the following form:

$$q_i(x) = \begin{cases} 0 & \text{if } x < \underline{\Delta} \\ \phi(x) \in [0, 1] & \text{if } x \in [\underline{\Delta}, \overline{\Delta}] \\ 1 & \text{if } x > \overline{\Delta} \end{cases}$$

where $\overline{\Delta} > \underline{\Delta}$. If agents follow strategy $q_i(x)$ then the expected number of investors who enter sector 1 conditional on a signal x is:

$$\begin{aligned} M_1(x, q) &= \int_{\underline{\Delta}}^{\overline{\Delta}} \phi(y)g(y-x)dy + \int_{\overline{\Delta}}^{\infty} g(y-x)dy \\ &= \int_{\underline{\Delta}}^{\overline{\Delta}} \phi(y)g(y-x)dy + (1 - G(\overline{\Delta} - x)) \end{aligned} \quad (3.8)$$

For $q_i(x)$ to be an equilibrium strategy the function $\phi(x)$ has to satisfy certain properties which will be highlighted in the following Proposition.

Proposition 18 (Mixed Strategy Equilibrium) *Assume there exists a function $\phi^*(x) \in [0, 1]$ $\forall x \in (\underline{\Delta}, \overline{\Delta})$, with $\underline{\Delta} > 0 > \overline{\Delta}$, $\phi^*(\overline{\Delta}) = 1$, $\phi^*(\underline{\Delta}) = 0$, $\phi'^*(\overline{\Delta}) = \phi'^*(\underline{\Delta}) = 0$ satisfying*

$$\begin{aligned} \int_{\underline{\Delta}}^{\overline{\Delta}} \phi^*(y)g(y-x)dy &= \frac{e^x}{1+e^x} - G(x-\overline{\Delta}), \forall x \in (\underline{\Delta}, \overline{\Delta}) \\ \int_{\underline{\Delta}}^{\overline{\Delta}} \phi^*(y)g(y-x)dy &\geq \frac{e^x}{1+e^x} - G(x-\overline{\Delta}), \forall x \geq \overline{\Delta} \\ \int_{\underline{\Delta}}^{\overline{\Delta}} \phi^*(y)g(y-x)dy &\leq \frac{e^x}{1+e^x} - G(x-\overline{\Delta}), \forall x \leq \underline{\Delta} \end{aligned} \quad (3.9)$$

⁴In the appendix I show that all mixed strategy equilibria of the game must have this form.

Then $q_i^*(x) = \begin{cases} 0 & \text{if } x < \underline{\Delta} \\ \phi^*(x) \in [0, 1] & \text{if } x \in [\underline{\Delta}, \overline{\Delta}] \\ 1 & \text{if } x > \overline{\Delta} \end{cases}$
is an equilibrium of the game.

Proof. In the Appendix. ■

The pure strategy equilibrium and the mixed strategy equilibria share a common feature. In both cases agents receiving very high (low) signals will enter sector 1 with probability 1. There is also a region of the signal space where the two forces behind a good signal balance out and investors are indifferent between the two sectors. For the pure strategy equilibrium case this occurs at exactly $x = 0$ while for the mixed strategy equilibrium this occurs in a range of signal values⁵.

Further analysis of the mixed strategy equilibrium for any general $g(\cdot)$ is a difficult task which goes beyond the scope of this paper⁶⁷. For this reason in the next section I will study the mixed strategy equilibrium of the game for a particular distribution $g(\cdot)$ which gives a closed form solution for $\phi^*(x)$. This special case will highlight some important characteristics of the model and will allow me to analyze the welfare implications of the model.

3.4 Equilibrium Characterization: Special Case

I will study the equilibrium of the entry game for the following distribution

$$f(x) = \frac{a}{2} e^{-a|x|}, a > 0 \quad (3.10)$$

This probability distribution will prove to be useful in solving explicitly the integral equation in (3.9). The parameter a , which will be crucial in characterizing the equilibrium, will determine the variance of the shock and will also determine the variance of the derived distribution $G(x)$ ⁸. With this distribution we can derive the distribution $g(x)$ using the transformation theorem. The next lemma characterizes the distribution of $\varepsilon_h - \varepsilon_j$ and the properties of this distribution.

Lemma 19 *If $\varepsilon_j, \varepsilon_h$ are each distributed according to (3.10), then the random variable $\varepsilon_j - \varepsilon_h$ has*

⁵As we will see in the next section, this range of indifference will potentially be the entire signal space.

⁶This is true for many commonly used distributions like the normal distribution.

⁷See Polyanin and Manzhirov (2008)

⁸In particular we can see that $E_f(x) = 0$ and $V_f(x) = \frac{2}{a^2}$.

a pdf

$$g(x) = \frac{a}{4}e^{-a|x|} + |x|\frac{a^2}{4}e^{-a|x|}$$

and a CDF

$$G(x) = \begin{cases} \frac{1}{2}e^{ax} \left(1 - x\frac{a}{2}\right) & \text{if } x < 0 \\ 1 - \frac{1}{2}e^{-ax} \left(1 + x\frac{a}{2}\right) & \text{if } x \geq 0 \end{cases}$$

Furthermore

$$\frac{\partial G(x)}{\partial a} > 0 (< 0) \text{ if } x > 0 (< 0)$$

Proof. In the Appendix ■

The properties of the distribution imply that there is a value a^* such that for $a < a^*$ we are in the parameter region where condition P holds. In fact, we can see that a necessary condition for condition P to be satisfied is $g(0) < \tilde{g}(0)$, which implies that $a^* = 1$.

I turn to the characterization of the mixed strategy equilibrium of the game. The next proposition shows that for the distribution of shocks that we have chosen, there exists a function ϕ that solves the integral equation of interest.

Proposition 20 *The strategy*

$$q^*(x) = \phi(x) \tag{3.11}$$

where

$$\phi(x) = \frac{e^x}{a^4(1+e^x)^5} \left(\begin{array}{l} a^4 e^{4x} + (4a^4 + 2a^2 - 1) e^{3x} + \\ (6a^4 + 2a^2 + 11) e^{2x} + \\ (4a^4 - 2a^2 - 11) e^x + (a^4 - 2a^2 + 1) \end{array} \right) \in (0, 1) \quad \forall x \in (-\infty, \infty)$$

is a mixed strategy equilibrium of the game if and only if $a > a^*$.

Furthermore, for $x > 0$, $\frac{\partial \phi(x)}{\partial a} < 0$, and $x < 0$, $\frac{\partial \phi(x)}{\partial a} > 0$

The above proposition shows that in the special case we are considering, the unique continuous and differentiable mixed strategy equilibrium when condition P is violated is $\phi(x)$ ⁹. This suggests that for low levels of a the equilibrium of the game is a pure strategy threshold equilibrium while for higher values of a the equilibrium is the mixed strategy equilibrium characterized above. The above proposition also highlights an important point: the probability that an investors enters the

⁹There may be other non-continuous or non-differentiable function which constitute a mixed strategy equilibrium.

sector for which he has the highest signal is decreasing in the precision of the signal. The above discussion is illustrated in Figure 1 in the appendix which shows how the equilibrium of the game varies as the precision of the signal (a) increases.

The mixed strategy equilibrium presented above relates closely to the previous literature of entry under incomplete information. Previous work by Jovanovic (1981) and Rob (1987), for example, have characterized a mixed strategy equilibrium for similar entry games as the one presented here. The main difference is that both these authors solve their models for a specific level of precision of the signal. This is precisely the main contribution of this paper, to highlight the interaction between agent's strategies and the precision of the signals that agents receive and to show the conditions for a mixed strategy equilibrium to emerge.

Having characterized the equilibrium of the game I turn back to the welfare analysis of the model. Section 2.2 pointed out that the entry game with complete information had constrained efficient levels of entry. The next section will analyze how this result changes once we introduce incomplete information at the entry stage. In particular I will show that the welfare costs of having imperfect signals at the moment of entry will vary with both the level of precision of the signals that agents get and with true level of relative demand of sector 1 with respect to sector 2.

3.4.1 Dispersed Information and Welfare

This section analyzes the effect of having uncertainty at the moment of entry on efficiency. In particular I will focus on how the equilibrium entry level and the optimal entry level differ using the special case for $f(\varepsilon)$ studied above. As I did in section 2.2, I will focus mainly on a constrained efficient analysis where the second stage monopoly power that firms have is taken as given.

Given the assumption that there is a continuum of entrants, the potential inefficiency in the equilibrium with dispersed information lies in the discrepancy between the equilibrium entry level and the social optimal entry level¹⁰. For this reason the welfare analysis of the model boils down to the comparison between the equilibrium entry level into the two sectors and the efficient entry level. As I pointed out previously the optimal entry level to industry i is $\alpha_i = A_i/(A_1 + A_2)$ which is a measure of how much consumers value good i consumption. The equilibrium level of entry to

¹⁰This comes from the fact that there is a continuum of entrants and for that reason the equilibrium entry level is a constant.

sector i on the other hand will be given by the following expression

$$M_1(a, \hat{\alpha}) = \int q^*(x) f(x - \hat{\alpha}, a)$$

where $q^*(x)$ is a threshold strategy for $a < a^*$ and $\phi(x)$ for $a \geq a^*$. I will start by analyzing the welfare properties when the precision of the signal is low ($a < 1$). such that there is a pure strategy threshold equilibrium. The next proposition shows that for a specific combination of $\hat{\alpha}$ and a the equilibrium entry level coincides with the socially efficient entry level.

Proposition 21 *If $a < 1$ the following holds:*

i) the equilibrium entry level is

$$M_1(a, \hat{\alpha}) = \begin{cases} \frac{1}{2} e^{a\hat{\alpha}} & \text{if } \hat{\alpha} < 0 \\ 1 - \frac{1}{2} e^{-a\hat{\alpha}} & \text{if } \hat{\alpha} \geq 0 \end{cases}$$

ii) For each level $\hat{\alpha}$ there exists a unique level of $a, \bar{a}(\hat{\alpha})$, such that $M_1(\bar{a}(\hat{\alpha}), \hat{\alpha}) = M_1^S(\hat{\alpha})$.

Furthermore we have the following

$$\frac{\partial |M_1^S(\hat{\alpha}) - M_1(a, \hat{\alpha})|}{\partial a} < 0 \text{ if } a < \bar{a}(\hat{\alpha}),$$

and

$$\frac{\partial |M_1^S(\hat{\alpha}) - M_1(a, \hat{\alpha})|}{\partial a} > 0 \text{ if } a > \bar{a}(\hat{\alpha})$$

iii) $\frac{\partial \bar{a}(\hat{\alpha})}{\partial \hat{\alpha}} < 0$ if $\hat{\alpha} < 0$, $\frac{\partial \bar{a}(\hat{\alpha})}{\partial \hat{\alpha}} > 0$ if $\hat{\alpha} > 0$

Finally if $\hat{\alpha} = 0$, $M_1^S(0) = M_1(a, 0)$.

The above proposition shows an important result of the paper. When the precision of the signals received by investor's is sufficiently low ($a \in (0, 1)$) there is a U-shaped relation between the absolute difference in the equilibrium number of entrants and the the socially efficient entry level ($|M_1^S(\hat{\alpha}) - M_1(a, \hat{\alpha})|$) and the precision of the signals, a . The proposition also suggests that if the social planner can choose a before entry takes place *and* he knows the value for $\hat{\alpha}$ when choosing a , then social efficient entry level can be achieved. This could arise if we added an extra stage in which the social planner invests in improving information dissemination among investors at some cost. One caveat to this conclusion arises when the social planner does not have full knowledge of $\hat{\alpha}$ at the moment of choosing a , in which case socially optimal entry is achieved with probability

zero. Furthermore, as can be seen from the proposition, the cost of mistargeting a can be very large.

The second thing to notice from the proposition is that uncertainty is inefficient as long as consumers value one sector more than the other. Furthermore, as can be seen in Figures 2 and 3, there is a non-monotonic relation between the discrepancy between the equilibrium and the efficient entry level and the relative demand of sector 1 with respect to sector 2.

Now I turn to the analysis when $a > 1$ and the equilibrium is a mixed strategy equilibrium. As I will show, when the precision is sufficiently high ($a > 1$) there are two cases under which the socially efficient level of entry is equivalent to the equilibrium level of entry: one is when the two sectors are equally profitable ($\hat{\alpha} = 0$) and the other is when the precision is sufficiently high ($a \rightarrow \infty$).

Proposition 22 *If $a \geq 1$ the following holds:*

- i) $\lim_{a \rightarrow \infty} M_1(a, \hat{\alpha}) = M_1^S(\hat{\alpha})$*
- ii) $M_1^S(0) = M_1(a, 0)$*

As was the case when $a < 1$, uncertainty is inefficient to the extent that sectors are not homogeneous. Furthermore, the above proposition suggests that as the signals become very precise, and uncertainty vanishes, the efficient entry level is achieved. Figures 4 and 5 complete the analysis for the case when $a > 1$. The first thing to notice is that for a given value of $\hat{\alpha}$ the discrepancy between the socially efficient entry level and equilibrium entry level is decreasing in a . This implies that when the equilibrium of the entry game is a mixed strategy equilibrium it is always socially beneficial to increase a . This goes in contrast to what occurs when $a < 1$ where small increases in the precision might be welfare detrimental. This suggests that if the social planner was given the choice to increase the precision of the signal before learning the value of $\hat{\alpha}$, he would prefer to set $a > 1$ and as large as possible. Another point to notice is that increasing the precision of the signal is particularly beneficial when α_1 has a value of 0.2 or 0.8. This suggests that the gains from increasing the precision of the signal received by investors are larger when one sector is more profitable than the other but the relative profitability is not too large.

3.5 Conclusion

I have presented a simple model of entry across sectors with incomplete information where I highlight the effect of both the precision of the signals received by agents and the relative demand of

one sector relative to the other on equilibrium entry and welfare. A crucial characteristic of the model presented is the tension between the direct effect of a high signal received by agents, a high relative demand, and the indirect effect of receiving a high signal, higher entry. The equilibrium of the game is driven by this tension and in particular by the precision of the signals received by agents. I show that when the precision of the signal is low, the direct effect of the signal dominates the indirect effect and the equilibrium of the game is a pure strategy equilibrium in which agents enter to the market for which they receive a higher signal. This is a consequence of the fact that when the precision is low an agent's signal is not too informative of other agents' signals. As the precision increases, signals become more informative and the indirect effect of signals becomes stronger. This gives place to the emergence of a mixed strategy equilibrium where agents randomize over the two industries.

The model presented also sheds light on the effect of dispersed information and information quality on competition, generated by agents' entry decisions, and efficiency. I show that the difference between the socially efficient level of entry and the equilibrium level of entry depends crucially on the precision of the signal and the true relative demand value. In particular I show that the effect of the signal's precision on welfare is non-monotonic. For low levels of precision there is a U-shaped relation in the discrepancy between equilibrium entry and the socially efficient level. Furthermore, there is a unique precision level for which the two entry levels coincide. Once precision increases, and the equilibrium of the game is a mixed strategy equilibrium, the difference in entry levels is decreasing in the precision level. This implies that when the precision level is low, small increases in precision might be welfare decreasing. The model finally shows that the relation between welfare and relative demand depends crucially on the precision level and is in general non-monotonic.

I conclude by discussing a couple of possible extensions to the model presented in this paper which could be interesting to explore in future work. First, the model presented studies a static entry game. One potential change to this assumption is to have incumbents making production and pricing decisions prior to the entry decision by investors. If this is the case the incumbents' actions become public signals of the sectoral demand and directly affect the entry decision by new firms¹¹. Adding this extra stage could add interesting implications to the welfare analysis (namely first period welfare and second period welfare) and the dynamic pricing decision of incumbents.

¹¹Harrington (1987) studies a similar idea in a linear demand/Cournot environment where firms face uncertainty of the cost of production and the elasticity of demand is known. Studying this case in a context as the one presented here could lead to a richer set of predictions and highlight the interaction of demand elasticity, entry deterrence and welfare.

Moreover, if we assume that there is learning in the production function the entry deterrence motive of incumbents can have important implications for cost reduction and growth. Second, the model assumes that the quality of the signals received by agents is exogenous. Furthermore, the paper emphasizes how the quality of the signals affects the equilibrium entry level in the economy. For this reason one interesting extension would be to endogenize the precision level and understand the private incentives of entrants to invest in higher quality signals. Related to this issue is the cost of information. In the model discussed here I have assumed that signals are costless. Analyzing a model with costly information can lead to interesting connections between credit constraints, entry and sectoral development. Finally, throughout the paper I have used specific functional forms which have allowed me to solve explicitly for entry levels and welfare. One important direction for future work is to generalize the present context and study if the results obtained hold under a more general framework.

3.6 References

- Angeletos, M. and Pavan, A. (2007). "Efficient use of Information and Social Value of Information." *Econometrica*, vol. 75, No.4.
- Bala. V. and Goyal. S (1994). "The birth of a new market." *The Economic Journal*, vol. 104, No. 423, pp. 282-290.
- Caplin, A. and Leahy, J. (1993). "Sectoral Shocks, Learning and Aggregate Fluctuations." *Review of Economic Studies*, vol. 60, No. 4, pp. 777-794.
- Dixit, A. and Stiglitz, J. (1977). "Monopolistic Competition and Optimum Product Diversity." *The American Economic Review*, vol. 67, No. 3, pp. 297-38.
- Harrington, J. (1987) "Entry Deterrence under Incomplete Information" *The RAND Journal of Economics*, vol. 18, No.2, pp.211-231.
- Jovanovic, B. (1981). "Entry with private information." *The Bell Journal of Economics*, vol. 12, pp. 649-660.
- Mankiw, G. and Whinston, M. (1986). "Free Entry and Social Inefficiency." *The RAND Journal of Economics*, vol. 17, No. 1, pp. 48-58.

- Morris, S. and Shin, H. (2003). "Global Games - Theory and Applications." in Advances in Economics and Econometrics, 8th World Congress of Econometric Society (M. Dewatripont, L. Hansen, and S Turnivsky, eds.), Cambridge University Press, Cambridge, Uk.
- Polyanin, A.D. and Manzhirov, A.V. (2008). "Handbook of Integral Equations." Second Edition, published by Chapman and Hall/CRC Press.
- Presscot, E. and Visscher, E. (1977). "Sequential Location among Firms with Foresight." The Bell Journal of Economics, vol. 8, pp. 378-393.
- Rob, R. (1986). "Entry, Fixed Costs and the Aggregation of Private Information." Review of Economic Studies, vol. 54, pp. 619-630.

3.7 Appendix 1: Proofs

3.7.1 Second Stage Equilibrium of the Game

From the log preferences of the consumer we have that demand for the consumption good i is:

$$P_i C_i = \frac{A_i}{A_1 + A_2} I, \forall i$$

Each consumption good producer will maximize profits, which gives the following intermediate good demand:

$$y_{ij} = Y_i \left(\frac{P_i}{P_{ij}} \right)^\sigma$$

The zero profits condition of the final good producer implies that

$$P_i = \left(\int_{j=1}^{M_i} P_{ij}^{1-\sigma} dj \right)^{1/(1-\sigma)}$$

which is also the marginal cost of producing one unit of consumption good i . Taking these demands into account, we know the intermediate good's profit is:

$$\pi_{ij} = Y_i P_i^\sigma \left(P_{ij}^{1-\sigma} - P_{ij}^{-\sigma} \frac{w}{q} \right)$$

which implies that the optimal price is

$$P_{ij}^* = \frac{\sigma}{\sigma - 1} \frac{w}{q}$$

Using the optimal price for each intermediate producer implies that the marginal cost of consumption good i will be

$$\begin{aligned} P_i^* &= \left(\int_{j=1}^{M_i} P_{ij}^{*1-\sigma} dj \right)^{1/(1-\sigma)} \\ &= P_{ij}^* M_i^{1/(1-\sigma)} = \frac{\sigma}{\sigma - 1} \frac{w}{q} M_i^{1/(1-\sigma)} \end{aligned}$$

Taking this into account we have that the profits of intermediate producer j in sector i will be

$$\pi_{ij}^* = \frac{Y_i M_i^{\sigma/(1-\sigma)} w}{q(\sigma - 1)}$$

We also know that in equilibrium we must have that there is market clearing which implies $C_i = Y_i$. Using the demand for the consumption goods we have:

$$\begin{aligned} y_{ij}^* &= \frac{\alpha_i q (\sigma - 1)}{\sigma w M_i} \left(w^* + \int \pi_{ij}^* \right) \\ l_{ij}^* &= \frac{\alpha_i (\sigma - 1)}{\sigma w M_i} \left(w^* + \int \pi_{ij}^* \right) \\ \pi_{ij}^* &= \frac{\alpha_i}{M_i} \left(w^* + \int \pi_{ij}^* \right) \end{aligned}$$

where $\alpha_i = A_i / (A_1 + A_2)$. We also know in equilibrium we must labor market clearing which implies

$$1 = M_1 l_1^* + M_2 l_2^*$$

where $l_i^* = \int_0^{M_i} l_{ij}^* dj$. Now, normalizing the wage rate to 1 and using the labor market clearing condition we have:

$$\int \pi_{ij}^* dj = \frac{1}{\sigma - 1}$$

Putting all this together we have that the equilibrium allocation in the second stage is:

$$\begin{aligned} y_{ij}^* &= \frac{\alpha_i q}{M_i}, \forall j \\ l_{ij}^* &= \frac{\alpha_i}{M_i}, \forall j \\ Y_i^* &= \alpha_i q M_i^{1/(\sigma-1)} \\ w^* &= 1 \\ P_{ij}^* &= \frac{\sigma}{\sigma - 1} \frac{w}{q} \\ P_i^* &= M^{1/(1-\sigma)} \frac{\sigma}{\sigma - 1} \frac{w}{q} \end{aligned}$$

3.7.2 Proof of Proposition 7

I proof proposition 7 in parts. I start by proving the following lemma.

Lemma 23 *In any pure strategy equilibrium where $q^*(x) = 1$ we must have that for $\varepsilon \rightarrow 0$, either $q^*(x + \varepsilon) = 1$ or $q^*(x - \varepsilon) = 1$.*

Proof. If we assume that this is not the case, then we would have

$$\begin{aligned} x + \varepsilon + \ln \left(\frac{1 - M_1(x + \varepsilon)}{M_1(x + \varepsilon)} \right) &< 0 \\ x - \varepsilon + \ln \left(\frac{1 - M_1(x - \varepsilon)}{M_1(x - \varepsilon)} \right) &< 0 \end{aligned}$$

$$2x + \ln \left(\frac{1 - M_1(x + \varepsilon)}{M_1(x + \varepsilon)} \right) + \ln \left(\frac{1 - M_1(x - \varepsilon)}{M_1(x - \varepsilon)} \right) < 0$$

If we take the limit when $\varepsilon \rightarrow 0$ of the expression above we have

$$x + \ln \left(\frac{1 - M_1(x)}{M_1(x)} \right) < 0$$

which violates the fact that $q^*(x) = 1$. ■

Then converse argument follows for the case when $q^*(x) = 0$. This implies that any pure strategy equilibrium of the game will have intervals where investors play the same strategy.

Next I show that any pure strategy equilibrium must be a threshold equilibrium.

Proposition 24 *Any pure strategy equilibrium must be a threshold equilibrium.*

Proof. From lemma 1 we know that the pure strategy equilibrium of the game will have intervals of investors playing the same strategy. This implies that

$$\begin{aligned} M_1(x) &= \int_{-\infty}^{\infty} q^*(y)g(y-x)dy \\ &= 1 - G(x_{n+1} - x) + (G(x_n - x) - G(x_{n-1} - x)) + \\ &\quad + (G(x_{n-2} - x) - G(x_{n-3} - x)) + \dots + G(x_1 - x) \end{aligned}$$

where $x_1 \leq x_2 \leq \dots \leq x_n \leq x_{n+1}$.

Given the assumption that $g(\cdot)$ is single peaked, we know that at most $M_1(x)$ will also be single peaked.. That is, either $M_1'(x) \geq 0 \forall x$, $M_1'(x) \leq 0 \forall x$, or there will be a unique finite x such that $M_1'(x) = 0$. Given this we have that all equilibria of the game have either two or three intervals. It is straightforward to see that a strategy where every player enters one industry is not an equilibrium. To see why there can't be three

intervals notice this would imply that either

$$M_1(x) = G(x_2 - x) - G(x_1 - x) \quad (3.12)$$

or

$$M_1(x) = G(x_1 - x) + 1 - G(x_2 - x) \quad (3.13)$$

Notice however that in (3.12) we must have

$$x_2 + \ln \left(\frac{\frac{1}{2} + G(x_1 - x_2)}{\frac{1}{2} - G(x_1 - x_2)} \right) = 0$$

which only holds if $x_2 < 0$ and hence $q^*(0) = 0$. But when $0 > x_2 > x_1$ we have that,

$$\ln \left(\frac{1 - M_1(0)}{M_1(0)} \right) > 0$$

which contradicts the assumption that $q^*(0) = 0$.

Now take (3.13), then we must have

$$x_1 + \ln \left(\frac{G(x_2 - x_1) - \frac{1}{2}}{\frac{3}{2} - G(x_2 - x_1)} \right) = 0$$

But notice that since $\frac{3}{2} - G(x_2 - x_1) > G(x_2 - x_1) - \frac{1}{2}$, we must have $x_2 > x_1 > 0$ which implies that $q^*(0) = 1$

However, in this case we have that

$$\ln \left(\frac{G(x_2) - G(x_1)}{1 - (G(x_2) - G(x_1))} \right) < 0$$

which contradict the assumption that $q^*(0) = 1$. ■

The above proposition states that we only have to consider threshold pure strategies as the potential equilibria of the game. One natural thing to predict is that under certain conditions the equilibrium of the game will have a threshold type strategy in which everyone follows their highest signal. It will turn out that the condition we need, to have a threshold pure strategy equilibrium, is:

$$\begin{cases} G(z) < \tilde{G}(z) & \text{if } z > 0 \\ G(z) > \tilde{G}(z) & \text{if } z < 0 \end{cases} \quad (3.14)$$

where

$$\tilde{G}(z) = \frac{\exp(z)}{1 + \exp(z)}$$

Finally I show that the threshold must be at 0 and that condition P must be satisfied.

Proof. From the above proposition we know that the only type of pure strategy equilibria of the game will be threshold strategy equilibria. First take the threshold strategies where agents with signal $x < k$ enter industry 1.. In this case we will have that the share of people in industry 1 is

$$\begin{aligned} \Pr(x_h < k|x_j) &= \Pr(x_j - \varepsilon_j + \varepsilon_h < k|x_j) \\ &= 1 - G(x_j - k) \end{aligned}$$

In this case the indifference condition for an individual with signal k is

$$k + \ln(1) = 0 \Leftrightarrow k = 0$$

But if the threshold is at $k = 0$, notice that for $x_j > 0$

$$\Delta\pi(x) = x + \ln\left(\frac{G(x)}{1 - G(x)}\right) > 0$$

which contradicts the argument that this is an equilibrium.

Now take the following proposed threshold strategy equilibrium where every one with signal difference $\Delta x = x_{1i} - x_{2i}$ enters industry 1 if $\Delta x > k$ and enters industry 2 otherwise. Notice that given the proposed equilibrium, the number of investors in industry 1 predicted by an investor with signal Δx is

$$\begin{aligned} \Pr(x_h > k|x_j) &= \Pr(x_j - \varepsilon_j + \varepsilon_h > k|x_j) \\ &= G(x - k) \end{aligned}$$

Given this, the differential payoff between following the strategy and deviating for an investor with signal difference Δx is

$$\Delta\pi(x) = x + \ln\left(\frac{1 - G(x - k)}{G(x - k)}\right)$$

For this to be an equilibrium we must have

$$\Delta\pi(k) = k + \ln\left(\frac{1 - G(0)}{G(0)}\right) = k = 0$$

Hence the only candidate threshold equilibrium of the game is the one where $k = 0$. Now, for this to be an equilibrium we need that for $\Delta x > 0$ the following holds

$$\Delta\pi(x) = x + \ln\left(\frac{1 - G(x)}{G(x)}\right) > 0 \iff G(x) < \frac{\exp(x)}{1 + \exp(x)}$$

and for $\Delta x < 0$

$$\Delta\pi(\Delta x) = x + \ln\left(\frac{1 - G(x)}{G(x)}\right) < 0 \iff G(x) < \frac{\exp(x)}{1 + \exp(x)}$$

which completes the proof. ■

Lemma 25 *The mixed strategy equilibrium can not be*

$$\widehat{q}_i(x) = \begin{cases} 1 & \text{if } x < \underline{\Delta} \\ \widehat{\phi}(\Delta x) \in [0, 1] & \text{if } x \in [\underline{\Delta}, \overline{\Delta}] \\ 0 & \text{if } x > \overline{\Delta} \end{cases}$$

Proof. Suppose the above structure is an equilibrium. This implies

$$\begin{aligned} M_1(x, \widehat{S}) &= \int_{-\infty}^{\underline{\Delta}} g(y - x) dy + \int_{\underline{\Delta}}^{\overline{\Delta}} \widehat{\phi}(y) g(y - x) dy \\ &= G(\underline{\Delta} - x) + \int_{\underline{\Delta}}^{\overline{\Delta}} \widehat{\phi}(y) g(y - x) dy < G(\overline{\Delta} - x) \end{aligned}$$

Notice that $M_1(\infty) = 0$ and $M_1(-\infty) = 1$, which contradicts the argument that $q^*(\infty) = 0$ is a best response for an agent with a signal $x \rightarrow \infty$ ■

This condition arises for the same reason as in the pure strategy case. For investors to stop entering the industry where they expect to have the higher demand it must be that the mass of investors entering that industry has to be sufficiently large, but this can not arise as an equilibrium..

3.7.3 Proof of Proposition 8

Proof. By the discussion of the pure strategy equilibrium we know that any candidate mixed strategy equilibrium must be of the form

$$\widehat{q}_i(x) = \begin{cases} 0 & \text{if } x < \underline{\Delta} \\ h(x) & \text{if } x \in [\underline{\Delta}, \overline{\Delta}] \\ 1 & \text{if } x > \overline{\Delta} \end{cases}$$

Notice that for the agents with signals $x \in (\underline{\Delta}, \overline{\Delta})$ the following indifference condition must hold:

$$\Delta\pi(x) = x + \ln\left(\frac{1 - M_1(x, \widehat{q})}{M_1(x, \widehat{q})}\right) = 0$$

where $M_1(x, \widehat{q})$ is defined in (3.8). The indifference condition implies

$$M_1(x) = \frac{e^x}{1 + e^x}$$

which using the definition of $M_1(x, \widehat{q})$ implies

$$\int_{\underline{\Delta}}^{\overline{\Delta}} h(y)g(y-x)dy + (1 - G(\overline{\Delta} - x)) = \frac{e^x}{1 + e^x}, \forall x \in [\underline{\Delta}, \overline{\Delta}] \quad (3.15)$$

We must have $\overline{\Delta} > 0 > \underline{\Delta}$. To see this notice that if $\overline{\Delta} > \underline{\Delta} > 0$, this would imply that

$$M_1(x) < 1 - G(\underline{\Delta}) < \frac{1}{2}$$

which contradicts the fact that $q^*(x) = 0$. A similar argument can be made if $\overline{\Delta} < \underline{\Delta} < 0$ since in this case $1 - G(\overline{\Delta}) > \frac{1}{2}$.

Differentiating (3.15) with respect to $\overline{\Delta}, \underline{\Delta}$ we obtain the following two conditions

$$\begin{aligned} (h(\overline{\Delta}) - 1)g(\overline{\Delta} - x) &= 0 \\ -h(\underline{\Delta})g(\underline{\Delta} - x) &= 0 \end{aligned} \quad (3.16)$$

which are satisfied if and only if $h(\overline{\Delta}) = 1$ and $h(\underline{\Delta}) = 0$.

Differentiating (3.15) with respect to $\bar{\Delta}, \underline{\Delta}$ a second time we obtain the following two conditions

$$\begin{aligned} h'(\bar{\Delta})g(\bar{\Delta} - x) + (h(\bar{\Delta}) - 1)g'(\bar{\Delta} - x) &= 0 \Rightarrow h'(\bar{\Delta}) = 0 \\ -h'(\underline{\Delta})g(\underline{\Delta} - x) - h(\underline{\Delta})g'(\underline{\Delta} - x) &= 0 \Rightarrow h'(\underline{\Delta}) = 0 \end{aligned}$$

Now, an agent with $x > \bar{\Delta}$ wants to enter industry 1 (2) if the following condition is met

$$\pi(\Delta x) = x + \ln \left(\frac{1 - M_1(x, \hat{q})}{M_1(x, \hat{q})} \right) \geq (\leq) 0$$

This condition implies that the following inequality must be satisfied:

$$\int_{\underline{\Delta}}^{\bar{\Delta}} h(y)g(y - x)dy \leq (\geq) \frac{e^x}{1 + e^x} - G(x - \bar{\Delta}), \forall \Delta x \in [\bar{\Delta}, \infty]$$

■

3.7.4 Proof of Lemma 9

Lemma 26 *If ε_j is distributed (3.10) the random variable $\varepsilon_j - \varepsilon_h$ has a pdf*

$$g(x) = \frac{a}{4}e^{-a|x|} + |x|\frac{a^2}{4}e^{-a|x|}$$

and a CDF

$$G(x) = \begin{cases} \frac{1}{2}e^{ax} \left(1 - x\frac{a}{2}\right) & \text{if } x < 0 \\ 1 - \frac{1}{2}e^{-ax} \left(1 + x\frac{a}{2}\right) & \text{if } x \geq 0 \end{cases}$$

Furthermore

$$\frac{\partial G(x)}{\partial a} > 0 (< 0) \text{ if } x > 0 (< 0)$$

Proof. First we know that $g(x)$ has mean zero since $E(\varepsilon_j - \varepsilon_h) = 0$ and it has variance $4/a^2$. So clearly a determines the dispersion of the distribution $G(x)$. Define the variables $u = h(x, y) = x - y, v = x$. We can rewrite everything in terms of v, u as: $x = v, y = v - u$. The Jacobian of the transformation is

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} = 1$$

Hence we have that the joint density of u and v is:

$$\begin{aligned}
h_{u,v}(u,v) &= f(v)f(v-u) \\
&= \frac{a^2}{4}e^{-a(|v|+|v-u|)}
\end{aligned}$$

Finally we want to find the distribution of the random variable u . We can find this by integrating over

$$\begin{aligned}
(u > 0) \int_{-\infty}^{\infty} \frac{a^2}{4}e^{-a(|v|+|v-u|)} dv &= \\
&= \int_{-\infty}^0 \frac{a^2}{4}e^{a(2v-u)} dv + \int_0^u \frac{a^2}{4}e^{-au} dv + \int_u^{\infty} \frac{a^2}{4}e^{-a(2v-u)} dv \\
&= \frac{a}{4}e^{-au} + u \frac{a^2}{4}e^{-au}
\end{aligned}$$

$$\begin{aligned}
(u < 0) \int_{-\infty}^{\infty} \frac{a^2}{4}e^{-a(|v|+|v-u|)} dv &= \\
&= \int_{-\infty}^u \frac{a^2}{4}e^{a(2v-u)} dv + \int_u^0 \frac{a^2}{4}e^{au} dv + \int_0^{\infty} \frac{a^2}{4}e^{-a(2v-u)} dv \\
&= \frac{a}{4}e^{au} - u \frac{a^2}{4}e^{au}
\end{aligned}$$

This can be rewritten as

$$g(x) = \frac{a}{4}e^{-a|x|} + |x| \frac{a^2}{4}e^{-a|x|}$$

which is precisely the claim. Now we want to find the CDF of g .

$$\begin{aligned}
(x < 0) \int_{-\infty}^x \left(\frac{a}{4}e^{-a|u|} + |u| \frac{a^2}{4}e^{-a|u|} \right) du &= \int_{-\infty}^x \left(\frac{a}{4}e^{au} - u \frac{a^2}{4}e^{au} \right) du \\
&= \frac{1}{4}e^{ax} - \int_{-\infty}^x u \frac{a^2}{4}e^{au} du \\
&= \frac{1}{4}e^{ax} - \left(x \frac{a}{4}e^{ax} - \int_{-\infty}^x \frac{a}{4}e^{au} du \right) \\
&= \frac{1}{2}e^{ax} \left(1 - x \frac{a}{2} \right)
\end{aligned}$$

$$\begin{aligned}
(x > 0) \quad & \frac{1}{2} + \int_0^x \left(\frac{a}{4} e^{-au} + u \frac{a^2}{4} e^{-au} \right) du = \\
& = \frac{1}{2} - \frac{1}{4} (e^{-ax} - 1) + \int_0^x u \frac{a^2}{4} e^{-au} du \\
& = \frac{1}{2} - \frac{1}{4} (e^{-ax} - 1) + \left(-x \frac{a}{4} e^{-ax} + \int_0^x \frac{a}{4} e^{-au} du \right) \\
& = \frac{3}{4} - \frac{1}{4} e^{-ax} - x \frac{a}{4} e^{-ax} - \frac{1}{4} (e^{-ax} - 1) \\
& = 1 - \frac{1}{2} e^{-ax} \left(1 + x \frac{a}{2} \right)
\end{aligned}$$

Hence we have that

$$G(x) = \begin{cases} \frac{1}{2} e^{ax} \left(1 - x \frac{a}{2} \right) & \text{if } x < 0 \\ 1 - \frac{1}{2} e^{-ax} \left(1 + x \frac{a}{2} \right) & \text{if } x \geq 0 \end{cases}$$

Notice that when $x < 0$ we have

$$\frac{\partial}{\partial a} \frac{1}{2} e^{ax} \left(1 - x \frac{a}{2} \right) = \frac{x}{2} e^{ax} - \frac{a}{4} e^{ax} - \frac{a}{4} e^{ax} x^2 < 0$$

and for $x > 0$ we have

$$\begin{aligned}
\frac{\partial}{\partial a} \left(1 - \frac{1}{2} e^{-ax} \left(1 + x \frac{a}{2} \right) \right) & = \frac{1}{2} x e^{-ax} - \frac{x}{4} e^{-ax} + x^2 \frac{a}{4} e^{-ax} \\
& = \frac{1}{4} x e^{-ax} + x^2 \frac{a}{4} e^{-ax} > 0
\end{aligned}$$

To finish notice that

$$G(x) = 1 - G(-x)$$

■

3.7.5 Proof of Proposition 10

Lemma 27 *The function*

$$\phi(x) = \frac{e^x}{a^4 (1 + e^x)^5} \begin{pmatrix} a^4 e^{4x} + (4a^4 + 2a^2 - 1) e^{3x} + \\ (6a^4 + 2a^2 + 11) e^{2x} + \\ (4a^4 - 2a^2 - 11) e^x + (a^4 - 2a^2 + 1) \end{pmatrix}$$

satisfies the integral equation

$$\int_{\underline{\Delta}}^{\overline{\Delta}} \phi(y) \left(\frac{a}{4} e^{-a|y-x|} + |y-x| \frac{a^2}{4} e^{-a|y-x|} \right) dy = \frac{e^x}{1+e^x} - (1 - G(\overline{\Delta} - x)), \forall x \in (\underline{\Delta}, \overline{\Delta}) \quad (3.17)$$

for any $\underline{\Delta}, \overline{\Delta}$. The equation satisfies $\phi(x) = 1 - \phi(-x)$, $f(0) = \frac{1}{2}$, $\phi'(0) > 0$, $\lim_{x \rightarrow \infty} \phi(x) = 1$, $\lim_{x \rightarrow -\infty} \phi(x) = 0$.

Proof. We start by solving the integral equation (3.17). This can be rewritten as:

$$\begin{aligned} & \int_{\underline{\Delta}}^x \phi(y) \left(\frac{a}{4} e^{-a(x-y)} + (x-y) \frac{a^2}{4} e^{-a(x-y)} \right) dy \\ & + \int_x^{\overline{\Delta}} \phi(y) \left(\frac{a}{4} e^{-a(y-x)} + (y-x) \frac{a^2}{4} e^{-a(y-x)} \right) dy \\ & = \frac{e^x}{1+e^x} - G(x - \overline{\Delta}) \end{aligned}$$

Differentiating with respect to x we get:

$$\begin{aligned} & \int_{\underline{\Delta}}^x \phi(y) \left(-(x-y) \frac{a^3}{4} e^{-a(x-y)} \right) dy + \int_x^{\overline{\Delta}} \phi(y) \left((y-x) \frac{a^3}{4} e^{-a(y-x)} \right) dy \\ & = \frac{e^x}{(1+e^x)^2} - g(x - \overline{\Delta}) \end{aligned}$$

Differentiating a second time with respect to x we get

$$\begin{aligned} & \int_{\underline{\Delta}}^x \phi(y) \left(-\frac{a^3}{4} e^{-a(x-y)} + (x-y) \frac{a^4}{4} e^{-a(x-y)} \right) dy + \\ & + \int_x^{\overline{\Delta}} \phi(y) \left((y-x) \frac{a^4}{4} e^{-a(y-x)} - \frac{a^3}{4} e^{-a(y-x)} \right) dy \\ & = e^x \frac{(1-e^x)}{(1+e^x)^3} - g'(x - \overline{\Delta}) \end{aligned}$$

From the original integral equation we have that:

$$a^2 \int_{\underline{\Delta}}^{\overline{\Delta}} \phi(y) \left(|y-x| \frac{a^2}{4} e^{-a|y-x|} \right) dy = a^2 \frac{e^x}{1+e^x} - a^2 G(x - \overline{\Delta}) - a^2 \int_{\underline{\Delta}}^{\overline{\Delta}} \phi(y) \frac{a}{4} e^{-a|y-x|} dy$$

So we can rewrite the second derivative as:

$$\int_{\underline{\Delta}}^x \phi(y) \frac{a}{4} e^{-a(x-y)} dy + \int_x^{\bar{\Delta}} \phi(y) \frac{a}{4} e^{-a(y-x)} dy = \frac{1}{2} \left(\frac{\frac{e^x}{1+e^x} - G(x - \bar{\Delta})}{-\frac{e^x(1-e^x)}{a^2(1+e^x)^3} + \frac{g'(x-\bar{\Delta})}{a^2}} \right) \quad (3.18)$$

which is again an integral equation. Differentiating this with respect to x we have:

$$\begin{aligned} & - \int_{\underline{\Delta}}^x \phi(y) \frac{a^2}{4} e^{-a(x-y)} dy + \int_x^{\bar{\Delta}} \phi(y) \frac{a^2}{4} e^{-a(y-x)} dy \\ &= \frac{e^x}{2a^2(1+e^x)^2} \left(a^2 - \frac{(1-4e^x+e^{2x})}{(1+e^x)^2} \right) - \frac{1}{2} \left(g(x - \bar{\Delta}) - \frac{g''(x - \bar{\Delta})}{a^2} \right) \end{aligned}$$

and finally differentiating a second time we get:

$$\begin{aligned} & -\frac{a^2}{2} \phi(x) + a^2 \int_{\underline{\Delta}}^{\bar{\Delta}} \phi(y) \frac{a}{4} e^{-a|x-y|} dy \\ &= \frac{e^x(1-e^x)}{2a^2(1+e^x)^5} \left(\left(a^2(1+e^x)^2 - (1-10e^x+e^{2x}) \right) \right) - \\ & -\frac{1}{2} \left(g'(x - \bar{\Delta}) - \frac{g'''(x - \bar{\Delta})}{a^2} \right) \end{aligned}$$

Replacing with (??) and solving for $f(x)$ we have

$$\begin{aligned} \phi(x) &= \frac{e^x}{a^4(1+e^x)^5} \left(\begin{array}{l} a^4 e^{4x} + (4a^4 + 2a^2 - 1) e^{3x} + \\ (6a^4 + 2a^2 + 11) e^{2x} + \\ (4a^4 - 2a^2 - 11) e^x + (a^4 - 2a^2 + 1) \end{array} \right) + \\ & + 2 \frac{g'(x - \bar{\Delta})}{a^2} - \frac{g'''(x - \bar{\Delta})}{a^4} - G(x - \bar{\Delta}) \end{aligned}$$

Notice that

$$\begin{aligned} G(x - \bar{\Delta}) &= \frac{1}{2} e^{a(x-\bar{\Delta})} - (x - \bar{\Delta}) \frac{a}{4} e^{a(x-\bar{\Delta})} \\ g'(x - \bar{\Delta}) &= -(x - \bar{\Delta}) \frac{a^3}{4} e^{a(x-\bar{\Delta})} \\ g'''(x - \bar{\Delta}) &= -\frac{a^4}{2} e^{a(x-\bar{\Delta})} - (x - \bar{\Delta}) \frac{a^5}{4} e^{a(x-\bar{\Delta})} \end{aligned}$$

This implies that $2\frac{g'(x-\bar{\Delta})}{a^2} - \frac{g'''(x-\bar{\Delta})}{a^4} - G(x-\bar{\Delta}) = 0$. Putting this together gives the solution to the integral equation

$$\phi(x) = \frac{e^x}{a^4(1+e^x)^5} \begin{pmatrix} a^4 e^{4x} + (4a^4 + 2a^2 - 1) e^{3x} + \\ (6a^4 + 2a^2 + 11) e^{2x} + \\ (4a^4 - 2a^2 - 11) e^x + (a^4 - 2a^2 + 1) \end{pmatrix}$$

We have that $\phi(x) \in (0, 1) \forall x$ if $a \geq 1$. This implies that for $a < 1$ the equilibrium is a pure strategy equilibrium.

Notice that

$$\begin{aligned} 1 - \phi(-x) &= 1 - \frac{e^{-x}}{a^4(1+e^{-x})^5} \begin{pmatrix} a^4 e^{-4x} + (4a^4 + 2a^2 - 1) e^{-3x} + \\ + (6a^4 + 2a^2 + 11) e^{-2x} + (4a^4 - 2a^2 - 11) e^{-x} \\ + (a^4 - 2a^2 + 1) \end{pmatrix} \\ &= 1 - \frac{1}{a^4(1+e^x)^5} \begin{pmatrix} a^4 + (4a^4 + 2a^2 - 1) e^x + (6a^4 + 2a^2 + 11) e^{2x} + \\ + (4a^4 - 2a^2 - 11) e^{3x} + (a^4 - 2a^2 + 1) e^{4x} \end{pmatrix} \\ &= \frac{e^x}{a^4(1+e^x)^5} \begin{pmatrix} a^4 e^{4x} + (4a^4 + 2a^2 - 1) e^{3x} + \\ (6a^4 + 2a^2 + 11) e^{2x} + \\ (4a^4 - 2a^2 - 11) e^x + (a^4 - 2a^2 + 1) \end{pmatrix} = \phi(x) \end{aligned}$$

We can see that for $a > 1$ we have

$$\frac{\partial \phi(x)}{\partial a} = -\frac{e^x(4(a^2 - 1)(e^{3x} - 1) + e^x(4a^2 + 44)(e^x - 1))}{a^5(1+e^x)^5} < 0$$

We can also check that:

$$\begin{aligned} \phi(0) &= \frac{1}{32a^4} \begin{pmatrix} (a^4 - 2a^2 + 1) + (4a^4 - 2a^2 - 11) + \\ (6a^4 + 2a^2 + 11) e^{2\Delta x} + a^4 + (4a^4 + 2a^2 - 1) \end{pmatrix} \\ &= \frac{1}{2} \end{aligned}$$

To finish the proof, notice that

$$\phi'(x) = \frac{e^x}{a^4(1+e^x)^6} \begin{pmatrix} (a^4 - 2a^2 + 1) e^{4x} + (4a^4 + 4a^2 - 26) e^{3x} + \\ (6a^4 + 12a^2 + 66) e^{2x} + (4a^4 + 4a^2 - 26) e^x + (a^4 - 2a^2 + 1) \end{pmatrix}$$

and that

$$\phi'(0) = \frac{1}{4a^4} (a^4 + a^2 + 1) > 0$$

$$\lim_{\Delta x \rightarrow -\infty} \phi(x) = \frac{0}{a^4} ((a^4 - 2a^2 + 1)) = 0$$

$$\begin{aligned} \lim_{\Delta x \rightarrow \infty} \phi(x) &= \lim_{\Delta x \rightarrow \infty} \frac{e^{5x}}{a^4 (1 + e^x)^5} \left(\begin{array}{c} a^4 + (4a^4 + 2a^2 - 1) e^{-x} + \\ (6a^4 + 2a^2 + 11) e^{-2x} + \\ (4a^4 - 2a^2 - 11) e^{-3x} + (a^4 - 2a^2 + 1) e^{-4x} \end{array} \right) \\ &= 1 \end{aligned}$$

■

3.7.6 Proof of Proposition 11

Proof. For $a < 1$ we have that agents play a threshold equilibrium. In this case the mass of agents

entering sector 1 will be $M_1(a, \hat{\alpha}) = F(\hat{\alpha}) = \begin{cases} \frac{1}{2} e^{a\hat{\alpha}} & \text{if } \hat{\alpha} < 0 \\ 1 - \frac{1}{2} e^{-a\hat{\alpha}} & \text{if } \hat{\alpha} \geq 0 \end{cases}$

Next, we can rewrite the socially optimal entry level to sector 1 as

$$M_1^S(\hat{\alpha}) = \alpha_1 = \frac{A_1}{A_1 + A_2} = \frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}}$$

where $\hat{\alpha} = \ln A_1 - \ln A_2$.

We can see that for $\hat{\alpha} < 0$ $M_1(0, \hat{\alpha}) = 1/2 > \frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}}$, while $M_1(1, \hat{\alpha}) - \frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}} = e^{\hat{\alpha}} \left(\frac{1}{2} - \frac{1}{1 + e^{\hat{\alpha}}} \right) < 0$.

We can also see that $\frac{\partial M_1(a, \hat{\alpha})}{\partial a} = \frac{1}{2} \hat{\alpha} e^{a\hat{\alpha}} < 0$. Putting this together with the continuity of $M_1(a, \hat{\alpha})$ with respect to a implies there is a unique $\bar{a} \in (0, 1)$ such that $M_1(a, \hat{\alpha}) = M_1^S(\hat{\alpha})$.

When $\hat{\alpha} > 0$, $M_1(0, \hat{\alpha}) = 1/2 < \frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}}$, while $M_1(1, \hat{\alpha}) - \frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}} = \frac{1}{2(1 + e^{\hat{\alpha}})} (1 - e^{-\hat{\alpha}}) > 0$. We can also check that $\frac{\partial M_1(a, \hat{\alpha})}{\partial a} = \frac{1}{2} \hat{\alpha} e^{-a\hat{\alpha}} > 0$, which together with the continuity of $M_1(a, \hat{\alpha})$ with respect to a implies there is a unique $\bar{a} \in (0, 1)$ such that $M_1(a, \hat{\alpha}) = M_1^S(\hat{\alpha})$.

We have that

$$\bar{a}(\hat{\alpha}) = \frac{1}{|\hat{\alpha}|} \ln \left(\frac{(1 + e^{|\hat{\alpha}|})}{2} \right)$$

and

$$\frac{\partial \bar{a}(\hat{\alpha})}{\partial |\hat{\alpha}|} = \frac{1}{|\hat{\alpha}|^2} \left(\frac{|\hat{\alpha}| e^{|\hat{\alpha}|}}{1 + e^{|\hat{\alpha}|}} - \ln \left(\frac{(1 + e^{|\hat{\alpha}|})}{2} \right) \right)$$

Now, using a Taylor expansion we have

$$\ln \left(\frac{(1 + e^0)}{2} \right) = 0 > \ln \left(\frac{(1 + e^y)}{2} \right) - y \frac{e^y}{1 + e^y}$$

which implies $\frac{\partial \bar{a}(\hat{\alpha})}{\partial |\hat{\alpha}|} > 0$.

Finally notice that the equilibrium number of entrants to sector 1 when $\hat{\alpha} = 0$ (or $\alpha_1 = 0.5$) is 0.5 independently of the value of a . ■

3.7.7 Proof of Proposition 12

Proof. Notice $\lim_{a \rightarrow \infty} \phi(x) = \frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}}$. Also when $a \rightarrow \infty$ all agents receive a signal $x = \hat{\alpha}$ and all agents play the randomizing strategy $\frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}}$ which implies that the number of agents entering sector 1 is $\frac{e^{\hat{\alpha}}}{1 + e^{\hat{\alpha}}} = M_1^s(\hat{\alpha})$.

Now, for $\hat{\alpha} = 0$ we have that the number of entrants to sector 1 is

$$\begin{aligned} \int \phi(x) f(x, a) dx &= [\phi(x) F(x, a)]_{-\infty}^{\infty} - \int \phi'(x) F(x, a) dx \\ &= 1 - \int \phi'(x) F(x, a) dx \\ &= 1 - \int_0^{\infty} \phi'(x) dx \\ &= 1/2 \end{aligned}$$

which shows that $M_1(a, 0) = M_1^s(0)$. ■

3.8 Appendix 2: Figures and Tables

Figure 1. Equilibrium Entry Probability as a Function of the Signal

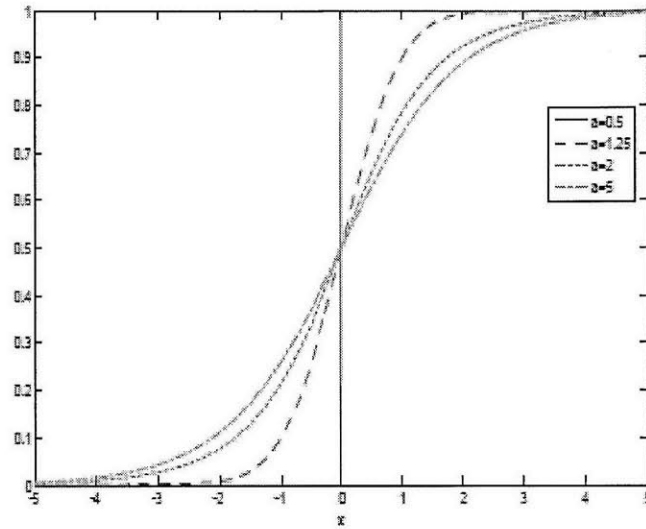


Figure 2. Equilibrium Entry as a Function of the Precision and $\alpha_1, a < 1$

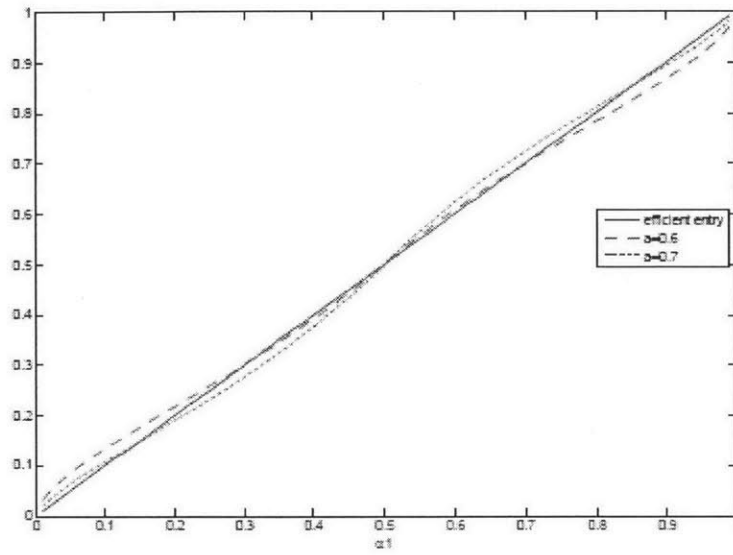


Figure 3. Difference between Equilibrium Entry And Socially Optimal Entry
as a Function of the Precision and $\alpha_1, a < 1$

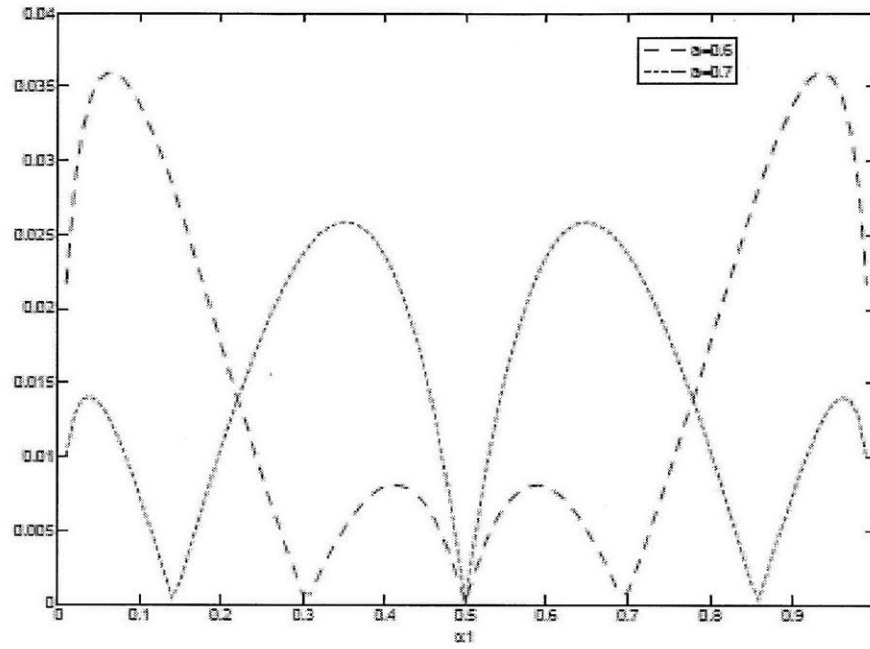


Figure 4. Equilibrium Entry as a Function of the Precision and $\alpha_1, a > 1$

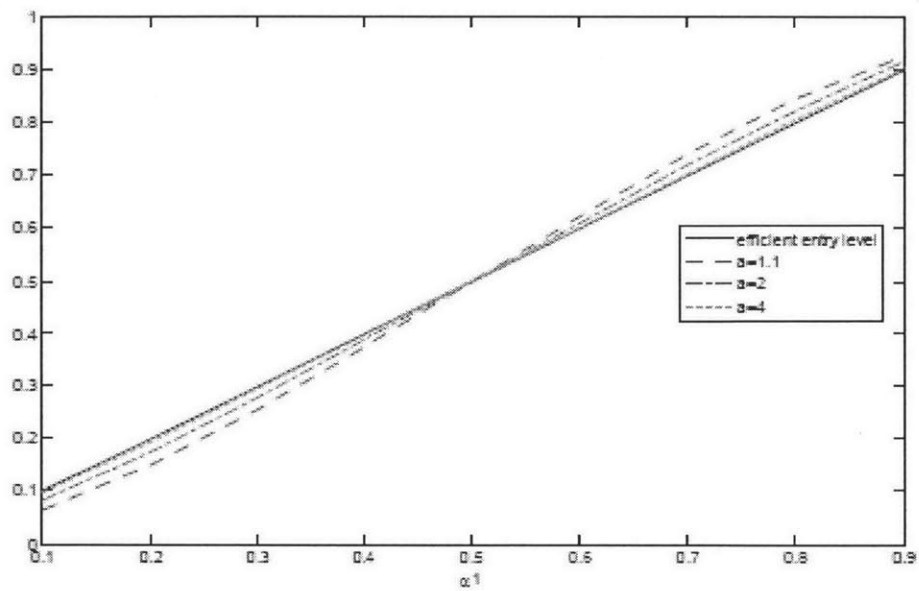


Figure 5. Difference between Equilibrium Entry And Socially Optimal Entry
as a Function of the Precision and $\alpha_1, a > 1$

