

TWO ESSAYS ON INCOME DISTRIBUTION IN  
A DEVELOPING ECONOMY

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ABSTRACT

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This thesis is a collection of two essays on the relationship between the distribution of income and the behavior of some crucial macro-variables in a developing economy.

The first essay purports to analyze, with reference to such an economy, the interaction between the distribution of income, education and the accumulation of capital. Given the distribution of income between different groups at any point of time and their accumulation functions of education and physical capital, it is shown how the incomes of these groups are determined at the next point of time through the interaction between education and physical capital, and therefore how the distribution of income changes over time. An analytical framework is constructed and on the basis of that a policy model developed. The crucial relationships of this model are estimable and these estimations have been made for two less developed countries, India and Colombia, to judge the egalitarian or non-egalitarian character of government policies in these countries over some specific periods of time.

The second essay is concerned with the interrelation between the distribution of income, the structure of markets, particularly of the credit market, and the problem of capital accumulation in a developing economy. The central idea is developed within the agricultural sector of such an economy, and then it is pointed out how this can be extended to cover the industrial sector as well. A model is built by taking into account the dualism that exists in such agriculture between the family and the capitalist farms, the distribution of income between these farms, the implication of that distribution on the structure of credit market and the decisions that these farms have to take on the use of inputs and the allocation of wealth under these circumstances. This model is then used to analyze the special problem of capital accumulation of this agriculture. It is found that given an unequal

initial distribution of income and the resulting imperfection of the credit market, such an agriculture can show a tendency to approach a state of zero rate of capital accumulation under very plausible conditions, and this can be accompanied by a process of immiserization of the family farms. Several ways of resolving this crisis are discussed including the solutions offered by technological progress as well as those offered by institutional changes.

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**Two Essays on the Distribution of Income  
in a Developing Economy**

**Asim K. Dasgupta**

**Essay 1: Income Distribution, Education and  
Capital Accumulation**

**Asim K. Dasgupta**

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## Income Distribution, Education And Capital Accumulation

Asim K. Dasgupta

### Introduction

The economic literature has a long tradition of thoughts on income distribution. It goes back to Ricardo, if not earlier. A remarkable feature of this tradition, however, is its preoccupation with the problem of income distribution among factors of production. Attempts have rarely been made to explain the distribution of income among individuals.<sup>1</sup> Understandably, the classical economists, in their days, might have had some justification in identifying any income group by the ownership of only one factor and therefore restricting their attention to the functional distribution of income. In the context of contemporary economic situation, however, it is the other problem — the distribution of income among individuals, which is the more meaningful and fundamental thing to look at.

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<sup>1</sup>Exceptions to this are the works of Champernowne, Mandelbrot and Stiglitz. See, D.G. Champernowne: "A Model of Income Distribution," Economic Journal, Vol. LXIII, 1953; B. Mandelbrot: "Stable Paretian Random Functions and the Multiplicative Variation of Income," Econometrica, Oct. 1961; J.E. Stiglitz: "Distribution of Income and Wealth Among Individuals," Econometrica, July, 1969.



Another important area where the literature on income distribution has definite gaps relates to the analysis of the role played by education. It is indeed somewhat surprising that, in spite of an increasingly wide recognition of education as a significant component in the explanation of national income,<sup>2</sup> when it comes to the related question of distribution of this income, there have been very few attempts to formulate an integrated structure to accommodate education into the existing theories of income distribution.

The purpose of this **essay** is, first, to develop a comprehensive theoretical framework to explain how education interacts with other factors of production, particularly physical capital, to determine the distribution of income among individuals, and then, estimate econometrically all the crucial relationships of the model in order to be able to point out the egalitarian or non-egalitarian tendencies of particular economies. The case studies will refer to the developing world and we have decided to choose one developing country from Asia and another from South America -- India and Colombia respectively -- to provide a variation in terms of socio-cultural background within the developing blocks.

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<sup>2</sup>See, T.W. Schultz: "Capital Formation by Education," Journal of Political Economy, Dec. 1960; E.F. Denison: The Sources of Economic Growth in the United States and the Alternatives Before Us, New York, 1962; Why Growth Rates Differ, The Brookings Institute, 1967. D.W. Jorgenson and Z. Griliches: The Explanation of Productivity Change, The Review of Economic Studies 1967. For results in the developing countries, see, M. Selowsky: Education and Economic Growth (Ph.D. Dissertation), University of Chicago.

Section 1 is a brief review of the existing literature on education and income distribution. With this as the background, the basic model is developed in Section 2. This model shows how the accumulations of human and physical capital by different income groups taken together determine the behavior of income distribution of an economy over time. Section 3 then is devoted to finding out the nature and implications of the conditions under which such an economy can converge to an egalitarian state. Since in any actual economy, the government usually plays a significant role in the educational process, it is important that this role of the government is properly accommodated in our analysis. With this in mind, a policy model has been developed by introducing government educational expenditure and taxes and then the nature of the optimal policies has been discussed (Section 4). It turns out that the convergence conditions and optimal policy rules involve essentially certain slope-characteristics of the relationships posited in the model. To be able to make any statement about the equalising or disequalising tendency of an economy, it is therefore necessary to estimate those relationships carefully. This is where the econometric estimation comes to play a significant role and we have done such estimation for India and Colombia and, on the basis of that, tried to reach some definite conclusions about the nature of income distribution and the incidence of government policies in these countries (Section 5). We conclude the essay by suggesting possible generalisations of our analysis and directions of future research (Section 6).

## 1. The Review

There are essentially three different ways in which education has been introduced into economic analysis. The most conventional among these is the so-called "human capital" doctrine where education is regarded as a productive factor and is supposed to play the most important role, independent of any other factor, in the determination of personal income. Then there is a second approach where education is still characterised as having a productive role, but, in the determination of income, other socio-economic characteristics are recognized to be at least as important as education. Finally, there is an interesting "job-access" hypothesis and education in this paradigm is looked upon just as a means of getting access to higher-paid jobs without necessarily having any connotation of productivity. In all these three approaches, there have been some attempts to relate education to the problems of income distribution and it is worthwhile to review them.

### The "Human Capital" Doctrine:

If one is historically minded, one can trace back the germs of this doctrine in the writings of classical economists. It is interesting to go through the relevant passages of Adam Smith<sup>3</sup> and John Stuart Mill<sup>4</sup> to see how very close they came to conceiving of

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<sup>3</sup>Adam Smith: The Wealth of Nations, Book I, Part 1, Ch. X.

<sup>4</sup>See, Collected Works of John Stuart Mill, University of Toronto Press, pp. 374-75, 385-87.

educational expenditure as a form of investment similar to any other physical investment. Their ideas were further perfected later on by Marshall<sup>5</sup> and Cannan.<sup>6</sup> This is a tradition which is interesting to trace. However, for the purpose of this brief review, we might have to start with a more contemporary presentation of the problem. The doctrine, in its modern form, has developed primarily out of the empirical works and theoretical formulations of the Chicago economists, although there are some contributions from other places as well.

The concept of human capital was first used symmetrically with physical capital by Schultz<sup>7</sup> in his empirical research and then this was followed up by Denison,<sup>8</sup> Aukrust<sup>9</sup> and others. These pioneering empirical works, however, were not very complete in their theoretical foundations. A complete theory of education as human capital gradually developed in the writings of Bowman<sup>10</sup> and particularly of Becker.<sup>11</sup>

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<sup>5</sup> Alfred Marshall, Principles of Economics, Vol. 1 edited by Guillebaud, p. 619.

<sup>6</sup> E. Cannan, Wealth, London, King, 1914, p. 199.

<sup>7</sup> See T.W. Schultz, op. cit. See Also his "Investment in Human Capital," American Economic Review, Vol. L1, 1961.

<sup>8</sup> E.F. Denison, op. cit. See also "Measuring the Contribution of Education (and the Residual) to Economic Growth" in The Residual Factor and Economic Growth, Paris, OECD, 1964.

<sup>9</sup> O. Aukrust: "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, Vol. 39, 1957.

<sup>10</sup> Mary J. Bowman: "Human Capital: Concepts and Measures in The Economics of Higher Education, (ed.) Mushkin, S., Washington, 1962.

<sup>11</sup> Gary S. Becker: Human Capital, New York, NBER, 1964.

This theory is now just too well known to need to be discussed in detail. We therefore present only a brief outline.

The doctrine essentially centers around the calculation of returns to expenditure on education in a way similar to the calculation of returns to physical investment. There are two kinds of returns one may be interested in. One is the "private rate of return" from the standpoint of an individual, the other is the "social rate of return" from the standpoint of a society. The private rate of return of any particular level of education is obtained by comparing the costs incurred by the individual and the returns received by him as a result of this education. The total costs are divided into monetary expenditures borne by him and opportunity costs. The returns, on the other hand, are the extra post-tax life time earnings associated with a particular level of education over some other level of education. The private rate of return of this particular level of education is the rate of discount which when applied to the stream of extra life time earnings just equates them to the total cost of that education. In principle, such private rates of return can be calculated for different types of education. The social rates of return are calculated in basically the same way with the difference that, in the calculation of social cost, government expenditure on education is added to private outlays, and the social returns are calculated from pre-tax rather than post-tax income differentials.

This theoretical framework has provided the basis of a stream, which at one time seemed to be never-ending, of empirical works, all

aiming at the calculation of these private and social rates of return from the same kind of age-education-earnings cross-section profiles and cost data for one country or another, and all stressing the importance of private rate of return in the explanation of individual decision-making in education and the social rate of return as the basis of educational expenditure policy of the State. The literature on this kind of empirical research is rather extensive.<sup>12</sup> However, we do not intend to digress on that, because the purpose of this review is not to survey the different approaches to economics of education in their every detail but to report on a more specific issue, namely, the analysis of the problem of income distribution within different approaches.

In the "human capital" approach, the problem of income distribution has been raised most notably by Gary Becker.<sup>13</sup> His views on

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<sup>12</sup>For a collection of the major works, see W. Lee Hansen (ed.), Symposium on Rates of Return to Investment in Education, 1967. For important works in the context of less developed countries, see C.A. Anderson and M.J. Bowman (eds.) Education and Economic Development. For particular countries, e.g. India and Colombia, see M. Blaug et. al., The Causes of Graduate Unemployment in India; M. Selowsky: Education and Economic Growth: Some International Comparisons, Center for International Affairs, Harvard University.

<sup>13</sup>To be bibliographically exact, however, one should mention that some of the problems about the relationship between the distribution of earnings and education were mentioned in an earlier work of Mincer (see, J. Mincer: "Investment in Human Capital and Personal Income Distribution," Journal of Political Economy, August 1958). But his ideas can be found in a more developed form in Becker, op. cit.

this question, however, seem to have changed over time. His initial position, presented in his book, 'Human Capital',<sup>14</sup> is this. Income of an individual is predominantly determined by earnings from his investment in education, and income of one individual differs from another because there is a difference in the rate of return and also in the amount of investment in human capital. This rate of return is higher for people with, what he calls, higher "ability" and that is why they invest more in education and accordingly the income distribution can very well tilt in their favor. He illustrates this point by suggesting that this can explain why white urban males with high I.Q.'s acquire more education than others. Although it looks incredible, this indeed is Becker's initial position and, as such, it certainly is a very incomplete theory of income distribution.

Fortunately, however, he revises his opinion within a period of three years and seems to recognise, as anyone should, also the supply side of the problem.<sup>15</sup> The optimal investment in human capital for a welfare-maximising individual, it is argued, is determined by the intersection of the demand curve for investment showing his marginal rate of return on an additional dollar of investment and the supply curve showing the marginal cost of financing the investment. The total amount invested in human capital therefore differs among persons

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<sup>14</sup>G.S. Becker: op. cit., pp. 52, 61-66.

<sup>15</sup>See G.S. Becker "Human Capital and the Personal Distribution of Income," Woytinsky Lecture, University of Michigan, 1967.

because of differences in either demand or supply conditions. Becker then goes on to discuss the possible causes of variations in the demand and supply curves among individuals and their effect on income distribution. Apart from formulating a theory, he has also carried out, along with Chiswick, an empirical study of income distribution for the United States, Canada, Mexico, Israel and Puerto Rico (treated as a country) and found that areas with greater income inequality have greater schooling inequality.<sup>16</sup> Chiswick later on did two more elaborate empirical works - one on the regional income distribution<sup>17</sup> and the other (this is done jointly with Mincer) on the intertemporal behavior of inequality in the United States.<sup>18</sup> Theoretical foundations of both these works are essentially Beckerite and empirical conclusions tend to confirm the direct relationship between the skewness of income and the skewness of schooling.

Although a special credit should go to Becker for initiating the discussion on income distribution within a "human-capital" framework, there are some major omissions in his analysis of the problem. In the first place, the model is essentially static. It only tells us how the demand and supply curves may be different among individuals

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<sup>16</sup>G.S. Becker and B.R. Chiswick: "Education and the Distribution of Earnings," American Economic Review, Vol. 56, No. 2 (May 1966).

<sup>17</sup>B.R. Chiswick: "An Interregional Analysis of Schooling and the Skewness of Income" in W. Lee Hansen (ed.), Education, Income and Human Capital.

<sup>18</sup>B.R. Chiswick and J. Mincer: "Time Series Changes in Personal Income Inequality in the U.S. from 1939, with Projections to 1985," Journal of Political Economy, 1972 (Special Issue).



to result in different possible distributions of income at a particular point of time. It does not consider the more interesting, and of course more complex, problem of how, starting with an initial distribution of resources and the demand and supply curves, different forces may interact with one another to characterise the behavior of this distribution over time. It should also be pointed out that even the theoretical framework used by Chiswick and Mincer<sup>19</sup> to study the time series of income distribution is essentially static. What has to be recognised, however, is a fundamental theoretical point – that whenever capital is introduced in a model, whether it is in the form of human capital or in any other form, at least two points (or, periods) of time get essentially linked up and therefore the model has to be unavoidably dynamic, and any problem of income distribution posed within such a model has to be intertemporal in nature.

Secondly, there is a lack of a proper awareness of the important co-existence of human capital with physical capital and their complementarity in production. The assumption that higher level of investment in human capital always means a lower rate of return has to be qualified significantly once the complementarity between the two kinds of capital is sufficiently understood. By focusing the attention only on human capital to the exclusion of physical capital, one runs the danger of repeating the same mistake which was made in the traditional capital theory by way of ignoring human capital.

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<sup>19</sup> Chiswick and Mincer, op. cit.

Finally, there is another incompleteness in the Beckerite analysis. Becker tells us about the differences in the demand and supply conditions among individuals, but, apart from making some casual observations such as the importance of parental gifts etc. in the financing of education, he never quite tries to relate these differences systematically with particular income groups and endogenise them in a complete theoretical framework. And, this affects his empirical analysis also. For instance, after observing a greater inequality in the distribution of schooling in the American South, he only speculates that this may be a consequence of less equal opportunity,<sup>20</sup> rather than actually pursuing it to find out in which way these opportunities are in fact unequal for different income classes. This is a very fundamental problem and, if one has to talk about income distribution at all, one should rather ask and try to answer this question than avoid it.

And, this leads us to an interesting work of Samuel Bowles<sup>21</sup> where this problem has been posed in a very direct and straightforward manner. The framework of analysis is again human-capital-theoretic and he uses it to understand the incidence of educational policy in capitalist less-developed countries. In such countries, he argues, the benefits from higher education tend to go to the elite groups and the benefits from primary education to the masses.

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<sup>20</sup>Becker and Chiswick, op. cit., p. 367.

<sup>21</sup>See his "Class Power and Mass Education," Harvard University Mimeographed (1971).

With a non-neutral government and its predictable bias, there is likely to be overspending on higher education compared to primary education. The evidence that he uses to support his hypothesis is that the returns to higher education are found to be lower than the returns to primary education indicating the suboptimality of resource allocation. This, according to Bowles, has been a deliberate policy decision in many less developed countries. This hypothesis of Bowles is remarkable in the sense that it perhaps is the first attempt within the tradition of human capital to link up educational policy to the class structure of a country.

However, there are some areas in which his analysis leaves scope of improvement. For instance, as has been pointed out by Bhagwati,<sup>22</sup> it does not exactly logically follow from the demonstration of a discrepancy between different rates of return that the relative inequality between the two groups has necessarily to go in a unique direction. The model has to be properly closed by a more complete set of assumptions on the expenditure-education and the education-income relationships. Again, on the questions of complementarity between human and physical capital and the intertemporal behavior of income distribution, Bowles' analysis seems to be vulnerable to the same kind of criticism which was found pertinent to the Becker-Chiswick Theory.

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<sup>22</sup> J. Bhagwati: "Education, Class Structure and Income Inequality," World Development, Vol. 1, May 1973, p. 24.

Another general shortcoming of the "human capital" doctrine has been its inflexibility, its failure to recognise and accommodate within its theoretical structure the importance of many socio-economic factors other than education. This exclusive reliance on education came to be increasingly disputed as the facts started telling a different story.

#### The Significance of Socio-Economic Factors

That the socio-economic background of an individual is an important determinant of his attainment of education and also of his income has been recognised for a long time. We have already referred to Marshall in connection with the "human capital" doctrine. It is interesting that one can again find passages in 'Principles of Economics' where he was careful enough to qualify and supplement his views on education as human capital with a proper acknowledgment of the significance of socio-economic influences.<sup>23</sup> One can also mention the early empirical works of Hollingshead and others.<sup>24</sup> But, for our purpose, a convenient starting point is the Coleman Report<sup>25</sup> which is usually regarded as a major source of data on the relationship of education to other social, psychological and

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<sup>23</sup> See R. Blandy: "Marshall on Human Capital, A Note," Journal of Political Economy, Vol. LXXVI, 1968.

<sup>24</sup> A. Hollingshead: Elmtown's Youth: The Impact of Social Classes on Adolescents; W. Warner, R. Havighurst, and M. Loeb: Who Shall Be Educated? New York, Harper, 1944.

<sup>25</sup> J.S. Coleman, et. al.: Equality of Educational Opportunity, Washington, Government Printing Office, 1966.

economic variables. The Report addressed itself to the question of inequality of educational opportunity among different racial groups in the U.S. and, from detailed statistical studies, concluded that the most important factor in educational attainment was the background of fellow students rather than teaching quality, curriculum and other facilities. After this publication, several other studies came to report similar findings. Hanoch,<sup>26</sup> for instance, found that the internal rate of return to increased schooling (except for graduate studies) was considerably lower for blacks than for whites. The work of Hanoch has been further extended by Harrison<sup>27</sup> and Weiss.<sup>28</sup> In all these works, however, it should be noted that the underlying implication was that socio-economic background played an important role in the determination of income indirectly through its effect on education.

Recently, Samuel Bowles in his works<sup>29</sup> has gone one step further. He has tried to establish that the socio-economic background variables, apart from influencing income through education, have a separate and direct impact on income. The specific econometric model he has used

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<sup>26</sup>G. Hanoch: "An Economic Analysis of Earnings and Schooling," Journal of Human Resources 2 (Summer 1967).

<sup>27</sup>B. Harrison: "Education and Earnings in the Urban Ghetto," Mimeo., University of Maryland, 1969.

<sup>28</sup>R. Weiss: "The Effect of Education on the Earnings of Blacks and Whites," Review of Economics and Statistics, May 1970.

<sup>29</sup>S. Bowles: "Schooling and Inequality from Generation to Generation," Journal of Political Economy, 1972 (Special Issue); "Unequal Education and the Reproduction of the Social Division of Labor," The Review of Radical Political Economics, Fall/Winter 1971.

to prove his point is a recursive one - the first relation specifies education as a function of socio-economic variables and, then, the second equation shows income as depending on education and the socio-economic variables. Bowles then argues in detail how the measurement errors with respect to the socio-economic vector were disproportionately large in earlier studies<sup>30</sup> and how a proper correction of these errors finally gives a higher coefficient to the background variables compared to education. He has also tried to trace down the source of the differences in socio-economic variables in the structure of a society and mentioned certain possible institutional arrangements which help to perpetuate these differences and through these the inequality in the distribution of income. Implicit in his analysis is a suggestion that the problem can be solved only by changing the structure of the society.

It is interesting to contrast this radical critique with a more liberal stand taken by Jencks.<sup>31</sup> In an extensive empirical study he has tried to find out the relationship between variations in income

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<sup>30</sup> P. Blau and O. Duncan: The American Occupational Structure, New York, Wiley, 1967. O. Duncan: "Ability and Achievement," Eugenics Q. (March, 1968). O. Duncan, D. Featherman and B. Duncan: Socio-economic Background and Occupational Achievement, U.S. Department of Health, Education and Welfare, May 1968.

<sup>31</sup> Christopher Jencks, Inequality, 1972 Basic Books, Inc. In this context also see J. Simmons, "Investment in Education: Alternative National Strategies," IBRD, Oct. 1973.

on the one hand and those in socio-economic background, educational attainment, cognitive skill and occupational status on the other. The general theme of this study is that the observed differences among individuals in all these attributes explain very little of the variations in their incomes. Therefore, it is in the unmeasured differences in motivation and especially in luck that the bulk of the explanation can be found. Since these unmeasured differences can not be controlled by policy, the best way of equalising income, Jencks concludes, is to redistribute income directly rather than going through the observable "explanatory" factors.

In this context it should be mentioned, however, that in a recent work Hall<sup>32</sup> has pointed out that there are some major statistical problems which were seemingly ignored by Jencks, but which can seriously limit his method of estimation of the significance of the observed attributes of the individuals in explaining the dispersion of income. As an alternative, Hall has suggested a statistically more satisfying method of estimation based on the mathematical theory of Tchebycheff system.

In another empirical study relating to the Indian data, Panchamukhi and Panchamukhi<sup>33</sup> have approached the problem in a somewhat

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<sup>32</sup>R.E. Hall: "On the Statistical Theory of Unobserved Components," M.I.T., Dept. of Economics, Working Paper No. 117, August 20, 1973.

<sup>33</sup>V.R. Panchamukhi and P.R. Panchamukhi, "Socio-Economic Variables and Urban Incomes" in H.N. Pandit (ed.) Measurement of Cost Productivity and Efficiency in Education, NCERT, New Delhi 1969.

different way. Using an ingenious statistical device they have succeeded in separating from the total concentration ratio of income distribution, the contributions made by the different socio-economic factors, namely, family income, occupation, education, sex and employment status. The conclusion they have reached is strikingly at variance with Jencks'; it shows that the concentration ratio of the total earnings can be significantly explained by the concentration ratio of the earnings due to socio-economic factors and that the imposition of residual factors can distort the distribution of income towards greater inequality only slightly.

It is clear therefore that from the standpoint of empirical confirmation, the issue about the significance of socio-economic factors is far from being settled. And, on the question of theoretical analysis, one can not avoid getting the feeling that all these studies have tended to fall short of providing a complete analytical structure. There is in each of these works an underlying implication of some particular theory of distribution. Unfortunately, however, this has rarely been made explicit in terms of a complete analysis of the interaction between income, education and other socio-economic variables. It is indeed difficult to find out in these studies a comprehensive economic model which is first spelled out and then econometrically tested to corroborate one hypothesis or other.



The Job-Access Model

Recently, a very different approach to economics of education has been suggested by Bhagwati.<sup>34</sup> In this approach, unlike in the first two approaches, education does not necessarily have any connotation of productivity. It is essentially a process of acquiring a credential for getting access to higher-paid jobs. An economy, under this hypothesis, is conceived of as consisting of a number of jobs with different price-tags attached to them. The effect of shifting the population from the uneducated into the educated category in such a situation is only to filter the educated down into the top jobs of the uneducated. Bhagwati elsewhere<sup>35</sup> has shown that it is indeed possible to build a general equilibrium model on the basis of such distribution of jobs with preassigned price tags.

There are reasons to believe that this "job-access" paradigm can be helpful in understanding the working of an actual educational process in a less developed country like India, particularly, in relating the educational process to the distribution of income among

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<sup>34</sup> Bhagwati, Jagdish, op. cit.

<sup>35</sup> See Bhagwati and Srinivasan: "Overqualification, Education and Welfare: A Theoretical Analysis," 1972, Mimeo. See also Bhagwati and Hamada, "The Brain Drain, International Integration of Markets for Professionals and Unemployment." A Theoretical Analysis, 1972. Mimeo.

different classes.<sup>36</sup> Bhagwati, for instance, has used this model to analyse the class distribution of the benefits from the government educational policy. The question raised is: which classes manage to get their children educated, so that they get access to the better jobs? The answer to this question depends to a great extent on the nature of the government policy and, on that, Bhagwati suggests this hypothesis: "For each class of education, the State (in capitalist LDC's) will subsidise the cost of education; the benefits of these subsidies will accrue disproportionately less to the poorer groups at each level of education; the higher the educational level being considered, the higher will be the average income-level of the groups

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<sup>36</sup>This paradigm has to be carefully distinguished from two other related paradigms. One is the so-called "screening" theory of Arrow (cf. K. Arrow, "Higher Education as a Filter," Stanford University, 1972, Mimeo.) where he argues that "Higher education ..... contributes in no way to superior economic performance; it increases neither cognition nor socialization. Instead, higher education serves as a screening device, in that it sorts out individuals of different abilities, thereby conveying information to the purchasers of labor ..... The screening or filter theory of higher education, as I shall call it, is distinct from the productivity-adding human capital theory but is not in total contradiction to it. From the view point of an employer, an individual certified to be more valuable is more valuable, to an extent which depends upon the nature of the production function." (pp. 2-3). Arrow, however, does not link up his theory to the problems of income distribution.

The "job-access" paradigm has also to be separated from Thurow's "job-competition" model which, as pointed out by Bhagwati, is really an adulterated form of the "job-access" paradigm bent in the direction of incorporating some productivity effects. (Cf. L. Thurow: "Education and Economic Equality," Public Interest, No. 28, Summer 1972).

to which the students belong; and the rate of governmental subsidisation to higher education will be greater than that to primary education" (see his Education, Class Structure and Income Equality, p. 24).

Bhagwati, in this paper, has not developed any formal model which can be econometrically tested to prove this hypothesis. But, he has indeed provided us with a wide variety of evidences on the basis of which the hypothesis does look very convincing.

In reviewing the literature, it is worthwhile to mention some individual works which do not quite fall into any one of the three schools of thoughts described above, but are interesting as such. A recent work by Fishlow<sup>37</sup> on the size-distribution of income in Brazil is worth mentioning in this sense. In analysing the factors responsible for shaping the income distribution, he has worked with an inequality index, originally developed from information theory by Theil, which is very useful from the standpoint of disaggregation. If, for example, it is deemed necessary to disaggregate the income classes along sectors and educational attainment, then this inequality index can be used to express total inequality as a sum of the differences among income classes, plus the variation of sectors within income classes, plus the variation of the means of the different education classes, within sector and income cells. There are reasons to believe<sup>38</sup> that in future

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<sup>37</sup> A. Fishlow: "Brazilian Size Distribution of Income," The American Economic Review, Papers and Proceedings, May 1972.

<sup>38</sup> R.S. Eckaus: "A Prospectus for the Modelling and Estimation of Personal Income Distributions in Less Developed Countries, Mass. Inst. of Technology, 1973, Mimeo.

research on income distribution this kind of disaggregation will have to be taken into account and in that context this inequality index will be a very natural thing to use. However, the problem with Fishlow's paper, and which we have already mentioned to be the problem with several other well-known studies on income distribution as well, is that there has not been enough of economic modelling. After decomposing the total inequality into the differences along useful categories, no attempt has been made to relate the variables of these categories with other economic variables or within themselves to provide an integrated structure of analysis. Yet, it is this structure which alone can provide an explanation of what we get to know through a useful device of statistical decomposition.

Another important wrinkle in the entire issue of the interaction between education and income distribution, and which has also become a subject of current interest, relates to the question of incidence of government tax-expenditure policy on education. For example, there is the well-known Hansen-Weisbrod-Pechman controversy<sup>39</sup> on the distribution

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<sup>39</sup>W. Lee Hansen and B.A. Weisbrod: Benefits, Costs, and Finance of Public Higher Education, Chicago, 1969; "The Distribution of Costs and Direct Benefits of Public Higher Education: The Case of California," Journal of Human Resources, Spring 1960; J.A. Pechman: "The Distributional Effects of Public Higher Education in California," Journal of Human Resources, Summer 1960. R.W. Hartman; "A Comment on the Pechman-Hansen-Weisbrod Controversy," Journal of Human Resources, Fall 1970. Also see, Hansen and Weisbrod: "On the Distribution of Costs and Benefits of Public Higher Education: Reply," Journal of Human Resources, Summer 1971.

of costs and benefits of public higher education in California. There is a recent World Bank research on public educational expenditure in Colombia,<sup>40</sup> and there are several others.<sup>41</sup> All these studies are of considerable interest so far as their compilation of data is concerned. But, unfortunately, there seems to be an important lapse in their basic analytical structure. Consider, for example, their common methodology. In all these studies, generally speaking, the following steps are consecutive. First, population groups are classified by income or race (Step 1); then, tax payment by each group is estimated (Step 2), followed by the computation of benefits arising from government expenditure on education, e.g., subsidies received by each group (Step 3) and, finally, the distributions of tax burden and benefits are compared (Step 4).

The third of these four steps does not seem to be convincing, and, for the following reason. Assuming that we want to measure the ultimate impact of a government expenditure policy on the distribution of income and that we want to do it by comparing tax burden with benefits, it is clear that if this tax burden is measured by the loss of income

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<sup>40</sup> J.P. Jallade: Public Expenditure on Education and Income Distribution in Colombia, (Mimeo.) World Bank, Washington.

<sup>41</sup> W. Norton Grubb: "The Distribution of Public Costs and Private Benefits in an Urban Public School System," National Tax Journal, March 1971. D.M. Windham: "The Redistributive Effects of Public Higher Education in Florida," Monograph Series F, No. 1, University of North Carolina. R.W. Judy: "The Income Redistributive Effects of Public Aid to Education in Canada," in Officer and Smith (eds.) Canadian Economic Problem and Policies.

through taxes, benefits should be correspondingly measured by the gain in income from educational subsidy. However, the gain in income is not really this immediate subsidy, as has been conveniently assumed in these empirical studies; it is the final gain in income resulting from a higher attainment of education made possible by the initial subsidy. To measure benefits, therefore, what has to be computed is a more complex thing; namely, the additional income generated for each class from the subsidies through the education-income relationship. A method of such computation will be developed later in this paper.<sup>42</sup> Surprisingly enough, there is a conspicuous lack of awareness of the significance of this education-income relationship in these empirical studies.

And, this brings us back to a recurring theme of this review, namely, the necessity of having a complete theoretical structure before starting with any empirical work. It is a very well-known methodological requirement and yet ignored in so many studies in this field. And, in a sense, this is somewhat ironical because over the last few years — precisely the period when these empirical studies were conducted — there has been a sudden flurry of theoretical interest on this question,

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<sup>42</sup> Please see Section 4 of this paper.

initiated by a suggestive work of Arrow.<sup>43</sup> But, unfortunately, these theoretical speculations in public finance had from the very beginning taken such an esoteric turn that they could not possibly serve the basis of any empirical work. The result is an intersection set which has remained nearly empty.

This, in short, is the review of the beaten track. It shows how different doctrines and different empirical works in economics of education have sought to tackle the problem of income distribution. It also shows where these attempts have remained incomplete. And, in that, it signals the direction in which our own research should go. It is clear now, for example, that for any meaningful research in this area, what is needed, first of all, is a comprehensive theoretical model which can explain how education interacts with other factors of production in shaping the distribution of income at any point of time and also over time. Since there is a controversy over the nature of the role of education in income distribution, attempts should be made to represent the alternative hypotheses through some appropriate characteristics of certain relationships of the model and then test

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<sup>43</sup>See K.J. Arrow: "A Utilitarian Approach to the Concept of Equality in Public Expenditures." Quarterly Journal of Economics, August 1971. See also J.R. Green and E. Sheshinski: "A Note on the Progressivity of Optimal Public Expenditures," Technical Report No. 65, Institute for Mathematical Studies in Social Sciences. Stanford University (August 1971); A.B. Atkinson: "How Progressive should the Income Tax Be," A.U.T.E. 1972; K. Hamada: Income Taxation and Income Subsidy, M.I.T. (Mimeo) 1973.

econometrically these characteristics to vindicate one hypothesis or another. Since econometric testing has to be an essential part of the research and there are some well-known restrictions imposed by the present state of econometric techniques and the availability of data, a proper care should be taken to simplify the equational structure of the model as far as permissible by economic theory. There is hardly any sense, at least at this stage of the game, to try to build yet another formally elegant model which can never be tested.

With all this in mind, we turn to Section 2.



## 2. The Analytical Model

In this section, we shall develop the basic theoretical framework of our analysis. Apropos of that, certain preliminary observations are necessary.

First, in this model, we propose to introduce education as a productive factor but along also with physical capital. As has been pointed out in the review, the productive role of education or physical capital, in isolation of each other, has never been underrecognized in the literature. What has never been clearly brought out, however, is the fact--and a very realistic fact at that--of coexistence and interaction of these two very important factors. We believe, and this will be shown, that it is the complementarity between education and physical capital that lies at the heart of the entire problem of income distribution.

Secondly, we have already seen<sup>44</sup> that certain socio-economic variables play an important role along with, and possibly also independent of, education in the determination of income distribution. A convenient and systematic way of endogenising these factors is to have them represented through physical capital owned by different income classes. There is evidence<sup>45</sup> to believe, for example, that in a less-developed country like India or Colombia the relative privileges enjoyed by different income classes due to these socio-economic factors go hand in hand with their ownership of land-capital.<sup>46</sup>

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<sup>44</sup>See Section 1, pp. 13-15.

<sup>45</sup>See Urban Income and Saving (1962), N.C.A.E.R., New Delhi; All India Rural Household Survey (1965), N.C.A.E.R., New Delhi; CEDE, Encuesta de Gastos e Ingresos Familiares, Mayo 1967-Mayo 1968.

<sup>46</sup>It is well known that for analytical purposes capital and land can be treated together as a composite factor.

Thirdly, although education is presented through a production structure, this characterisation is not essential for our purpose. This model can be very easily modified, and without any basic change in the conclusion, to accommodate the case where education has no productive connotation, i.e., the 'job-access' paradigm. In fact, our model will turn out to be a rather general analytical framework which can include all the relevant hypotheses--the human-capital doctrine, the doctrine of socio-economic factors, the job-access paradigm and even the neoclassical Stiglitz-type capital-theoretic model--as special cases.

Finally, at some point of our analysis, we might have to make the assumption of perfect competition. However, this is strictly a simplifying and analytically inessential assumption. All the results carry over even if market imperfections are introduced.

Consider now an economy with  $Y$  as the aggregate income,  $L$  the total population and  $\bar{y} \left( = \frac{Y}{L} \right)$  the per capita income. The total population can be partitioned into two groups, to be called 1 and 2, such that every member of Group 1 has income  $\geq \bar{y}$  and every member of Group 2 has income  $< \bar{y}$ . Let  $Y_1$ ,  $Y_2$  and  $L_1$ ,  $L_2$  be the total incomes and populations of Group 1 and Group 2 respectively. The average incomes of the respective groups therefore are  $y_1 \left( = \frac{Y_1}{L_1} \right)$  and  $y_2 \left( = \frac{Y_2}{L_2} \right)$ . Clearly then,  $y_1 > y_2$ . And, this leads us to define the coefficient of inequality as

$$(2.1) \quad x = \frac{y_1}{y_2}$$

The production process in this economy is characterised by a neoclassical constant returns to scale production function

$$(2.2) \quad Y = F(K, N, L) ,$$

where  $Y$  is the aggregate output,  $K$  the aggregate stock of physical capital,  $L$  the "raw" labour and  $N$  the aggregate human capital measured by the number of school years. In an alternative production-theoretic characterisation of education, labour is divided into several categories  $L_0, L_1, \dots, L_n$  where  $L_0$  is the "raw" labour and  $L_i$  corresponds to the labour force with the  $i$ -th level of education. These two types of formulations lead to similar end-results. However, we find it easier to capture the essence of the problem in terms of the former approach.

Most of the time, we shall work with the intensive form of the production function and, using the assumption of constant returns to scale, (2.2) can be written as

$$(2.3) \quad y = f(k, n) ,$$

where  $y$  is the output per man,  $k$  the physical capital per man and  $n$  the human capital per man. Each factor is paid its marginal product. Therefore, if  $r$  is the rental on physical capital,  $h$  the rental on human capital and  $w$  the wage rate, we can write

$$(2.4) \quad r = \frac{\partial f(k, n)}{\partial k} , \quad h = \frac{\partial f(k, n)}{\partial n}$$

and

$$w = f(k, n) - k \frac{\partial f(k, n)}{\partial k} - n \frac{\partial f(k, n)}{\partial n} .$$

We assume generalized diminishing returns to factors and complementarity between them. Therefore,

$$(2.5) \quad \frac{\partial r}{\partial k} < 0, \quad \frac{\partial h}{\partial n} < 0,$$

$$\frac{\partial r}{\partial n} > 0, \quad \frac{\partial h}{\partial k} > 0, \quad \frac{\partial w}{\partial k} > 0, \quad \frac{\partial w}{\partial n} > 0.$$

Let  $K_i$ ,  $N_i$  and  $L_i$  be the physical capital, human capital and labour of the  $i$ -th Group ( $i = 1, 2$ ), so that

$$(2.6) \quad K = K_1 + K_2, \quad N = N_1 + N_2 \quad \text{and} \quad L = L_1 + L_2.$$

The idea is that the factors of production owned by the two groups are brought together and used in one production function.<sup>47</sup> Rewriting (2.6), we have

$$(2.7) \quad k = a_1 k_1 + a_2 k_2 \quad \text{and} \quad n = a_1 n_1 + a_2 n_2$$

where  $k_i = \frac{K_i}{L_i}$  and  $a_i = \frac{L_i}{L}$ . We assume that the rate of growth of population does not vary significantly from one income group to another. This assumption is made because (a) in the less developed countries, particularly in the context of India, although there is some relationship between birth rate and income class (to be exact, even this statement needs qualification in view of the inverse J-shaped appearance of the birthrate-income curve),<sup>48</sup> there is in fact no such systematic relationship between income group and the rate of growth of population, presumably because higher birth rates among the poor tend to be matched by correspondingly higher death rates; and, (b) the incidence of intergroup

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<sup>47</sup>There is an underlying assumption of full employment of every factor. This again is a simplifying assumption. The possibility of unemployment, particularly of labour, can be incorporated in our analysis. Since the available data indicate that the incidence of unemployment of labour is higher among the poor (cf. Bhagwati: "Education, Class Structure and Income Inequality," World Development, May 1973), a more complete characterization of the problem along this line should, as will be evident later, support our final conclusion.

<sup>48</sup>See, National Sample Survey, Tables with Notes on Family Planning (1960-61), Indian Statistical Institute.

marriage can again be supposed not to be very significant.<sup>49</sup> All these imply that  $a_1$  and  $a_2$  can be treated as approximately constant.

The average income of the  $i$ -th Group can now be written as

$$(2.8) \quad y_i = w + rk_i + hn_i \quad (i = 1, 2) .$$

We shall call this the income-generation function of Group  $i$ , making it distinct from the production function. This function shows how the income of any one group is related to its own resources and the factor prices which, in their turn, are determined by the production technology and the accumulation of resources by the two groups. Any difference in the average income, as defined by (2.8), of the respective groups will be supposed to be due to a corresponding difference in the average ownership of physical and human capital, i.e., it will be assumed that

$$y_1 > y_2 \implies k_1 > k_2 \quad \text{and} \quad n_1 > n_2 .$$

How do the accumulations of two types of capital take place for the two groups? Let  $S_i$ , the saving per man of Group  $i$ , be a fixed proportion,  $s_i$ , of its income, so that

$$(2.9) \quad S_1 = s_1 y_1 \quad \text{and} \quad S_2 = s_2 y_2 .$$

The extensive household surveys, conducted both in India and Colombia,<sup>49</sup> clearly show that saving-income ratio monotonically increases from lower to higher income classes. Therefore, if per capita income is chosen to be the cutoff point, it follows that  $s_1 > s_2$ . Let  $\lambda_1^i$  and  $\lambda_2^i$  be the proportions of the savings of Group  $i$  used for accumulation of physical capital and expenditure on

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<sup>49</sup> See, Urban Income and Saving, op. cit., p. 78; All India Rural Household Survey, Vol. II, p. 96; CEDE, Encuesta de Gastos e Ingresos Familiares.

education respectively, so that

$$\begin{aligned}
 \dot{k}_1 &= \lambda_1^1 s_1 y_1 - \gamma k_1 & \dot{k}_2 &= \lambda_1^2 s_2 y_2 - \gamma k_2 \\
 (2.10) & & \text{and} & \\
 \dot{E}_1 &= \lambda_2^1 s_1 y_1 - \gamma E_1 & \dot{E}_2 &= \lambda_2^2 s_2 y_2 - \gamma E_2
 \end{aligned}$$

where  $\dot{E}_i$  is the cumulated expenditure per man by Group i and overdot, as usual, denotes derivative with respect to time. It should be noted that since there are two kinds of capital and they are not produced by the same production function, a notion of relative price unavoidably comes in. Therefore, expenditure on education should be considered as being measured in terms of the price of education relative to physical capital, i.e.,  $E = Pn$ , where  $P = P(n)$  is this relative price. We shall ignore any "price effect" and assume  $\dot{P} = 0$ , so that  $\dot{E} = P\dot{n}$ . The physical capital good in this analysis, of course, is considered the same as the "unique" commodity (since they are produced by the same production function) which is taken as the numeraire.

$\gamma$  is the rate of growth of population and, because of the reasons already mentioned,<sup>50</sup> is assumed to be the same for both groups. Nothing is essentially lost, but simplicity gained, if we now put  $\gamma = 0$ . This is because finally we will be interested only in the comparison between  $\frac{\dot{E}_1}{y_1}$  and  $\frac{\dot{E}_2}{y_2}$ , and in this comparison, particularly for the countries we are interested in, the difference between  $s_1$  and  $s_2$  usually overwhelmingly dominates the difference between the "capital-widening" terms,

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<sup>50</sup> ibid. pp. 25-26.

$$\gamma \frac{E_1}{y_1} \text{ and } \gamma \frac{E_2}{y_2} \quad 51$$

The question now is: how are these  $\lambda_1^i$  and  $\lambda_2^i$  determined for each income group? This is essentially a problem of portfolio choice and there are some interesting issues connected with this problem. It is known, for instance, that if the transformation frontier between two types of investment goods is linear and if there is no uncertainty, then either there is a "corner" solution implying that all savings will go to only one form of investment or else there is an indeterminacy with the allocation ratios lying anywhere in the closed interval  $[0, 1]$ .<sup>52</sup>

However, these problems do not arise in our model, because, as will be seen in Section 5, the supply price of education increases with the level of education, i.e.,  $P'(n) > 0$  and also  $P'(\dot{n}) > 0$  and that ensures the required curvature of the transformation frontier. The equation of the transformation frontier for any Group  $i$  is

$$\phi_i(k, n) = \dot{k}_i + P\dot{n}_i - s_i y_i = 0 \quad (\text{setting } \gamma = 0)$$

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<sup>51</sup>In the case of India, for example, it will be found that  $s_1 \approx 30\%$  and  $s_2 \approx 0$  (Table 6),  $\gamma = 2.5\%$ ,  $\frac{E_1}{y_1} = 0.112$  and  $\frac{E_2}{y_2} = 0.037$  (Table 5).

<sup>52</sup>The problem of portfolio allocation in growth-theoretic models with heterogeneous capital goods was first raised by Hahn and then followed up by Shell and Stiglitz. See, F.M. Hahn: "Equilibrium Dynamics with Heterogeneous Capital Goods," Quarterly Journal of Economics, November, 1966; K. Shell and J.E. Stiglitz: "The Allocation of Investment in a Dynamic Economy," Quarterly Journal of Economics, Nov. 1967. Also see D. Foley and M. Sidrauski: Monetary and Fiscal Policy in a Growing Economy.

Since  $P'(\dot{n}) > 0$ , it is clear that

$$\frac{d\dot{k}_i}{d\dot{n}_i} = -P, \quad \text{and} \quad \frac{d^2\dot{k}_i}{d\dot{n}_i^2} < 0$$

Therefore, the transformation frontier for both groups will have concave shape from the origin. However, one curve will not be a radial "blow-up" of the other, and for the following reason.

$\dot{E}_1^{\max}$  (for  $\lambda_1^1 = 0$ )  $>$   $\dot{E}_2^{\max}$  (for  $\lambda_1^2 = 0$ ). Correspondingly,  $\dot{n}_1^{\max} > \dot{n}_2^{\max}$ . But  $P'(\dot{n}_i) > 0$ . Therefore the relative difference between the intercepts of the transformation frontiers of the two income groups will not be the same on the two axes. A possible situation is shown in Fig. 1 where  $T_1T_1$  and  $T_2T_2$  are the transformation frontiers of Group 1 and Group 2 respectively.

We assume that, confronted with the transformation locus, a typical member of each group in any period allocates his savings between the accumulation of two types of capital in such a way as to maximize his total rental income in that period. In so doing, he treats the rental rates,  $r$  and  $h$ , on the physical and human capital as given. These rental rates refer to the aggregate stocks of the two kinds of capital and can be shown to be equal to the corresponding internal rates of return.

Of the two, however, the internal rate of return on human capital (i.e.,  $h$ ) needs a more careful analysis. Although for the economy as a whole there is in any period only one value of  $h$  referring to the total stock of human capital, there is at the same time a difference in the value of internal rate of return on education as between



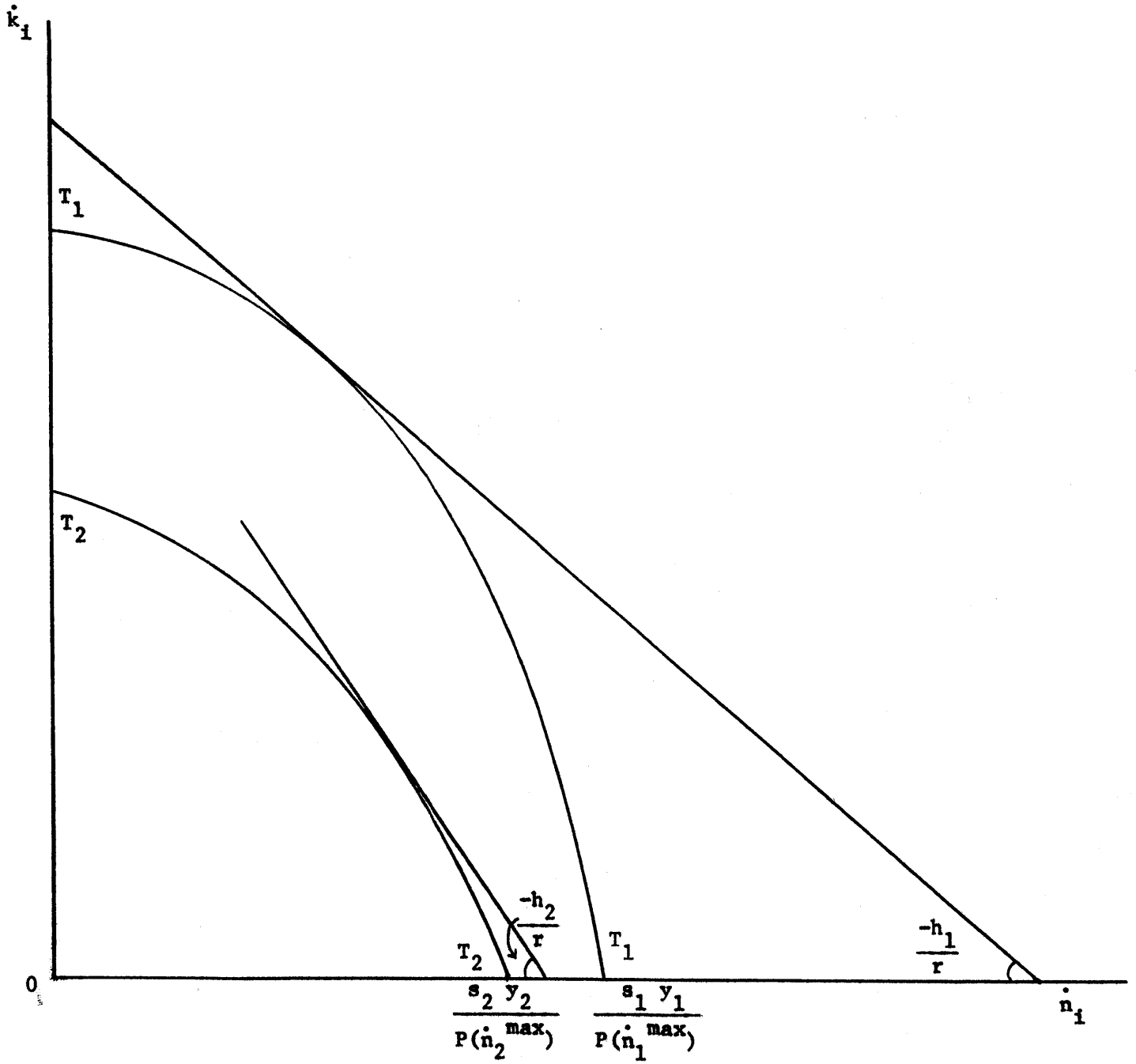


Figure 1

the two income groups. Denoting the rates for Group 1 and Group 2 by  $h_1$  and  $h_2$  respectively,  $h$  should be looked upon as the weighted average of  $h_1$  and  $h_2$ , i.e.,  $h = \frac{N_1}{N}h_1 + \frac{N_2}{N}h_2$  where  $N_1$ ,  $N_2$  and  $N$  are the total stocks of human capital for Group 1, Group 2 and the economy as a whole. The difference between  $h_1$  and  $h_2$  can be explained in the following way.

The rate of return to any educational investment is calculated by equating the discounted sum of future returns from the additional education with the cost of that education. A special characteristic of this educational cost is that, in addition to the direct expenditure, it also includes the opportunity cost which is the income foregone while attaining this additional education. Now, since Group 1 has on the average higher level of educational attainment and there is a positive correlation between educational level and income earnings, it is clear that this opportunity cost will be higher for Group 1. Hence, there will be a corresponding difference between  $h_1$  and  $h_2$ . To repeat, therefore, although for the economy as a whole there will be in any period a unique 'h' as an average concept, the value of the rate of return on education will be different when viewed from the standpoint of the two income classes taken separately. And in making the portfolio decision, the typical individual of each class will consider its own rate of return,  $h_1$  or  $h_2$ , as the relevant parameter.

It is also clear that since there is no similar problem of opportunity cost in the accumulation of physical capital, there will be no inter-class difference in the value of  $r$ .

The portfolio problem for a typical individual of Group 1 then is to choose  $\dot{k}_1$  and  $\dot{n}_1$  such that the rental income<sup>53</sup>

$$\hat{y}_1 = r\dot{k}_1 + h_1\dot{n}_1 \text{ is maximum}$$

subject to

$$\phi_1(\dot{k}_1, \dot{n}_1) = \dot{k}_1 + p\dot{n}_1 - s_1 y_1 = 0$$

The first-order condition for the constrained maximum requires that

$\frac{h_1}{p} = r$ , which is the standard perfect arbitrage condition in the absence of any Wicksellian "price-effect." The second-order condition is ensured by the concavity of the transformation frontier. Since  $h_1 \neq h_2$ , it is clear from the first-order condition that there will be some difference between  $\lambda^1_s$  and  $\lambda^2_s$  - the ratios in which savings are allocated between two kinds of investments by the two groups. A possible situation is shown in Fig. 1.

Denoting now the proportions of income spent by Group 1 on the accumulation of physical and human capital by  $\beta_i$  and  $e_i$  respectively, where  $\beta_i = \lambda^i_s s_i$  and  $e_i = \lambda^i_n s_i$  ( $i = 1, 2$ ), we can write

$$\begin{aligned} \dot{k}_1 &= \beta_1 y_1 & \dot{k}_2 &= \beta_2 y_2 \\ (2.11) \quad & & \text{and} & \\ \dot{E}_1 &= e_1 y_1 & \dot{E}_2 &= e_2 y_2 \end{aligned}$$

We shall see that in the countries we are interested in  $s_1$  is so overwhelmingly large compared to  $s_2$  that it tends to outweigh any

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<sup>53</sup>We have ruled out any consumption benefit from these investment and hence the objective function has not been characterised in terms of utility function.

conceivable variation in the allocation ratios. As a result, the effective proportions of income spent by Group 1 in both types of investment tend to be larger than Group 2, i.e.,  $\beta_1 > \beta_2$  and  $e_1 > e_2$ . For instance, in the case of India it has already been mentioned that  $s_1 \approx 30\%$  and  $s_2 \approx 0$ , so that as long as  $\omega > h_1 > 0$  and  $\omega > r > 0$ , it follows that  $\beta_1 > \beta_2$  and  $e_1 > e_2$ .

Now, the existing stock of expenditure of Group  $i$  on human capital at any point of time can be written as

$$E_i(t) = \int_{-\infty}^t E(\tau) d\tau$$

The average educational attainment of the group, denoted by  $n_i(t)$  for any point of time  $t$ , is a function of this accumulated educational investment per man over all the past years up to  $t$ . Or, in other words, it is a function of the existing stock of (cumulated) expenditure per man, i.e.,

$$(2.12) \quad n_i(t) = n(E_i(t)) \quad i = 1, 2.$$

This function along with other crucial relationships will be estimated in Section 5.

The equation (2.1)-(2.12) define the model of an economy, the framework of our analysis. The problem which is now posed within this framework is: starting with an initial nonegalitarian situation, as characterised by  $x = \frac{y_1}{y_2} > 1$ , does this economic system converge to an egalitarian state? To answer this question, it is necessary, first of all, to spell out the meaning of convergence more precisely and find out the conditions for such convergence. This is what we propose to do in the next section.

### 3. Meaning and Conditions of Convergence

By convergence to an egalitarian state, we shall mean that, starting with a value  $> 1$ , the time function  $x(t)$  should tend to 1 asymptotically and monotonically, i.e.,

$$(3.1) \quad \lim_{t \rightarrow \infty} x(t) = 1, \quad \text{with } \dot{x}(t) < 0 \text{ for every } t \in (0, \infty)$$

The convergence has to be asymptotic because, given the definition of  $x$  which is conditional upon a particular scheme of partitioning,<sup>54</sup>  $x(t)$  can never attain the value 1 exactly, it can only approach that asymptotically.

We also insist on the monotonicity of convergence because, for one thing, it may be highly desirable, on very pertinent non-economic grounds, to rule out the arbitrariness of a process which leads to a fall in equality at one point of time and rise at another; it is preferable to be interested in a process which assures "mere equality" at every iteration. For another, in the case of non-monotonic convergence, it is not possible to find out a necessary or a sufficient condition which is valid for every point of time. The conditions then can only be of an asymptotically restrictive nature and therefore uninteresting from the standpoint of economic policy.

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<sup>54</sup> See p. 27.

Given this meaning of convergence, the conditions for its attainment can be stated in the following way:

Proposition 1: A sufficient condition for  $\lim_{t \rightarrow \infty} x(t) = 1$  subject to  $\dot{x}(t) < 0$  for every  $t \in (0, \infty)$  is that there exists a number  $\epsilon (< 0)$  such that

$$(3.2) \quad \frac{dx(t)}{dt} / \frac{dx(t)-1}{x(t)-1} \leq \epsilon < 0 \quad \text{for every } t \in (0, \infty)$$

Proof: Define  $x^*(t) = x(t) - 1$ , so that the given problem is equivalent to finding out the sufficient condition for  $\lim_{t \rightarrow \infty} x^*(t) = 0$  with  $\dot{x}^*(t) < 0$  for every  $t \in (0, \infty)$ . Now, for any  $t$ , we can write

$$\frac{dx^*(t)}{dt} / \frac{dx^*(t)}{x^*(t)} = -\lambda(t) \quad \text{where} \quad \lambda(t) > 0$$

so that

$$x^*(t) = x^*(0) e^{-\int_0^t \lambda(\tau) d\tau}$$

Clearly then, the necessary and sufficient condition for

$$\lim_{t \rightarrow \infty} x^*(t) = 0 \quad \text{is} \quad \int_0^{\infty} \lambda(t) dt = \infty$$

It would have been most desirable if this necessary and sufficient condition could be transformed into an equivalent condition presentable in terms of  $\lambda(t)$  and holding for every  $t$ , because such a characterisation is very important from the standpoint of economic interpretation. Unfortunately, however, it is possible to prove that there does not exist any such condition which is both necessary and sufficient

and, at the same time, capable of characterising  $\lambda(t)$  for every  $t$ . It is only possible to derive such an "equivalent" condition in an asymptotic sense, i.e., valid only for  $t \rightarrow \infty$ . Any asymptotic condition, however, in the context of our model does not have any interesting economic interpretation or practical use.

We shall therefore derive one sufficient condition in terms of  $\lambda(t)$ , valid for every  $t$ , and one necessary condition in terms of  $\lambda(t)$ , again valid for every  $t$ . These two conditions will have to be unavoidably distinct, the dichotomy being the result of an equivalence of the basic condition of convergence to the unboundedness of an improper integral.

From this requirement of unboundedness of  $\int_0^t \lambda(t) dt$  subject to the condition of monotonicity, it follows that a sufficient condition for this is:  $\lambda(t) (> 0)$  should be bounded away from zero. But, by definition,

$$\lambda(t) = - \frac{dx^*(t)}{dt} / x^*(t) = - \frac{dx(t)}{dt} / x(t) - 1$$

Hence the proposition.

**Remark:** That this condition is only sufficient can be verified by setting  $\lambda(t) = \frac{1}{t^p}$ , where  $p \leq 1$

Quite independent of this sufficient condition, it is also possible to state a necessary condition for convergence in the following way:

Proposition 2: A necessary condition for  $\lim_{t \rightarrow \infty} x(t) = 1$  subject to  $\dot{x}(t) < 0$  for every  $t \in (0, \infty)$  is

$$(3.3) \quad \frac{dy_1(t)}{dt} / y_1(t) < \frac{dy_2(t)}{dt} / y_2(t) \quad \text{for every } t \in (0, \infty)$$

The proposition is self-evident from the definition of monotonicity and  $x(t)$ .

Usually, one would have considered the necessity of supplementing this condition by the following restriction:

$$\dot{x}(t) = 0 \quad \text{for } t \geq \bar{t} \quad \text{when } x(\bar{t}) = 1,$$

so that any danger of an inequality being opened up in the opposite direction is ruled out. However, this is not necessary in our case because, as pointed out earlier,  $x(t)$  can approach 1 only asymptotically and, by definition,  $x(t) \neq 1$ .

For our analysis, we shall only make use of the condition stated in Proposition 2.

Now the question which naturally arises is: how can this condition, formal as it is, be used to study the behavior of income distribution in an actual economy? We suggest the following procedure to answer this question.

In many economies, particularly in Colombia and India, it is possible to obtain data on the relationship between educational attainment and income earnings of the individuals. A curve can therefore be fitted to such a scatter.



However, a curve fitted in this way will not show the (partial) relationship between only income and education, because there are other relevant variables which have not been controlled and which have come into this scatter. This will be a curve showing the relationship between income and education when all other relevant variables are changing. Although on a first thought it may sound strange, it is precisely this curve--this total relationship between income and education--that we really want and we will presently see why.

It should also be noted that there is not one curve but actually two curves, one for each group, which should be fitted. This is quite clear from the income-generation functions (2.8) which were specified separately for the two groups. Once such curves are carefully regressed, on appropriate econometric specification of "best-fit", it is possible to estimate the values of  $\frac{dy_1}{dn_1}$  and  $\frac{dy_2}{dn_2}$  corresponding to the average values of  $n_i$  for each group. These  $\frac{dy_i}{dn_i}$  ( $i = 1, 2$ ) should be interpreted as the total derivative of  $y_i$  with respect to  $n_i$  or, alternatively, as the ratio of the total differentials,  $dy_i$  and  $dn_i$ . Hence they stand for the entire following expression:

$$\frac{dy_i}{dn_i} = \frac{1}{dn_i} \left[ \left( \frac{\partial w}{\partial k} dk + \frac{\partial w}{\partial n} dn \right) + \left( \frac{\partial r}{\partial k} dk + \frac{\partial r}{\partial n} dn \right) k_i + rdk_i + \left( \frac{\partial h}{\partial k} dk + \frac{\partial h}{\partial n} dn \right) n_i + rdn_i \right] \quad i = 1, 2.$$

It is also possible to obtain data on the expenditure at different levels of education. From these data, the expenditure-education function,  $n_i = n(E_i)$ , can be estimated and the values of  $\frac{dn_1}{dE_1}$  and  $\frac{dn_2}{dE_2}$  determined corresponding to the average educational expenditure of the respective groups. In general, this expenditure-education relationship is found to be concave from below.

Finally, from the household surveys, the data are available on the saving-income ratios and educational expenditure-income ratios for different income groups. From these, one can estimate the values of  $e_1 \left( = \frac{\dot{E}_1}{Y_1} \right)$  and  $e_2 \left( = \frac{\dot{E}_2}{Y_2} \right)$ .

Now, collect these three sets of informations and put them side by side to obtain the values of these chain derivatives:

$$\frac{dy_1}{dn_1} \cdot \frac{dn_1}{dE_1} \cdot \frac{dE_1}{dt} \cdot \frac{1}{Y_1} \quad \text{and} \quad \frac{dy_2}{dn_2} \cdot \frac{dn_2}{dE_2} \cdot \frac{dE_2}{dt} \cdot \frac{1}{Y_2} .$$

However, these chain derivatives are nothing but  $\frac{dy_1}{dt}/Y_1$  and  $\frac{dy_2}{dt}/Y_2$ , the two crucial expressions in terms of which the conditions of convergence have been stated. Therefore, one can now compare the values of these two expressions to ascertain the behaviour of the distribution of income--its egalitarian or non-egalitarian tendency--in any economy.

This is a simple procedure and our motivation in suggesting such a procedure has been this. The distribution of income is determined by the interaction of several variables; this is obvious from expressions such as  $\frac{dy_i}{dn_i}$ . However, very often it is not possible to obtain data on the detailed inter-relationship of all the variables. Let us therefore approach the problem in a slightly different way. It may be possible to select carefully a few variables on which the data are available and then it may also be possible by using their observed relationships to extract information about the nature of the underlying non-observable interactions. The procedure outlined above is fundamentally based on this intuitive idea.

And, such a procedure has a remarkable advantage. Since it purports to bring out the implications of the underlying interactions and integrate them with the conditions of convergence, it can provide a very important insight

into the working of different kinds of opposing forces, their nature and mutual strength, all of which taken together make the distribution of income what it is at any point of time. Let us illustrate this. It is generally found, and this will be corroborated for India and Colombia in Section 5, that  $\frac{dy_1}{dn_1} > \frac{dy_2}{dn_2}$ , the derivatives being evaluated at the average level of educational attainment of the respective groups. It is also found that  $e_1 \left( = \frac{\dot{E}_1}{y_1} \right) > e_2 \left( = \frac{\dot{E}_2}{y_2} \right)$ . However, on the other hand, given the usual concavity (from below) of the expenditure-education relationship, it is expected that  $\frac{dn_1}{dE_1} < \frac{dn_2}{dE_2}$ , the derivatives being again evaluated at the average expenditure points. Therefore, it is not clear whether  $\frac{dy_1}{dt}/y_1 > \frac{dy_2}{dt}/y_2$ ; it depends on the relative magnitudes of the respective slope-characteristics and ratios. Postponing a complete analysis of how these relative magnitudes can be known in any particular economy for a later discussion, let us consider at the moment a situation where it is supposed, for the purpose of illustration, that these magnitudes are such that

$$\frac{dy_1}{dt}/y_1 \left( = \frac{dy_1}{dn_1} \cdot \frac{dn_1}{dE_1} \cdot \frac{dE_1}{dt} \cdot \frac{1}{y_1} \right) > \frac{dy_2}{dt}/y_2 \left( = \frac{dy_2}{dn_2} \cdot \frac{dn_2}{dE_2} \cdot \frac{dE_2}{dt} \cdot \frac{1}{y_2} \right)$$

which means that the necessary condition for convergence to an egalitarian state is violated. Now, why can this happen? What are the forces responsible for it?

Differentiating  $y_1$  and  $y_2$  totally with respect to time, it can be found that

$$\frac{dy_1}{dt}/y_1 > \frac{dy_2}{dt}/y_2 \text{ means}$$

$$\begin{aligned}
 & \dot{k} \left( \frac{1}{y_1} \frac{\partial h}{\partial k} n_1 - \frac{1}{y_2} \frac{\partial h}{\partial k} n_2 \right) + \dot{n} \left( \frac{1}{y_1} \frac{\partial r}{\partial n} k_1 - \frac{1}{y_2} \frac{\partial r}{\partial n} k_2 \right) + r \left( \frac{k_1}{y_1} - \frac{k_2}{y_2} \right) + h \left( \frac{\dot{n}_1}{y_1} - \frac{\dot{n}_2}{y_2} \right) \\
 (3.4) \quad & > \dot{k} \left( \frac{1}{y_1} \left| \frac{\partial r}{\partial k} \right| k_1 - \frac{1}{y_2} \left| \frac{\partial r}{\partial k} \right| k_2 \right) + \dot{n} \left( \frac{1}{y_1} \left| \frac{\partial h}{\partial n} \right| n_1 - \frac{1}{y_2} \left| \frac{\partial h}{\partial n} \right| n_2 \right) + \dot{\omega} \left( \frac{1}{y_2} - \frac{1}{y_1} \right)
 \end{aligned}$$

It is interesting to interpret this inequality. There are two kinds of forces working in this system - one tending to equalise incomes of the two classes and the other having a disequalising effect. In disentangling these forces, we shall suppose that Group 1 has more of both physical and human capital per person relative to its average level of income,

$$\text{i.e., } \frac{k_1}{y_1} > \frac{k_2}{y_2} \quad \text{and} \quad \frac{n_1}{y_1} > \frac{n_2}{y_2} . \quad ^{55}$$

With the accumulation of both physical and human capital per man, the rate of return on either capital as related to its own stock tends to fall (i.e.,  $\dot{k} \frac{\partial r}{\partial k} < 0$ ,  $\dot{n} \frac{\partial h}{\partial n} < 0$ ) because of generalised diminishing returns. This affects Group 1 more unfavorably than Group 2 because, by supposition, Group 1 has more of both physical and human capital per person relative to its income level. Therefore, this is an equalising force and its absolute value is shown by the first two terms on the R.H.S. of (3.4). Also, as the wage rate goes up with the accumulation of human and physical capital (note that  $\dot{\omega} = \frac{\partial \omega}{\partial k} \dot{k} + \frac{\partial \omega}{\partial n} \dot{n}$  and  $\frac{\partial \omega}{\partial k}, \frac{\partial \omega}{\partial n} > 0$ ), This again affects the average income of Group 1 more

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It is possible to consider other possible situations where this supposition is not true. However, the basic logic behind this interpretation still remains the same, only the terms which come under the equalising and disequalising forces interchange their positions.

unfavorably relative to its previous level of income since  $y_1 > y_2$  ; the last term on the R.H.S. captures this. The R.H.S., as a whole, therefore shows the absolute value of the equalising forces in the system.

On the other hand, as the accumulation of physical capital goes on, it tends to increase the return on human capital through the effect of complementarity (i.e.,  $k \frac{\partial h}{\partial k} > 0$ ). The same is true of the effect of the accumulation of human capital on the return on physical capital (i.e.,  $n \frac{\partial r}{\partial n} > 0$ ). This complementarity effect tends to favor Group 1 more than Group 2 because again Group 1 has more of both kinds of capital per man relative to its income level. This is therefore a disequalising force and is captured by the first two terms on the L.H.S. of (3.4). In addition to this, if it is found that Group 1 is accumulating both types of capital at a faster rate in relation to its income (i.e.,  $\frac{\dot{k}_1}{y_1} > \frac{\dot{k}_2}{y_2}$  and  $\frac{\dot{n}_1}{y_1} > \frac{\dot{n}_2}{y_2}$ ), then, so long as  $r$  and  $h$  are positive, this also has a disequalising effect as shown by the last two terms on the L.H.S.

Now it is clear that when the L.H.S. of (3.4) exceeds the R.H.S., the disequalising forces in the economy are stronger in absolute value than the equalising forces and that explains why and how " $\frac{dy_1}{dt}/y_1 > \frac{dy_2}{dt}/y_2$ " implies an increase in inequality over time.

We want to emphasize the important of the coexistence of physical and human capital and their complementarity in the explanation of this inequality. In the absence of any one of these two kinds of capital

(say education), diminishing returns tend to be a very dominant force<sup>56</sup> and on the basis of that, under certain assumptions, one can indeed visualise a built-in equalising device in an economy. It is interesting to point out that the model of income distribution suggested by Stiglitz turns out to be essentially a special case of our model when human capital is ignored. Similarly, the traditional human-capital doctrine is again another polar case when physical capital is ignored. Small wonder that in either approach there is a suggestion of natural equalisation of income.<sup>57</sup>

However, the moment we introduce education along with physical capital, a potentially disequalising force which works through the complementarity between the two kinds of capital comes into operation. And, now it becomes possible for any income group to postpone the otherwise disequalising effect of diminishing returns to either form of capital indefinitely by simultaneously investing in both of them. This simultaneity of investment in two (or more) types of capital which are mutually complementary in raising the productivity of each other acts as a safety valve against the pressures of "the falling rate of profit" due to diminishing returns. We believe that the nature of the distribution of income in any economy is determined in this way,

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<sup>56</sup> In this case, the first two terms on the L.H.S. of (3.4) showing the disequalising complementary effects disappear; only  $r \left( \frac{\dot{k}_1}{y_1} - \frac{\dot{k}_2}{y_2} \right)$  remains. Similarly, the second term on the R.H.S. disappears.

<sup>57</sup> See Stiglitz, op. cit.

by the comparative strength of these two mutually opposing forces of diminishing returns and complementarity, along with the relative endowment of resources and their rates of accumulation by the respective income classes. This is a simple point, but still worth making.

We also believe that in any economy the government plays a very significant role in this entire process, and it is time that we accommodate this role of the government in our analysis. This leads us to the formulation of a policy model.

#### 4. The Policy Model

In this section, we propose to develop a policy model, corresponding to our basic analytical structure by introducing taxes and government educational expenditure.

Consider a system where incomes of both groups are taxed and then tax proceeds are spent by the government, among other things, on education for both classes. Let  $z_1$  and  $z_2$  be the over-all tax-rates and  $Y_1^d$  and  $Y_2^d$  the total disposable incomes of the rich and the poor respectively. Then, in any period,

$$(4.1) \quad Y_1^d = (1 - z_1)Y_1, \quad Y_2^d = (1 - z_2)Y_2$$

or

$$(4.1)' \quad y_1^d = (1 - z_1)y_1, \quad y_2^d = (1 - z_2)y_2,$$

where  $y_1^d$  and  $y_2^d$  are the average disposable incomes of the two groups. The coefficient of inequality should now be redefined as

$$(4.2) \quad x = \frac{y_1^d}{y_2^d}.$$

The production characteristics are the same as in the analytical model and therefore the relationships (2.2)-(2.7) simply carry over. For the same reason, the pre-tax income-generation functions are also exactly like (2.8).

The saving per man of either group should of course be considered now as depending on its average disposable income, so that

$$(4.3) \quad S_1 = s_1 y_1^d, \quad S_2 = s_2 y_2^d.$$



These private savings are used by the individuals of both groups to accumulate physical capital and education in the same manner as before. The difference is that now, in addition to private expenditure on education, there will be government educational expenditure for both groups as well.<sup>51</sup> Therefore, we can write

$$(4.4) \quad \dot{E}_1 = \dot{E}_1^p + \dot{E}_1^g, \quad \dot{E}_2 = \dot{E}_2^p + \dot{E}_2^g$$

where  $\dot{E}_i$ ,  $\dot{E}_i^p$  and  $\dot{E}_i^g$  are the total, private and government expenditure on education per man of Group  $i$  ( $i = 1, 2$ ). The private expenditure, as has already been mentioned, are determined in the same way as before so that

$$(4.5) \quad \dot{E}_1^p = e_1 y_1^d, \quad \dot{E}_2^p = e_2 y_2^d.$$

How are  $\dot{E}_1^g$  and  $\dot{E}_2^g$  determined? These government expenditures are related to the total budget and, through that, to taxes. Taxes, in their turn, are related to the incomes of the two groups and the entire system therefore can be causally closed.

The total tax proceeds,  $T$ , in any period is

$$(4.6) \quad T = z_1 y_1 L_1 + z_2 y_2 L_2.$$

Let  $\alpha$  be the fraction of the total budget,  $B$ , which is financed by taxes and  $\delta$  the portion of the total budget allotted to educational expenditure. Hence, denoting by  $\dot{E}^*$  the total expenditure on education, we have  $\dot{E}^* = \delta B = \frac{\delta}{\alpha} T$ . This total expenditure in any period is divided between Group 1 and Group 2

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<sup>51</sup>In this policy model, on the expenditure side, we are interested only in the educational expenditure policies, although the model can be easily extended to include expenditure policies on physical capital.

in the ratio of, say,  $v_1$  and  $v_2$ , so that government educational expenditure per man for each income group can be written as

$$(4.7) \quad \dot{E}_1^G = v_1 \frac{\dot{E}^*}{L_1} = v_1 \frac{\delta}{\alpha} \frac{T}{L_1} = \lambda_1 \frac{T}{L_1},$$

and

$$\dot{E}_2^G = v_2 \frac{\dot{E}^*}{L_2} = v_2 \frac{\delta}{\alpha} \frac{T}{L_2} = \lambda_2 \frac{T}{L_2},$$

where  $\lambda_i = v_i \frac{\delta}{\alpha}$  so that  $\lambda_1 > \lambda_2 \iff v_1 > v_2$

Finally, there is the expenditure-education relationship:

$$(4.8) \quad n_i = n(E_i) \quad (i = 1, 2)$$

where  $n_1$  has the same meaning as before, but  $E_1$  is now the sum of the existing stocks of private educational expenditure per man made by Group 1 and also government educational expenditure made per man for Group 1.

Equations (4.1)-(4.8) and the relevant equations which are taken unaltered from the analytical structure define the policy model. The question is: what is the necessary requirement for tax and expenditure policy to ensure the objective of convergence to an egalitarian state? To be more precise, what should be the nature of  $\lambda_1, \lambda_2, z_1, z_2$  at any point of time to ensure  $\lim_{t \rightarrow \infty} x(t) = 1$ ?

Consider a necessary condition which has to be satisfied. By proposition 2 of Section 3 and (4.2), it follows that a necessary condition for this convergence is

$$(4.9) \quad \frac{dy_1^d(t)}{dt} / y_1^d(t) - \frac{dy_2^d(t)}{dt} / y_2^d(t) \leq 0$$

for every  $t \in (0, \infty)$ . Using (4.1)', (4.4), (4.5), (4.7) and (4.8), (4.9)

can be written as

$$(4.10) \quad \frac{dz_1}{1-z_1} - \frac{dz_2}{1-z_2} + \left[ \frac{dy_1}{dn_1} \frac{dn_1}{dE_1} e_1(1-z_1) + \frac{\lambda_1 T}{L_1 y_1} \right] - \left[ \frac{dy_2}{dn_2} \frac{dn_2}{dE_2} e_2(1-z_2) + \frac{\lambda_2 T}{L_2 y_2} \right] \stackrel{\Delta}{=} 0$$

This should hold for every point of time; time-subscript is skipped for convenience.

We propose to estimate all the parameters and coefficients involved in (4.10) for both India and Colombia and then use this condition to find out whether at least a necessary requirement for convergence has been satisfied in these two countries in a particular period. The purpose of this paper is to enquire into the egalitarian or nonegalitarian character of government policies on taxation and educational expenditure for these two countries, and, to reach a definite conclusion in this matter, we shall see, it is only this necessary condition that we need to use.

It should be pointed out, however, that it is also possible to derive a sufficient condition for convergence with reference to this policy model. By using Proposition 1 of Section 3, such a sufficient condition can be written as

$$(4.11) \quad \frac{dy_1^d(t)}{dt} / y_1^d(t) - \frac{dy_2^d(t)}{dt} / y_2^d(t) \leq \xi \left( 1 - \frac{1}{x(t)} \right)$$

for  $\xi (< 0)$  and  $t \in (0, \infty)$ . Using again (4.1)', (4.4), (4.5), (4.7) and (4.8), (4.11) can be expressed as

$$(4.12) \quad \left[ \frac{dz_1}{1-z_1} - \frac{dz_2}{1-z_2} \right] + \frac{dy_1}{dn_1} \frac{dn_1}{dE_1} \left[ e_1(1-z_1) + \frac{\lambda_1 T}{L_1 y_1} \right] - \frac{dy_2}{dn_2} \frac{dn_2}{dE_2} \left[ e_2(1-z_2) + \frac{\lambda_2 T}{L_2 y_2} \right] = \xi \left( 1 - \frac{1}{x(t)} \right)$$

This again should hold for every  $t$ .

This sufficient condition can be used to find out the nature of optimal policy instruments. With distributional objective as the only target, this policy model is obviously underdetermined in the Tinbergenesque sense with degrees of freedom left on the choice of policy instruments. There are different ways of closing these degrees of freedom. We can choose to parameterise the taxation side of the problem and then derive the optimal structure of expenditure at a point of time and also over time, or we can parameterise the expenditure part and in a symmetrical way find out the optimal evolution of tax structure. Or, better still, we can use the degree of freedom to generate the optimal trade-off equation between the taxation and expenditure instruments. This will help us in determining to what extent one set of policy variables can be substituted by another at a point of time as well as over time. However, this analysis of and the derivation of the optimal policy paths is another important subject by itself to which we do not want to enter right now because it is not strictly essential for the purpose of this paper and also nothing short of another complete research work can do justice to that subject.

Coming back to this paper, to the question which we want to answer, it is enough, as has been mentioned before, to use only the necessary condition and compare the actual characteristics of tax-expenditure policies of India and Colombia with the necessary characteristics as given by (4.10).

It is to these comparisons and empirical results that we now turn.

### 5. Empirical Results

It is clear from (4.9)-(4.11) that to be able to make a definitive statement about the equalising or disequalising tendency of an economy and the nature of government policies in any period, it is necessary to estimate the income-education relationship and the expenditure-education relationship for each income group and collect information about the over-all incidence of the tax structure as well as the educational expenditure-income ratios (both private and government) for each class. We have done these estimations and tried to obtain all the other relevant informations for both India and Colombia. As is understandable for this kind of empirical work, the data were not always available in a very obliging form and therefore at times we had to make certain simplifying assumptions. Our conclusions are valid only subject to these assumptions.

There is another point which should be clarified at the outset in order to avoid any possible misunderstanding. Since we are asking the question about convergence of an economic system, we are basically interested in its dynamic behavior. However, by a suitable characterisation of the problem, as discussed in detail in Section 3, we have been able to reduce the crucial condition for this convergence to an expression which has to hold for any particular point of time. As a result, it is now possible to judge the dynamic tendency of an economy just by looking at its behavior at one point of time. We shall, therefore, estimate the relevant relationships and collect information about the required parameters for the two countries only for a particular point (or, period)

of time, because that is all what we need in order to be able to use the necessary condition (4.10) and conclude about possible convergence, or the lack of it, in any economy. Therefore, the data we are looking for are the cross-section data; we do not have to use the time-series data.

This method which enables one to study the dynamic behavior of a system by looking at its property only at one point of time is a very useful and efficient device of extracting all the relevant informations about the laws of motion of the system, particularly when the time-series data are not available which, incidentally, also happens to be the case for the two countries we are interested in. We have devoted so much time in explaining the logic of this method because we do not want it to be misunderstood as a static or pseudo-dynamic analysis in the Hicksian sense.

### India

#### The Income-Education Relationships:

Before reporting the empirical findings on these relationships, it is appropriate to make certain observations:

1. The data from which the income-education relationships are measured relate to individual educational attainment and individual income for both income groups. However, the postulated relationships such as (2.8) relate to the average income and the average educational attainment of the respective groups. It is therefore important that we establish a link between the two.

Let  $Y_{ij}$  be the income of the  $j$ -th member of Group  $i$  ( $i = 1, 2$ ),  $\bar{y}_i$  the average income of the group,  $N_{ij}$  the educational attainment of its  $j$ -th member.

and  $n_i$  its average educational attainment. Then writing the total relationship between individual income and individual educational attainment of any Group  $i$  as  $Y_{ij} = \Psi_i(N_{ij})$ , we have, upon expanding  $\Psi_i(N_{ij})$  in Taylor's series around  $N_{ij} = n_i$ ,

$$(5.1) \quad Y_{ij} = \Psi_i(n_i) + (N_{ij} - n_i) \Psi_i'(n_i) + \frac{(N_{ij} - n_i)^2}{2!} \Psi_i''(n_i) + \frac{(N_{ij} - n_i)^3}{3!} \Psi_i'''(n_i) .$$

We have made cubic approximation and ignored higher-order derivatives. Summing both sides over every individual of Group  $i$  and dividing each term by the population  $L_i$ , it follows that

$$(5.2) \quad y_i = \Psi_i(n_i) + \frac{1}{2} \text{var}(N_{ij}) \Psi_i''(n_i) + \frac{1}{6L_i} \sum_j (N_{ij} - n_i)^3 \Psi_i'''(n_i) .$$

Therefore,

$$(5.3) \quad \frac{dy_i}{dn_i} = \Psi_i'(n_i) + \frac{1}{2} \text{var}(N_{ij}) \Psi_i'''(n_i) + \frac{1}{2} \sum_j (N_{ij} - n_i)^2 (-1) \Psi_i'''(n_i) , \text{ ignoring higher-order derivatives}$$

$$= \Psi_i'(n_i)$$

From the available data relating to individual income and educational attainment, it is possible to estimate the R.H.S. of (5.3) and therefore  $\frac{dy_i}{dn_i}$ .

2. As it has already been explained, we seek to understand through this estimation the total relationship between the average income and the average education of each group when all other relevant variables have been deliberately left uncontrolled. We want all the underlying interactions to work themselves out and are interested only in the end result as represented by this total relationship between income and education.

3. One limitation of the data on the income-education relationship is that they are available in a systematic form only for urban India and therefore,

so far this particular relationship is concerned, the estimation had to be based on the urban data. The standard sources for such data are the Urban Income Survey and the relevant issues of Technical Manpower.<sup>59</sup> However, it was not possible to obtain from these sources the "raw" data in a very detailed form. The data are usually available in a summarized form as presented in Table 1 and our analysis had to be based on that. It should be pointed out in this context that all the well-known estimates of the rates of return on education in India have been made on the basis of this particular tabular information.<sup>60</sup>

4. It should also be noted that the data on the relationship between income and education for individuals, such as obtained from Table 1, really refer to earners rather than to individuals as such. So far, in our theoretical framework, we have not made any distinction between the two but, given this restriction of the data, such a distinction may now be deemed necessary. And, we can introduce that quite easily into our analysis and accommodate the resulting changes in the following way.

Let us express the average income  $y_i$  of Group  $i$  as

$$(5.4) \quad y_i = \frac{Y_i^e}{L_i} \cdot \frac{L_i^e}{L_i} = y_i^e l_i^e$$

where  $L_i^e$  is the total number of earners,  $y_i^e$  the average income per earner and  $l_i^e$  the proportion of earners in the population of Group  $i$ . Denoting now by  $N_i^e$  the

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<sup>59</sup>Urban Income Survey, *op. cit.*; Technical Manpower (March, 1961; Dec. 1963, Aug. 1964, Oct. 1964, March 1965, June 1965, June 1966).

<sup>60</sup>See A.M. Nall Gounden, "Investment in Education in India", *Journal of Human Resources*, 1967. Blaug *et al.*, *op. cit.*, Chs. 7, 9.



Table 1: Average monthly earnings of workers: by education level,  
Urban India, 1960/61 (Rs. per month)

Education level	Illiterate	Primary	Middle	Matric	Graduate	Engineering Graduate
Actual no. of years to completion	0	4.6	7.7	10.8	14.8	16.8
Income ↓						
	11	16				
	14	16				
	16	18				
	21	23	29			
	24	25	36			
	30	34	41			
	30	40	41	41		
	34	41	42	83		
	37	53	75	98		
	38	57	90	117		
	39	69	98	121	123	
	47	71	100	121	141	
	47	74	104	123	147	309
	63	104	125	163	243	362
	72	125	156	217	337	497
	86	139	180	241	372	647
	85	161	209	249	403	782
	69	132	212	278	426	918
	61	116	162	259	450	1,134
	62	102	144	208	351	1,266

Sources: Urban Income Survey, N. C. A. E. R., New Delhi.

Technical Manpower, op. cit.

total educational attainment of the earners, we can write

$$(5.5) \quad N_i^e = \mu_i N_i$$

where  $\mu_i$  ( $0 < \mu_i \leq 1$ ) shows the proportion of the total education of Group  $i$  that is remunerative. Clearly,  $\mu_i = 1$  when there is no educated unemployment. Dividing both sides of (5.5) by  $L_i^e$ , we have

$$(5.6) \quad n_i^e = \mu_i \frac{n_i}{l_i}$$

where  $n_i^e$  is the average educational attainment of the earners of Group  $i$ .

Now, differentiation of (5.4) with respect to  $n_i$  gives

$$(5.7) \quad \frac{dy_i}{dn_i} = \frac{dy_i^e}{dn_i^e} \frac{dn_i^e}{dn_i} l_i^e + y_i^e \frac{dl_i^e}{dn_i}$$

$$= \frac{dy_i^e}{dn_i^e} \left( \frac{d\mu_i}{dn_i} n_i + \mu_i \right) + y_i^e \frac{dl_i^e}{dn_i}, \quad \text{using (5.6).}$$

This equation (5.7) provides the required link between  $\frac{dy_i}{dn_i}$  and the estimable  $\frac{dy_i^e}{dn_i^e}$ , given the other variables and derivatives.

About these other variables and derivatives, it is possible to make some qualitatively comparative statements on their values for the two income groups. For one thing, by definition,  $y_1^e > y_2^e$ . For another, it is possible to refer to some interesting evidences showing how an increase in educational attainment can affect in very different ways the chances of getting employment for the typical individuals of the two groups, Group 1 having. not with a

surprisingly, a distinct edge over Group 2.<sup>54</sup> For our purpose, this can be taken to imply that in all these magnitudes, namely,  $\frac{dl_1^e}{dn_1}$ ,  $\mu_1$ ,  $\frac{d\mu_1}{dn_1}$ , Group 1 has a comparative advantage, i.e.,

$$\frac{dl_1^e}{dn_1} > \frac{dl_2^e}{dn_2}, \quad \mu_1 > \mu_2 \quad \text{and} \quad \frac{d\mu_1}{dn_1} > \frac{d\mu_2}{dn_2}.$$

Supposing further that  $\frac{d\mu_i}{dn_i}$ ,  $\frac{dl_i^e}{dn_i} > 0$ , and using the fact that  $n_1 > n_2$ , we can write

$$(5.8) \quad \frac{dy_1}{dn_1} = \theta_1 \frac{dy_1^e}{dn_1^e} \quad \text{and} \quad \frac{dy_2}{dn_2} = \theta_2 \frac{dy_2^e}{dn_2^e}$$

with  $\theta_1 > 0$  and  $\theta_1 > \theta_2$ .

For the estimation of  $\frac{dy_i^e}{dn_i^e}$  from the observed data relating to individual earners, of course, we shall use a relationship similar to (5.3), i.e.

$$(5.9) \quad \frac{dy_i^e}{dn_i^e} = \Psi_i^{e'}(n_i^e)$$

With all these in mind, let us start with the data of Table 1. First of all, we partition the observations into two sets, Set 1 and Set 2, such that Set 1 contains all the values of income, corresponding to different educational levels, which are  $\geq$  the average income per earner in urban India in 1960-61

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<sup>54</sup> Cf. D.G.E. and T., Employment Survey of the Alumni of Delhi University (1962). This survey has revealed that till 1960, the most important method (around 40%) of obtaining employment by graduates has been "personal contacts". A similar study with the data related to the State of Gujrat for the year 1970 has again confirmed the importance of family and class-links in the recruitment pattern (cf. Hommes and Trivedi: "The Market for Graduates - A Field Report", Econ. & Pol. Weekly, Dec. 11, 1971). It should not be wrong to suppose that these personal or family contacts are more likely to be formed among people belonging to the same income class rather than across the classes. Starting therefore with an unequal distribution of income and concentration of economic power among Group 1, a typical member of Group 1, with certain educational qualification, will have an easier access to employment as compared with a typical member of Group 2 with the same qualification. sticity",

(which was approximately Rs. 100 per month), and Set 2 similarly contains all the values of income < Rs. 100. Since the data relate to the earners, the average income per earner is the appropriate cutoff point; it corresponds, with adjustments for the earner-population ratio, to the per capita income. Therefore, Set 1 can be supposed to represent in the sample the education-income combinations of Group 1 and Set 2 those of Group 2.

Polynomials of increasing order (up to 3rd order) are fitted, first, by the method of ordinary least squares to both sets and the results are summarized in Tables 2 and 3. Consider the results for Group 2 (the poor) in Table 2.  $R^2$  is quite low in each case, but since we have deliberately excluded the other explanatory variables,  $R^2$  is not the thing we are interested in. Our concern is with the values of the coefficients and their associated t-statistics. Judging by this criterion, the linear form appears to be the best among the three. However, because of unequal variations of the conditional distributions of income for different values of education, the existence of heteroscedasticity could be suspected. Therefore, we performed an F-test, suggested by Goldfeld and Quandt<sup>62</sup>, by arranging the observations according to an ascending order of the levels of education and then finding out the ratio between the least squares residuals of higher- and lower-numbered observations. Given the nature of the data--particularly, its concentration at some specific levels of education--certain middle observations had to be omitted (as permitted by the test). The resulting F-statistic

$$F(12, 12) = \frac{.122141 \text{ E } 07}{.113664 \text{ E } 07}$$

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<sup>62</sup>S.M. Goldfeld and R.E. Quandt (1965), "Some Tests for Homoscedasticity", JASA, 60, pp. 539-547.

Table 2

Regressions of Income on Education for the Earners  
of Group 2, Urban India, 1960-61\*

Linear:

$$Y_{2j}^e = 503.665 + 26.6069 N_{2j}^e$$

$$t: (7.83169) (2.09521)$$

No. of obs. = 46  $R^2 = 0.0907$

Quadratic:

$$Y_{2j}^e = 528.592 - 12.9481 N_{2j}^e + 4.59702 (N_{2j}^e)^2$$

$$t: (7.78743) (-0.345148) (1.12012)$$

No. of obs. = 46  $R^2 = 0.1165$

Cubic:

$$Y_{2j}^e = 531.6 - 61.2584 N_{2j}^e + 18.0486 (N_{2j}^e)^2 - 0.8631 (N_{2j}^e)^3$$

$$t: (7.72401) (-0.546439) (0.608295) (-0.457845)$$

No. of obs. = 46  $R^2 = 0.1209$

\*To maintain uniformity with the educational expenditure data, the income figures in these regressions have been considered annually.

is not significant to reject the null hypothesis of homoscedasticity.

To sum up, therefore, the simple linear form

$$Y_{2j}^e = 503.665 + 26.6069 N_{2j}^e$$

seems to be the best fit for the income-education relationship for the earners of Group 2. This implies, by (5.9), that

$$\frac{dy_2^e}{dn_2} = \Psi^{e'}(n_2) = 26.6069 \quad \text{since} \quad \Psi^{e'''} = 0.$$

Consider next Group 1 (the rich). The results of ordinary least squares regressions of income on education are shown in Table 3. It is clear that judging by the criterion of the significance of t-statistics of the relevant coefficients the choice of admissible functional forms is now wider, since all the three cases -- linear, quadratic and cubic -- display acceptable t-statistics. It should be pointed out that the t-statistics associated with the constant terms are not particularly relevant because eventually we are interested in the derivatives of the functions. For the same reason as before, we have carried out the F-test for heteroscedasticity and the relevant F-statistic is found to have the value

$$F = \frac{0.144447 \text{ E } 09}{0.405084 \text{ E } 07}$$

which clearly falls in the critical region. A correction for heteroscedasticity is therefore needed, and we have done that in terms of two alternative specifications, by assuming that the variance of the error term is proportional, first, to the level of education, and then, to the square of the level of education. The corresponding results of weighted least squares regressions are shown by equations 1 and 2 for each functional form in Table 4.

Table 3

Regressions of Income on Education for the Earners of Group 1,  
Urban India, 1960-61 (Income: Rupees per annum)

Linear:

$$Y_{1j}^e = -2036.98 + 498.528 N_{1j}^e$$

$$t: (-1.82532) \quad (5.42126)$$

No. of obs. = 43

Estimated Variance-Covariance Matrix:

$$R^2 = 0.4175$$

$$\begin{bmatrix} 0.125 \text{ E } 07 & -0.964 \text{ E } 05 \\ -0.964 \text{ E } 05 & 0.946 \text{ E } 04 \end{bmatrix}$$

Quadratic:

$$Y_{1j}^e = 6473.19 - 1316.53 N_{1j}^e + 82.7727(N_{1j}^e)^2$$

$$t: (2.53157) \quad (-2.5813) \quad (3.60437)$$

No. of obs. = 43

Estimated Variance-Covariance Matrix:

$$R^2 = 0.5603$$

$$\begin{bmatrix} 0.654 \text{ E } 07 & -0.126 \text{ E } 07 & 0.542 \text{ E } 05 \\ -0.126 \text{ E } 07 & 0.260 \text{ E } 06 & -0.116 \text{ E } 05 \\ 0.542 \text{ E } 05 & -0.116 \text{ E } 05 & 0.527 \text{ E } 03 \end{bmatrix}$$

Cubic:

$$Y_{1j}^e = -10995.1 + 4785.82 N_{1j}^e - 546.454(N_{1j}^e)^2 + 19.7097(N_{1j}^e)^3$$

$$t: (-1.69524) \quad (2.21239) \quad (-2.49783) \quad (2.88967)$$

No. of obs. = 43

Estimated Variance-Covariance Matrix:

$$R^2 = 0.6379$$

$$\begin{bmatrix} 0.421 \text{ E } 08 & -0.138 \text{ E } 08 & 0.136 \text{ E } 07 & -0.412 \text{ E } 05 \\ -0.138 \text{ E } 08 & 0.468 \text{ E } 07 & -0.470 \text{ E } 06 & 0.144 \text{ E } 05 \\ 0.136 \text{ E } 07 & -0.470 \text{ E } 06 & 0.479 \text{ E } 05 & -0.149 \text{ E } 04 \\ -0.412 \text{ E } 05 & 0.144 \text{ E } 05 & -0.149 \text{ E } 04 & 0.465 \text{ E } 02 \end{bmatrix}$$

Table 4  
Income-Education Relationships for the Earners of Group 1,  
Urban India (1960-61): Weighted Least Squares Regressions.

Equations	Estimated Variance-Covariance Matrix
<u>Linear:</u>	
(1) $\frac{Y_{1j}^e}{\sqrt{N_{1j}^e}} = -987.85 \frac{1}{\sqrt{N_{1j}^e}} + 406.537 \sqrt{N_{1j}^e}$ $t: (-1.31458) \quad (5.63903)$ No. of obs. = 43 $R^2 = 0.2385$	$\begin{bmatrix} 0.565 \text{ E } 06 & -0.495 \text{ E } 05 \\ -0.495 \text{ E } 05 & 0.520 \text{ E } 04 \end{bmatrix}$
(2) $\frac{Y_{1j}^e}{N_{1j}^e} = -297.387 \frac{1}{N_{1j}^e} + 334.059$ $t: (-0.591483) \quad (5.69769)$ No. of obs. = 43 $R^2 = 0.0025$	$\begin{bmatrix} 0.232 \text{ E } 06 & -0.265 \text{ E } 05 \\ -0.265 \text{ E } 05 & 0.345 \text{ E } 04 \end{bmatrix}$
<hr style="border-top: 1px dashed black;"/>	
<u>Quadratic:</u>	
(1) $\frac{Y_{1j}^e}{\sqrt{N_{1j}^e}} = 4902.5 \frac{1}{\sqrt{N_{1j}^e}} - 921.534 \sqrt{N_{1j}^e} + 67.406(N_{1j}^e)^{\frac{3}{2}}$ $t: (2.926) \quad (-2.059) \quad (3.916)$ No. of obs. = 43 $R^2 = .4419$	$\begin{bmatrix} 0.281 \text{ E } 07 & -0.500 \text{ E } 06 & 0.273 \text{ E } 05 \\ -0.500 \text{ E } 06 & 0.176 \text{ E } 06 & -0.643 \text{ E } 04 \\ 0.273 \text{ E } 05 & -0.643 \text{ E } 04 & 0.313 \text{ E } 03 \end{bmatrix}$
(2) $\frac{Y_{1j}^e}{N_{1j}^e} = 3933.37 \frac{1}{N_{1j}^e} - 753.157 \sqrt{N_{1j}^e} + 56.3006 N_{1j}^e$ $t: (3.415) \quad (-2.699) \quad (3.922)$ No. of obs. = 43 $R^2 = 0.2979$	$\begin{bmatrix} 0.133 \text{ E } 07 & -0.313 \text{ E } 06 & 0.152 \text{ E } 05 \\ -0.313 \text{ E } 06 & 0.778 \text{ E } 05 & -0.391 \text{ E } 04 \\ 0.152 \text{ E } 05 & -0.391 \text{ E } 04 & 0.203 \text{ E } 03 \end{bmatrix}$
<hr style="border-top: 1px dashed black;"/>	
<u>Cubic:</u>	
(1) $\frac{Y_{1j}^e}{\sqrt{N_{1j}^e}} = -8679.00 \frac{1}{\sqrt{N_{1j}^e}} + 3276.76 \sqrt{N_{1j}^e} - 463.03(N_{1j}^e)^{\frac{3}{2}} + 17.09(N_{1j}^e)^{\frac{5}{2}}$ $t: (-1.830) \quad (2.376) \quad (-2.628) \quad (3.025)$ No. of obs. = 43 $R^2 = 0.5479$	$\begin{bmatrix} 0.225 \text{ E } 08 & -0.786 \text{ E } 07 & 0.810 \text{ E } 06 & -0.254 \text{ E } 05 \\ -0.786 \text{ E } 07 & 0.280 \text{ E } 07 & -0.293 \text{ E } 06 & 0.926 \text{ E } 04 \\ 0.810 \text{ E } 06 & -0.293 \text{ E } 06 & 0.310 \text{ E } 05 & -0.991 \text{ E } 03 \\ -0.254 \text{ E } 05 & 0.926 \text{ E } 04 & -0.991 \text{ E } 03 & 0.319 \text{ E } 02 \end{bmatrix}$
(2) $\frac{Y_{1j}^e}{N_{1j}^e} = -6934.61 \frac{1}{N_{1j}^e} + 3303.39 \sqrt{N_{1j}^e} - 390.98 N_{1j}^e + 14.77 (N_{1j}^e)^2$ $t: (-1.879) \quad (2.471) \quad (-2.69) \quad (3.09)$ No. of obs. = 43 $R^2 = .4280$	$\begin{bmatrix} 0.132 \text{ E } 08 & -0.483 \text{ E } 07 & 0.517 \text{ E } 06 & -0.167 \text{ E } 05 \\ -0.483 \text{ E } 07 & 0.179 \text{ E } 07 & -0.193 \text{ E } 06 & 0.628 \text{ E } 04 \\ 0.517 \text{ E } 06 & -0.193 \text{ E } 06 & 0.211 \text{ E } 05 & -0.692 \text{ E } 03 \\ -0.167 \text{ E } 05 & 0.628 \text{ E } 04 & -0.692 \text{ E } 03 & 0.229 \text{ E } 02 \end{bmatrix}$



The efficiency of the estimators is indicated by the magnitude of the principal diagonal terms of the estimated variance-covariance matrix. From the standpoint of efficiency, it is obvious from Table 4 that the second equation under each functional form gives a better fit, and therefore we have calculated, by using (5.9), the value of  $\frac{dy_i^e}{dn_i}$  for each of these second equations. The results are summarised below.

Linear:

$$\frac{dy_i^e}{dn_i} = \psi_i^{e'} = 334.059$$

Quadratic:

$$\begin{aligned} \frac{dy_i^e}{dn_i} &= \psi_i^{e'}(n^e) = -753.157 + 2(56.39)(11.4) \\ &= 532.535 \text{ (since } n_i^e = 11.4 \text{ approx.)} \end{aligned}$$

Cubic:

$$\begin{aligned} \frac{dy_i^e}{dn_i} &= \psi_i^{e'}(n^e) = 3303.39 + 2(-390.98)(11.4) + 3(14.77)(11.4)^2 \\ &= 147.573. \end{aligned}$$

We can work in terms of any one of the three values of  $\frac{dy_i^e}{dn_i}$ , and we have checked that the final conclusion remains the same in each case. However, after ensuring efficiency of the estimators within each functional form, if we now decide, as we have done before, on the criterion of the significance of t-statistics of the relevant coefficients in choosing among the three functional forms, then the linear case appears to be the most acceptable. For this reason we shall work with  $\frac{dy_i^e}{dn_i} = 334.059$ .

To sum up our findings on the income-education relationships for Group 1 and Group 2, we can write, by using (5.8), that

$$\begin{aligned} \frac{dy_i}{dn_i} &= \theta_1 \frac{dy_i^e}{dn_i} = \theta_1 334.059 \text{ and} \\ (5.10) \quad \frac{dy_i}{dn_i} &= \theta_2 \frac{dy_i^e}{dn_i} = \theta_2 26.6069 \end{aligned}$$

opportunity

in the

As has been already mentioned, one limitation of this empirical analysis is that, except for urban India, the data are just not available in a form which can serve the basis of any meaningful regression analysis, and this is in spite of the existence of an All-India Rural Household Survey. This raises a problem, because most of the other relevant information on the educational expenditure and government policies are obtainable only on the national level. Therefore, we had to assume that the relative difference between  $\frac{dy_1}{dn_1}$  and  $\frac{dy_2}{dn_2}$ , as calculated from urban data, provides a reasonable approximation for the country as a whole. Although there is undeniably an arbitrariness associated with this assumption, it should be emphasized that the final conclusion is "robust" with respect to this assumption; it remains insensitive to a wide range of possible variations of  $\frac{dy_1}{dn_1}$  and  $\frac{dy_2}{dn_2}$ . This will be clearer as we go along.

The Education-Expenditure Relationship:

This relationship, as expressed in (4.8), shows how the different levels of education are attainable corresponding to different amounts of educational expenditure per person. In a sense, this is the counterpart of the production function and is assumed to be the same for both groups. Educational expenditure, as mentioned before, has two components: (a) private expenditure such as expenditure on books, stationery and tuition, and (b) government expenditure in the form of current expenditure, costs of inspection, rent on buildings, etc.

We have decided not to include opportunity cost in this expenditure, because the relationship we are interested in is only about the expenditure actually made and educational level correspondingly attained. It is true that opportunity cost, along with other factors, does play an important role, but only in the

basic portfolio decision, namely, in the determination of how much of saving should be allotted to expenditure on education. We have discussed that problem in Section 2. But educational expenditure, after being determined in this way, is itself only the direct expenditure; it does no longer include opportunity cost within itself. And, to repeat, we are now interested strictly in the relationship between the amounts of this direct expenditure actually made and the consequent attainment of educational levels.

Coming back to the two components of direct expenditure, the data on the private component can be obtained from the Education Commission Report<sup>63</sup> for the year 1965-66. We have deflated these data by the rate of increase in the cost level to get the corresponding figures for 1960-61, making them contemporaneous with the income-education data. For the data on government expenditure, we relied on the work of Blaug and his associates.<sup>64</sup> All these informations are summed up in Table 5.

To estimate the expenditure-education relationship column 1 has been regressed on column 4 and since, unlike in the income-education relationship, one can avoid the zero value of the variables, the logarithmic form can be chosen as an appropriate specification of function (4.8), that is,

$$n = AE^b$$

where A is some positive constant and b captures the concavity (from below) of the function. Ordinary least squares regression gives the following estimate of this function:

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<sup>63</sup> Report of the Education Commission, Govt. of India (1966), p. 468.

<sup>64</sup> Blaug et al., Ch. 8.

Table 5

Educational Expenditure and Levels of  
Educational Attainment, India, 1960-61

Educational level (No. of school years)	Private Expenditure per student (Rs. per annum)	Total (Private & Govt.) Expenditure per student (Rs. per annum)	Cumulative Total Expenditure per student (Rs. per annum)
1	2	3	4
1	1.2	33.2	33.2
2	1.2	33.2	66.4
3 Primary	8.0	40.0	106.4
4	11.7	43.7	150.1
5	12.0	44.0	194.1
6	25.4	91.4	285.5
7 Middle	28.4	94.4	379.9
8	44.1	110.1	490.0
9	89.1	239.1	729.1
10 Secondary	93.0	243.0	972.1
11	92.8	242.8	1214.9
13 Intermediate	256.0	606.0	2426.9
15 Graduate (B. A., B. Sc., B. Comm.)	377.0	919.0	4264.9
17 Engineering Graduate	290.0	1564.0	8682.9

Sources: Report of the Education Commission, Govt. of India (1966), p. 468.

Blaug et al., Ch. 8.

$$\log n = -1.14198 + 0.48239 \log E$$

$$t : (-4.03944) (10.8325)$$

$$\text{No. of obs.} = 14 \quad R^2 = 0.9072 .$$

Noting that the average educational attainment of Group 1 is 11.4 school years with the corresponding annual expenditure per person at Rs. 1214.9 (approx.), the derivative of the function evaluated at the average expenditure point for the rich is

$$\frac{dn_1}{dE_1} = A (.012) .$$

Similarly, with the corresponding figures--the average educational attainment equaling 3.6 school years and the average annual expenditure per person approximately Rs. 150.1--the relevant value of the derivative for the poor is

$$\frac{dn_2}{dE_2} = A (.036) .$$

Now, combining these estimates with the estimates of the income-education relationships for the respective groups as shown in (5.10), we have

$$\frac{dy_1}{dn_1} \frac{dn_1}{dE_1} = \theta_1 A (334.059) (.012) = A_1 (4.008) , \quad \text{and}$$

(5.11)

$$\frac{dy_2}{dn_2} \frac{dn_2}{dE_2} = \theta_2 A (26.6069) (.036) = A_2 (0.9578)$$

with  $A_1 > A_2$  .

The Educational Expenditure-Income Ratios:

It is not possible to obtain directly from the Household Surveys of India any quantitative information about the educational expenditure-income ratios of the two income groups. However, there are some reliable data available on the

saving-income relationship as shown in Tables 6 and 7, and we can use the analytical framework outlined in Section 2 to extract from these data the required qualitative information about the educational expenditure-income ratios of the two groups. From Tables 6 and 7 it is abundantly clear that if the average income is chosen to be the cutoff point with appropriate adjustments for the average size of households in both urban and rural India, then not only  $s_1$  (the saving-income ratio of Group 1) is significantly higher than  $s_2$  (the saving-income ratio of Group 2) but also  $s_2 \leq 0$ . This implies by our analysis of the portfolio decision problem in Section 2 that  $e_1 > e_2$ . Therefore, using (5.11), it becomes obvious that

$$(5.12) \quad \frac{dy_1}{dn_1} \frac{dn_1}{dE_1} e_1 > \frac{dy_2}{dn_2} \frac{dn_2}{dE_2} e_2 .$$

Now refer back to (4.10). If there were no government intervention in the form of allocation of educational expenditure between the two groups or taxation, then (5.12) could have been taken to imply that the educational process left to the private sector was acting as a disequaliser of income in India in the early sixties. However, as we know, we should bring the government in the picture. A pertinent question then is: what should have been the nature of government policies at that time to counteract the disequalising forces of the private sector and make the Indian economy move toward a more equal situation?

#### Government Policies on Educational Expenditure and Taxes:

Let us take one policy at a time. As indicated before in Section 4, let us first parameterise the taxation side of the problem by supposing that as if the over-all tax structure were proportional and expected to remain so over time, i.e.,  $z_1 = z_2$  and  $\frac{dz_1}{dt} = \frac{dz_2}{dt}$ .

Table 6

Saving and Income of Urban Households  
by Income Class, India

Disposable Income	Weighted per cent of household	Average disposable income per household Rs.	Average saving per household Rs.	Saving income ratio (per cent)
Under 1,000	42.5	608	-125	-20.6
1,000 - 1,999	32.5	1,390	- 83	- 6.0
2,000 - 2,999	10.7	2,387	- 24	- 1.0
3,000 - 3,999	5.8	3,476	15	0.4
4,000 - 5,999	4.6	4,818	467	9.7
6,000 - 9,999	2.4	7,420	843	11.4
10,000-14,999	0.8	12,292	3,019	24.6
15,000-24,999	0.5	18,867	6,227	33.0
25,000 and over	0.2	40,452	18,017	44.5
All households	100.0	1,862	62	3.3

Source: Urban Income and Saving, N. C. A. E. R. (1962), p. 78.

Table 7

Saving and Income of Rural Households  
by Income Class, India

Income Class (Rs. per annum)	Weighted percentage of households %	Weighted average saving per household Rs.	Weighted average income per household Rs.	Saving income ratio
Up to 360	6.52	-13	222	-5.9
361 - 480	7.12	-36	426	-8.4
481 - 600	9.67	-26	545	-4.8
601 - 720	9.87	-24	663	-3.6
721 - 900	13.42	- 4	810	0.5
901-1200	16.02	- 3	1043	0.3
1201-1800	18.94	40	1464	2.7
1801-2400	8.27	124	2078	6.0
2401-3600	5.76	252	2886	8.7
3601-4800	2.30	400	4105	9.7
4801-7200	1.23	1088	5727	19.0
Above 7200	0.88	1993	12370	16.1
<b>All Income Classes</b>	100.00	63	1328	4.7

Source: All India Rural Household Survey, Vol. II, N.C.A.E.R. (1965),  
p. 96.



Then it follows from (4.11) that to improve the distribution of income, the government policy on the allocation of educational expenditure, as determined by the choice of  $v_1$  and  $v_2$  and therefore of  $\lambda_1$  and  $\lambda_2$ , should have been such that

$$\frac{dy_1}{dn_1} \frac{dn_1}{dE_1} \left[ e_1(1-z) + \frac{\lambda_1 T}{L_1 Y_1} \right] < \frac{dy_2}{dn_2} \frac{dn_2}{dE_2} \left[ e_2(1-z) + \frac{\lambda_2 T}{L_2 Y_2} \right].$$

But, given (5.12), for this inequality to hold, it was necessary to have

$$\frac{\lambda_1 T}{L_1 Y_1} < \frac{\lambda_2 T}{L_2 Y_2}.$$

We now like to refer to Table 8<sup>65</sup> where, choosing the per capita income again as the cutoff point between the two groups, we have found that

$$100 \frac{L_1}{L} y_1 = \text{Rs. } 1194.74 \quad \text{and} \quad 100 \frac{L_2}{L} y_2 = \text{Rs. } 1375.36$$

so that  $L_1 y_1 < L_2 y_2$ . Therefore, for the educational expenditure policy to have any egalitarian impact it was required that  $\lambda_1 < \lambda_2$  which by (4.7) would imply

$$(5.13) \quad v_1 < v_2.$$

With this requirement in mind let us study the nature of policies of the Indian Government on the allocation of educational expenditure between Group 1 and Group 2 during the early sixties in particular and over the planning period in general. There is, of course, a problem of obtaining data identifying any particular kind of educational expenditure with a particular income group in a country like India which is ostensibly committed to freedom of opportunity and similar democratic ideals. This information can be obtained only in indirect ways.

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Although these data refer to 1963-64, there are reasons to believe that, for our purpose, the situation was not significantly different in 1960-61.

Table 8  
Percentage Distribution of Persons by Size-class  
of Per Capita Income Per 30 Days

Per Capita Income Rs.	Mean Per Capita Income Rs.	Percentage of Persons
0 - 10	8.45	8.46
10 - 15	12.8	20.64
15 - 20	16.93	22.33
20 - 25	23.35	16.62
25 - 30	27.22	10.52
30 - 40	34.25	9.75
40 - 50	44.43	4.39
50 - 60	54.52	2.33
60 - 70	64.59	1.38
70 - 80	74.68	0.88
80-100	88.83	1.02
100-150	119.8	0.97
150-200	170.2	0.33
200-300	239.8	0.22
300-500	374.1	0.11
500-	449.8	0.06

Source: Ahmed and Bhattacharya, "Size Distribution of Per Capita Personal Income in India", Econ. and Pol. Weekly, Special No., August, 1972.

Looking, for example, at the data provided by the All-India Survey of Graduates (presented in Table 9) relating to the family incomes of the students going to the university or technical education and comparing those with the figures for the average family and per capita income (obtainable from Tables 6, 7 and 8), it becomes clear that higher education is mostly, if not wholly, meant for Group 1. An extreme situation is found to prevail at the Indian Institutes of Technology where the class background of the students is really conspicuously elitistic (Table 10). Although it has not been possible to find these informations exactly for the year 1960-61 which is our general reference point, we believe that it may not be too unreasonable to suppose that the class structure of higher education did not change drastically over the late fifties and early sixties.

In the primary and secondary education, the correspondence between the stage of education and any particular class is not so complete. Still, certain facts should be noted. From the Report of the Education Commission<sup>66</sup> it can be found that in the secondary education in 1960-61, percentage of students paying fee was around 65% and the average annual fee per student was as high as Rs. 55.6. Combining these data with the contemporary figure for per capita annual income which was Rs. 360 and noting at the same time that the saving-income ratio of Group 2 was generally nonpositive (Tables 6 and 7), one can conclude that the majority of students in the secondary schools would again come from Group 1. It was only in the primary education where the private expenditure per student was considerably low (Table 5) that a significant participation of Group 2 could have been possible along, of course, with Group 1.

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<sup>66</sup>  
Op. cit., p. 110.

Table 9

Distribution of Families of 1954 Graduates and of All Families by Average Monthly Family Income (per cent)

Average monthly income of family Rs.	Percentage of graduates' families	Percentage of all families	
		Urban	Rural
500 and above	23.3	4.0	0.9
200 - 499	45.7	15.7	8.5
Below 200	29.1	80.3	90.6

Source: Blaug et al., op. cit., p. 131.

Table 10

Parental Income of Students Entering an I.I.T., 1970

Income of parent Rs. per month	Students entering I.I.T. (Percentage)
151 - 200	2
201 - 500	18
500-1000	29
1000 and above	51

Source: A.D. King, "Elite Education and the Economy", Econ. & Pol. Weekly, Aug. 29, 1970.

With this approximate correspondence between the types of education and the class background of the students, let us have a look at the data<sup>67</sup> on the government expenditure on different levels of education over the period of planning in general. It is found that the percentage of total educational expenditure on primary education steadily dropped from 55% in the First Plan (1951-1956) to 35% in the Second Plan (1956-1961) and to 30% in the Third Plan (1961-1966). The percentage of expenditure on secondary education rose from 13% in the First Plan to 18% in the Second Plan and then remained steady in the Third Plan. The proportional increase in expenditure was most conspicuous in higher education: from 9% in the First Plan to 17% in the Second Plan and a monotonic increase then on. Similarly, the percentage of expenditure on technical education rose from 13% in the First Plan to 17% in the Second Plan and then to 25% in the Third Plan. From all these data, which are mainly suggestive in nature, one has reason to suspect that the condition (5.13) might not have been satisfied in India in the early sixties.

This conclusion becomes stronger if we now bring taxes into the picture and study the over-all incidence of government policies. So long, for purely illustrative convenience, we have made a kind of "as if" hypothesis, namely, that the over-all tax structure had remained proportional in the early sixties. However, the actual situation was somewhat different and it is time now to turn to the real facts, which, for our purpose, can be briefly summed up in the following way.

1. Throughout the period of planning, the proportion of direct taxes in the total revenues of the Central and State Governments has steadily fallen

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<sup>67</sup> For these data see Third Five-Year Plan, Govt. of India, Chs. xxxix, xxx.

from 41% in 1951-52 to 30.7% in the Third Plan period as shown in Table 11. This is remarkable because one would have expected that in a period when the national income was rising the ratio between direct and indirect taxes would go up.

2. It is also interesting, if not surprising, to observe that not only there has been a diminishing importance of direct taxes, but considering the nature of progression of direct taxes, particularly of income taxes, it is found that the average rate of income tax as measured by income tax as a percentage of total assessed income of all taxpaying individuals has also declined from 18.6% in 1950-51 to 12.1% in 1963-64.<sup>68</sup> Of course, there have been variations in the rate of taxation for some income groups.<sup>69</sup> But all these variations took place within those income ranges which would fall, in terms of our classification, under Group 1. In fact, the members of Group 2 would not in general come under income taxation at all. From the standpoint of income taxes, the relevant group is only Group 1 and therefore the important thing to watch is not what was happening to the tax rates within this group but the average rate of taxation for this group as a whole. And, we have seen that this average rate was falling.

3. How would then the members of Group 2 come under taxation at all? This is where indirect taxes come in. From Table 11 it has already become clear that throughout the planning period indirect taxes as compared with direct taxes have become increasingly more important in the total revenue. In the early sixties, in particular, indirect taxes have provided nearly two-thirds

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<sup>68</sup> Cf. A. Bagchi, "Dissecting the Tax Structure", Econ. and Pol. Weekly, Jan. 15, 1972.

<sup>69</sup> This has been studied in great detail by Gupta. See his "Income Distribution, Tax Yield and Income Tax", Econ. and Pol. Weekly, Oct. 14, 1972.

Table 11

Direct Taxes as a Proportion of Centre and State Revenue

Year	Direct taxes as Percent of Central Revenue	Direct taxes as Percent of State Revenue	Direct taxes as Percent of total (Centre & State) Revenue
1951-52	36.3	47.1	41.0
First Plan Period	30.4	48.3	38.4
Second Plan Period	29.0	41.5	34.8
Third Plan Period	28.9	33.5	30.7

Sources: Government of India, Ministry of Finance, Explanatory Memorandum on the Budget of the Central Government for 1967-68.

Reserve Bank of India Bulletin, "Finances of State Governments, 1967-68", Aug. 1967.

of the total revenue. Although some of these indirect taxes were on luxury items, most of them were on items which were essential consumption goods for the poor, such as kerosene, sugar, cotton fabrics, etc. The unmistakably regressive nature of such taxation in India has been pointed out in some recent studies.<sup>70</sup> It should also be borne in mind that at very low level of income, consumption expenditure can be equated with income itself (see Tables 6 and 7) and therefore most of the indirect taxes were nothing but income tax from the poor.

The total picture which emerges out of these considerations is that in the early sixties the over-all Indian tax-structure was regressive and was to become more regressive. In terms of our policy model, this can be taken to imply that

$$(5.14) \quad z_1 < z_2 \quad \text{and} \quad \frac{dz_1}{dt} < \frac{dz_2}{dt}$$

in the period of time we are interested in.

To understand the over-all incidence of government policies, let us bring together the policies on educational expenditure and those on taxes which were to help finance this expenditure. The important inequality to which we have to refer now is (4.10). Using (5.11), (5.14), the conclusion on expenditure policies and also noting that  $L_1Y_1 < L_2Y_2$  and  $e_2 \simeq 0$ , we find that the crucial condition for convergence as given by (4.10) was violated in the early sixties, if not throughout the planning period, in India. We also understand that government policies on educational expenditure and taxes

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<sup>70</sup> S.L. Shetty, "A Note on Distribution of Indirect Tax Burden by Grades of Income in Farm and Non-Farm Sectors", The Indian Econ. Journal, July-Sept., 1971, p. 93, Table 1.



could have been used to make the economy move toward an egalitarian society by counteracting the disequalising forces which were found inherent in the private sector of the economy. However, the policies which were actually adopted were very different. They not only failed to counterbalance the forces of the private sector but in fact were instrumental in deteriorating the distribution of income further. Education, along with other factors, was made to serve as a disequaliser of income.

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#### Colombia

We now want to see how far our findings on India are comparable with the situation in Colombia. For this we shall go through the same kind of empirical exercise, estimating the income-education and the expenditure-education relationships and collecting the required informations on the educational expenditure-income ratios and government policies. The methodology to be followed will be almost the same as before and therefore the corresponding descriptions will not be repeated except when there is some variation. It will be found that because of the better availability of data and possibility of referring to some existing research works, the empirical analysis this time will be easier.

#### The Income-Education Relationships:

These relationships will be estimated from the data on income and educational attainment of the earners in Bogota, a major urban centre in Colombia, for the years 1963-66. One advantage here, as compared to India, is that these data are available in a more detailed form (Table 12) over a sample size of 10,715 people. Following the same method as before, we

Table 12

Bogota, Males and Femals: Hourly Wages by Schooling, 1963-66  
(In Pesos of 1966)

Income											
Edu- cational levels & no. of years											
Illiteracy	0	0.10 (8)	0.17 (120)	0.59 (133)	0.50 (117)	0.96 (112)	1.12 (127)	0.94 (66)	1.47 (81)	1.32 (56)	2.36 (44)
Primary	1	0.35 (18)	0.15 (64)	0.82 (74)	1.38 (59)	2.12 (47)	1.93 (46)	1.54 (32)	2.33 (26)	2.18 (17)	4.75 (8)
Primary	2.5	0.14 (108)	0.93 (573)	1.46 (509)	1.95 (369)	2.18 (278)	2.22 (254)	2.99 (192)	2.51 (156)	3.12 (99)	1.78 (51)
Primary	5	0.54 (32)	1.62 (483)	3.22 (591)	3.39 (518)	3.92 (422)	4.41 (351)	4.44 (258)	5.50 (197)	5.21 (148)	4.35 (73)
Bachillerato	6.5	0.35 (3)	3.78 (184)	4.05 (268)	4.96 (204)	5.19 (147)	5.15 (142)	5.71 (69)	5.92 (51)	5.61 (32)	6.67 (15)
Bachillerato	8.5		3.90 (111)	5.47 (259)	6.63 (221)	8.48 (133)	8.53 (130)	10.75 (89)	10.52 (65)	11.76 (46)	12.42 (33)
Bachillerato	11		6.00 (28)	6.80 (174)	10.52 (163)	14.33 (130)	21.92 (110)	16.40 (83)	19.70 (59)	21.50 (61)	19.34 (41)
University	12.5		6.10 (4)	10.17 (47)	14.02 (41)	26.26 (19)	18.35 (17)	15.00 (2)	29.70 (8)	23.60 (3)	
University	14.5			10.00 (28)	13.95 (60)	22.65 (44)	18.78 (28)	24.09 (27)	33.00 (9)	31.20 (9)	18.60 (5)
University	16.5			15.88 (35)	20.90 (88)	23.00 (91)	31.10 (69)	29.16 (31)	27.70 (39)	31.50 (34)	21.27 (11)

Source: Muestras de Desempleo CEDE

Note: Figures in parenthesis represent number of observations.

partition the observations into Set 1 and Set 2 such that Set 1 contains all the values of income, corresponding to different education levels, which are  $\geq$  the average income per earner, and Set 2 similarly contains all the values of income  $<$  the average income. Then polynomials (up to third order) are fitted, first, by the method of ordinary least squares to both sets, and the results are shown in Tables 13 and 14.

An interesting point to note here is that although the sample size looks unmanageably large, the distinct pairs of income and education levels are not that many in number, and corresponding to each distinct pair  $(Y_{ij}^e, N_{ij}^e)$  for any Group  $i$  there exists a frequency to be denoted by  $f_{ij}$ . A convenient way of carrying out least squares regression in such a situation is to define a transformation on the original values such that the regression equation, say, in the linear case, takes the form:

$$(5.15) \quad Y_{ij}^e \sqrt{f_{ij}} = \alpha \sqrt{f_{ij}} + \beta N_{ij}^e \sqrt{f_{ij}},$$

where  $f_{ij}$  is the frequency corresponding to a distinct pair  $(Y_{ij}^e, N_{ij}^e)$ . It can be easily proved that the least squares estimators remain invariant with respect to this transformation.

The results of such regressions for Group 2 (the poor) are summarized in Table 13. Going again by the criterion of the significance of  $t$ -statistics of the coefficients, the linear form seems to be the most acceptable. To vindicate the hypothesis of homoscedasticity the Goldfeld-Quandt test was performed to yield the F-ratio<sup>71</sup>

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<sup>71</sup>The degrees of freedom written here are only indicative of the numbers of distinct observations.

Table 13

Regressions of Income on Education for the Earners  
of Group 2, Bogota, Colombia, 1963-66\*

Linear:

$$Y_{2j}^e \sqrt{f_{2j}} = 1603.49 \sqrt{f_{2j}} + 1436.47 N_{2j}^e \sqrt{f_{2j}}$$

$$t: \quad (2.3016) \quad (8.6384)$$

$$R^2 = 0.8063 \quad \text{No. of distinct pairs} = 46$$

Quadratic:

$$Y_{2j}^e \sqrt{f_{2j}} = 1833.68 \sqrt{f_{2j}} + 1231.26 N_{2j}^e \sqrt{f_{2j}} + 29.4916 (N_{2j}^e)^2 \sqrt{f_{2j}}$$

$$t: \quad (1.9992) \quad (2.2351) \quad (0.3911)$$

$$R^2 = 0.8070 \quad \text{No. of distinct pairs} = 46$$

Cubic:

$$Y_{2j} \sqrt{f_{2j}} = 2525.19 \sqrt{f_{2j}} - 293.89 N_{2j} \sqrt{f_{2j}} + 562.58 (N_{2j})^2 \sqrt{f_{2j}} - 47.93 (N_{2j})^3 \sqrt{f_{2j}}$$

$$t: \quad (2.5881) \quad (-0.2905) \quad (1.8235) \quad (-1.7793)$$

$$R^2 = 0.8205 \quad \text{No. of distinct pairs} = 46$$

\*The income figures are considered annually.

$$F(28, 14) = \frac{0.240537 E 11}{0.131696 E 11}$$

which does not fall in the critical region. Therefore, the ordinary linear form

$$Y_{2j}^e \sqrt{f_{2j}} = 1603.49 \sqrt{f_{2j}} + 1436.47 N_{2j}^e \sqrt{f_{2j}}$$

turns out to be the best for the income-education relationship for the earners of Group 2, and this implies by (5.9) that

$$\frac{dy_2^e}{dn_2} = \Psi^{e'}(n_2^e) = 14.36.47 \quad \text{since} \quad \Psi^{e'''} = 0,$$

or, by (5.8),

$$(5.16) \quad \frac{dy_2^e}{dn_2} = \theta_2(1436.47) \quad \text{where} \quad \theta_2 > 0.$$

Going through the same routine for Set 1 we find from Table 14 that in the case of ordinary least squares the linear form is again the only acceptable specification among the three (using the criterion of the significance of the t-statistic). However, as it was true for India so it is also here, the F-ratio designed to test heteroscedasticity

$$F(11, 30) = \frac{0.274112 E 12}{0.309921 E 11}$$

is significant enough to warrant some corrections of heteroscedasticity. These corrections have been introduced through weighted least squares regressions which were to be carried out this time only for the linear case. The two alternative specifications of heteroscedasticity are the same as before and the corresponding results are shown in terms of equations (1) and (2) in Table 15. Using again the usual criterion of efficiency of the estimates as indicated

Table 14

Regressions of Income on Education for the Earners  
of Group 1, Bogota, Colombia, 1963-66  
(Income figures in pesos per annum)

Linear:

$$Y_{1j}^e \sqrt{f_{1j}} = -17608.3 \sqrt{f_{1j}} + 4830.63 N_{1j}^e \sqrt{f_{1j}}$$

$t: \quad (-2.9396) \quad (8.9241)$

$R^2 = 0.6021$                       No. of distinct pairs = 46

Quadratic:

$$Y_{1j}^e \sqrt{f_{1j}} = 773.83 \sqrt{f_{1j}} + 1254.28 N_{1j}^e \sqrt{f_{1j}} + 158.84 (N_{1j}^e)^2 \sqrt{f_{1j}}$$

$t: \quad (0.0426) \quad (0.3713) \quad (1.072)$

$R^2 = 0.6125$                       No. of distinct pairs = 46

Cubic:

$$Y_{1j}^e \sqrt{f_{1j}} = 35459.2 \sqrt{f_{1j}} - 9973.79 N_{1j}^e \sqrt{f_{1j}} + 1277.57 (N_{1j}^e)^2 \sqrt{f_{1j}} - 34.51 (N_{1j}^e)^3 \sqrt{f_{1j}}$$

$t: \quad (0.6464) \quad (-0.5837) \quad (0.7628) \quad (-0.6706)$

$R^2 = 0.6166$                       No. of distinct pairs = 46

Table 15

Weighted Least Squares Regressions of Income on Education  
for the Earners of Group 1, Bogota, Colombia, 1963-66  
(Income figures in pesos per annum)

$$(1) \frac{Y_{1j}^e \sqrt{f_{1j}}}{\sqrt{N_{1j}^e}} = -14506.5 \frac{\sqrt{f_{1j}}}{\sqrt{N_{1j}^e}} + 4537.42 \sqrt{N_{1j}} \sqrt{f_{1j}}$$

$$t: \quad (-2.7981) \quad (8.8064)$$

$R^2 = 0.4840$                       No. of distinct pairs = 46

Estimate of Variance-Covariance Matrix:

$$\begin{bmatrix} 0.269 \text{ E } 08 & -0.254 \text{ E } 07 \\ -0.254 \text{ E } 07 & 0.265 \text{ E } 06 \end{bmatrix}$$

$$(2) \frac{Y_{1j} \sqrt{f_{1j}}}{N_{1j}^e} = -10829.0 \frac{\sqrt{f_{1j}}}{N_{1j}^e} + 4153.18 \sqrt{f_{1j}}$$

$$t: \quad (-2.4492) \quad (8.5188)$$

$R^2 = 0.4298$                       No. of distinct pairs = 46

Estimate of Variance-Covariance Matrix:

$$\begin{bmatrix} 0.195 \text{ E } 08 & -0.204 \text{ E } 07 \\ -0.204 \text{ E } 07 & 0.238 \text{ E } 06 \end{bmatrix}$$

Estimate of Variance-Covariance Matrix in the linear case  
under ordinary least squares:

$$\begin{bmatrix} 0.359 \text{ E } 08 & -0.310 \text{ E } 07 \\ -0.310 \text{ E } 07 & 0.293 \text{ E } 06 \end{bmatrix}$$

by the magnitudes of the principal diagonal terms of the variance-covariance matrix of the estimated coefficients, the natural choice seems to be equation (2):

$$\frac{Y_{1j}^e \sqrt{f_{1j}}}{N_{1j}^e} = -10829.0 \frac{\sqrt{f_{1j}}}{N_{1j}^e} + 4153.18 \sqrt{f_{1j}}$$

which implies by (5.9) that

$$\frac{dy_1^e}{dn_1} = \Psi^{e'}(n_1) = 4153.18 \quad \text{since} \quad \Psi^{e'''} = 0$$

or, by (5.8),

$$(5.17) \quad \frac{dy_1}{dn_1} = \theta_1(4153.18) .$$

Comparing (5.17) with (5.16) and noting that  $\theta_1 > \theta_2$ , it immediately follows that

$$\frac{dy_1}{dn_1} > \frac{dy_2}{dn_2} ,$$

a qualitative result which was also true in the case of India.

#### The Expenditure-Education Relationship:

In estimating this relationship we have made the same distinction as before between private expenditure on schooling materials and government expenditure per student. The data on both kinds of expenditure and the corresponding levels of education have been summarized in Table 16. For the data on private expenditure we have relied on a study reported by Schultz.<sup>72</sup> The government expenditure figures have been calculated from a research work of Selowsky<sup>73</sup> after making

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<sup>72</sup>T. Schultz, Returns to Education in Bogota, Colombia, pp. 25-27.

<sup>73</sup>M. Selowsky, The Effect of Unemployment and Growth on the Rate of Return to Education: The Case of Colombia. Harvard University.



an adjustment for the incidence of this expenditure between the students of the public and private schools.

With the specification of logarithmic form of the expenditure-education relationship (4.8), column 1 (of Table 16) was regressed on column 4, and the regression results can be presented in the following way:

$$\log n = -2.1796 + 0.6623 \log E$$

$$t: (-10.8672) (15.9690)$$

$$R^2 = 0.9480 \quad \text{No. of obs.} = 16$$

The average educational attainment of Group 1 is 10.6 schooling years and the average educational expenditure approximately 15015.0 pesos. Therefore, the derivative of the expenditure-education function at the average expenditure point of Group 1 is

$$\frac{dn_1}{dE_1} = A(.026)$$

where A is a positive constant. Similarly, the value of the derivative corresponding to the average expenditure of Group 2 (which is approximately 4150.5 pesos) is

$$\frac{dn_2}{dE_2} = A(.039) .$$

Combining these informations with (5.16) and (5.17), we have

$$(5.18) \quad \frac{dy_1}{dn_1} \frac{dn_1}{dE_1} = A \theta_1(4153.18)(.026) = A_1(107.98) \quad \text{and}$$

$$\frac{dy_2}{dn_2} \frac{dn_2}{dE_2} = A \theta_2(1436.47)(.039) = A_2(56.022) .$$

With  $A_1 > A_2$ , it is obvious that  $\frac{dy_1}{dn_1} \frac{dn_1}{dE_1} > \frac{dy_2}{dn_2} \frac{dn_2}{dE_2}$ . The qualitative similarity between this result and that implied by (5.11) should be noted.

Table 16

Expenditure Per Student: By Schooling Years  
Urban Colombia, 1965-66 (in pesos)

Years of Schooling (1)	Private Expenditure (2)	Total (Private & Public) Expenditure (3)	Cumulative Total (Private & Public) Expenditure (4)
<u>Primary</u>			
1	583.6	926.4	926.4
2	651.8	994.6	1921.0
3	728.5	1071.3	2992.3
4	815.4	1158.2	4150.5
5	911.4	1254.2	5404.7
<u>Bachillerato</u>			
1	945.6	1356.2	6760.9
2	1063.4	1474.0	8234.9
3	1149.4	1560.0	9794.9
4	1240.4	1651.0	11445.9
5	1327.3	1737.9	13183.8
6	1420.6	1831.2	15015.0
<u>University</u>			
1	1522.0	7451.0	22466.0
2	1643.0	7572.0	30038.0
3	1772.0	7701.0	37739.0
4	1914.0	7843.0	45582.0
5	2067.0	7996.0	53578.0

Sources: T. Schultz, op. cit.; M. Selowsky, op. cit.

The Educational Expenditure-Income Ratios:

For Colombia, these ratios can be directly calculated from the data of the Household Surveys. However, before we use these data a few preliminary observations should be made.

1. In these calculations and also in the estimation of government policies on taxation and expenditure to be done later, the relevant informations are obtained from the data relating to households rather than to individuals. The reason why we use these data, and without further adjustments, is because there are reasons to believe that in Colombia the distribution of income among households is not at all significantly different from the distribution of income among individuals, presumably due to the absence of any systematic variation of the household size over income groups.

2. The data on educational expenditure-income ratios refer to the year 1967-68 and those on the incidence of government policies are available only for 1970, and therefore they are not exactly contemporaneous with the data from which the income-education relationships (1963-66) and the expenditure-education relationship (1965) were estimated. However, it should be noted that these income-education and expenditure-education relationships are basically structural relationships and are not likely to change appreciably within a period of four or five years. Therefore, an assumption of structural invariance may not be very unrealistic in this context, and we also derive comfort from the fact that our final conclusions are not sensitive to minor variations in these structural relationships.

3. We should also mention that since these structural relationships could be estimated only for urban Colombia and since all the other relevant informations

are available separately for the urban and the rural sector (which was not true in the case of India), we have decided to restrict our analysis to the urban sector of the economy.

Let us now return to the calculation of the educational expenditure-income ratios for both Group 1 and Group 2. From Table 18 it can be found that the average household income in urban Colombia is near the midpoint of the income bracket 36000-48000 (pesos). Using this as the cutoff point in Table 17 and then weighting the educational expenditure-income ratio of every income bracket by the corresponding number of households (see Table 18), we have calculated that  $e_1$ , the value of this ratio for Group 1 as a whole, is .05. The value of  $e_2$  similarly calculated is .0225. Combining these values with (5.18), it follows that

$$(5.19) \quad \frac{dy_1}{dn_1} \frac{dn_1}{dE_1} e_1 > \frac{dy_2}{dn_2} \frac{dn_2}{dE_2} e_2$$

Comparing this result with (5.12) we can come to a conclusion which seems to be remarkably similar between India and at least urban Colombia that the educational process left to the private sector acts as a dis-equaliser of income. The question which we had raised at this point in the case of India and which we shall now ask of Colombia is: what has been the role of government policies in counteracting these disequalising forces of the private sector?.

#### Government Policies on Educational Expenditure and Taxes:

For Colombia it is much easier to get the relevant data on the incidence of government policies due to some reliable empirical works by

Table 17

Educational Expenditure and Income of  
Urban Households, Colombia, 1967-68

Income Bracket Billions of pesos/year	Educational expenditure as proportion of income (in %)
0 - 12000	2.9
12000 - 24000	1.4
24000 - 36000	2.4
36000 - 48000	3.7
48000 - 60000	4.1
60000 - 84000	4.5
84000-108000	5.5
108000-180000	5.6
180000-240000	6.4
240000-	7.2

Source: CEDE, Encuesta de Gastos e Ingresos Familiares,  
Mayo 1967 - May 1968.

Table 18

Distribution of Income Among Urban Households, Colombia, 1970  
 (Number of households in thousands, income in billions of Pesos)

Income bracket (Pesos/Year)	Number of Households	Total Income	Cumulative percentage of	
			Households	Income
0 - 6000	43.3	0.20	2.1	0.2
6000 - 12000	257.8	2.38	14.7	2.9
12000 - 18000	308.9	4.56	29.9	8.0
18000 - 24000	309.2	6.39	45.0	15.1
24000 - 30000	204.9	5.44	55.0	21.2
30000 - 36000	153.6	5.03	62.5	26.8
36000 - 48000	193.1	7.91	72.0	35.6
48000 - 60000	148.3	7.87	79.3	44.4
60000 - 72000	89.7	5.87	83.6	51.0
72000 - 84000	74.9	5.76	87.2	57.4
84000-120000	121.0	12.10	93.1	70.9
120000-180000	75.2	10.65	96.8	82.8
180000-240000	38.1	6.51	98.7	90.1
Over 240,000	24.6	8.81	100.0	100.0
<b>Total</b>	<b>2,042.6</b>			

Sources: DANE, Special Tabulations of Household Survey (EH2) and Charles McLure, op. cit., Table 2.  
 J.P. Jallade, op. cit., Table 5.

Jallade and McLure.<sup>74</sup> To make better use of these data, we shall rewrite the crucial condition for convergence (4.10) in an alternative form:

$$(4.10)' \quad \left[ \frac{dz_1}{dt} - \frac{dz_2}{dt} \right] + \frac{dy_1}{dn_1} \frac{dn_1}{dE_1} \left[ e_1(1 - z_1) + \frac{v_1 \dot{E}^*}{y_1 L_1} \right] - \frac{dy_2}{dn_2} \frac{dn_2}{dE_2} \left[ e_2(1 - z_2) + \frac{v_2 \dot{E}^*}{y_2 L_2} \right] \leq \epsilon$$

where  $\dot{E}^*$  is the total government expenditure on education and  $v_1$  and  $v_2$  are the ratios in which this expenditure is allocated between Group 1 and Group 2.

Now using the average household income as the cutoff point we have calculated from Table 19 the total public educational expenditure for the two groups and found that

$$v_1 \dot{E}^* = 1390 \text{ (million pesos)} \text{ and } v_2 \dot{E}^* = 2423 \text{ (million pesos)} .$$

With the same kind of partitioning it is found from Table 10 that

$$y_1 L_1 = 57.57 \text{ (billion pesos)} \text{ and } y_2 L_2 = 31.91 \text{ (billion pesos)}$$

so that

$$\frac{v_1 \dot{E}^*}{y_1 L_1} = .024 \quad \text{and} \quad \frac{v_2 \dot{E}^*}{y_2 L_2} = .076 .$$

We can also calculate from Table 19 the values of  $z_1$  and  $z_2$ , the average rates of over-all taxation for Group 1 and Group 2 respectively, by weighting the tax rates of the respective income brackets by the corresponding number of households, and the results are

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<sup>74</sup> Cf. J.P. Jallade, *op. cit.*; Charles McLure, "The Incidence of Colombian Taxes: 1970", (mimeo).

Table 19

Allocation of Taxes and Public Subsidies for Education  
Among Urban Households, Colombia, 1970

Income bracket Billions of Pesos/year	Effective rates of taxation (Direct & Indirect)	Total subsidies for education (Millions of Pesos)
0 - 6000	11.5	71
6000 - 12000	7.7	217
12000 - 18000	9.6	370
18000 - 24000	10.3	530
24000 - 30000	10.3	413
30000 - 36000	10.9	345
36000 - 48000	12.0	477
48000 - 60000	10.5	410
60000 - 72000	11.5	301
72000 - 84000	10.7	163
84000-120000	12.4	196
120000-180000	12.2	189
180000-240000	14.2	62
240000 -	27.5	69

Source: J.P. Jallade, op. cit., Tables 8 and 16.



$$z_1 = 12.28 (\%) \quad \text{and} \quad z_2 = 10.02 \%$$

From the available informations there are reasons to believe that the over-all tax structure with its mildly progressive nature was not changing significantly in the late sixties so that we can roughly assume

$$\frac{dz_1}{dt} = \frac{dz_2}{dt} = 0 .$$

Feeding all these informations along with (5.18) and the values of  $e_1$  and  $e_2$  into (4.10)', it is found that

$$\frac{dy_1}{dn_1} \frac{dn_1}{dE_1} \left[ e_1(1 - z_1) + \frac{v_1 \dot{E}^*}{y_1 L_1} \right] = A_1 \quad (7.325)$$

and

$$\frac{dy_2}{dn_2} \frac{dn_2}{dE_2} \left[ e_2(1 - z_2) + \frac{v_2 \dot{E}^*}{y_2 L_2} \right] = A_2 \quad (5.392)$$

which means that the crucial condition of convergence has not been satisfied. <sup>75</sup>

Although this final result indicating the failure of government policies to counteract the disequalizing forces inherent in the private educational system is the same for both India and Colombia, there is at the same time an important difference. In the case of India, government policies not only failed to counteract the forces of the private sector but in effect also supported and reinforced them, whereas in the case of Colombia, as evident from the values of  $\frac{v_i \dot{E}^*}{y_i L_i}$  and  $z_i$ , there was at least an attempt made by the Government to go against these disequalizing forces, but it was evidently not strong enough. Still the fact remains that of the two the latter is a less unfortunate situation.

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<sup>75</sup> For reasons already mentioned, we restricted this empirical analysis to urban Colombia. However, if the basic structural relationships--the income-education and the expenditure-education relationships--which were estimable only for the urban sector could be used for the entire economy, then it is possible to study the incidence of government policies in rural Colombia. We have checked this and found that the conclusion with respect to (4.10)' remains invariant.

Before closing this section on empirical analysis, we want to mention again that the data which were used in carrying out these numerous calculations were not always available (and this was particularly true of India) in a suitable form and therefore we were forced to make some drastically simplifying assumptions which we have always tried to make explicit. However, with a full awareness of these unavoidable limitations, we believe that the major qualitative thrust of the conclusions is quite independent of all these restrictions of data.

## 6. Generalisations

In this final section we want to mention some of the possible generalisations of our analysis.

1. It should be pointed out that the development of our analytical structure in terms of a two-class economy was a descriptive simplification; an extension to any finite number of classes, on some appropriate criterion for classifying the population, can be made without much difficulty and with a definite gain of insight into the complex movements of all the income brackets relative to one another. It may be useful, for example, to divide the population by the quartile values of income.

2. Human capital in our analysis has been presented as an aggregate stock and, as such, it has the same heuristic justification as provided by the concept of homogeneous physical capital in the literature on capital theory. The assumption of homogeneity, however, is not essential for our analysis. It is possible to disaggregate labor into different categories according to the embodiment of different levels of education and write the production function as  $y = f(k, L_0, L_1, \dots, L_n)$  where  $L_i$  is the labor with  $i$ -th level of education.  $L_0$  is the "raw" labor; it corresponds to  $L$  of our previous characterisation. The portfolio theory which will show how different classes will allocate their savings in accumulating physical capital and augmenting supplies of different categories of educated labor will now be more complex. But so long as one income class has an overwhelming advantage in terms of ability to save, its educational expenditure-income ratio will tend to dominate pointwise that of the other class. And, if

it is assumed that different categories of labor are imperfect substitutes of one another, and that higher is the level of education embodied in any category of labor higher is its marginal productivity as well as its "complementarity effect" with physical capital, then it can be proved that our previous conclusion about the disequalising tendency in an economy gets only stronger. This is a generalisation which can be completed without much difficulty.

3. Technical progress was ignored in the original analysis, but it can be accommodated. Considering technical progress to be of an endogenous variety, an appropriate way of characterising it is to have it included as another factor of production (e.g., the biochemical inputs in agriculture) which is accumulable and complementary to both physical and human capital. If therefore one class has, by virtue of its higher saving propensity, a relative advantage in the accumulation of this factor along with the two other kinds of capital, the conclusion of our original model will again be reinforced. This extension in fact is a natural corollary of the second generalisation mentioned above.

4. We had suggested towards the end of Section 4 a possibility of characterising the optimal policy paths, and that needs elaboration. The scope of this essay was, first, to develop an analytical structure and, then, use that structure with appropriate econometric estimations to comment on a specific question, namely, whether the policies on taxation and educational expenditure adopted by the governments of two particular countries were to help equalise incomes or not. In answering

that question, it was found adequate to use only a necessary condition as given by (4.10) and conclude on the basis of that. However, once this question has been answered, it is quite natural to be interested in another closely related question. Supposing that the actual policies in an economy are found to violate the necessary condition for convergence, what, then, are the alternative optimal policies that an economist can recommend?

To answer this question, one has to use the sufficient condition as given by (4.12), and on the basis of that two kinds of attempts can be made. In the first place, it may be worthwhile to find out the qualitative properties of the optimal policies on taxation and expenditure and the nature of the trade-off between the two, at a point of time and also over time. Choosing the absolute differences, namely,  $(z_1 - z_2)$  and  $(\lambda_2 - \lambda_1)$  as the indicators of the degree of progression in the tax and expenditure structure, one convenient way of attacking this problem will be to verify certain pertinent conjectures, e.g., whether both  $(z_1 - z_2)$  and  $(\lambda_2 - \lambda_1)$  are quadratic functions of time, increasing initially to a finite maximum and then declining monotonically and asymptotically. Secondly, it is also possible on the basis of the estimated values of the crucial parameters involved in (4.12) to generate numerical solutions for the optimal policies. These econometric estimations, we have seen, are not very complex and, therefore, depending on the availability of data, it is at least possible to indicate the range of values that the policy instruments should take for a particular economy over, say, the next five years.

5. Another important generalisation will be to introduce the consideration of economic growth along with that of income distribution and see to what extent the previous qualitative conclusions on the policy paths will change because of this additional objective. This problem can be analysed in the following way. Supposing that there is a shift in the education policy, say, a shift in the government educational expenditure in favor of Group 1 and against Group 2 (with  $T$  unchanged), one can find out, on the one hand, its effect on equality as indicated by  $1/x$  and, on the other, given the curvature property of  $n_1 = n(E_1)$ , its effect on the aggregate  $\pi$  and through that on  $Y$ , the total output, and its rate growth,  $\dot{Y}/Y$ . From these two changes (considered in relative sense) it is possible to derive the elasticity of  $\dot{Y}/Y$  with respect to  $1/x$ . If this elasticity turns out to be positive, then there is of course no problem of choice involved and the previous results on the policy paths will remain unaltered. If, however, this elasticity is negative, then there is a tradeoff between growth and equality, and to resolve that a social welfare function with  $\dot{Y}/Y$  and  $1/x$  as the arguments will have to be brought in to introduce the appropriate ordering. The nature of the optimal policy in this case will depend on the magnitude of the elasticity of  $\dot{Y}/Y$  with respect to  $1/x$  and on the specification of relative weights of  $\dot{Y}/Y$  and  $1/x$  in the social welfare function. If, for example, the elasticity of  $\dot{Y}/Y$  with respect to  $1/x$  is found to be less than 1 and if an equal weight is given to  $\dot{Y}/Y$  and  $1/x$  in the social

welfare function (e.g., a Cobb-Douglas specification of the utility function with the same exponent for both  $\dot{Y}/Y$  and  $1/x$ ), then clearly the previous results on the optimal policy paths will carry over.

6. The analytical structure in this paper had a distinct neo-classical character, with its reliance on the well-behaved production function and marginal productivity theory. This was a deliberate choice because we wanted to prove certain results staying within the neoclassical framework. However, the neoclassical framework is by no means essential for our results. It is possible to snap any systematic link between the production function and the rates of return to factors and start instead with the income-generation functions as the primitive concept and build the entire structure from that. It may be noted that an advantage in this kind of approach is that here it is much easier to accommodate different types of market imperfections and other institutional characteristics. And, if there are reasons to believe that these institutional characteristics are themselves conditioned by the distribution of income, then it can be shown that the conclusion on disequalising tendencies of income distribution which were derived within a neoclassical framework will follow a fortiori once these institutional considerations are brought into the picture.

**Essay 2: Income Distribution, Market Imperfections  
and Capital Accumulation in a  
Developing Economy**

**Asim K. Dasgupta**



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Income Distribution, Market Imperfections and  
Capital Accumulation in a Developing Economy

Asim K. Dasgupta

1. Introduction

There are different ways in which one can try to explain the problem of insufficient capital accumulation in a less developed country. In the conventional analyses this explanation is often found exogenized in terms of factors such as socio-cultural attitudes towards saving and investment, irrationality of peasant behavior, technological facts of externalities on demographic parameters. In this essay I intend to offer an alternative explanation in terms of the distribution of income. It will be shown that given the existing distribution of income and the resulting market imperfections, especially credit market imperfection, in many developing countries, the income groups which can and in fact do save may not use their saving for capital accumulation, not necessarily because of any esoteric cultural reasons, technological or demographic obstacles, but simply because that will go against the maximization of their, very rational, objective function relating to net income or utility. One advantage of this kind of explanation is that it goes a longer way in endogenizing the problem of capital accumulation in terms of economic variables. It also draws our attention to a different kind of

constraints on economic development, different from the ones suggested by the conventional analyses.

I shall develop the main thesis within the agricultural sector of a less developed country and then point out how this can be extended to cover the industrial sector as well. In Section 2 the major characteristics of such an agriculture are described, stressing particularly the dualism that exists between the family and the capitalist farms, the distribution of income between them and the implication of that distribution on the structure of rural credit market. Given these characteristics, a model is developed in Section 3 by deriving the decision rules that the family and the capitalist farms will adopt about the use of inputs and allocation of wealth on the basis of some well defined maximizing objective. In Section 4 this model is then used to analyze the special problem of capital accumulation of this agriculture. It is found that given an unequal initial distribution of income and the associated imperfection of credit market, such an agriculture can show a tendency to approach a state of zero rate of capital accumulation under very plausible conditions, and this can be accompanied by a process of immiserization of family farms. The importance of the distribution of income and the structure of credit market as factors responsible for this crisis is brought out more precisely in Section 5 where the results of this model are compared with those of a hypothetical situation involving a more equal distribution of income and a perfect credit market. In Section 6

several ways of resolving this crisis are discussed, including particularly the solution that is offered by technical progress. Here, it is found that the issues connected with a special kind of technical progress, namely, the Green Revolution, as well as those connected with some other solutions based on institutional changes, can be given an interesting interpretation. In Section 7 the conclusions of this model are compared with other existing results in the literature. Finally, several ways of generalizing the basic model are suggested (Section 8).

It needs also to be pointed out at the outset that certain assumptions of our model, made particularly about the nature of market imperfections, are based primarily on the characteristics prevailing in the Indian agriculture. But, in these respects, the Indian situation may not be very atypical of peasant agriculture of many other less developed countries in Asia, Africa and Latin America.

## 2. Characteristics of the Economy

Consider an economy with an agricultural and an industrial sector. Although the primary concern of this paper is with the agricultural sector, it is worthwhile in the beginning to comment very briefly on the structure of industrial sector as well, particularly its links with the agricultural sector, so that results derived within the agricultural sector can be viewed from the perspective of the entire economy.

The industrial sector is partitioned into a private sector producing a luxury consumption good, to be consumed partly in the industrial sector and partly in the agricultural sector, and a government sector producing a capital good to be used again in both sectors.<sup>1)</sup> The agricultural sector, in its turn, produces a necessary consumption good — a part of it is consumed within agriculture and another part goes to industry. The other link between the two sectors is through the labor market. The credit markets of the two sectors, however, are mutually isolated and there is no significant intersectoral exchange of saving.<sup>2)</sup> This relationship between agriculture and industry — their links as well as isolation — is a characteristic common to many developing countries.

Given this structure of the entire economy, we shall, as indicated before, concentrate on the agricultural sector. In order to be able to do that, we choose, for most of this paper, not to go into the problems of interaction between agriculture and industry. It will

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- 1) One can also consider the government sector as providing other intermediate goods, like fertilizers. This additional category of inputs can be included in our analysis without much difficulty.
  - 2) For empirical evidence on the structure of these markets in India, see R.K. Hazari, The Structure of the Corporate Private Sector (1966), pp. 308-322; T.N. Krishnan, "Taxation of Property and Net Wealth in India: A Note", Econ. and Pol. Weekly, Jan. 1, 1972; All India Rural Debt and Investment Survey, Reserve Bank of India Bulletin, June 1965.

be assumed that the agricultural output can be sold at a fixed (money) price within the sector and also to industry, and so can be the luxury consumption good produced by industry. Capital goods are also available to the agricultural sector at a fixed price from the industrial sector and migration of labor from agriculture to industry is not significant. It will be mentioned later on how all these assumptions can be relaxed and results generalized, but to start with they help us to focus our attention on the agricultural sector.

Within the agricultural sector, an important feature observed in many less developed countries is the coexistence of the family and the capitalist farms. The distinction between the two is based on the significance of hired labor in the total labor force used in the respective farms. The family farm uses labor mostly of its family members whereas the capitalist farm is dependent primarily on the wage-labor coming from the family farms.<sup>3)</sup> For the sake of simplicity, we will assume in our analysis that the family farm uses only the family labor and the capitalist farm only the wage-labor from the family farms.

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3) In agriculture, in addition to these two classes, there is also a class of landless labor. In India, for example, according to the National Sample Survey (19th Round) data, landless agricultural labor households constituted 12.2 per cent of the total number of rural households in 1964-65. To begin with, this landless labor will not be considered in our analysis, but it will be shown later how its existence can be naturally accommodated into the basic model without causing any change in analysis.

The distribution of land between these two types of farms is given at any point of time, and there does not exist any significant market for land. By this it is meant that there does not exist any market for voluntary exchange of land. One important reason for this is that in a society exposed to various kinds of risk, and with few means of insurance known, land is a highly attractive asset to hold. In particular, to a farmer on the margin of subsistence, who is most likely to be the potential seller of land, the risk of parting with land is often one of starvation and land prices rarely fully reflect this risk as evaluated by the farmer.<sup>4),5)</sup> However, although there does not exist any voluntary exchange of land, "distress sale" of land does take place. In fact, it will be shown later that it is through such a mechanism that the capitalist farm can take over the ownership of land from the family farms in some special situations, such as default of loan by the latter. But until a family farm is driven to such an extreme situation, the total amount of land owned by a family does not get voluntarily exchanged.

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4) See P.K. Bardhan, "Size, Productivity and Returns to Scale: An Analysis of Farm Level Data in Indian Agriculture", Journal of Pol. Ec. Vol. 81, No. 6, 1973; K.N. Raj, "Ownership and Distribution of Land", Indian Econ. Rev., April, 1970.

5) In this context, one should also mention that the factors which are considered important in preventing any substantial leasing out of land by the big farmer are the tenancy and rent control legislations in vogue in many less developed countries.

Now, the size of this land holding of a family farm is usually quite small compared to that of a capitalist farm.<sup>6)</sup> Not only is its size of land holding small, its average income, the inverse relationship between size of land holding and productivity of land notwithstanding,<sup>7)</sup> is also very low in absolute magnitude and in comparison with that of the capitalist farm.<sup>8)</sup>

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- 6) In India, for instance, it is found from the 1961 Census data that the farms with 93.8 per cent of family labor in the total labor force have less than one acre of land holding; those with 87.4 per cent of family labor have 5 acres or less; and then the size of land holding gets larger as the proportion of hired labor in total labor force increases and it is found that the farms which use 80 per cent or more of hired labor in their labor force have land holdings of 30 acres or more.
- 7) This inverse relationship has been widely observed in many less developed countries. See, in particular, Government of India, Ministry of Food and Agriculture, Farm Management Studies.
- 8) If, for example, the figure of 87.4 per cent of family labor in labor force, and therefore 5 acres of holding size, is taken as an approximate cutoff point between the family and the capitalist farms, then by the NCAER estimate of the distribution of agricultural income by the size of land holding, the median figure for the annual income of a family farm is around Rs 747 which is one-fourth of the corresponding figure for a capitalist farm. To give a more complete description of this distribution of income, it has a Lorenz ratio = 0.35, with the bottom 20 per cent of the population, consisting of the family farms, having 7 per cent of total income while the top 20 per cent, consisting of the capitalist farms, having 44 per cent of the share. It is also interesting to note that the income share rises relatively slowly as one goes from the bottom deciles upwards, but from the ninth to the topmost decile there is a sudden increase of nearly 150 per cent, pointing to the extreme concentration at the top. This seems to be a characteristic common to many poor countries. See, National Council of Applied Economic Research: All India Consumer Expenditure Survey, 1966, 1967.



This distribution of income between the family and the capitalist farms — the significant disparity between their average incomes as well as the low absolute value of the family farm's income — is to be taken as the description of the initial state in our analysis. And, as we shall presently see, this has an important implication on the structure of agricultural credit market. For that, one has to look into, among other things, the nature of the production process in agriculture.

The production process in agriculture can be best described by the Bohm-Bawerkian continuous input-point output technology. The entire process takes place over an interval of time which can be called an agricultural "year" and can be taken to be equal to a "period" in our analysis. Within each such period, starting from the beginning point and spread over the entire interval, labor and capital are applied by both the family and the capitalist farms to their given amounts of land, and then output is obtained at the end point of the period. The production function is assumed to be neoclassical showing constant returns to scale and diminishing returns to factors, and is the same for both the farms. However, the decisions they have to take on the use of labor and capital, though related, are not exactly the same.

Consider, first, the family farm. It starts any period with a certain amount of family labor and a net income obtained from the previous period. Of this family labor, a part is to be used in its

own production and the rest to be sent away to work on the capitalist farm for wage which, we assume, is paid post facto. By the net income of the previous period is meant the gross income of that period which, because of the nature of agricultural production and of wage payment, was obtained at the end point of the period, less the amount of loan that was taken in that period and had to be paid back. As already documented, the average gross income of the family farm is very low and hence its average net income is even lower. From the available empirical evidence, we find it reasonable to assume that from this level of average income it is not possible for the family farm to save anything<sup>9)</sup>. The family farm, therefore, does not own any stock of capital; it has to take production loan for using capital. Not only is the average net income of the family farm low to rule out saving, very often it is also inadequate to meet the per head consumption needs of the family over the entire production period. Since wage is paid at the end of the period, this implies that the family farm has

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9) Strictly speaking, for our analysis, it is not necessary for the saving to be zero, it is only necessary to have a situation where the family farm can not save enough so that it has to take loans. Still, we choose to stick to the assumption of zero saving by the family farm, because given the empirical evidence, at least in the case of India, and our choice of the cutoff point between the family and the capitalist farms (see, footnote 8), we believe that this is a more accurate description of the actual saving behavior of a typical family farm. See, the NCAER Rural Household Survey (1965), Tables 33 and 36.

to take loan also for consumption purposes<sup>10)</sup>.

All these loans are taken from the capitalist farm and under conditions of an imperfect credit market. The cause and the nature of this imperfection will be explained shortly. What needs to be carefully mentioned here is that after a certain amount of loan has been taken at a given rate of interest by the family farm, it has to allocate this loan between the uses for consumption and production, and this allocation can only be done with respect to a well defined objective function. This will be precisely shown in Section 3.

Using these loans, the family farm produces its output at the end of the period. This output, evaluated at the fixed market price, together with the wage earned from the capitalist farm determines the gross income of the family farm for this period. The net income is then obtained by deducting from the gross income the loans which have been taken in this period and which, in our analysis, are always supposed to be paid back at the end of the period. It is with this net income, the total and the corresponding average, that the family farm starts the next period. Along with the net income, there is also a different size of labor force supplied in the next period, and the

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10) The analysis does not change in any essential way if wage is considered to be paid in advance. Then an interest is charged on this wage and therefore, in effect, wage becomes a part of the consumption loan. It can be checked that the conclusions of this paper are invariant with respect to the nature of wage payment.

rate of growth of this labor force is to be considered as exogenously given.

We like to point out now that, to begin with, it is helpful to suppose that the average net income of the family farm, though small, is positive.<sup>11)</sup> This means that although the family farm could not save and had to take loans because its average net income at the beginning point of the period was small, and the output and wage earnings were not to be available until the end of the period, and during this period the family had to take care of its consumption needs as well as keep the production going with rented capital, yet when the output is finally obtained and wage income received, it can indeed pay back those loans and is left with some positive average net income with which it can start the next period. That is, in the beginning,

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11) Nothing is altered in our basic analysis or in the final conclusion if the net income of the family farm is nonpositive to start with. It will be demonstrated in Section 4 that, under certain plausible conditions, a dualistic agriculture can show an inherent tendency to approach a limiting state with respect to capital accumulation and impoverishment of the family farm. A situation of nonpositive net income of the family farm simply means, as will be evident later on, that the agriculture in question is at an advanced stage of this tendency. From the standpoint of analysis, this situation is even simpler to tackle since in this case one can skip certain intermediate steps. We think, however, that it is not enough to analyze only this terminal stage as it may relate to a dualistic agriculture, it is also necessary to understand and explain the historical process by which such an agriculture is actually brought to this terminal stage. That is why we have decided to start with an initial situation which is somewhat away from this terminal stage, being characterized by a positive net income for the family farm. The situation with a nonpositive net income of the family farm will then come to be analyzed incidentally as a part of the more complete analysis of the evolutionary process.

there is no problem of defaulting to worry about. The interesting question, then, is: what happens over time? Does this average net income increase or stay constant, and therefore remain positive? Or, does it fall over time, threatening a bankruptcy of the family farm? How does the capitalist farm react to that situation? The purpose of this paper is precisely to answer these questions, by analyzing the intertemporal behavior of the average net income of the family farm vis-a-vis the capitalist farm and then relating that to the entire question of capital accumulation.

Let us now turn to the capitalist farm. Like the family farm, the capitalist farm also starts any period with a given number of family members and a net income from the previous period. But, there are two important differences. First, the members of the capitalist family do not work and labor is hired for production from the family farm. Secondly, the average net income of the capitalist farm is much higher than that of the family farm, and with this higher level of income the capitalist farm can both consume and save. Its consumption is on the agricultural product as well as on the luxury consumption good from industry, both of which are assumed to be available at fixed prices. More important than consumption is the fact that the capitalist farm can save, something which the family farm could not do, and this saving when added to the pre-existing stock of wealth gives the total wealth of the capitalist farm for the present period. The capitalist farm can keep this wealth in two forms: (a) capital to be used in its

own production, and (b) loan to be given to the family farm<sup>12)</sup>.

This choice of portfolio, of course, has to be made with respect to a well defined objective function, and this will be shown in Section 3.

The capitalist farm, thus, combines two operations at the same time — production and lending, and it is to be noted that in the market for the latter there exists an imperfection. This imperfection in the credit market arises primarily because of the special nature of the distribution of income and wealth already mentioned, whereby there are numerous family farms with a low level of average income and wealth, and therefore in need of credit, and a relatively few capitalist farms with a much higher level of average income and wealth, and in a position to supply that credit. These relatively few capitalist farms, again, are found to be spread over the entire agricultural sector with the result that within a local credit market there exists a typical situation of many family farms facing one (or very few, but homogeneous enough to be considered one) capitalist farm as the money lender<sup>13)</sup>.

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12) It should be noted that the capitalist farm has control only over the amount of loan to be given to the family farm at a certain rate of interest. Beyond that, it does not have any control on the final allocation of that loan between production and consumption. That allocation is done only by the family farm and in accordance with its own objective function, as has already been mentioned.

13) It is an interesting exercise to prove how starting with an initial distribution of income such as has been considered here, the relatively few capitalist farms will find it most profitable to have themselves spread over the entire sector so that each one can enjoy a monopolistic hedge in its local credit operation.

This monopolistic position of the capitalist farm in the credit market is also reinforced by the lack of any serious competition from the conventional commercial banks. This is because there are some special problems connected with assessing the credit worthiness of the family farms, arising mainly from their low income and wealth position, and the commercial banks, located as they are in the urban areas, are at a serious disadvantage in handling these problems. Very often, therefore, it is found that the participation of the commercial banks in the agricultural credit market is practically negligible<sup>14)</sup>. This is a job which the local capitalist farmer, due to his intimate knowledge of the economic positions of the family farmers, is uniquely suited to perform, and, here, he can outcompete not only the urbanized commercial banks but also the other capitalist farmers who are not strictly local.

An appropriate stylized way of characterizing the agricultural credit market is therefore to describe it in terms of a representative set which is sufficiently localized and consists of several family farms and one capitalist farm, with the latter enjoying a virtual monopoly in the local lending activity. And, the agricultural sector can then be visualized as the union of numerous such sets which are not only

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14) In India, for instance, it is found that in the year 1961-72 the commercial banks had accounted for only 0.4 per cent of the total credit received by the agricultural households. See, All India Rural Debt and Investment Survey, 1961-62, Reserve Bank of India Bulletin, Sept. 1965, pp. 1299-1301.

significantly insulated from the credit market of the industrialized urban sector, but are also nonintersecting among themselves so far as credit operations are concerned. It should be noted, however, that this nonintersection is meant to apply only to the credit market. With respect to the labor market, for example, there is no such isolation, the relevant market being the entire agricultural sector itself<sup>15)</sup>.

Given this structure, the capitalist farm has, at the end of the period, two sources of income — one is the value of output produced with its own capital and hired labor, and the other is the interest earnings from loan. These two kinds of income can be added up to get the gross income of the capitalist farm, and its net income is then obtained by deducting from this the wages to be paid to the hired family farmers. The capitalist farm begins the next period with this net income, the total and the corresponding average. Meanwhile, the size of its family has grown over the period, the rate of growth, as in the case of the family farm, being determined exogenously.

This is a description of a dualistic agriculture with the family and the capitalist farms, their initial distribution of income, the implication of that distribution on the structure of credit market and the general nature of the decisions they have to take on the use of

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15) Later on, in Section 8, we shall discuss briefly the possibility where imperfection of the credit market may also imply a monopsony in the labor market whereby the family farmers may be forced to work only for the local capitalist-cum-money lender.



inputs and allocation of wealth. The purpose of this essay is to derive these decision rules in a precise form and analyze them in relation to the particular question of capital accumulation. For that, the objectives of the two farms are to be stated precisely, and, in this context, we assume that both the farms in making their allocation decisions are guided by the objective of maximizing the happiness of their respective family members, not only within one period, but also over a certain span of periods, and express this objective as a discounted sum of utility defined over a stipulated time horizon and relating to consumption per head of the family members of the respective farms. This intertemporal characterization of the objective, it should be noted, is essential if the decision rules with respect to saving and accumulation of wealth are to be accounted for. It should also be noted that although on grounds of analytical completeness we shall work with this Ramsey-type intertemporal objective functional and derive the decision rules subject to that, most of these rules can also be derived, as will be shown later in Section 3, from a somewhat simpler specification of the objective, namely, that the family and the capitalist farms try to maximize their net income (i.e., profit) in any period with an additional intertemporal requirement that the net income of any period should not fall below that of the previous period.

Decision making with reference to an objective function, specified in either of these two forms, can be regarded as the usual expression of rational behavior in economic analysis. And, as pointed out at the very

outset, our intention in this essay is to offer an alternative explanation of the agricultural stagnation of a less developed economy on the basis of such a framework of rational behavior on the part of both the family and the capitalist farms, but as applied to the very special objective circumstances of a dualistic agriculture which arise primarily from its state of distribution of income and the related structure of the credit market.

### 3. The Model

As indicated in the last section, the agricultural sector can be considered as divided into numerous sets consisting of the family and the capitalist farms and these sets can be regarded as non-intersecting in their credit operations. Suppose, for the sake of simplicity, that there are  $m$  such identical sets ( $m$  is sufficiently large, but finite) and within each set there are  $n$  identical family farms and one capitalist farm (or, a few of them, homogeneous enough to be regarded as one unit). With this notion of aggregation, we now proceed to derive the allocational decision rules, first for the family farm and then for the capitalist farm, taking into account all the structural characteristics as already mentioned. It is these rules that will define the model of our analysis.

#### The Family Farm:

Given the nature of agricultural production and the relationship between the processes of income generation for the two types of farms,

as outlined in the last section, the total income of a family farm belonging to such a representative set in any period  $t$  can be written as

$$(3.1) \quad Y_1(t) = \bar{P}F(\bar{T}_1, K_1(t), L_1(t)) + w(t) L_2(t) - i(t)\bar{P}_k K_1(t) \\ - (1 + i(t))\bar{P}C_1^l(t) ,$$

where  $Y_1(t)$  is the total net income of the family farm in period  $t$ ,  $\bar{P}$  is the fixed money price of the agricultural output which is produced subject to a neoclassical production function  $F$  showing constant returns to scale and diminishing returns to factors,  $\bar{T}_1$  is the given amount of land which remains unchanged except in the case of default of loan,  $K_1(t)$  is the amount of capital rented from the capitalist farm,  $L_1(t)$  is the labor of family members used in its own production,  $L_2(t)$  is the family labor sent away for work in the capitalist farm and  $w(t)$  is the money wage rate thereof. The first two terms on the R.H.S. of (3.1) add up to give the total gross income of the family farm.  $\bar{P}_k$  is the fixed price of capital,  $\bar{P}C_1^l(t)$  the money value of the consumption loan and  $i(t)$  the rate of interest for period  $t$ , so that the last two terms of (3.1) are the rental payment on the production loan and interest-plus-principal payment on the consumption loan respectively.

There are certain issues in connection with the production and the consumption loan which are worth clarifying at the outset. In the first place, there is a difference in the way the two loans are paid back in each period. Since the services of capital can be rented per period, the payment of production loan in any period in the absence of depreciation, is just the payment of rental. The consumption loan, on the other hand, is like wages fund; it can not be used without its being exhausted and hence the payment of consumption loan includes both principal and interest. Secondly, given the continuous input-point output technology, capital needs to be rented in the beginning of the period and used in production over the entire period. The consumption loan, on the other hand, need not be taken right in the beginning of a period. Depending on the amount of net income available from the previous period, it can be taken at any time within the period, but naturally before the end point when the output is again available. Since we have used the same rate of interest for both kinds of loan, it should be understood that an initial adjustment has been made for the rate of interest on the consumption loan, so that it can refer to the entire period. Finally, it should be noted that we shall very often add up  $\bar{P}_k K_1(t)$  and  $\bar{P}C_1^l(t)$  to define the total loan of the family farm in any period  $t$ , and there is no stock-flow contradiction involved. Note that the total loan of the family farm in any period  $t$  is:

$$\sum_{\tau=-\infty}^{\tau=t} \bar{P}C_1(\tau) + \sum_{\tau=-\infty}^{\tau=t-1} \bar{P}_k \Delta K_1(\tau)$$

But, in our analysis, it is assumed that loans are paid back at the end of each year, so that

$$\sum_{\tau=-\infty}^{\tau=t-1} \bar{P} C_1^l(\tau) = 0$$

Therefore with  $K_1(t)$  denoting the capital stock covering the entire period  $t$ , the total loan of the family farm in any period  $t$  can be written as:  $\bar{P}_k K_1(t) + \bar{P} C_1^l(t)$ .

Given the total net income of the family farm, as defined in (3.1), its average net income in period  $t$  is:

$$(3.2) \quad y_1(t) = \frac{Y_1(t)}{\bar{L}_1(t)} = \frac{\bar{P} F(\bar{T}_1, K_1(t), L_1(t)) + w(t)L_2(t)}{\bar{L}_1(t)} \\ - \frac{i(t)\bar{P}_k K_1(t) + (1+i(t))\bar{P} C_1^l(t)}{\bar{L}_1(t)}$$

where  $\bar{L}_1(t)$  is the size of the family and, for the sake of simplicity, is also taken to be its total labor force<sup>16)</sup>.

$$(3.3) \quad L_1(t) + L_2(t) = \bar{L}_1(t)$$

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16) Alternatively, one can assume that the labor force is a certain fixed proportion  $0 < \alpha < 1$  of  $\bar{L}_1(t)$ .

It is assumed that  $\bar{L}_1(t)$  grows at an exogenously fixed rate  $g$  :

$$(3.4) \quad \bar{L}_1(t) = \bar{L}_1(0)(1 + g)^t, \quad g > 0$$

As already explained in detail, the net income with which the family farm starts any period is low so that it cannot save and has to rent capital and also take consumption loan to meet the consumption requirement<sup>17)</sup>. Therefore, denoting by  $c_1(t)$  the consumption per head of the family in real terms, we can write

$$(3.5) \quad c_1(t) = \frac{y_1(t-1)}{\bar{P}(1+g)} + \frac{C_1^l(t)}{\bar{L}_1(t)}$$

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17) We have already mentioned it before (cf. p. 11n), and we repeat it here, that for our analysis and final conclusions it is not essential that saving of the family farm be zero and the amount of its consumption loan positive. What we need is a situation where, because of the existing distribution of income, the family farm cannot save enough and it has to take some loan from the capitalist farm, be it consumption loan or renting of capital (in other words, the credit market should be allowed to remain in the picture). Given such an upperbound on saving on the part of the family farm properly defined, it can be shown just by using the property of imperfection of the credit market and the stated objectives of the farms that, under very plausible conditions, the system will evolve over time in such a way that after a certain period of time the saving of the family farm will in fact drop to a negligible amount and that it will also have to take consumption loan. And, the present analysis applies from then on. Therefore, the assumptions of zero saving and positive consumption loan on the part of the family farm are not analytically essential. We assume it only because there is evidence to believe that it is a more empirically appropriate way of describing the reality in a less developed country. In the context of India, for example, it is clear from the NCAER Rural Household Survey (1965), Tables 33 and 36, that, given our choice of the cutoff point between the family and the capitalist farms in terms of land holding, the saving of the median family farm is indeed negligible. It is also evident from the All

These are the definitions of the relevant variables as applied to the family farm and the definitional equations involving them. The question, now, is: how does the family farm make its choice about the value of these variables,  $L_1$ ,  $K_1$  and  $C_1^l$ , when its objective, as mentioned at the end of the last section, is to maximize a discounted sum of utility relating to per capita consumption of its family members over some stipulated time horizon, i.e., to maximize

$$(3.6) \quad \sum_0^{T_1} \lambda_1^{-t} U(c_1(t))$$

where  $T_1$  is the length of time horizon for the family farm,  $\lambda_1 (>1)$  is the discount factor for its time preference,  $U$  is its instantaneous utility function with required concavity and  $c_1(t)$  is defined by (3.5).

This is essentially a discrete analogue of the generalized Ramsey problem, and the Euler conditions for maximum<sup>18)</sup> in this discrete-time case are obtained by constructing the following sum of two adjacent terms of the utility functional,

17) CONT'D. India Rural Debt and Investment Survey (op.cit.), and on the same choice of the cut off point, that nearly 70 per cent of the total loan taken by the family farms in the year 1961-62 has been for consumption purposes.

18) The second-order Legendre condition is satisfied by the concavity of utility function.

$$\begin{aligned}
 (3.7) \quad Z_1 = & \lambda_1^{-t} U \left[ \frac{y_1(t-1)}{\bar{P}(1+g)} + \frac{C_1^l(t)}{\bar{L}_1(t)} \right] + \lambda_1^{-(t+1)} U \left[ \frac{F(\bar{T}_1, K_1(t), L_1(t))}{\bar{L}_1(t+1)} \right. \\
 & + \frac{w(t)(\bar{L}_1(t) - L_1(t))}{\bar{P} \bar{L}_1(t+1)} - \frac{i(t)\bar{P}_k K_1(t)}{\bar{P} \bar{L}_1(t+1)} \\
 & \left. - \frac{(1+i(t)) C_1^l(t)}{\bar{L}_1(t+1)} + \frac{C_1^l(t+1)}{\bar{L}_1(t+1)} \right]
 \end{aligned}$$

and then setting the partial derivatives of  $Z_1$  with respect to the relevant arguments,  $L_1(t)$ ,  $K_1(t)$  and  $C_1^l(t)$  equal to zero<sup>19)</sup>:

$$(3.8) \quad \bar{P} F_{L1}(\bar{T}_1, K_1(t), L_1(t)) = w(t)$$

$$(3.9) \quad \frac{\bar{P}}{\bar{P}_k} F_{K1}(\bar{T}_1, K_1(t), L_1(t)) = i(t)$$

$$\begin{aligned}
 (3.10) \quad \lambda_1 U' \left[ \frac{y_1(t-1)}{\bar{P}(1+g)} + \frac{C_1^l(t)}{\bar{L}_1(t)} \right] - \left( \frac{1+i(t)}{1+g} \right) U' \left[ \frac{F(\bar{T}_1, K_1(t), L_1(t))}{\bar{L}_1(t)(1+g)} \right. \\
 + \frac{w(t)(\bar{L}_1(t) - L_1(t))}{\bar{P} L_1(t)(1+g)} - \frac{i(t)\bar{P}_k K_1(t)}{\bar{P} \bar{L}_1(t)(1+g)} \\
 \left. - \frac{(1+i(t))C_1^l(t)}{\bar{L}_1(t)(1+g)} + \frac{C_1^l(t)}{\bar{L}_1(t)(1+g)} \right] = 0,
 \end{aligned}$$

19) For a discussion of the Euler conditions in the discrete-time case, see P.A. Samuelson, "A Turnpike Refutation of the Golden Rule in a Welfare-Maximizing Many-Year Plan" in (R.C. Metron ed.) The Collected Scientific Papers of Paul A. Samuelson, Vol. 3, pp. 108-110.



where

$$F_{L1} = \frac{\partial F(\ )}{\partial L_1} , \quad F_{K1} = \frac{\partial F(\ )}{\partial K_1} \quad \text{and} \quad U'(\ ) = \frac{d U (c_1(\ ))}{dc_1(\ )} .$$

Note that (3.8) and (3.9) are the static optimality conditions which give the family farm's decision rules with respect to the use of labor and capital respectively. In making these decisions, the family farm takes  $w$  and  $i$  as parameters. The wage rate is determined by the aggregate supply of labor from all the family farms and the aggregate demand for labor from all the capitalist farms of the agricultural sector taken together, while the rate of interest is set within a representative set monopolistically by the capitalist farm. An individual family farm acting alone can not affect either  $w$  or  $i$ .

The condition (3.10), on the other hand, is the dynamic optimality condition (an analogue of the Ramsey rule for the problem of the family farm) which has to hold for any pair of adjacent periods ( $t, t+1$ ), and it gives us the demand function of the family farm for the consumption loan,  $\bar{P}C_1^l$ . It is a second-order difference equation embedded in the optimal time profile of  $\bar{P}C_1^l$  and it is known that such a profile is uniquely fixed by the initial and the terminal condition relating to  $\bar{P}C_1^l$ . We choose to specify these conditions by two constants, to be denoted by  $B_1$  and  $B_2$ .

Given these specifications, for any period t, the value of  $\bar{P}C_1^l$  obtained from the previous period (which, incidentally, is zero because of the assumption that the loan of any period is to be paid back in that period) as well as that related to the next period,  $\bar{P}C_1^l(t+1)$ , can be taken as predetermined, and it is then possible to characterize the demand function for  $\bar{P}C_1^l$  for any period t as:

$$(3.11) \quad \bar{P}C_1^l(t) = \psi(i(t), w(t), y_1(t-1), \bar{L}_1(t); \lambda_1, g, B_1, B_2)$$

where the variables,  $K_1(t)$  and  $L_1(t)$ , are eliminated by virtue of (3.8) and (3.9), and  $\lambda_1, g, B_1$  and  $B_2$  are the given constants. Now, by using the implicit function rule with respect to (3.10), it can be easily seen, as is also intuitively expected, that

$$\psi_1 = \frac{\partial \bar{P}C_1^l(t)}{\partial i(t)} < 0, \quad \psi_2 = \frac{\partial \bar{P}C_1^l(t)}{\partial w(t)} > 0, \quad \psi_3 = \frac{\partial \bar{P}C_1^l(t)}{\partial y_1(t-1)} < 0$$

$$\text{and } \psi_4 = \frac{\partial \bar{P}C_1^l(t)}{\partial \bar{L}_1(t)} > 0.$$

In the same way, it can also be verified that the elasticities of  $\bar{P}C_1^l(t)$  with respect to  $i(t)$  and  $w(t)$ , to be denoted by  $e_{\psi, i}$  and  $e_{\psi, w}$ , are inversely related with the value of the discount factor,  $\lambda_1$ , and those with respect to  $y_1(t-1)$  and  $\bar{L}_1(t)$ , to be denoted by  $e_{\psi, y_1}$  and  $e_{\psi, \bar{L}_1}$ , are directly related with  $\lambda_1$ .

Of particular importance for our later analysis is the comparison between  $e_{\psi,i}$  and  $e_{\psi,y_1}$ . At a low level of income, when the consumption is more of a necessity than luxury, it is reasonable to expect that in any period the elasticity of  $\bar{PC}_1^L$  with respect to the net income available in that period is significantly higher than that with respect to the rate of interest to be paid on the loan<sup>20)</sup>. A good way of presenting this phenomenon in terms of our analytical framework is through an appropriate valuation of  $\lambda_1$ . Since at a very low level of income, an individual is expected to be specially concerned about its immediate, rather than future, consumption, one can consider  $\lambda_1$  of the family farm as having a significantly high value. And, given the qualitative nature of the relationship of  $e_{\psi,i}$  and  $e_{\psi,y_1}$  with  $\lambda_1$  as just mentioned, such a high value of  $\lambda_1$  can then be taken to imply a correspondingly high value of  $e_{\psi,y_1}$  compared to  $e_{\psi,i}$ .

We now want to make a short digression on a related issue, which is of some concern in the literature on development, bearing on the decision of the family farm with respect to  $L_1$  (and, therefore, also  $L_2$ ) and a possible imperfection of the labor market. It is often mentioned that there exists a positive gap between the wage rate at

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20) For similar reason,  $e_{\psi,y_1}$  will also dominate  $e_{\psi,w}$  since wage is supposed to be received at the end of the period.

which labor can be hired from the family farm and the marginal product of labor in the family farm<sup>21)</sup>. It is interesting to see that this situation can be easily accommodated in terms of our framework of analysis. One important reason behind the existence of this wage gap, it is believed, is the fact that when the members of the family farm, particularly the women, work in their own farm they can coordinate and combine farm work with domestic chores, something which they are unable to do when at work as a hired labor on the capitalist farm<sup>22)</sup>. What this means in terms of our analytical framework is that there is an opportunity cost associated with  $L_2$  being sent away to work at the capitalist farm. If  $\mu(t)$  is taken to denote this opportunity cost per unit of  $L_2(t)$ , then (3.2) can be rewritten as

$$(3.2)' \quad y_1(t) = \frac{\bar{P} F(\bar{T}_1, K_1(t), L_1(t)) + (w(t) - \mu(t))L_2(t)}{\bar{L}_1(t)} - \frac{i(t)\bar{P}_k K_1(t)}{\bar{L}_1(t)} - \frac{(1+i(t))\bar{P}C_1^l(t)}{\bar{L}_1(t)}$$

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- 21) See, J. Bhagwati and S. Chakravarty, "Contributions to Indian Economic Analysis: A Survey", Am. Ec. Review, 59, No. 2, Suppl. (Sept. 1969); D. Majumdar: "Size of Farm and Productivity: A Problem of Indian Peasant Agriculture", Economica, 32, May 1965; A.K. Sen: "Peasants and Dualism with or without Surplus Labor", Jr. of Pol. Ec., Oct., 1966.
- 22) See P.K. Bardhan, Loc. cit. pp. 1379-1381. For empirical evidence in the Indian context, see P. Visaria, "The Farmers' Preference for Work on Family Farms", in Report of the Committee of Experts on Unemployment Estimates, New Delhi, Govt. of India, 1970.

and (3.8) as

$$(3.8)' \quad \bar{P} F_{L_1}(\bar{T}_1, K_1(t), L_1(t)) = w(t) - \mu(t)$$

There will be a similar modification of (3.10) so that (3.11) can be rewritten by including  $\mu(t)$  as another argument:

$$(3.11)' \quad \bar{P} C_1^l(t) = \psi(i(t), w(t), y_1(t-1), \bar{L}_1(t), \mu(t); \lambda_1, g, B_1, B_2)$$

with  $\bar{P} C_1^l(t)/\partial\mu(t) < 0$ . It is now clear from (3.8)' that so long as  $\mu(t) > 0$ ,  $w(t) > \bar{P} F_{L_1}$  and therefore the wage gap<sup>23</sup>.

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23) There is an alternative explanation of the wage gap, due to Lewis (cf. his "Economic Development with Unlimited Supplies of Labour", Manchester School of Econ. and Soc. Studies, May 1954), which suggests that the peasant leaving his family to work outside loses his income from the farm, equal to the average product per person, and the wage rate outside must compensate for this. This explanation can also be accommodated in our analytical framework. Note that for this argument to be valid, it is necessary to assume that the outgoing peasant cannot rent out or sell his share in the land held by the joint family, the family refuses to subsidize him with remittances and that he does not remit back his wages. What all this means is that when the peasant goes out in this way, he, in effect, ceases to be a member of the family. To capture this situation, therefore, the wage term in the expression of net income of the family should be dropped, and then the wage rate of the outgoing peasant indeed becomes equal to the average net income of the family farm. It should be emphasized, however, that this explanation of the wage gap, based as it is on a particular kind of relationship between the outgoing peasant and the family, is more appropriate for the rural-urban migration than for the allocation of family labor between its own farm and the capitalist farm within agriculture. In this context see also, J. Stiglitz: "Rural-Urban Migration, Surplus Labour and the Relationship between Urban and Rural Wages", East African Economic Review, Dec. 1969, and "Wage Determination and Unemployment in L.D.C.'S" The Quarterly Journal of Economics, May 1974.

The value of  $\mu(t)$  can be considered as depending on  $L_2(t)$  and  $\bar{L}_1(t)$ :

$$\mu(t) = \mu(L_2(t), \bar{L}_1(t))$$

where  $\partial\mu(t)/\partial L_2(t) > 0$ , since the opportunity cost increases as more of family labor goes out to work in the capitalist farm, and  $\partial\mu(t)/\partial\bar{L}_1(t) < 0$ , since there are economics of scale associated with a larger size of the family. But,  $d\bar{L}_1/dt > 0$  and, also generally,  $dL_2/dt > 0$ . Hence the sign of  $d\mu/dt$  is ambiguous. We start by assuming that the two effects tend to cancel out each other so that  $\mu$  can be taken not to change over time, and then see later on how the results will have to be qualified if  $\mu$  is considered to change in one way or other. For the purpose of our immediate analysis, therefore, (3.11)' will be written as:

$$(3.11)'' \quad \bar{P} C_1^l(t) = \psi(i(t), w(t), y_1(t-1), \bar{L}_1(t); \mu, \lambda_1, g, B_1, B_2) ,$$

with  $\mu$  treated as a constant.

It should be carefully noted in this context that the existence of a wage gap, as described above, is quite consistent with the imperfections of both the land market and the capital market which we have previously specified<sup>24)</sup>. With the imperfection of labor market thus accommodated

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24) For an analysis of this question, see R.S. Eckaus: The Market Structure and Performance of Traditional Agriculture, Mass. Inst. Of Technology, 1969, Mimeo.

and the Equations (3.2), (3.8) and (3.11) accordingly modified by (3.2)', (3.8)' and (3.11)", the optimal decision rules for the family farm are given by (3.8)', (3.9) and (3.11)".

Let us now turn to the capitalist farm to find out its corresponding optimal decision rules.

The Capitalist Farm:

Recollecting that there are  $n$  identical family farms and one capitalist farm within a representative set (of the family and the capitalist farms), the total net income of the capitalist farm in any period  $t$  can be written as:

$$(3.12) \quad Y_2(t) = \bar{P} F(\bar{T}_2, K_2(t), n L_2(t)) - w(t) n L_2(t) + i(t) M(t) ,$$

where  $Y_2$  is the total net income,  $\bar{P}$  is the fixed money price at which the agricultural product can be sold by both the capitalist and the family farm,  $F$  is the production function available to both of them,  $\bar{T}_2$  is the given amount of land which again remains unaltered until the capitalist farm takes over the land of the family farm in the event of a default of loans,  $K_2(t)$  is the capital stock owned by the capitalist farm and used in its own production, and  $M(t)$  is the total loan given by the capitalist farm to  $n$  identical family farms, i.e.,

$$(3.13) \quad M(t) = n \bar{P}_k K_1(t) + n \bar{P} C_1^k(t)$$

Note that while the family farm's repayment of the consumption loan, for reasons already mentioned, has to include both the principal and interest and therefore the amount to be deducted from its gross income on this account is  $(1 + i(t))\bar{P} C_1^l(t)$ , the definition of the capitalist farm's flow of income in any period, on the other hand, can include only the interest earnings on the loan, whether the loan is for consumption or production.

The average net income of the capitalist farm can then be written as:

$$(3.14) \quad y_2(t) = \frac{Y_2(t)}{\bar{L}_2(t)} = \frac{\bar{P} F(\bar{T}_2, K_2(t), nL_2(t)) - w(t)nL_2(t)}{\bar{L}_2(t)} + \frac{i(t) M(t)}{\bar{L}_2(t)}$$

where  $\bar{L}_2(t)$  is the size of the capitalist family which, like that of the family farm, grows at an exogenously fixed rate  $g$  :

$$(3.15) \quad \bar{L}_2(t) = \bar{L}_2(0)(1 + g)^t, \quad g > 0.$$

Unlike the family farm, however, the capitalist farm can save and this saving in period  $t$ , when added to its wealth already existing from the previous period,  $A_2(t-1)$ , defines the total wealth of the capitalist farm in period  $t$ ,  $A_2(t)$ . Therefore, consumption per head



of its family members in real terms for period  $t$  can be written as:

$$(3.16) \quad c_2(t) = \frac{y_2(t-1)}{\bar{P}(1+g)} - \frac{A_2(t)}{\bar{P} \bar{L}_2(t)} + \frac{A_2(t-1)}{\bar{P} \bar{L}_2(t)}$$

Now, we know that for any value of  $A_2(t)$ , however determined, the capitalist farm can hold this wealth in terms of two kinds of assets: capital to be owned and used in its own production and loans to be given to the family farms, so that

$$(3.17) \quad A_2(t) = \bar{P}_k K_2(t) + M(t) .$$

These are the definitions of the relevant variables for the capitalist farm and the definitional equations involving them. Given these, the problem of the capitalist farm is to choose the values of the variables  $L_2(t)$ ,  $K_2(t)$  and  $M(t)$  (and, given (3.17), also  $A_2(t)$ ) so as to maximize the discounted sum of utility relating to the per head consumption of its family members over a stipulated time horizon, i.e., maximize

$$(3.18) \quad \sum_0^{T_2} \lambda_2^{-t} U(c_2(t))$$

where  $T_2$  and  $\lambda_2$  are the time horizon and the discount factor for the capitalist farm, and  $c_2(t)$ , its real per head consumption, is defined by (3.16). Note that the instantaneous utility function,  $U$ , has been

considered to be the same (with required concavity) for both the family and the capitalist farm.

The Euler conditions for maximum are then obtained by setting the partial derivatives of the sum of typical adjacent terms of the series<sup>25)</sup>,

$$\begin{aligned}
 (3.19) \quad z_2 = & \lambda_2^{-t} U \left[ \frac{y_2(t-1)}{\bar{P}(1+g)} - \frac{\bar{P}_k K_2(t) + M(t)}{\bar{P} \bar{L}_2(t)} + \frac{A_2(t-1)}{\bar{P} \bar{L}_2(t)} \right] \\
 & + \lambda_2^{-(t+1)} U \left[ \frac{F(\bar{T}_2, K_2(t), nL_2(t))}{\bar{L}_2(t)(1+g)} - \frac{w(t)nL_2(t)}{\bar{P} \bar{L}_2(t)(1+g)} \right. \\
 & + \frac{i(t) M(t)}{\bar{P} \bar{L}_2(t)(1+g)} - \frac{A_2(t+1)}{\bar{P} \bar{L}_2(t)(1+g)} \\
 & \left. + \frac{\bar{P}_k K_2(t) + M(t)}{\bar{P} \bar{L}_2(t)(1+g)} \right]
 \end{aligned}$$

with respect to  $L_2(t)$ ,  $K_2(t)$  and  $M(t)$  respectively equal to zero:

$$(3.20) \quad \bar{P} F_{L_2}(\bar{T}_2, K_2(t), nL_2(t)) = w(t)$$

$$(3.21) \quad \lambda_2^{-t} U'(c_2(t)) \left[ -\frac{\bar{P}_k}{\bar{P} \bar{L}_2(t)} \right] + \lambda_2^{-(t+1)} U'(c_2(t+1))$$

$$\left[ \frac{F_{k2}(\bar{T}_2, K_2(t), nL_2(t))}{\bar{L}_2(t)(1+g)} + \frac{\bar{P}_k}{\bar{P} \bar{L}_2(t)(1+g)} \right] = 0$$

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25) The second-order conditions are again taken care of by the concavity of U.

$$(3.22) \quad \lambda_2^{-t} U'(c_2(t)) \left[ -\frac{1}{\bar{P} \bar{L}_2(t)} \right] + \lambda_2^{-(t+1)} U'(c_2(t+1))$$

$$\left[ \frac{i(t) \left(1 - \frac{1}{e(t)}\right) + 1}{\bar{P} \bar{L}_2(t) (1+g)} \right] = 0$$

where  $F_{L_2} = \frac{\partial F(\ )}{\partial nL_2}$  ,  $F_{k_2} = \frac{\partial F(\ )}{\partial K_2}$  and  $e = -\frac{\partial M}{\partial i} \frac{i}{M}$

is the elasticity of the aggregate demand for loan with respect to the rate of interest, the aggregate demand being obtained by adding up the demand for consumption and production loan over all the family farms in the representative set.

Clearly, (3.20) is the optimal rule for choosing  $L_2(t)$  , while (3.21) and (3.22) can be combined to yield:

$$(3.23) \quad \frac{\bar{P}}{\bar{P}_k} F_{k_2}(\bar{T}_2, K_2(t), nL_2(t)) = i(t) \left(1 - \frac{1}{e(t)}\right)$$

or,

$$\theta(t) \frac{\bar{P}}{\bar{P}_k} F_{k_2}(\bar{T}_2, k_2(t), nL_2(t)) = i(t) \quad ,$$

where  $\theta(t) = \frac{1}{1 - \frac{1}{e(t)}}$  , and this gives the capitalist farm's rule of

allocating any given amount of  $A_2(t)$  between  $M(t)$  and  $\bar{P}_k K_2(t)$ .

The optimal rule for choosing the amount of  $A_2(t)$  can then be derived in the following way. Given (3.17),  $\partial z_2/\partial A_2$  can be expressed as a linear combination of  $\partial z_2/\partial k_2$  and  $\partial z_2/\partial M$ :

$$(3.24) \quad \frac{\partial z_2}{\partial A_2} = \frac{\partial z_2}{\partial M} + \frac{1}{\bar{P}_k} \frac{\partial z_2}{\partial k_2}$$

Now, substituting the values of  $\partial z_2/\partial M$  and  $\partial z_2/\partial k_2$  from (3.21) and (3.22),  $\partial z_2/\partial A_2$  can be set equal to zero to obtain:

$$(3.25) \quad \lambda_2 U' \left[ \frac{y_2(t-1)}{\bar{P}(1+g)} - \frac{A_2(t)}{\bar{P} \bar{L}_2(t)} + \frac{A_2(t-1)}{\bar{P} \bar{L}_2(t)} \right] \\ - \left[ \frac{\frac{i(t)}{\theta(t)} + 1}{1+g} \right] U' \left[ \frac{F(\bar{T}_2, K_2(t), nL_2(t))}{\bar{L}_2(t)(1+g)} - \frac{w(t)nL_2(t)}{\bar{P} \bar{L}_2(t)(1+g)} \right. \\ \left. + \frac{i(t)M(t)}{\bar{P} \bar{L}_2(t)(1+g)} - \frac{A_2(t+1)}{\bar{P} \bar{L}_2(t)(1+g)} + \frac{A_2(t)}{\bar{P} \bar{L}_2(t)(1+g)} \right] = 0$$

which is a second-order difference equation embedded in the optimal time profile of  $A_2$ <sup>26)</sup>. It is known that this profile is uniquely fixed by the initial and the terminal condition relating to  $A_2$ , and we shall specify these by two constants, to be denoted by  $D_1$  and

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26) The Equation (3.25) can be regarded as the analogue of the Ramsey rule for the problem of the capitalist farm.

$D_2$  . With these specifications, for any period  $t$  , the values of  $A_2(t-1)$  and  $A_2(t+1)$  can be taken as given, subsumed in these specifications, and then (3.25) can be used to characterize the capitalist farm's holding of  $A_2$  in the period  $t$  as:

$$(3.26) \quad A_2(t) = f\left(\frac{i(t)}{\theta(t)}, y_2(t-1), \bar{L}_2(t); \lambda_2, g, D_1, D_2\right)$$

where the other variables in (3.25) are eliminated by virtue of (3.20) and (3.23), and  $\lambda_2, g, D_1$  and  $D_2$  are the given constants. Note that because of the monopolistic position of the capitalist farm in the credit market, its decision to hold  $A_2(t)$  depends, among others, on the marginal rate of return,  $\frac{i(t)}{\theta(t)}$  , rather than on the average rate of return,  $i(t)$  . Clearly, under a competitive situation,  $\theta(t) = 1$  and these two rates of return would be the same.

By using the implicit function rule to (3.25), one can verify what one intuitively expects about the signs of the partial derivatives of  $f$  , i.e.,

$$f_1 = \frac{\partial A_2(t)}{\partial \frac{i(t)}{\theta(t)}} > 0, \quad f_2 = \frac{\partial A_2(t)}{\partial y_2(t-1)} > 0,$$

and

$$f_3 = \frac{\partial A_2(t)}{\partial \bar{L}_2(t)} > 0.$$

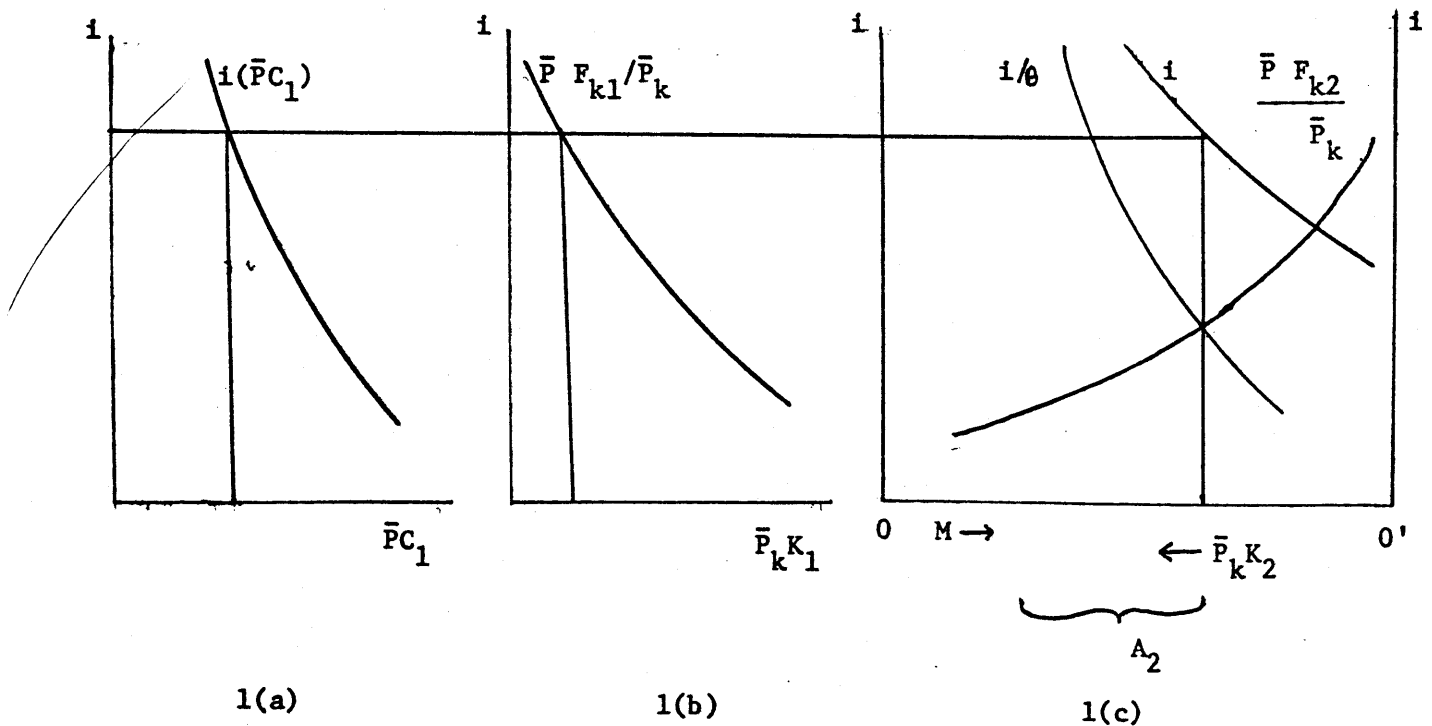
In the same way it can also be found that the elasticity of  $A_2(t)$  with respect to  $i(t)/\theta(t)$  ,  $e_{f, i/\theta}$  is inversely related with

$\lambda_2$  and those with respect to  $y_2(t-1)$  and  $\bar{L}_2(t)$ ,  $e_{f,y_2}$  and  $e_{f,\bar{L}_2}$ , are directly related to  $\lambda_2$ . Of particular importance for our later analysis is the comparison between the difference of the income and the rate of return elasticities of  $A_2$  for the capitalist farm,  $(e_{f,y_2} - e_{f,i/\theta})$ , and the difference of the corresponding elasticities of  $\bar{PC}_1$  for the family farm,  $(e_{\psi,y_1} - e_{\psi,i})$ . Since the average level of income of the capitalist farm is significantly higher than that of the family farm, and accordingly the consumption of the capitalist farm is less determined by the consideration of necessity, it is reasonable to expect that the difference between  $e_{f,y_2}$  and  $e_{f,i/\theta}$  for the capitalist farm will be significantly smaller than the corresponding difference between  $e_{\psi,y_1}$  and  $e_{\psi,i}$  for the family farm.

A good way of presenting this phenomenon in terms of our analytical framework is again through an appropriate stipulation of  $\lambda_2$  in relation to  $\lambda_1$ . Since  $y_2(t-1)$  is significantly higher than  $y_1(t-1)$  and the standard of living of the capitalist farm is way above the state of existence of the family farm, the preference pattern of the capitalist farm will be significantly less biased for the needs of immediate consumption than what it was for the family farm. In other words, one can stipulate  $\lambda_1 > \lambda_2$ , and, given the relationship of  $e_{\psi,y_1}$  and  $e_{\psi,i}$  with  $\lambda_1$  and that of  $e_{f,y_2}$  and  $e_{f,i/\theta}$  with  $\lambda_2$  and the structural similarity between  $\psi$  and  $f$ , this difference between  $\lambda_1$  and  $\lambda_2$  can be taken to imply a corresponding difference between  $(e_{\psi,y_1} - e_{\psi,i})$  and  $(e_{f,y_2} - e_{f,i/\theta})$ , i.e.,

$$(3.27) \quad (e_{\psi, y_1} - e_{\psi, i}) > (e_{f, y_2} - e_{f, i/\theta})$$

We find therefore that the capitalist farm's decision to hold its total wealth,  $A_2(t)$ , is given by (3.26), and its decision to allocate that wealth between  $M(t)$  and  $\bar{P}_k K_2(t)$ , which is taken simultaneously with the decision to hold  $A_2(t)$ , is given, as already mentioned, by (3.23). This allocation of  $A_2(t)$  between  $M(t)$  and  $\bar{P}_k K_2(t)$  along with the consequent determination of the rate of interest and, given that rate of interest, the final allocation of this loan,  $M(t)$ , by each family farm between the uses of consumption and production - all are shown in Figures 1(a), 1(b) and 1(c).



In going through these figures, it should be kept in mind that this is a depiction of the working of only the asset-cum-credit market of a dualistic agriculture. This is not a full general equilibrium picture, because, to keep the diagrams simple, we have not shown the interactions with the labor market explicitly. From (3.11)" it is clear that, given other arguments, the demand of a typical farm for the consumption loan can be related with the rate of interest as shown in terms of the  $i(\bar{P} C_1^L)$  curve in Figure 1(a). Similarly, the demand for production loan can be obtained from (3.9) and, given other variables, its relationship with the rate of interest can be depicted as shown by the  $\bar{P} F_{k2}/\bar{P}_k$  curve in Figure 1(b). Horizontally adding up the curves  $i(\bar{P} C_1^L)$  and  $\bar{P} F_{k2}/\bar{P}_k$  and multiplying the sum by  $n$ , the market demand curve for loan,  $M$ , is obtained, and it is shown, as mapped against the rate of interest, by the curve  $i$  in Figure 1(c) where  $M$  is measured along  $00'$  with  $0$  as the origin and the rate of interest is measured along the vertical axis. Note that given the monopolistic position of the capitalist farm in the credit market, the aggregate demand curve for loan facing the capitalist farm has to be necessarily downward sloping. The curve  $i/\theta$  is then obtained from this aggregate demand curve by using the average-marginal relationship.

The demand of the capitalist farm for  $\bar{P}_k K_2$  can be derived from (3.23) and its relationship to the rate of interest is shown in terms of the curve  $\bar{P}/\bar{P}_k F_{k2}$  in Figure 1(c), where  $\bar{P}_k K_2$  is measured along  $0'0$  with  $0'$  as the origin and  $i$  is measured along the corresponding



vertical axis. The length of  $OO'$  is equal to the total amount of wealth,  $A_2(t)$ , that the capitalist farm has chosen to hold in this period. From the intersection of the curves  $i/\theta$  and  $\bar{P}_k F_{k2}/\bar{P}_k$  in Figure 1(c), the equilibrium rate of interest is determined along with the allocation of  $A_2$  between  $M$  and  $\bar{P}_k F_{k2}/\bar{P}_k$  by the capitalist farm. Given this rate of interest, each family farm decides on the amounts of consumption loan and production loan it will take, as shown in Figures 1(a) and 1(b).

It is clear from (3.23) that for an interior solution to this problem of allocation between  $M$  and  $\bar{P}_k F_{k2}/\bar{P}_k$  it is necessary to have  $e > 1$ . If  $e \leq 1$ , then the solution, as known from the standard theory of monopoly, tends to be in the neighborhood of a corner with the capitalist farm trying to charge an infinitely high rate of interest for an infinitesimally small amount of loan. We are therefore led to distinguish between two possible situations:

- (1) The level of the average net income of the family farm is low and it is taking loans for both consumption and production, but the income is still above that level at which the family farm has to take consumption loan to meet the biologically minimum subsistence needs. In other words, the consumption needs can still be made flexible in the event of a sufficiently high rate of interest, implying thereby that  $e \neq 1$  for the entire range of the aggregate demand curve for loan.
- (2) The other possibility is that the level of the average net income of the family farm is in fact so low that consumption loan is

taken by the family farm for subsistence needs. Then the value of  $e$  may very well be below 1 over the entire range of the aggregate demand curve for loan<sup>26)</sup> with the result that the capitalist farm can really charge a high enough rate of interest until the family farm becomes totally impoverished and is forced to sell his land and join the ranks of landless labor at a subsistence wage<sup>26a)</sup>. In this case, the solution is self evident and we have nothing more to say about it by way of analysis, apart from mentioning that this situation actually represents the terminal state of a process relating to the behavior of capital accumulation and the impoverishment of the family farm in a dualistic agriculture, and when the system comes to this state then the complete impoverishment of the family farm becomes imminent.

We shall come back to this situation (2) later on. But it needs to be pointed out here, as was also mentioned once in Section 2, that the purpose of this essay is not simply to describe this terminal state, although it may very well be the case with some of the present-day dualistic agriculture, but also to try to explain and understand the historical process by which a dualistic agriculture is actually brought to this terminal state, the tendencies which are inherent in this system and make it move in a particular direction. To be able to do that, it

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26) This special situation is likely to arise particularly in the event of some unpredictable needs in consumption or production, and then the family farm can indeed find itself placed in a vulnerable position.

26a) The process can not go beyond this point, because it is to the obvious interest of the capitalist farm to keep the family farmer alive in order to get the supply of labor.

is important to start from a situation which is somewhat away from the terminal state, and therefore we choose the situation (1) as the description of the initial state and develop an analysis of the entire process of evolution from that point onward. It will be seen in the course of this analysis that the situation (2) will in fact come to appear as a part of that evolutionary process.

As an offshoot of this discussion, one can consider the value of  $e(t)$  in any period  $t$  as directly related to the level of average net income available to the family farm in that period, i.e.,  $y_1(t-1)$  and, since  $\theta(t) = \frac{1}{1 - \frac{1}{e(t)}}$ , one can also write,

$$(3.28) \quad \theta(t) = \theta(y_1(t-1)) \quad , \quad \text{with} \quad \frac{d\theta}{d y_1(t-1)} = \theta' < 0 \quad .$$

\* \* \*

To sum up, given the objective of maximizing the sum of discounted utility relating to per head consumption of the family members over a stipulated time horizon, the optimum decision rules for the family farms with respect to the relevant variables are given by (3.8)', (3.9), (3.11)", and those of the capitalist farm by (3.20), (3.23) and (3.26). These rules, taken together with the definitional equations, define the basic equational structure of our model of the dualistic agriculture for any particular period. For convenience of later reference, let us collect the equations in one place:

The Family Farm:

$$(3.2)' \quad y_1(t) = \frac{\bar{P} F(\bar{T}_1, K_1(t), L_1(t)) + (w(t) - \mu) L_2(t)}{\bar{L}_1(t)} - \frac{i(t) \bar{P}_k K_1(t) + (1 + i(t)) \bar{P} C_1^l(t)}{\bar{L}_1(t)}$$

$$(3.3) \quad L_1(t) + L_2(t) = \bar{L}_1(t)$$

$$(3.8)' \quad \bar{P} F_{L1}(\bar{T}_1, K_1(t), L_1(t)) = w(t) - \mu$$

$$(3.9) \quad \frac{\bar{P}}{\bar{P}_k} F_{K1}(\bar{T}_1, K_1(t), L_1(t)) = i(t)$$

$$(3.11)'' \quad \bar{P} C_1^l(t) = \psi(i(t), w(t), y_1(t-1), \bar{L}_1(t); \mu, \lambda_1, g, B_1, B_2)$$

The Capitalist Farm:

$$(3.14) \quad y_2(t) = \frac{\bar{P} F(\bar{T}_2, K_2(t), nL_2(t) - w(t) nL_2(t))}{\bar{L}_2(t)} + \frac{i(t) M(t)}{\bar{L}_2(t)}$$

$$(3.13) \quad M(t) = n \bar{P}_k K_1(t) + n \bar{P} C_1^l(t)$$

$$(3.20) \quad \bar{P} F_{L2}(\bar{T}_2, K_2(t), nL_2(t)) = w(t)$$

$$(3.23) \quad \theta(y_1(t-1)) \frac{\bar{P}}{\bar{P}_k} F_{k2}(\bar{T}_2, K_2(t), nL_2(t)) = i(t)$$

$$(3.26) \quad A_2(t) = \bar{P}_k K_2(t) + M(t) \equiv f\left(\frac{i(t)}{\theta(t)}, y_2(t-1), \bar{L}_2(t); \lambda_2, g, D_1, D_2\right)$$

Clearly, given  $B_1, B_2, D_1, D_2, g, \mu, \lambda_1$  and  $\lambda_2$  as constants and  $y_1(t-1), y_2(t-1), \bar{L}_1(t)$  and  $\bar{L}_2(t)$  as parameters, we have, in any period  $t$ , as unknowns:  $y_1(t), K_1(t), L_1(t), w(t), L_2(t), i(t), C_1^L(t), K_2(t), M(t)$  and  $A_2(t)$ , and the number of unknowns equals the number of equations. Another way of looking at this structure of equations is that, given the initial and the terminal condition as captured by the constants  $B_1, B_2, D_1$  and  $D_2$ , and given other structural constants,  $g, \mu, \lambda_1$  and  $\lambda_2$ , the optimal time profiles of  $\bar{P} C_1^L(t)$  and  $A_2(t)$  and, associated with them, the profiles of all other variables are uniquely defined (except of the singular cases). The set of equations mentioned above is nothing but the characterization of these profiles in a particular period of time. And, in this characterization, the parameters clearly are  $y_1(t-1), y_2(t-1), \bar{L}_1(t)$  and  $\bar{L}_2(t)$ ; they change over time driving the system to the next period. To know the intertemporal behavior of the system, which is the next step of our analysis, it is therefore essential to know the direction of changes in these four parameters.

It needs to be mentioned here that in finding out the qualitative nature of these parametric changes as well as in deriving many other

subsequent results, for the manoeuvrability of a differential operator, we shall work in terms of time derivatives rather than in terms of time differences. However, the underlying period analytic structure of our model, which was described in Section 2 and formalized in this section, will always be implied and, once the derivations are over, we shall interpret the results by coming back to this framework of period analysis.

With this in mind, our problem now is to find out the signs of the time derivatives of  $\bar{L}_1$ ,  $\bar{L}_2$ ,  $y_1$  and  $y_2$ . Of these four parameters, the signs of  $\bar{L}_1$  and  $\bar{L}_2$  are already known to be positive by (3.4) and (3.15). The signs of the remaining two,  $dy_1/dt$  and  $dy_2/dt$ , will be given by the following propositions.

Proposition 1: Given the objective (3.18) if the rate of capital accumulation in the capitalist farm does not exceed the golden rule value, then  $dy_2/dt \geq 0$ , except for the case when the system is self-destructive.

Proof: Given the objective (3.18), it is clear from (3.21) that if

$$(3.29) \quad \frac{\bar{P}}{\bar{P}_k} F_{k2}(\bar{T}_2, K_2(t), nL_2(t)) \geq \lambda_2(1+g)$$

i.e., if the rate of capital accumulation in the capitalist farm of the underdeveloped dualistic agriculture does not exceed the golden rule - catenary turnpike level (an assumption which can be made without straining any credibility, at least in the beginning of the process),

then it follows from the concavity of  $U$  that

$$(3.30) \quad \Delta c_1(t) > 0$$

The same result can be stated in continuous time with the objective (3.18) expressed as

$$(3.18)' \quad \text{Max} \int_0^{T_2} e^{-\rho_2 t} U(c_2(t)) dt$$

where  $\rho_2$  is the rate of time preference of the capitalist farm and  $c_2$  is to be written as

$$(3.16)' \quad c_2 = \frac{y_2}{\bar{P}} - \frac{\bar{P}_k \frac{dK_2}{dt} + \frac{dM}{dt}}{\bar{P} \bar{L}_2}$$

The continuous analogue of (3.21) is:

$$(3.21)' \quad e^{-\rho_2 t} U' \left[ \frac{F_{k2}}{\bar{L}_2} \right] = \frac{d}{dt} \left[ e^{-\rho_2 t} U' \frac{\bar{P}_k}{\bar{P} \bar{L}_2} \right]$$

or,

$$-\frac{dU'/U'}{dt} = \frac{\bar{P} F_{k2}}{\bar{P}_k} - (\rho_2 + g)$$

which shows that if  $\bar{P} F_{k2} / \bar{P}_k \geq \rho_2 + g$ , the continuous counterpart of (3.29), then

$$(3.30)' \quad \frac{d c_2}{dt} \geq 0, \quad \text{by the concavity of } U.$$

Next, treating  $A_2 (= \bar{P}_k K_2 + M)$  as one variable, we derive the corresponding Euler equation and then, multiplying both sides of the equation by  $dA_2/dt$ , express it in the following alternative form<sup>27)</sup>:

$$(3.31) \quad \frac{d}{dt} [e^{-\rho_2 t} U + \frac{dA_2}{dt} e^{-\rho_2 t} U] = -\rho_2 e^{-\rho_2 t} U$$

or,

$$(3.32) \quad U' \frac{dc_2}{dt} + \frac{d^2 A_2}{dt^2} \frac{U'}{\bar{P} \bar{L}_2} - \frac{dA_2}{dt} \frac{U' g}{\bar{P} \bar{L}_2} \\ + \frac{dA_2}{dt} \frac{1}{\bar{P} \bar{L}_2} [U'' \frac{dc_2}{dt} - U' \rho_2] = 0$$

We can now distinguish between the two cases depending on whether (1)  $dA_2/dt \geq 0$  or (2)  $dA_2/dt < 0$ .

Case 1 ( $dA_2/dt \geq 0$ ): In this case, given (3.30)' and the concavity of  $U$ , it follows from (3.32) that

$$(3.33) \quad \frac{dc_2}{dt} + \frac{d^2 A_2}{dt^2} \frac{1}{\bar{P} \bar{L}_2} - \frac{dA_2}{dt} \frac{g}{\bar{P} \bar{L}_2} \geq 0$$

which, by (3.16)', is equivalent to

$$(3.34) \quad \frac{dy_2}{dt} \geq 0 .$$

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27) See I.M. Gelfand and S.V. Forrin, Calculus of Variations, Prentice-Hall (1963), pp 18-19.



Case 2 ( $dA_2/dt < 0$ ): Here, one can again have two possibilities:

(i) the absolute value of  $[U'' \frac{dc_2}{dt} - U'\rho_2]$  in (3.32) is not high enough so that (3.33) continues to hold and we have the same result as (3.34), or (ii) the absolute value of  $[U'' \frac{dc_2}{dt} - U'\rho_2]$  is high enough to make

$$(3.33)' \quad \frac{dc_2}{dt} + \frac{d^2A_2}{dt^2} \frac{1}{\bar{P} \bar{L}_2} - \frac{dA_2}{dt} \frac{g}{\bar{P} \bar{L}_2} < 0 \quad , \quad \text{or}$$

$$(3.34)' \quad \frac{dy_2}{dt} < 0 \quad .$$

But, given  $dc_2/dt > 0$ , this also means that  $d^2A_2/dt^2 < 0$  along with  $dA_2/dt < 0$ , implying that the system is self-destructive over a finite time horizon.

Comment: Given the objective of maximizing  $\int_0^{T_2} e^{-\rho_2 t} U(c_2(t)) dt$ , the last situation, the possibility (ii) under the case 2, is naturally ruled out if either the stipulated time horizon is considered to be sufficiently long or the terminal rate of growth of wealth is required not to fall below a certain positive number. Thus one can conclude that under quite general conditions, (3.18) can be taken to imply  $dy_2/dt \geq 0$ .

Proposition 2: Given the objective (3.6) and that the rate of capital accumulation in the family farm does not exceed the golden rule value,  $dy_1/dt > 0$  if  $d \bar{P} C_1^l/dt \leq 0$ , and  $dy_2/dt \leq 0$  implies  $d \bar{P} C_1^l/dt > 0$ .

Proof: Given the objective (3.6), it is clear from (3.10) that if

$$(3.35) \quad 1 + i(t) \geq \lambda_1(1+g)$$

i.e., noting from (3.9) that  $i(t) = \bar{P} F_{k1}(\cdot) / \bar{P}_k$ , if the rate of growth of capital accumulation in the family farm does not exceed the golden rule level, then

$$(3.36) \quad \Delta c_1(t) \geq 0 .$$

Formulating the problem in continuous time, it can again be shown, exactly in the same way as in the proof of Proposition 1, that

$$(3.36)' \quad \frac{dc_1}{dt} \geq 0 \quad \text{if} \quad \frac{\bar{P} F_{k1}}{\bar{P}_k} > \rho_1 + g ,$$

where  $\rho_1$  is the rate of time preference of the family farm and  $c_1$  is to be written as:

$$(3.5)' \quad c_1 = \frac{y_1}{\bar{P}} + \frac{C_1^L}{\bar{L}_1}$$

From this expression of  $c_1$  it is immediate that

$$(3.37) \quad \frac{1}{\bar{P}} \frac{dy_1}{dt} = \frac{dc_1}{dt} - \frac{dC_1^L}{dt} \frac{1}{\bar{L}_1} + g \frac{C_1^L}{\bar{L}_1} ,$$

so that by using (3.36)' it follows that if  $d \bar{P} C_1^l / dt \leq 0$  then  $dy_1 / dt > 0$ , and  $dy_1 / dt \leq 0$  implies  $d \bar{P} C_1^l / dt > 0$ .

**Proposition 3:** If the weighted average of the rates of capital accumulation in the family and the capitalist farms, weights being the rentals on capital used in the respective farms, is not high enough to exceed the rate of growth of labor force by an amount, defined by the rate of growth of labor force, the amount and the rate of change of the consumption loan and the shares of land and capital, then  $dy_2 / dt \geq 0$  implies  $dy_1 / dt < 0$ , and  $dy_1 / dt \geq 0$  implies  $dy_2 / dt < 0$ .

**Proof:** From (3.2) and (3.14) the expressions of  $dy_1 / dt$  and  $dy_2 / dt$  can be derived as:

$$(3.38) \quad \frac{dy_1}{dt} = \frac{1}{\bar{L}_1} [\bar{P} F_{k1} \frac{dK_1}{dt} + (w-\mu)g \bar{L}_1 + \frac{dw}{dt} L_2 - \frac{d}{dt} \{i \bar{P}_k K_1 + (1+i) \bar{P} C_1^l\} - g Y_1]$$

and,

$$(3.39) \quad \frac{dy_2}{dt} = \frac{1}{\bar{L}_2} [\bar{P} F_{k2} \frac{dK_2}{dt} - \frac{dw}{dt} nL_2 + \frac{d}{dt} (i M) - g Y_2]$$

Now, quite generally, it is true that

$$(3.40) \quad g Y_2 - \bar{P} F_{k2} \frac{dK_2}{dt} > n[\bar{P} F_{k1} \frac{dK_1}{dt} + (w-\mu) g \bar{L}_1 - \frac{d\bar{P}C_1^l}{dt} - g Y_1]$$

if,

$$g[(n \bar{P} F_{k1} K_1 + \bar{P} F_{k2} K_2) + (n \bar{P} F_{T1} \bar{T}_1 + \bar{P} F_{T2} \bar{T}_2)] \\ + n \frac{d\bar{P} C_1^\ell}{dt} > n \bar{P} F_{k1} \frac{dK_1}{dt} + \bar{P} F_{k2} \frac{dK_2}{dt} ,$$

(where  $F_{Ti} = \partial F(\ ) / \partial T_i \Big|_{T_i = \bar{T}_i}$ ,  $i = 1, 2$ , and use has been made of  $\mu > 0$  and the homogeneity property of  $F$ )

i.e., if,

$$(3.41) \quad g + \left[ g \frac{n \bar{P} F_{T1} \bar{T}_1 + \bar{P} F_{T2} \bar{T}_2}{n \bar{P} F_{k1} K_1 + \bar{P} F_{k2} K_2} + \frac{\frac{d\bar{P} C_1}{dt} n \bar{P} C_1^\ell}{\bar{P} C_1^\ell n \bar{P} F_{k1} K_1 + \bar{P} F_{k2} K_2} \right] \\ > \frac{n \bar{P} F_{k1} K_1 G_{k1} + \bar{P} F_{k2} K_2 G_{k2}}{n \bar{P} F_{k1} K_1 + \bar{P} F_{k2} K_2} ,$$

where  $G_{ki} = dK_i/dt/K_i$ ,  $i = 1, 2$ . Clearly, (3.41) is the statement of the condition that the weighted average of the rates of capital accumulation in the family and the capitalist farms, weights being the rentals on capital used in the respective farms, does not exceed the rate of growth of labor force by an amount defined by the rate of growth of labor force, the rate of change in the consumption loan and the ratio between the shares of land and capital, and that between the value of consumption loan and share of capital.

From (3.39) it is evident that  $dy_2/dt \geq 0$  implies

$$(3.42) \quad \frac{d}{dt} (i M) \geq \frac{dw}{dt} nL_2 - \bar{P} F_{k2} \frac{dK_2}{dt} + g Y_2$$

Now, if (3.41) holds, then by using (3.40) and (3.42), it further follows that

$$(3.43) \quad \frac{d}{dt} (i M) > n[\bar{P} F_{k1} \frac{dK_1}{dt} + (w-\mu) g\bar{L}_1 + \frac{dw}{dt} L_2 - g Y_1]$$

or, by transferring  $d/dt (iM)$  on the R.H.S. and then dividing both sides by  $n$ , we have

$$(3.44) \quad \frac{dy_1}{dt} < 0 .$$

In a symmetric manner it can be proved that, given (3.41),  $dy_1/dt \geq 0$  implies  $dy_2/dt < 0$ .

Comment: We shall henceforth assume that the agricultural sector of a less developed economy, such as we are interested in, cannot, to begin with, accumulate capital at a rate so much faster than the rate of growth of labor force that (3.41) gets violated. Since in such an agriculture the share of land is generally more dominant than the share of capital on the amount of consumption loan, and the value of  $g$  is significantly high, this is indeed a reasonable assumption to make about the initial state of this agriculture.

The import of Proposition 3 is that in an underdeveloped dualistic agriculture when the rate of capital accumulation is not taking place at a sufficiently fast rate, it is not possible for the capitalist farm to have  $dy_2/dt \geq 0$  and the family farm to have  $dy_1/dt \geq 0$  at the same time. Only one of the two groups can make it. And, in a situation where, given the distribution of income, the capitalist farm enjoys a monopolistic position in the credit market, it has a prior advantage of choosing its plan of saving and accumulation of  $A_2(t)$  in an optimal fashion (i.e., satisfying (3.26)), so that  $i$  is made to change over time in a way that  $d/dt (iM)$ , the increase in earnings from loan, and  $\bar{P} F_{k2} dK_2/dt$ , the increase in earnings from capital accumulation in its own farm, taken together becomes larger than  $dw/dt nL_2 + gY_2$  implying by (3.39) that  $dy_2/dt \geq 0$ . The family farm, so long as it operates atomistically in the labor and in the credit market and therefore takes  $w$  and  $i$  as parameters, has no such prior advantage. It acts as a follower after the decision has been taken by the capitalist farm with respect to the savings plan. And, when such a decision is taken by the capitalist farm so that  $dy_2/dt \geq 0$ , and the situation is not one of a sufficiently fast rate of accumulation, then, as shown in Proposition 3, the family farm ends up with  $dy_1/dt < 0$ . We are thus led to this following proposition:

Proposition 4: Given the bias in the distribution of income in favor of the capitalist farm and its consequent monopolistic position in the credit market, it has the advantage over the family farm in ensuring  $dy_2/dt \geq 0$ , and as the rate of capital accumulation to begin with satisfies (3.41), this implies, by Proposition 3, that  $dy_1/dt < 0$ .

With the signs of  $\frac{dy_1}{dt}$  and  $\frac{dy_2}{dt}$  thus known,<sup>28</sup> and the signs of  $\frac{d\bar{L}_1}{dt}$  and  $\frac{d\bar{L}_2}{dt}$  already given, we shall now proceed to derive, by totally differentiating the system of equations with respect to time, the qualitative properties of the time derivatives of all the relevant variables, and then, by analyzing these derivatives, conclude about the intertemporal behavior of the system.

At this point, a comment on methodology seems pertinent. In following this method of solution, what we are doing, in effect, is that we are observing the equations of motion of the system over a short range and, from such an observation of the local qualitative character of the phase space, inferring about the global tendencies of the system. We are deliberately choosing this method as against the usual method of global

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<sup>28</sup> As pointed out at the end of Section 2, most of the decision rules of the family and the capitalist farms as well as the signs of the time derivations of the parameters could have been derived from a simpler specification of the objective, where both the farms are trying to maximize their respective net income (i.e., profit) in any period with an additional intertemporal requirement that this net income of any period should not fall below that of the last period. In the case of the family farm, for example, the maximization of  $y_1$  subject to (3.5) with a given value of  $c_1$  yields the decision rules with respect to the use of  $L_1$  and  $K_1$  which are the same as (3.8) and (3.9). Similarly, for the capitalist farm, the maximization of  $y_2$  subject to (3.7) with a given value of  $A_2$  gives the decision rules with respect to the use of  $L_2$  and the allocation of  $A_2$  between  $M$  and  $P_k K_2$  which are again the same as (3.20) and (3.23). The only problem about this kind of specification of the objective function, however, is connected with the derivation of the demand function for  $\bar{P}C_1^l$  and  $A_2$ . The question of the demand function for  $\bar{P}C_1^l$  can still be settled at least in our case, by specifying the level of per-head consumption,  $c_1$ , to some predetermined minimum level although that is not always the best way of explaining the consumption decision. But the problem is more serious with respect to the determination of the capitalist farm's decision on  $A_2$ . With an open-ended specification of the inter-temporal objective, such as  $\frac{dy_2}{dt} \geq 0$ , the decision on  $A_2$  also gets characterized by inequality and thus remains somewhat ill defined. And, to dodge the issue by saying that  $A_2$  is a certain fixed proportion of, say,  $y_2$  is not really explaining an important dimension of the choice problem of the capitalist farm with respect to  $A_2$ . This choice problem can be analyzed only in terms of the type specification of the objective function such as we have been working with.

phase construction, because if the system is known to have sufficient monotonicity, this is indeed a valid and at the same time a simpler substitute for the global technique. And, in the case of our model, we shall see that, because of the nature of the time functions of  $y_1$ ,  $y_2$ ,  $\bar{L}_1$ , and  $\bar{L}_2$ , there exists enough monotonicity in the system to warrant such a global qualitative inference from a local analysis.

#### 4. Behavior of the System Over Time

From the standpoint of intertemporal analysis, the equations of the model, mentioned in the last section, can be further condensed as

$$(4.1) \quad F_{L1}(\bar{T}_1, K_1(t), L_1(t)) + \frac{\mu}{P} = F_{L2}(\bar{T}_2, K_2(t), nL_2(t)) = \frac{W}{P}$$

$$(3.3) \quad L_1(t) + L_2(t) = \bar{L}, (t)$$

$$(3.11) \quad \bar{P}C_1^l(t) = \psi(i(t), w(t), y_1(t-1), \bar{L}_1(t); \mu, \lambda_1, g, G_1, B_2)$$

$$\psi_1 < 0, \psi_2 > 0, \psi_3 < 0, \psi_4 > 0.$$

$$(4.2) \quad F_{K1}(\bar{T}_1, K_1(t), L_1(t)) = \theta(t) F_{K2}(\bar{T}_2, K_2(t), nL_2(t)) = i \frac{\bar{P}_k}{P}$$

$$(3.36) \quad A_2(t) = f\left(\frac{i(t)}{\theta(t)}, y_2(t-1), \bar{L}_2(t); \lambda_2, g, D_1, D_2\right) =$$

$$n\bar{P}C_1^l(t) + n\bar{P}_k K_1(t) + \bar{P}_k K_2(t),$$

$$f_1 > 0, f_2 > 0, f_3 > 0.$$

In calculating the changes of the variables over time, for reasons of convenience of working with time derivatives as mentioned before, we shall continue to work in the framework of continuous time,



although the underlying period analytic structure should again be kept in mind, and we shall refer back to it for the purposes of interpreting the results. For simplifying calculations, in the beginning we shall also hold  $\theta$  constant and relax it later on. Now, totally differentiating these five equations with respect to time, and then eliminating  $\frac{di}{dt}$  and  $\frac{dw}{dt}$ , we get

$$(4.3) \quad \frac{\partial F_{L1}}{\partial K_1} \frac{dK_1}{dt} - \frac{\partial F_{L2}}{\partial K_2} \frac{dK_2}{dt} + \left( \frac{\partial F_{L1}}{\partial L_1} + n \frac{\partial F_{L2}}{\partial nL_2} \right) \frac{dL_1}{dt} = n \frac{\partial F_{L2}}{\partial nL_2} \frac{d\bar{L}_1}{dt}$$

$$(4.4) \quad \frac{\partial F_{k1}}{\partial K_1} \frac{dK_1}{dt} - \theta \frac{\partial F_{k2}}{\partial K_2} \frac{dK_2}{dt} + \left( \frac{\partial F_{k1}}{\partial L_1} + n\theta \frac{\partial F_{k2}}{\partial nL_2} \right) \frac{dL_1}{dt} = n\theta \frac{\partial F_{k2}}{\partial nL_2} \frac{d\bar{L}_1}{dt}$$

$$(4.5) \quad \left\{ (n\psi_1 - \frac{f_1}{\theta}) \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial K_1} + n\psi_2 \frac{\partial F_{L1}}{\partial K_1} + n\bar{P}_k \right\} \frac{dk_1}{dt} + \frac{\bar{P}}{\bar{P}_k} \frac{dk_2}{dt} \\ + \left\{ (n\psi_1 - \frac{f_1}{\theta}) \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial L_1} + n\psi_2 \bar{P} \frac{\partial F_{L1}}{\partial L_1} \right\} \frac{dL_1}{dt} = \frac{dx}{dt} ,$$

where

$$(4.6) \quad \frac{dx}{dt} = f_2 \frac{dy_2}{dt} + f_3 \frac{d\bar{L}_2}{dt} - n\psi_3 \frac{dy_1}{dt} - n\psi_4 \frac{d\bar{L}_1}{dt} .$$

The Jacobian:

$$\Delta = \begin{vmatrix} \frac{\partial F_{L1}}{\partial K_1} & - \frac{\partial F_{L2}}{\partial K_2} & \left( \frac{\partial F_{L1}}{\partial L_1} + n \frac{\partial F_{L2}}{\partial nL_2} \right) \\ \frac{\partial F_{k1}}{\partial K_1} & - \theta \frac{\partial F_{k2}}{\partial K_2} & \left( \frac{\partial F_{k1}}{\partial L_1} + n\theta \frac{\partial F_{k2}}{\partial nL_2} \right) \\ \left( n\psi_1 - \frac{f_1}{\theta} \right) \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial K_1} + n\psi_2 \frac{\partial F_{L1}}{\partial K_1} + n\bar{P}_k & \bar{P}_k & \left( n\psi_1 - \frac{f_1}{\theta} \right) \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial L_1} \\ & & + n\psi_2 \bar{P} \frac{\partial F_{L1}}{\partial L_1} \end{vmatrix}$$

$$\begin{aligned}
&= \left\{ \bar{P}_k + \theta(n\psi_1 - \frac{f_1}{\theta}) \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k2}}{\partial K_2} + n\psi_2 \bar{P} \frac{\partial F_{L2}}{\partial K_2} \right\} \left\{ \frac{\partial F_{k1}}{\partial K_1} \frac{\partial F_{L1}}{\partial L_1} - \frac{\partial F_{k1}}{\partial L_1} \frac{\partial F_{L1}}{\partial K_1} \right\} \\
&+ \theta n^2 \left\{ \bar{P}_k + \frac{1}{n}(n\psi_1 - \frac{f_1}{\theta}) + \psi_2 \bar{P} \frac{\partial F_{L1}}{\partial K_1} \right\} \left\{ \frac{\partial F_{k2}}{\partial K_2} \frac{\partial F_{L2}}{\partial nL_2} - \frac{\partial F_{k2}}{\partial nL_2} \frac{\partial F_{L2}}{\partial K_2} \right\} \\
&+ n\bar{P}_k \left\{ \frac{\partial F_{k1}}{\partial K_1} \frac{\partial F_{L2}}{\partial nL_2} - \frac{\partial F_{k1}}{\partial L_1} \frac{\partial F_{L2}}{\partial K_2} \right\} + \theta n\bar{P}_k \left\{ \frac{\partial F_{L1}}{\partial L_1} \frac{\partial F_{k2}}{\partial K_2} - \frac{\partial F_{L1}}{\partial K_1} \frac{\partial F_{k2}}{\partial nL_2} \right\},
\end{aligned}$$

by grouping the terms appropriately. On inspecting this expression, it follows that

$$(4.7) \quad \Delta > 0,$$

given the usual properties of the partial derivations of a neoclassical production function, the signs of the partial derivatives of the functions  $\psi$  and  $f$ , the second-order conditions of maximum of (3.6) and (3.18), and assuming that the forces of diminishing returns are stronger than those of complementarity; i.e.,

$$\left| \frac{\partial F_{k1}}{\partial k_k} \right| > \left| \frac{\partial F_{k1}}{\partial L_1} \right| \quad \text{etc.}$$

Using Cramer's rule, we then have

$$(4.8) \quad \frac{dK_1}{dt} = \frac{1}{\Delta} \begin{vmatrix} n \frac{\partial F_{L2}}{\partial nL_2} \frac{d\bar{L}_1}{dt} & - \frac{\partial F_{L2}}{\partial K_2} & (\frac{\partial F_{L1}}{\partial L_1} + \frac{\partial F_{L2}}{\partial nL_2}) \\ n\theta \frac{\partial F_{k2}}{\partial nL_2} \frac{d\bar{L}_1}{dt} & - \theta \frac{\partial F_{k2}}{\partial K_2} & (\frac{\partial F_{k1}}{\partial L_1} + n \frac{\partial F_{k2}}{\partial nL_2}) \\ \frac{dx}{dt} & \bar{P}_k & (n\psi_1 - \frac{f_1}{\theta}) \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial L_1} + n\psi_2 \bar{P} \frac{\partial F_{L1}}{\partial L_1} \end{vmatrix}$$



Given, again, the second-order conditions of maximum and that the forces of diminishing returns are stronger than those of complementarity, it follows by using (4.1) and (4.2) that

$$n \frac{dK_1}{dt} + \frac{dK_2}{dt} < 0 \text{ if}$$

$$(4.11) \quad \frac{dx}{dt} - n\psi_1 \frac{\partial i}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} + \frac{f_1}{\theta} \frac{\partial i}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} - n\psi_2 \frac{\partial w}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} < 0$$

which obviously also implies

$$(4.12) \quad \frac{dx}{dt} < 0 .$$

By using the definition of  $\frac{dx}{dt}$ , given by (4.6), the condition (4.11)

can be written as

$$(4.13) \quad \frac{f_1}{\theta} \frac{\partial i}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} + f_2 \frac{dy_2}{dt} + f_3 \frac{d\bar{L}_2}{dt} < n[\psi_1 \frac{\partial i}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} + \psi_2 \frac{\partial w}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} + \psi_3 \frac{dy_1}{dt} + \psi_4 \frac{d\bar{L}_1}{dt}]$$

It is clear from (3.11)" and (3.36) that the L.H.S. of (4.13) shows nothing but the total change in  $A_2$  overtime following the changes in the relevant parameters and the R.H.S. the total change in  $\bar{PC}_1^L$  of all the family farms taken together following similar changes in its relevant parameters.

So long, for the sake of simplification,  $\theta$  has been held constant.

It is known, however, from (3.28) that  $\theta = \theta(y_1)$  with  $\frac{d\theta}{dy_1} < 0$  and accordingly

$\theta$  is expected to change over time due to changes in  $y_1$ . It is appropriate

now to incorporate this change into our analysis.

There will be essentially two additional changes in the system following this variation in  $\theta$ .

- (1) There will be a change in  $\frac{1}{\theta}$  and, through that, an additional variation in  $A_2$ , to be captured by  $f_1 \frac{\partial(1/\theta)}{\partial \theta} \frac{\partial \theta}{\partial y_1} \frac{dy_1}{dt}$ . It needs to be

pointed out that this change in  $\frac{1}{\theta}$  is inversely related with  $\theta$ . To see this, consider a situation where  $\theta$  has fallen, and, for convenience of illustration, consider it where  $\theta$  has fallen all the way to the value 1; i.e., the interest elasticity of the aggregate demand for Loan,  $M$ , has increased to infinity implying thereby a change from imperfect to perfect credit market.<sup>29</sup>

Then,

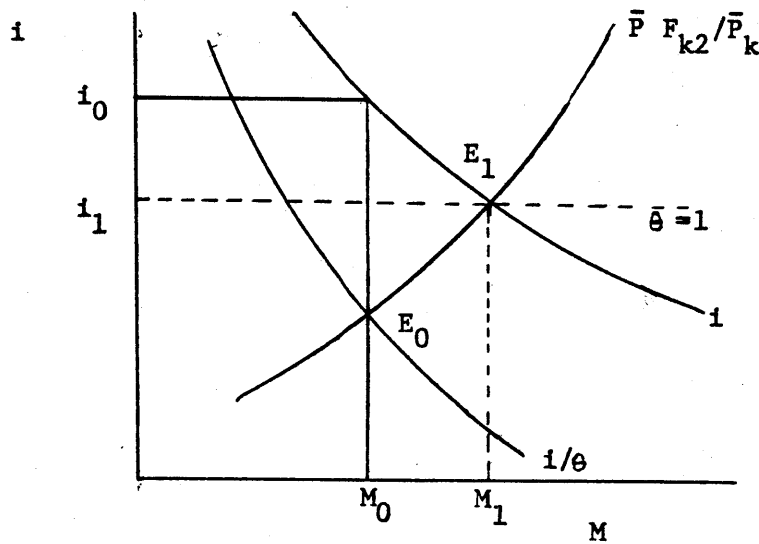


Figure 2

referring to Fig. 2, it is clear that as  $\theta$  falls to the value 1 and the curve  $\frac{1}{\theta}$  becomes horizontal at  $i_1$ , the system moves from its old equilibrium  $E_0$  to the new equilibrium  $E_1$  and the value of  $\frac{1}{\theta}$  increases from  $E_0H_0$  to  $E_1M_1$ . It should be noted here that although the value of  $\frac{1}{\theta}$  increases, and the inverse relationship between  $\theta$  and  $\frac{1}{\theta}$  is thus demonstrated, the value of  $i$  actually falls from  $i_0$  in the old equilibrium to  $i_1$  in the new equilibrium.

<sup>29</sup>This case will come to be very relevant for our discussion in Section 5.

(2) This variation in  $i$ , consequent upon a change in  $\theta$ , will now produce the second change in the system by generating an additional variation in  $\bar{PC}_1^{\ell}$ . This can be expressed as  $\psi_1 \frac{\partial i}{\partial \theta} \frac{\partial \theta}{\partial y_1} \frac{dy_1}{dt}$ , where  $\frac{\partial i}{\partial \theta} > 0$ .

Accommodating these two changes, in  $A_2$  and  $\bar{PC}_1^{\ell}$ , caused by the change in  $\theta$ , the condition (4.13) can be generalized as

$$(4.14) \quad f_1 \frac{\partial(\frac{1}{\theta})}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} + f_1 \frac{\partial(\frac{1}{\theta})}{\partial \theta} \frac{d\theta}{dy_1} \frac{dy_1}{dt} + f_2 \frac{dy_2}{dt} + f_3 \frac{d\bar{L}_2}{dt} < n[\psi_1 \frac{\partial i}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} + \psi_1 \frac{\partial i}{\partial \theta} \frac{\partial \theta}{\partial y_1} \frac{dy_1}{dt} + \psi_2 \frac{\partial w}{\partial \bar{L}_1} \frac{d\bar{L}_1}{dt} + \psi_3 \frac{dy_1}{dt} + \psi_4 \frac{d\bar{L}_1}{dt}$$

With a little effort, this condition can also be expressed in an alternative way, in terms of the elasticities of the relevant variables:

$$(4.15) \quad A_2 [e_{f, \frac{1}{\theta}} e_{\frac{1}{\theta}, \bar{L}_1} g + e_{f, \frac{1}{\theta}} e_{\frac{1}{\theta}, \theta} e_{\theta, y_1} \frac{dy_1}{dt} / y_1 + e_{f, y_2} \frac{dy_2}{dt} / y_2 + e_{f, \bar{L}_2} g] < n \bar{PC}_1^{\ell} [-e_{\psi, i} e_{i, \bar{L}_1} g + e_{\psi, i} e_{i, \theta} e_{\theta, y_1} \frac{dy_1}{dt} / y_1 - e_{\psi, w} e_{w, \bar{L}_1} g - e_{\psi, y} \frac{dy_1}{dt} / y_1 + e_{\psi, \bar{L}} g]$$

$$\text{where } e_{f, \frac{1}{\theta}} = \frac{\partial A_2}{\partial \frac{1}{\theta}} \frac{\frac{1}{\theta}}{A_2}, \quad e_{f, y_2} = \frac{\partial A_2}{\partial y_2} \frac{y_2}{A_2} \quad \text{and} \quad e_{f, \bar{L}_2} = \frac{\partial A_2}{\partial \bar{L}_2} \frac{\bar{L}_2}{A_2}$$

are the elasticities of  $A_2$  with respect to  $\frac{1}{\theta}$ ,  $y_2$  and  $\bar{L}_2$ ,

$$e_{\psi, i} = - \frac{\partial \bar{PC}_1^{\ell}}{\partial i} \frac{i}{\bar{PC}_1^{\ell}}, \quad e_{\psi, w} = \frac{\partial \bar{PC}_1^{\ell}}{\partial w} \frac{w}{\bar{PC}_1^{\ell}}, \quad e_{\psi, y_1} = - \frac{\partial \bar{PC}_1^{\ell}}{\partial y_1} \frac{y_1}{\bar{PC}_1^{\ell}} \quad \text{and}$$

$$e_{\psi, \bar{L}_1} = \frac{\partial \bar{PC}_1^{\ell}}{\partial \bar{L}_1} \frac{\bar{L}_1}{\bar{PC}_1^{\ell}} \quad \text{are the elasticities of } \bar{PC}_1^{\ell} \text{ with respect to } i, w, y_1$$

and  $\bar{L}_1$ ,  $e_{\frac{i}{\theta}, \bar{L}_1} = \frac{\partial \frac{i}{\theta}}{\partial \bar{L}_1} \cdot \frac{\bar{L}_1}{\frac{i}{\theta}}$  and  $e_{i/\theta, \theta} = - \frac{\partial \frac{i}{\theta}}{\partial \theta} \cdot \frac{\theta}{i/\theta}$  are the elasticities of  $\frac{i}{\theta}$  with respect to  $\bar{L}_1$  and  $\theta$ ,  $e_{i, \bar{L}_1} = \frac{\partial i}{\partial \bar{L}_1} \cdot \frac{\bar{L}_1}{i}$  and  $e_{i, \theta} = \frac{\partial i}{\partial \theta} \cdot \frac{\theta}{i}$  are the elasticities of  $i$  with respect to  $\bar{L}_1$  and  $\theta$ , and finally,

$e_{\theta, y_1} = - \frac{\partial \theta}{\partial y_1} \cdot \frac{y_1}{\theta}$  and  $e_{w, \bar{L}_1} = - \frac{\partial w}{\partial \bar{L}_1} \cdot \frac{\bar{L}_1}{w}$  are the elasticity of  $\theta$  with respect to  $y_1$  and that of  $w$  with respect to  $\bar{L}$ , respectively. All these elasticities are defined, as usual, in terms of their absolute magnitudes.

Now, for the purpose of final interpretation, we like to express this condition, (4.14) or (4.15), by going back, as suggested at the outset, to our original framework of period analysis. Then, (4.14), for example, will have to be written as

$$(4.14)' \quad f_1 \frac{\partial(\frac{i}{\theta})}{\partial \bar{L}_1} \Delta \bar{L}_1(t) + f_1 \frac{\partial(\frac{i}{\theta})}{\partial \theta} \frac{d\theta}{dy_1} \Delta y_1(t-1) + f_2 \Delta y_2(t-1) + f_3 \Delta \bar{L}_2(t) \\ < n[\psi_1 \frac{\partial i}{\partial \bar{L}_1} \Delta \bar{L}_1(t) + \psi_1 \frac{\partial i}{\partial \theta} \frac{d\theta}{dy_1} \Delta y_1(t-1) + \psi_2 \frac{\partial w}{\partial \bar{L}_1} \Delta \bar{L}_1(t) + \psi_3 \Delta y_1(t-1) \\ + \psi_4 \Delta \bar{L}_1(t)]$$

where all the partial derivatives, by the mean value theorem, are to be considered as evaluated at some interior points of the relevant intervals.

In a similar way, (4.15) will be expressed as

$$(4.15)' \quad A_2(t) [e_{f, \frac{i}{\theta}, \bar{L}_1} e_{\frac{i}{\theta}, \bar{L}_1} g + e_{f, \frac{i}{\theta}, \theta} e_{\frac{i}{\theta}, \theta} e_{\theta, y_1} \frac{\Delta y_1(t-1)}{y_1(t-1)} + e_{f, y_2} \frac{\Delta y_2(t-1)}{y_2(t-1)} + e_{f, \bar{L}_2} g] \\ < n \bar{P} C_1^2(t) [- e_{\psi, i, \bar{L}_1} e_{i, \bar{L}_1} g + e_{\psi, i, \theta} e_{i, \theta} e_{\theta, y_1} \frac{\Delta y_1(t-1)}{y_1(t-1)} - e_{\psi, w} e_{w, \bar{L}_1} g \\ - e_{\psi, y_1} \frac{\Delta y_1(t-1)}{y_1(t-1)} + e_{\psi, \bar{L}_1} g]$$

where all the elasticities are to be considered as arc, rather than point, elasticities.

This condition, (4.14)' or (4.15)', can now be interpreted in the following way. There are, as we know, four parameters,  $\bar{L}_1(t)$ ,  $\bar{L}_2(t)$ ,  $y_1(t-1)$ , and  $y_2(t-1)$ , the changes in which make the system move. The direction of changes in these parameters are also known to us--  $\bar{L}_1(t)$ ,  $\bar{L}_2(t)$  and  $y_2(t-1)$  increase, while  $y_1(t-1)$  falls, over time. As all of these parameters change at the same time, and in these directions, they produce a combined effect on each variable of the system. The condition, (4.14)' or (4.15)' is nothing but a description and a comparison of such combined effects or total changes in two crucial variables, the wealth of the capitalist farm,  $A_2(t)$ , and the aggregate consumption loan of all the family farms,  $n\bar{PC}_1^l(t)$ . It is important to focus one's attention, among others, on these two variables, because by looking at their changes it is possible to conclude about the direction of aggregate capital accumulation.

Consider, first, the L.H.S. of (4.14)'. It shows the total change in  $A_2(t)$  when all the parameters are changing. An increase in  $\bar{L}_1(t)$ , going through the complementarity between  $K_1(t)$  and  $L_1(t)$ , and between  $K_2(t)$  and  $nL_2(t)$ , increases the value of  $i(t)$  and therefore also of  $\frac{i(t)}{\theta(t)}$ , and with  $f_1 > 0$  that increases  $A_2(t)$  (the first term). On the other hand, as  $y_1(t-1)$  falls over time, the value of  $\theta_1(t)$  increases and, therefore, for reasons already considered, the value of  $\frac{i(t)}{\theta(t)}$  falls, producing a dampening effect on  $A_2(t)$  (the second term). Then, the third and the fourth term of the L.H.S. show the positive effect on  $A_2(t)$  of the changes in  $y_2(t-1)$  and  $\bar{L}_2(t)$ , respectively.



The R.H.S. of (4.14)' shows the total change in  $\bar{PC}_1^l(t)$  due to the variations in all the relevant parameters. Such parameters for  $\bar{PC}_1^l(t)$  are  $\bar{L}_1(t)$  and  $y_1(t-1)$ . The increase in  $\bar{L}_1(t)$  affects  $\bar{PC}_1^l(t)$  in three different ways: (a) by going through the complementarity between capital and labor and increasing thereby the value of  $i(t)$ , it tends to lower  $\bar{PC}_1^l(t)$  (the first term); (b) by reducing the value of  $w(t)$  because of diminishing returns, it tends to reduce  $\bar{PC}_1^l(t)$  (the third term); and finally (c) by itself, given that  $\psi_4 > 0$ , it increases  $\bar{PC}_1^l(t)$  (the last term). The fall in  $y_1(t-1)$ , the other parameter, affects  $\bar{PC}_1^l(t)$  in two opposite directions. On the one hand, such a fall is known to increase the demand for consumption loan (the fourth term); on the other, because of a consequent increase in  $\theta(t)$  and  $i(t)$ , a reduction in  $\bar{PC}_1^l(t)$  is also expected (the second term). Adding up all these, we get on the R.H.S. of (4.14)' the net total change in  $\bar{PC}_1^l(t)$ .

Now, (4.14)', or its equivalent formulation in terms of elasticities, (4.15)', describes a special situation where the elasticities of  $f$  and  $\psi$  with respect to the relevant variables are such that, following a simultaneous change in all the parameters, the total increase in  $\bar{PC}_1^l(t)$  is greater than the total increase  $A_2(t)$ . But, since  $\Delta A_2(t) - n\Delta\bar{PC}_1^l(t) = n\Delta\bar{P}_k K_1(t) + \Delta\bar{P}_k K_2(t)$ , that immediately means that in such a situation the total capital accumulation in agriculture will be negative. Referring back to the Figures 1(a), 1(b), and 1(c), this situation will be depicted by the  $i(\bar{PC}_1^l)$  curve in 1(a) shifting at a rate faster than the rate of expansion of the length  $00'$  in 1(c), forcing thereby a shrinking back

of the demand curve for  $\bar{P}_k K_1$  (of all the family farms) and  $\bar{P}_k K_2$  taken together.

The purpose of emphasizing this particular condition, this apparently special situation, is really to draw attention to an important, and a very generally plausible, tendency of the path of capital accumulation in a dualistic agriculture. For this type of agriculture, we have seen that the initial state can be taken to be characterized as one where the average income of the family farm is very low and, relative to that, the average income of the capitalist farm is significantly high. Given this distribution of income, the income elasticity of the family farm's demand for  $\bar{P}_k K_1$  is likely to be much higher than its interest elasticity, and then the gap between these elasticities,  $(e_{\psi, y_1} - e_{\psi, i})$  is also likely to be much wider than the gap between the corresponding elasticities,  $(e_{f, y_2} - e_{f, i/\theta})$ , for the capitalist farm. We have established all this in Section 3, through an appropriate characterization of the values of the discount factors,  $\lambda_1$  and  $\lambda_2$ . Now, if we inspect the terms on the R.H.S. and the L.H.S. of (4.15)' and recollect what we have known, again from our analysis in Section 3, about the relative significance of the elasticities of  $\bar{P}_k K_1$  and  $A_2$  with respect to their different arguments (for example, the dominance of  $e_{\psi, y_1}$  over  $e_{\psi, w}$  or  $e_{\psi, i}$ ), and then judge the relative weights of the different terms on both sides of (4.15)', it becomes clear that given the values of  $\frac{\Delta y_1(t-1)}{y_1(t-1)} (> 0)$ ,  $\frac{\Delta y_2(t-1)}{y_2(t-1)} (> 0)$ ,  $g$  and other elasticities, if the difference between  $(e_{\psi, y_1} - e_{\psi, i})$  and  $(e_{f, y_2} - e_{f, i/\theta})$  is sufficiently high, then the condition (4.15)' will always come to hold. Therefore,

in a dualistic economy where the distribution of income is known to be unequal and therefore the difference between  $(e_{\psi,y_1} - e_{\psi,i})$  and  $(e_{f,y_1} - e_{f,i/\theta})$  is known to be significant, the situation, implied by (4.15)' is not a special situation, but a pointer towards a very general and indeed a real, possibility.

Here, we can distinguish between two types of situations: (1) It may happen that in the case of a particular dualistic agriculture, the initial state itself is characterized by a value of  $y_1$  which is so low both in absolute value and in relation to  $y_2$  and therefore the difference between  $(e_{\psi,y_1} - e_{\psi,i})$  and  $(e_{f,y_2} - e_{f,i/\theta})$  is so great that (4.15)' comes to hold right in the beginning. This has the implication that this agriculture will never be able to come out of the initial stagnation. This is a case essentially similar to the one we have touched upon before<sup>30)</sup>. (2) Alternatively, and perhaps more typically, the value of  $y_1$ , to start with, may not be that low and the value of  $y_2$  that high so that (4.15)' may not hold in the beginning, and therefore there may be some capital accumulation going on. But, then, referring back to (4.10) it is clear that although  $n\Delta K_1(t) + \Delta K_2(t)$  may be positive to start with, its algebraic value is related inversely with the difference  $[(e_{\psi,y_1} - e_{\psi,i}) - (e_{f,y_1} - e_{f,i/\theta})]$ . Therefore, what happens over time is that as the value of  $y_1$ , for reasons already mentioned, falls and that of  $y_2$  increases monotonically,  $(e_{\psi,y_1} - e_{\psi,i})$  keeps on increasing relative to  $(e_{f,y_2} - e_{f,i/\theta})$ , and as a result the

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30) See the discussion of the case where the elasticity of  $M$  with respect to  $i$  is less than one, pp 41-43.

rate of capital accumulation starts falling, and there is again a definite tendency for the system to approach a state described by (4.15)'.

We thus find that in a dualistic agriculture with an unequal distribution of income, there may exist, under very plausible conditions, an inherent tendency, either in the beginning or eventually, for capital accumulation to slow down, stop, or even become negative in the net sense. Given this tendency, the question which then naturally arises is: What are the ultimate limits of this capital path and, associated with it, the path of average income of the family farm?

Consider, first, the limit of the path of capital. If the production function in agriculture can be supposed to be such that a certain minimum amount of every factor, and in particular of capital, is essential for producing positive output; i.e.,

$$F(\bar{T}_1, K_1, L_1) = 0 \text{ for } K_1 < K^*$$

$$> 0 \text{ for } K_1 \geq K^*$$

where  $K^*$  is the essential requirement of capital and it is assumed that the essential quantities of other factors are available, then, given the general tendency of capital accumulation as mentioned before, the limit of the capital path is to end up with this minimum amount, and nothing more.

For the limit of  $y_1(t)$ , given that  $\Delta y_1(t-1) \leq 0$  for all  $t$  (by Proposition 4), there are two possibilities:

(1)  $\lim_{t \rightarrow \infty} y_1(t) = A$ , where  $A(> 0)$  is some constant. In this case, with the asymptote of  $y_1(t)$  defined by a nonnegative constant, although the net income of the family farm falls monotonically over time, there is no defaulting of loans. The relationship between the processes of income generation for the two types of farms, therefore, remains unchanged, and so also is the qualitative behavior of the system over time. Only the family farm gets increasingly immiserized, and the inequality between the incomes of the capitalist and the family farm widens.

(2) If, on the other hand,  $y_1(t) = 0$  for some finite  $t$ , say,  $t^*$ , then there is a problem, because at the next period of time, as the capitalist farm, in trying to fulfill its objective (3.18), wants to ensure  $\Delta y_2(t^*) \geq 0$  (Proposition 1),  $\dot{y}_1(t^* + 1)$  becomes negative.

Given that

$$y(t) = \frac{\overline{PF}(\overline{T}_1, K_1(t), L_1(t)) + (w(t) - \mu)L_2(t)}{\overline{L}_1(t)} - \frac{i\overline{P}_k K_1(t) + (1 + i(t))\overline{PC}_1^l(t)}{\overline{L}_1(t)}$$

this implies that the loan cannot be totally repaid from the family farm's gross income coming from wages and the value of output. Something has to give, and the way the system accommodates this situation is through the mechanism of "distress sale" of land by the family farm to the capitalist farm. Such a transfer of land is supposed to take place in the event of any failure on the part of the family farm to repay its loan. However, since in the next period the capitalist farm will again want to ensure  $\Delta y_2(t^* + 1) \geq 0$ , the "distress sale

of land" continues<sup>31)</sup>. And, as it continues, a time may eventually come when all the land originally owned by the family farm will be taken over by the capitalist farm, and the family farmer will be reduced to the position of landless labor with wages earned from working in the capitalist farm as its only source of income.

But the process need not stop here if this wage is found to be above the subsistence level. Writing down the expressions of  $\Delta y_1(t-1)$  and  $\Delta y_2(t-1)$  which are just the discrete counterparts of (3.38) and (3.39),

$$(3.38)' \quad \Delta y_1(t-1) = \frac{1}{\bar{L}_1(t-1)} [\bar{P} F_{k1} \Delta K_1(t-1) + (w(t-1) - \mu) g \bar{L}_1(t-1) \\ + \Delta w(t-1) L_2(t-1) - \Delta \{i(t-1) \bar{P}_k K_1(t-1) \\ + (1 + i(t-1) \bar{P} C_1^0(t-1))\} - g Y_1(t-1)]$$

$$(3.39)' \quad \Delta y_2(t-1) = \frac{1}{\bar{L}_2(t-1)} [\bar{P} F_{k2} \Delta K_2(t-1) - \Delta w(t-1) n L_2(t-1) \\ + \Delta \{i(t-1) M(t-1)\} - g Y_2(t-1)] ,$$

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31) It should be noted that as a result of any increase in  $T_2$  and fall  $T_1$ , there is an increase in the marginal product of labor in the capitalist farm and a fall of it in the family farm, implying a reallocation of  $L_1$  and  $L_2$ . Similarly, there is also an increase in the marginal product of capital in the capitalist farm and a fall of it in the family farm, again implying a reallocation of  $K_1$  and  $K_2$ . But, although there is a change in the composition of the demand, the behavior of the total amount of the demand for capital,  $n \bar{P}_k K_1 + \bar{P}_k K_2$ , relative to the demand for  $\bar{P} C_1^0$ , does not change, following the usual decrease in  $y_1$ .

and knowing that when the family farm is dispossessed of its land,  $K_1(t - 1) = 0$ ,  $\Delta K_1(t - 1) = 0$ ,  $M(t - 1) = n\bar{P}C_1^l(t - 1)$  and  $\bar{L}_1(t) = L_2(t)$ , and that also, with (4.15)' holding,  $\Delta K_2(t - 1) \leq 0$ , we can see that the capitalist farm in trying to ensure, as a part of his objective,  $\Delta y_2(t - 1) \geq 0$ , will still find it possible to increase the value of  $\Delta\{i(t - 1) M(t - 1)\}$  and then ensure its repayment by deducting the corresponding amount from the payment of wage at the end of the period. The process finally stops when the wage rate in this way is reduced to the subsistence level.

We are therefore led to the following conclusion. If the agricultural sector of a less-developed country is found to have the characteristics mentioned in Section 2, most importantly, if it is characterized by an unequal distribution of income between the family and the capitalist farms, with the capitalist farms combining the operations of production and lending at the same time, and enjoying a monopolistic position in the latter, then, in the absence of any other exogenous factor, it is possible for the system to have an inherent tendency to remain in or approach a state of stagnation in capital accumulation. And, this is also likely to be accompanied by a process of immiserization of the family farms with the possibility of an eventual polarization between the capitalist farmers on the one hand and the family farmers, dispossessed of their land and reduced to the level of landless laborers at the subsistence level, on the other. Whether these inherent tendencies will in fact be realized in a particular situation will, of course, depend on the relative significance of other exogenous factors present in that

situation and also on how closely the characteristics of the situation conform to the ones assumed in our analysis. We shall return to this question later on, in Section 6, when we shall consider the existence of such exogenous factors in terms of various types of technical progress and also the possibility of some variation in institutional characteristics, and see to what extent they may or may not prevent these tendencies from being realized in some real-life situations.

But, before that, we like to point out in a more precise form the significance of the distribution of income and the structure of credit market in generating these tendencies in a dualistic agriculture. The arguments to this effect have already been given in general terms, but, because of their importance to the central hypothesis of this essay, we like to put these arguments in a more precise manner.



5. Significance Of The Distribution Of Income And  
The Structure Of Credit Market

To understand the importance of the unequal distribution of income and the imperfection of credit market as the factors responsible for these tendencies toward stagnation in capital accumulation and immiserization of the family farms in a dualistic agriculture, it is important to single out the implications of these two factors from those of other forces in the system. We therefore propose to carry out a comparative analysis where the situation so long assumed in our model will be compared, from the standpoint of the question of capital accumulation, with another situation which will have all other characteristics, particularly, the per capita income of the entire agricultural sector and the rate of growth of population the same as before, with the only exceptions that it will have a more equal distribution of income and a perfect credit market.

It should be pointed out here that we are considering these two crucial variations, equalization of incomes and perfection of the credit market, as essentially interconnected. This is because we have seen in Section 2 that the factor crucially responsible for the imperfection of the credit market is the initial distribution of income whereby there are numerous family farms with a very low level of average income and therefore in need of credit, and a relatively few capitalist farms with a significantly higher level of average income

and in a position to supply that credit. It is not possible to remove this imperfection without at the same time improving the distribution of income<sup>32)</sup>. The two issues of equalization of incomes and perfection of credit market are therefore to be considered together and their implications studied jointly.

Consider, first, the existing situation in a dualistic agriculture with unequal distribution of income and imperfect credit market. Let  $\bar{y}_1(t-1)$  be the net per capita income of this agriculture in the beginning of period  $t$  :

$$(5.1) \quad \bar{y}_1(t-1) = \ell_1(t-1) y_1(t-1) + \ell_2(t-1) y_2(t-1)$$

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32) We have noted in this connection that there are certain administrative problems connected with credit operation in the rural areas which help preserve the monopoly power of the local capitalist farm as the money lender. But, we have also seen that these administrative problems are again fundamentally due to the family farms having a low level of income and small amount of asset (land). Hence, any attempt to perfect the rural credit market by focusing attention only on the administrative problems and without any regard to the fundamental cause of these problems is likely to be self-defeating. In India, for example, the attempts to solve the problem by creating the cooperative banks, unaccompanied by any change in the basic income and asset position of the family farms, have often ended up diverting funds in favor of the capitalist farms. To solve the problem, therefore, it is essential to think in terms of improving the average income (and asset holding) of the family farm. But improving the average income of the family farm will also imply, in a situation of not sufficiently high rate of capital accumulation and in the absence of any significant exogenous change, a reduction in the average income of the capitalist farm (by Proposition 3) and therefore a redistribution of income, at least in the beginning of the process.

where  $l_1(t-1)$  and  $l_2(t-1)$  are the proportions of the family and the capitalist farms in the total rural population. Let  $\bar{x}(t-1) = y_1(t-1)/y_2(t-1)$  be the index of inequality in the distribution of income and  $g$ , as before, the rate of growth of population in both the family and the capitalist farm.

Consider, next, a new situation where the values of  $\bar{y}_1(t-1)$  and  $g$  are the same as before, but where instead of letting  $y_1(t-1)$  and  $y_2(t-1)$  change according to the previous manner, a policy intervention is made, through, say, a measure of land reform or an agricultural income tax-cum-subsidy, which has the effect of redistributing a definite amount of income from the capitalist to the family farm over the period  $t$ . This is a case of pure income redistribution without any overall change in  $\bar{y}_1(t-1)$ , so that

$$(5.2) \quad \Delta y_1(t-1) = - \frac{l_2(t-1)}{l_1(t-1)} \Delta y_2(t-1) > 0$$

In this new situation, following the increase in  $y_1(t-1)$  there will also be a reduction, for reasons already mentioned, in the value of  $\theta(t)$  implying a lessening of imperfection in the credit market. In fact, if the equalization of income is sufficiently complete, the value of  $\theta(t)$  will tend to fall to 1 which is the state of perfect credit market.

The question, now, is: what is the effect of this move from the original to the new situation, of this redistribution of income and

perfection of credit market, on the rate of capital accumulation? To be noted that so far the changes in the parameters in the new situation are concerned, the changes in  $\bar{L}_1(t)$  and  $\bar{L}_2(t)$  are the same as before, but the changes in  $y_1(t-1)$  and  $y_2(t-1)$  are now exactly in the opposite direction, with  $\Delta y_1(t-1)/y_1(t-1) > 0$  and  $\Delta y_2(t-1)/y_2(t-1) < 0$ . To analyze the effect of these new qualitative changes in  $y_1(t-1)$  and  $y_2(t-1)$ , as brought about by the redistribution of income, on the rate of aggregate capital accumulation we have to refer to the crucial condition (4.15)'.

On observing the terms in (4.15)', it becomes clear that following these changes in  $y_1(t-1)$  and  $y_2(t-1)$  and a consequent fall in  $\theta(t)$  there will be changes in both sides of (4.15)' as compared to the original situation. On the R.H.S., as a result of an increase in  $y_1(t-1)$ , there will be, on the one hand, a fall in  $\bar{PC}_1^l(t)$  and hence a change in the fourth term; on the other hand, due to a consequent fall in  $\theta(t)$  and  $i(t)$ , there will be an increase in  $\bar{PC}_1^l(t)$  and therefore a change in the second term. Summing up these two changes, we get the total change in  $\bar{PC}_1^l(t)$ :  $n PC_1^l(t) [e_{\psi, i} e_{i, \theta} e_{\theta, y_1} \Delta y_1(t-1)/y_1(t-1) - e_{\psi, y_1} \Delta y_1(t-1)/y_1(t-1)]$ , as a result of moving to the new situation.

Similarly, on the L.H.S. there will be two kinds of changes as compared to the original situation. Since there is a fall in  $y_2(t-1)$ , this by itself, will mean a fall in  $A_2(t)$  (a change in the third term). At the same time, because of a fall in  $y_1(t-1)$  and a fall in  $\theta(t)$ ,

and therefore a rise in  $i(t)/\theta(t)$ , there will also be a positive effect on  $A_2(t)$  (a change in the first term). Adding these two changes, we get

$$A_2(t) [e_{f,i/\theta} e_{i/\theta, \theta} e_{\theta, y_1} \Delta y_1(t-1)/y_1(t-1) + e_{f, y_2} \Delta y_2(t-1)/y_2(t-1)]$$

Now, if the resulting total change in the L.H.S. exceeds the corresponding total change in the R.H.S., i.e., if

$$(5.3) \quad A_2(t) [e_{f,i/\theta} e_{i/\theta, \theta} e_{\theta, y_1} \Delta y_1(t-1)/y_1(t-1) + e_{f, y_2} \Delta y_2(t-1)/y_2(t-1)]$$

$$> n \bar{P}C_1^k(t) [e_{\psi, i} e_{i, \theta} e_{\theta, y_1} \Delta y_1(t-1)/y_1(t-1)]$$

$$- e_{\psi, y_1} \Delta y_1(t-1)/y_1(t-1)]$$

i.e. by using (5.2) so that  $\Delta y_2(t-1) = -\frac{\lambda_1(t-1)}{\lambda_2(t-1)} \Delta y_1(t-1)$  and also the definition of  $\bar{x}(t-1)$ , if

$$(5.4) \quad A_2(t) [e_{f,i/\theta} e_{i/\theta, \theta} e_{\theta, y_1} - e_{f, y_2} \bar{x}(t-1) \frac{\lambda_1(t-1)}{\lambda_2(t-1)}]$$

$$> n \bar{P}C_1^k(t) [e_{\psi, i} e_{i, \theta} e_{\theta, y_1} - e_{\psi, y_1}] ,$$

then we can conclude that, as a result of the redistribution of income and consequent lessening of imperfection in the credit market,  $A_2(t)$  in the new situation will tend to increase faster than  $\bar{P}C_1^{\ell}(t)$  in algebraic value implying thereby that there will be a definite increase in the rate of capital accumulation.

Now, we already know from our analysis in Section 3 that when the distribution of income between the family and the capitalist farm is such that the family farm has a level of average income which is very low both in absolute amount and in relation to the capitalist farm, then the difference between the income and the interest elasticity of the demand of the family farm for consumption loan is always significantly greater than the corresponding difference between the income and the rate of return elasticity of holding of wealth by the capitalist farm. Therefore, in the context of the present comparison when the distribution of income is known to be unequal to start with, i.e., given the value of  $\ell_1(t-1)/\ell_2(t-1)$ ,  $x(t-1)$  is known to be sufficiently low, the difference  $(e_{\psi, y_1} - e_{\psi, i})$  is expected to be significantly greater than the difference  $(e_{f, y_2} - e_{f, i/\theta})$  and therefore, given the ratio between  $A_2(t)$  and  $\bar{P}C_1^{\ell}(t)$  and other elasticities, it is clear that in such a situation (5.4) is very likely to hold. In other words, if the distribution of income is significantly unequal and the credit market imperfect, it is quite possible to promote capital accumulation by redistributing that income and breaking the imperfection in the credit market.

It is of some importance to compare this conclusion with a well known traditional wisdom which has always tended to uphold inequality as an argument for promoting capital accumulation. For the purpose of this comparison it is useful to rewrite (5.4) as

$$(5.4)' \quad \left[ \frac{\partial A_2}{\partial i/\theta} \frac{\partial i/\theta}{\partial \theta} \frac{\partial \theta}{\partial y_1} - n \frac{\partial \bar{P}C_1^l}{\partial i} \frac{\partial i}{\partial \theta} \frac{\partial \theta}{\partial y_1} \right]$$

$$> \left[ \frac{\partial A_2}{\partial y_2} \frac{l_1(t-1)}{l_2(t-1)} + n \frac{\partial \bar{P}C_1^l}{\partial y_1} \right]$$

which is obtained by substituting the definitions of the elasticities and transferring the terms between the two sides. By the mean value theorem once again, the derivatives in this expression are to be considered as evaluated at some appropriate interior points of the respective intervals.

The crux of the traditional argument is that the marginal propensity to save of the poorer income group can be taken to be lower than that of the richer group, and therefore any equalization of incomes will lower the amount of aggregate saving and reduce the rate of capital accumulation. Now, translating this argument in terms of our analytical framework, where the wealth holding of the capitalist farm is to be taken as the equivalent of the saving of the rich and the consumption loan of the family farm the (negative) saving of the poor, we find that the basic contention of this traditional hypothesis has the effect of

rendering the R.H.S. of (5.4)' positive and, by implication, since the interaction between income and interest has not been considered in this hypothesis, the L.H.S. zero. With the R.H.S. thus exceeding the L.H.S., it is clear from (5.4)', or its equivalent formulation (5.4), that one can then get a conclusion by which any equalization of incomes will appear as detrimental to capital accumulation.

There is, however, a crucial assumption relating to the effect of the process of income redistribution that underlies the core of this traditional argument. The assumption, it seems, is that the redistribution of income is a neutral phenomenon so far its effects on the institutional structure of the economy is concerned; apart from immediately affecting the income terms in the saving function, it does not affect the structure of the economy at all, it does not for example, affect the structure of any market.

This assumption of neutrality, however, need not always be true, and it is particularly not true for a dualistic agriculture of a less developed country. In such an agriculture we have seen that the structure of the credit market is crucially connected with the existing state of the distribution of income between the family and the capitalist farms, with the result that a redistribution of income in favor of the former always has the effect of lessening the imperfection of this structure. Under this situation, therefore, any equalization of incomes, in addition to having an "income effect" which may tend to reduce the supply of aggregate wealth (the R.H.S. of (5.4)')



and which alone was considered in the traditional argument, will also have, through the perfection of credit market, an important "interest effect" which will be seen in terms of a fall in the rate of interest and a rise in the marginal rate of return on wealth (the L.H.S. of (5.4)'), and which will tend to increase the availability of wealth for capital accumulation. And, if this interest effect of income redistribution dominates its income effect, and we have explained that there are plausible conditions under which it very well may, then the final effect on capital accumulation will be very different from what was suggested in the traditional argument.

Finally, there is an important dynamic implication of this perfection of credit market. We have seen that although both the family and the capitalist farm want to ensure, as a part of their objective, that  $\Delta y_1(t-1) \geq 0$ , under imperfect credit market, it is only the capitalist farm which succeeds in achieving it because then it has a prior advantage of choosing the amount of saving through which it can affect the value of the rate of interest. However, once the imperfection of credit market is removed, the capitalist farm will no longer have any advantage to ensure  $\Delta y_2(t-1) \geq 0$ . As a result, it can now be equally possible for  $\Delta y_1(t-1) \geq 0$ , and should that happen, it will also become possible, because of the nature of the elasticities of  $\bar{P}C_1^k(t)$  and  $A_2(t)$ , for capital accumulation to keep on increasing. Thus, the effect of equalization of incomes and perfection of credit market initiated in any particular period need not be restricted to

that period only, it can indeed open up the possibility of increase in capital accumulation on a permanent basis.

When capital accumulation keeps taking place in this way, a time may eventually come when it will be possible for the system to cross that threshold value of capital accumulation subject to which Proposition 3 was found valid. And if Proposition 3 is rendered ineffective, it will then be possible for both  $y_1$  and  $y_2$  to increase over time and we will have a situation where not only the blocks on capital accumulation will be removed, but the tendency toward the immiserization of the family farm will also be reversed.

Thus in a dualistic agriculture comparing the existing situation of unequal distribution of income and imperfection of the credit market with a situation of more equalized incomes and perfected credit market, and observing how the possibilities of significant increase in capital accumulation can be opened up by moving toward the latter situation, one can come to understand the crucial importance of the existing state of income distribution and the structure of credit market as the factors responsible for aborting these possibilities and perpetuating instead a tendency toward stagnation.

\* \* \* \* \*

The central idea of this paper, that the insufficiency of capital accumulation in a dualistic agriculture can be explained in terms of the existing distribution of income and the imperfection of

credit market, needs to be carefully distinguished from some other hypotheses in the literature. It has to be distinguished, for example, from the usual "vicious circle of poverty" hypothesis which, in essence, suggests that an underdeveloped country tends to remain underdeveloped because, given its small per capita income, it can hardly generate any significant amount of saving at the aggregate level. And, ruling out the possibility of any large-scale inflow of foreign capital except in some special situations, this limitation on the aggregate saving also implies a corresponding limitation on the accumulation of capital, and hence the economy is trapped in a kind of low level equilibrium. The problem is further compounded, it is added, by the fact that most of these underdeveloped countries are also in their second phase of demographic evolution, experiencing a high rate of population growth.

What we have shown, on the other hand, is that it is possible to offer an alternative explanation of the phenomenon of underdevelopment by shifting the focus of analysis, which in these traditional hypotheses has only been on the central tendency of the distribution of income, to the dispersion of the distribution and the structure of credit market that results from this dispersion. We have shown that given the existing per capita income and the rate of growth of population as they are in a less developed country, it may be possible, just by redistributing income more equally and breaking down imperfection of the credit market, to generate enough saving from which

capital accumulation can be initiated. One can then further argue that if this capital accumulation and therefore the growth of income is sustained long enough, that by itself may lead to a demographic reversal.

Our argument needs also to be contrasted with a sociological hypothesis according to which the failure of a less developed country to generate capital accumulation is to be explained in terms of the lack of appropriate socio-cultural factors. We think, however, that one may not necessarily have to go for this kind of exogenization of explanation. It is possible, as we have shown in the context of a dualistic agriculture, to explain this phenomenon of stagnation in basic economic terms, in terms of the decisions taken by the family and the capitalist farm to satisfy their economic objective under the special circumstances produced by the unequal distribution of income and the imperfection of credit market. It is shown that with the distribution of income and the structure of credit market as they are, the capitalist farmer will always find it worthwhile to restrict the amount of saving as well as its allocation to productive use to a certain level, determined, among others, by the interest elasticity of the market demand for loan, not necessarily because of any cultural inhibition but because given his economic objective, that is the most profitable thing to do.

## 6. Different Ways of Resolving the Crisis

Given the tendency of a dualistic agriculture to approach a state of stagnation in capital accumulation, the question which naturally arises is: Are there ways in which this tendency can be reversed and the system lifted out of this impasse? The following possibilities are suggested.

1. Suppose that the agricultural sector has reached the state of stagnation where (4.15)' holds and where all land of the family farms has been taken over by the capitalist farm and the wage rate has been reduced to the subsistence level. When the system is actually pushed to this extreme situation, interestingly enough, it also acquires a potentially redeeming feature. This is because, with all land of the family farms taken over and the wage at the subsistence level, it is no longer possible for the capitalist farm to ensure  $\Delta y_2(t - 1) \geq 0$  (which, as we know from Proposition 1, is necessary to fulfill its basic objective (3.18)), by either increasing the interest earnings or reducing the wage payment. In other words, with these two channels closed, there is no way for the capitalist farm to increase, or hold constant, its per capita net income at the expense of the other group. To ensure  $\Delta y_2(t - 1) \geq 0$  in this situation, it is clear from (3.39)' (since the second and the third term on the R.H.S. are reduced to zero), that the capitalist farm will now have to start accumulating capital. As a result, we can have two possibilities:

(a) This rate of accumulation of capital in the very first iteration may be so high and therefore, given the complementarity with labor, the increase in the wage rate and through that the increase in

$y_1(t - 1)$  so significant that, with the elasticities of  $\overline{PC}_1^l(t)$  and  $A_2(t)$  with respect to different arguments as they are, there may be a reversal in the direction of inequality in (4.15)'. If this happens, then, of course, a breakthrough will be initiated, and the stagnation will turn out to be self-correcting.

(b) More typically, however, the rate of capital accumulation in the very first instance may not be that high and the increase in the wage rate and  $y_1(t - 1)$  not that significant so that (4.15)' may continue to hold. This implies that, with the initial capital accumulation, as the wage rate is only increased from its previous subsistence level, the capitalist farm at the next iteration will find it again most profitable to be able to ensure  $\Delta y_2(t) \geq 0$  by increasing the value of interest earnings, and then getting it repaid by subtracting the corresponding amount from the wage payment, until the wage rate again falls back to the subsistence level. In other words, stagnation of a dualistic agriculture can be stable in the small and unstable only in the large. It is interesting that we come to this well-known result in the development literature,<sup>33)</sup> but for very different reasons.

If, therefore, there are reasons to believe that from a state of stagnation the system may not always self-initiate capital accumulation at a rate high enough to disturb the local stability, then one has to think in terms of some change in the institutional structure or in terms of exogenous factors to dislodge the system from its low-level equilibrium and bring about global instability in the right direction.

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33)

H. Leibenstein, Economic Backwardness (1957), Chs. 1-4.

2. We shall first take up the question of institutional change, and here we shall start by considering the possibility of such a change in the structure of the labor market. Suppose that we think of a different situation in the labor market where, unlike what has been assumed so far, the family farmers or, in the extreme case, the landless laborers organize themselves in each set and act as a group rather than as atomistic individuals in supplying labor to the capitalist farm. The implication of this institutional change is that corresponding to the monopolistic situation on the supply side of the credit market, there is now a monopoly also on the supply side of the labor market. In our analysis, so far, only the capitalist farm had the power to ensure  $\Delta y_2(t-1) \geq 0$ , because, given its monopolistic control in the credit market, it had a prior advantage of choosing the amount of saving and, through that, adjusting the interest earnings appropriately. But, now, with a corresponding monopoly power in the labor market, the family farms or landless laborers, as the case may be, have a similar advantage in their choice of the amount of labor to be supplied by which they can control the wage earnings and thus also ensure  $\Delta y_1(t-1) \geq 0$  (cf. the equation (3.38)'). Therefore, depending on the balance of monopoly power in the two markets, it is now quite possible to have a situation where  $\Delta y_1(t-1) \geq 0$ . And, once that happens, we have seen in Section 5 that it also becomes possible, given the elasticities of  $\overline{PC}_1^{\theta}(t)$  and  $A_2(t)$  with respect to the relevant arguments, for capital accumulation to be initiated on a permanent basis. This is an interesting example of how the stagnation in agriculture can be resolved through an institutional change in the labor market. However, since this solution implies

that at least in the initial stage of capital accumulation  $\Delta y_2(t - 1) < 0$ , in suggesting this solution one should also be aware of the type of resistance that is to come from the capitalist farms against the implementation of this kind of change.

3. Another kind of institutional change which is more usually considered, and we also have mentioned it in Section 5, is a policy of land reforms which can change the ownership of land in favor of the family farms. To the extent that such a policy can be implemented at a significant scale, it has the same qualitative effect on  $y_1$  as is obtained from a change in the structure of labor market mentioned above, and, therefore, it has the same kind of potential for breaking the stagnation in capital accumulation. But, here too, one should keep in mind the implication of a decline in the average income of the capitalist farm that is inevitable in the initial periods of this change.

4. From these considerations of internal institutional changes let us now pass on to the question of the so-called exogenous changes in the system and see how the crisis in capital accumulation can be resolved through them. Of all the vehicles of such changes, the one which is most commonly considered is technological progress. Conceived exogenously, this technological progress can be represented as a function of time:

$$(6.1) \quad \lambda = \lambda(t)$$

where  $\lambda$  is the indicator of technological progress. Here, for the convenience of working with differential operator, we are again working in terms of continuous time. Reinterpretation of the results in terms of period analysis should be immediate.



Assuming that the incidence of this technological progress is the same on both the family and the capitalist farms,  $\lambda$  can be introduced as another factor in the production function of both the farms,<sup>34)</sup> and (3.2)' and (3.14) be rewritten as

$$(6.2) \quad y_1 = \frac{\bar{P}F(\bar{T}_1, K_1, L_1; \lambda) + (w - \mu)L_2}{\bar{L}_1} - \frac{i\bar{P}_k K_1 + (1 + i)\bar{P}C_1^k}{\bar{L}_1}$$

and

$$(6.3) \quad y_2 = \frac{\bar{P}F(\bar{T}_2, K_2, nL_2; \lambda) - wnL_2}{\bar{L}_2} + \frac{iM + n\bar{P}C_1^k}{\bar{L}_2},$$

where  $w$  and  $i$  will now depend also on  $\lambda$ ; i.e.,

$$(6.4) \quad w = \bar{P}F_{L1}(\bar{T}_1, K_1, L_1; \lambda) + \mu = \bar{P}F_{L2}(\bar{T}_2, K_2, nL_2; \lambda), \quad \text{and}$$

$$(6.5) \quad i = \frac{\bar{P}}{\bar{P}_k} F_{k1}(\bar{T}_1, K_1, L_1; \lambda) = \theta \frac{\bar{P}}{\bar{P}_k} F_{k2}(\bar{T}_2, K_2, nL_2; \lambda).$$

Clearly, then,

$$(6.6) \quad \left. \frac{dw}{dt} \right|_{\lambda} = \frac{dw}{dt} + \bar{P} \frac{\partial F_{L1}}{\partial \lambda} \frac{d\lambda}{dt} = \frac{dw}{dt} + \bar{P} \frac{\partial F_{L2}}{\partial \lambda} \frac{d\lambda}{dt},$$

where  $\left. \frac{dw}{dt} \right|_{\lambda}$  is the total change in the wage rate with respect to time taking into account the effect of technological progress and  $\frac{dw}{dt}$  is the change in the same in the absence of technological progress, i.e., the kind of change we had so long been considering. Similarly,

$$(6.7) \quad \left. \frac{di}{dt} \right|_{\lambda} = \frac{di}{dt} + \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial \lambda} \frac{d\lambda}{dt} = \frac{di}{dt} + \theta \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k2}}{\partial \lambda} \frac{d\lambda}{dt}.$$

34)

Note that we are representing technical progress in its most general form, avoiding, for example, its representation in terms of the factor-augmenting form which is essentially a restrictive case. See, E. Burmeister and A. Dobell, Mathematical Theories of Economic Growth, pp. 67-77.

Furthermore, using the property of homogeneity of degree one of  $F$  in  $T_1$ ,  $K_1$  and  $L_1$  and, therefore, homogeneity of degree zero of the partial derivatives of  $F$ , and also the property of continuity of the second-order partial derivatives of  $F$ , we have <sup>35)</sup>

$$(6.8) \quad \frac{\partial F}{\partial \lambda} (\bar{T}_1, K_1, L_1; \lambda) = \bar{T}_1 \frac{\partial F_{T1}}{\partial \lambda} \frac{d\lambda}{dt} + K_1 \frac{\partial F_{k1}}{\partial \lambda} \frac{d\lambda}{dt} + L_1 \frac{\partial F_{L1}}{\partial \lambda} \frac{d\lambda}{dt}$$

and

$$(6.9) \quad \frac{\partial F}{\partial \lambda} (\bar{T}_2, K_2, L_2; \lambda) = \bar{T}_2 \frac{\partial F_{T2}}{\partial \lambda} \frac{d\lambda}{dt} + K_2 \frac{\partial F_{k2}}{\partial \lambda} \frac{d\lambda}{dt} + nL_2 \frac{\partial F_{L2}}{\partial \lambda} \frac{d\lambda}{dt},$$

so that, taking into account technological progress in this most general form, i.e., allowing for the possibility of technological progress affecting the marginal product of every factor, the original expressions for  $\frac{dy_1}{dt}$  and  $\frac{dy_2}{dt}$ , given by (3.22) and (3.23), can be rewritten as

$$(6.10) \quad \frac{dy_1}{dt} = \frac{1}{L_1} \left[ \bar{P} \frac{dK_1}{dt} + \bar{T}_1 \bar{P} \frac{\partial F_{T1}}{\partial \lambda} \frac{d\lambda}{dt} + K_1 \bar{P} \frac{\partial F_{k1}}{\partial \lambda} \frac{d\lambda}{dt} + L_1 \bar{P} \frac{\partial F_{L1}}{\partial \lambda} \frac{d\lambda}{dt} + (w-\mu)g\bar{L}_1 \right. \\ \left. + \left( \frac{dw}{dt} + \bar{P} \frac{\partial F_{L2}}{\partial \lambda} \frac{d\lambda}{dt} \right) L_2 - \frac{d}{dt} (i\bar{P}_k K_1 + (1+i)\bar{P}C_1^k) - \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial \lambda} \frac{d\lambda}{dt} \bar{P}C_1^k - g Y_1 \right]$$

35)

Proof: By the property of homogeneity of degree one of  $F$  with respect to  $T_1$ ,  $K_1$  and  $L_1$ , we have, for example, in the case of the family farms

$$F(\bar{T}_1, K_1, L_1; \lambda) = \bar{T}_1 F_{T1}(\bar{T}_1, K_1, L_1; \lambda) + K_1 F_{k1}(\bar{T}_1, K_1, L_1; \lambda) + L_1 F_{L1}(\bar{T}_1, K_1, L_1; \lambda)$$

Now, totally differentiating both sides with respect to  $t$ , we get, upon cancelling out the common terms and regrouping the terms,

$$\frac{\partial F}{\partial \lambda} \frac{d\lambda}{dt} = \frac{dK_1}{dt} \left[ \bar{T}_1 \frac{\partial F_{T1}}{\partial K_1} + K_1 \frac{\partial F_{k1}}{\partial K_1} + L_1 \frac{\partial F_{L1}}{\partial K_1} \right] + \frac{dL_1}{dt} \left[ \bar{T}_1 \frac{\partial F_{T1}}{\partial L_1} + K_1 \frac{\partial F_{k1}}{\partial L_1} + L_1 \frac{\partial F_{L1}}{\partial L_1} \right] \\ + \left[ \bar{T}_1 \frac{\partial F_{T1}}{\partial L_1} + K_1 \frac{\partial F_{k1}}{\partial L_1} + L_1 \frac{\partial F_{L1}}{\partial L_1} \right].$$

By the property of continuity of the second-order partial derivatives and homogeneity of degree zero of the partial derivatives, the first two terms of the R.H.S. are zero. Hence the required result.

and,

$$(6.11) \quad \frac{dy_2}{dt} = \frac{1}{L_2} [\bar{P} \bar{F}_{k2} \frac{dk_2}{dt} + \bar{T}_2 \bar{P} \frac{\partial F_{T2}}{\partial \lambda} \frac{d\lambda}{dt} + K_2 \bar{P} \frac{\partial F_{k2}}{\partial \lambda} \frac{d\lambda}{dt} + nL_2 \bar{P} \frac{\partial F_{L2}}{\partial \lambda} \frac{d\lambda}{dt} \\ - (\frac{dw}{dt} + \bar{P} \frac{\partial F_{L2}}{\partial \lambda} \frac{d\lambda}{dt}) nL_2 + \frac{d}{dt} (iM) + \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial \lambda} \frac{d\lambda}{dt} M - g Y_2]$$

For the purpose of our analysis, we now make a distinction between (a) labor-using technological progress which will be defined by

$$(6.12) \quad n_i L_i \frac{\partial F_{Li}}{\partial \lambda} \frac{d\lambda}{dt} > K_i \frac{\partial F_{ki}}{\partial \lambda} \frac{d\lambda}{dt} + \bar{T}_i \frac{\partial F_{Ti}}{\partial \lambda} \frac{d\lambda}{dt}$$

and (b) non-labor using technological progress which will be defined as

$$(6.13) \quad n_i L_i \frac{\partial F_{Li}}{\partial \lambda} \frac{d\lambda}{dt} \leq K_i \frac{\partial F_{ki}}{\partial \lambda} \frac{d\lambda}{dt} + \bar{T}_i \frac{\partial F_{Ti}}{\partial \lambda} \frac{d\lambda}{dt},$$

$$i = 1, 2, \text{ and } n_i = 1 \text{ for } i = 1.$$

(a) In the case of labor-using technological progress, if the increase in the productivity of labor is sufficiently significant, particularly, relative to the increase in the productivity of capital, so that

$$(6.14) \quad \bar{P} \left( \frac{\partial F_{L1}}{\partial \lambda} L_1 + \frac{\partial F_{L2}}{\partial \lambda} L_2 \right) \frac{d\lambda}{dt} > \frac{\bar{P}}{\bar{P}_k} \frac{\partial F_{k1}}{\partial \lambda} \frac{d\lambda}{dt} \bar{P} C_1^k - \bar{T}_1 \bar{P} \frac{\partial F_{T1}}{\partial \lambda} \frac{d\lambda}{dt} - \frac{dw}{dt}$$

$$- (w-\mu)g\bar{L}_1 + \frac{di}{dt} (\bar{P}_k K_1 + \bar{P} C_1^k) + (1+i) \frac{d\bar{P} C_1^k}{dt} + gY_1$$

then, as evident from (6.10), it is possible for  $y_1$  of the family farm to increase over time. And, so far the capitalist farm is concerned,

in the beginning it is also possible for them to ensure the nonnegativity of  $\frac{dy_2}{dt}$  as before, because no matter how significant the bias of technological progress in favor of labor is, any increase in  $w$  due to technological progress is compensated by an equivalent increase in the productivity of labor (see (6.11)). Now, with  $y_1$  increasing and  $y_2$  nondecreasing, there will be both a downward pull on  $\overline{PC}_1^L$  and an upward pull on  $A_2$  (since there will be an additional positive effect through the increase in  $\frac{i}{\theta}$ ), and it is clear from (5.3) that in such a situation there is bound to be an increase in capital accumulation. And, if this situation is maintained, the stagnation in agriculture can indeed be overcome.

However, there is a different problem which is likely to arise in this case from the standpoint of the capitalist farm and for the following reason. As capital keeps accumulating and  $y_1$  increasing, a time may eventually come when it will be possible for the family farm to self-finance its consumption as well as production needs, thus getting rid of the imperfect credit market altogether. But, this will also mean a total loss of one source of income for the capitalist farm, as will be shown by the disappearance of the two terms,  $\frac{d}{dt} (iM)$  and  $\frac{\overline{P}}{P_k} \frac{\partial F_{k1}}{\partial \lambda} \frac{d\lambda}{dt} M$ , on the R.H.S. of (6.11). And, in a situation where

$$K_2 \overline{P} \frac{\partial F_{k2}}{\partial \lambda} \frac{d\lambda}{dt} + \overline{T}_2 \overline{P} \frac{\partial F_{T2}}{\partial \lambda} \frac{d\lambda}{dt}$$

is not significant, this may indeed imply  $\frac{dy_2}{dt} < 0$  eventually. Thus, the capitalist farms may have more to lose than to gain from a labor-using technological progress. And, therefore, to the extent that they, as a group have any control over the introduction of this technological progress, it is possible that they will effectively resist that introduction, although

it is clear that this kind of technological progress, if introduced, can indeed solve the problem of agricultural stagnation.

In real life, technological progress of this kind is best illustrated by education, particularly, by a productivity-oriented primary education in the rural areas. And, it may be interesting to explain and interpret, in the light of the analysis just made, the kind of bias that is found to exist in the government policy against primary education in some of the less-developed countries which have agricultural situation similar to the one being discussed here.<sup>36)</sup>

(b) Turning now to non-labor using technological progress, we find that even here it is possible for  $y_1$  to keep increasing because, as is clear from (6.13), there can always be some increase in the marginal product of labor, and therefore there is again a danger of an eventual loss of credit market and interest earnings from the standpoint of the capitalist farm. However, it is now possible to distinguish between two types of situations depending on whether

$$(6.15) \quad K_2 \bar{P} \frac{\partial F_{k2}}{\partial \lambda} \frac{d\lambda}{dt} + \bar{T}_2 \bar{P} \frac{\partial F_{T2}}{\partial \lambda} \frac{d\lambda}{dt} < \left( \frac{dw}{dt} + \bar{P} \frac{\partial F_{L2}}{\partial \lambda} \frac{d\lambda}{dt} \right) nL_2 - \bar{P} F_{k2} \frac{dK_2}{dt} + gY_2$$

$$(6.16) \quad K_2 \bar{P} \frac{\partial F_{k2}}{\partial \lambda} \frac{d\lambda}{dt} + \bar{T}_2 \bar{P} \frac{\partial F_{T2}}{\partial \lambda} \frac{d\lambda}{dt} \geq \left( \frac{dw}{dt} + \bar{P} \frac{\partial F_{L2}}{\partial \lambda} \frac{d\lambda}{dt} \right) nL_2 - \bar{P} F_{k2} \frac{dK_2}{dt} + gY_2$$

at a time  $t^*$  where the source of interest income may have been totally lost.

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36)

In the context of India, for example, one finds that the government policy has in fact been systematically biased against primary education throughout the period of planning. See, J. Bhagwati: "Education, Class Structure and Income Inequality," World Development, Vol. 1, May 1973, p. 24; A. Dasgupta: "Income Distribution, Education and Capital Accumulation," World Bank Working Paper, May 1974. For similar phenomenon in other less-developed countries, see S. Bowels: "Class Power and Mass Education," Harvard University, 1971,

In the first case, it is possible for  $y_1$  and, in the beginning, also for  $y_2$  to increase and therefore, for reasons already mentioned, there will be again a good possibility of achieving a breakthrough in capital accumulation. But, once again, this situation will not be acceptable to the capitalist farm, because although it is a case of technological progress which is biased in favor of land and capital, it is still possible for  $y_1$  to increase sufficiently to cause a loss of interest earnings for the capitalist farm without the increases in the marginal products of capital and land being significant enough to compensate for that loss. It is only when the bias in technological progress for capital and land is sufficiently high to assure the capitalist farm an overcompensating gain through the increases in the marginal products of capital and land in the event of a possible loss of credit market that the capitalist farm will be found motivated to adopt the technological progress. This situation is shown by the second case (6.16).

There are two interesting implications that come out of this analysis. In the first place, it is clear that in a dualistic agriculture, where the capitalist farm, because of the existing distribution of income, is more likely to have the effective power in adopting any technological change, the existence of knowledge of a technological progress, by itself, is not enough for its implementation, no matter how powerful it may be to dislodge the agriculture from its stagnation. Technological progress has to be of a particular type, specially biased

in favor of capital and land in order to be adopted in the system. The type of technological progress which in real life comes close to this description is the so-called Green Revolution, which is supposed to increase productivity of every factor, but proportionately more of land and capital<sup>37)</sup>.

Secondly, even with the knowledge of a non-labor using technological progress like the Green Revolution, its acceptance will be easier when the opportunity cost of adopting such a change is not significant for the capitalist farm, where the opportunity cost is measured by the interest earnings to be foregone in the event of a loss of the credit market. This opportunity cost, in its turn, depends on the degree of jointness with which the two operations — capitalist farming and money lending — are performed by a single group. Therefore, the less identified these activities are with one economic group, the less is the opportunity cost and higher is the prospect for adoption of any given kind of non-labor using technological progress.

It is of some importance to mention this last point, because it may help to explain the differential impact that the Green Revolution is found to have on different parts of the dualistic agriculture of a country like India. Apart from the fact that the knowledge of this technological progress is itself more developed in certain crop pattern

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37) For an empirical confirmation of this point, in the context of India, see D.P. Chaudhri, 'Factors Affecting Productivity On Different Size Class of Farm Holdings in India', May 1974 (mimeo).

than others (which in terms of our model, means that different values of  $(\partial F_{Ti}/\partial\lambda d\lambda/dt + \partial F_{Ki}/\partial\lambda d\lambda/dt)$  are available for different regions specializing in particular crops), it should also be carefully noted that the success of the Green Revolution has been found to be weakest in those parts of India (the eastern region) where the operations of money lending and capitalist farming are closely identified with one single group, and found to be strongest in those parts (the northern region) where, for some interesting historical and institutional reasons, this identification is far less complete<sup>38)</sup>.

We conclude, therefore, by observing that there do exist quite a few potential solutions to the problem of agricultural stagnation in a less developed country. However, it is also found that unless the institutional structure happens to be particularly propitious (with an appropriate separation between the money lending and the farming activities) or technological progress of a specially biased nature (a sufficiently high value of  $\partial F_{Ti}/\partial\lambda d\lambda/dt + \partial F_{Ki}/\partial\lambda d\lambda/dt$ ), many of these solutions (such as the possibilities (2), (3) and 4(a)) call for a change which is likely to go against the interest of the capitalist farmers. And, in the context of a society where the political power may exist in the hands of these farmers, this analysis draws our attention to a kind of constraints on economic development, very different from the ones suggested in the conventional analyses.

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38) See, A. Bhaduri, 'A Study in Agricultural Backwardness under Semi-Feudalism', Economic Journal, March 1973, pp. 120-122.



### 7. Some Other Results In The Literature

Although much has been written about the dualism between the agricultural and the industrial sector of an underdeveloped economy, the literature on the dualism that exists within the agricultural sector is not very extensive. The existence of a dualistic agriculture was mentioned by Sen<sup>39)</sup>, but a more complete analysis of the problem of resource allocation within this structure came to be developed in the writings of Eckaus<sup>40)</sup> and also of Anderson<sup>41)</sup>. On the questions of income distribution and capital accumulation in such agriculture, the issues which are of more immediate concern to us, the two important works are by Bardhan<sup>42)</sup> and Bhaduri<sup>43)</sup>.

Bardhan has considered a model of dualistic agriculture by taking into account the imperfections of the labor and the credit market. Imperfection of the labor market in his analysis arises due to

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39) A.K. Sen, 'Peasants and Dualism with or without Surplus Labor', Journal of Pol. Ec., Vol. 74, Oct. 1966.

40) R.S. Eckaus, 'The Market Structure and Performance of Traditional Agriculture', August 1969 (Mimeo).

41) K.P. Anderson, Peasant and Capitalist Agriculture in the Developing Country, MIT Thesis (1968).

42) P.K. Bardhan, 'A Model of Growth of Capitalism in a Dual Agrarian Economy' in Bhagnati & Eckaus (eds): Essays in Honour of Rosenstein-Rodan (1972).

43) A. Bhaduri, op. cit.

the gap that exists between the wage rate at the capitalist farm and the marginal product of labor at the family farm, and this imperfection can be regarded as working in favor of the family farm. Imperfection of the credit market, on the other hand, is reflected in the fact that it is possible to get cheaper credit with the increase in the wealth position of the borrower, and wealth here is identified with a composite factor of production, called land-capital. The source of this credit has been left exogenous in this model, but it is clear that the capitalist farm has a comparative advantage in the credit market because, unlike the family farm, it can save and this saving is always turned into the accumulation of land-capital (a consequence of the assumption that land-capital is the only asset), thus increasing the base for cheaper credit. This credit is then used by the capitalist farm to buy a category of inputs which is the vehicle of land-augmenting technological progress, so that the imperfection of credit market is finally reflected in the capitalist farm having a cheaper access to technological progress. This advantage of imperfect credit market, because of the continuous accumulation of land-capital, is essentially a dynamic one and it is shown how, under certain assumptions on the production function and the nature of technical progress, this dynamic advantage can dominate the purely static advantage of the labor market imperfection over time and thus make the income of the capitalist farm grow relatively faster than the income of the family farm.

This model of Bardhan is remarkably elegant in its formal structure, but it leaves certain questions unanswered. Apart from certain minor points, such as, the apparent sensitivity of the conclusions to the particular form (namely, Cobb-Douglas) of the production function considered, the more serious question relates to the assumption of the existence of only one asset in the form of land-capital. Since the implication of this assumption is that whatever is saved is automatically turned into accumulation of capital, one can not, under such assumption, pose any problem of capital accumulation. What Bardhan has done, therefore, really amounts to sidetracking the question of stagnation in capital accumulation in a dualistic agriculture and considering instead the question of the distribution of income in the context of a hypothetical growth process of such an agriculture. However, in view of the persisting problem of insufficiency of capital accumulation in many dualistic agriculture, of evidence to the effect that saving by certain groups of farmers has not automatically been transformed into capital accumulation<sup>44)</sup>, we have found it more important to address ourselves to this existing problem of stagnation and see to what extent it is related to the question of income distribution.

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44) In the context of India, for example, see, the NCAER Rural Household Survey (1965), Tables 33 and 36; All-India Rural Debt and Investment Survey, Reserve Bank of India Bulletin, June 1965.

In posing this problem we have developed a comprehensive model of dualistic agriculture by taking into account all its major characteristics relating, among others, to the state of its distribution of income and the structure of markets, and including, in particular, the possibility of existence of two forms of holding wealth in terms of capital and loan. We have seen that in terms of this model it is possible to provide an explanation of the problem of inadequate capital accumulation of this kind of agriculture, an explanation which also brings out the crucial importance of income distribution in this matter.

Turning to Bhaduri's paper, one should note first of all, that it does not strictly relate to the problem of a dualistic agriculture in the sense we have defined it. It relates to what he calls 'a semi-feudal' agriculture which is characterized in his analysis by the existence of two income groups — the sharecroppers and the landowners. The sharecropper works on the land owned by the landowner for a fixed share of the harvest, and he also takes consumption loan from the same landowner. The problem of portfolio choice that is inherent in such a situation has not been clearly stated in the model, but, through the specification of technical progress, Bhaduri has made the landowner face a problem of conflict between the two sources of income, interest earnings and the income from harvest, which is similar in spirit to the problem encountered by the capitalist farm in the presence of technical progress in our model. This concept of conflict is then used by Bhaduri to interpret in a very interesting and insightful way the historical forces that may be taking shape in certain parts of India.

The difficulty which we have found with Bhaduri's paper, however, is that there are certain crucial issues which are never explained in economic terms. It is not clear, for example, why the sharecropper will never go for production loan even if his income is above the subsistence level and the productivity of land-capital is increasing due to technical progress. It should be noted that the final conclusion of the paper about the inevitability of a conflict between the two sources of income of the landowner is not quite independent of the way this issue is resolved. To say that such issues are settled politically is perhaps to leave them economically underexplained. The more interesting line of enquiry may be to try to find out whether there are any basic economic forces at work which make such supposedly political solutions what they are. In fact, in the case of this corner solution involving the consumption loan, we have seen that it is indeed possible to give an explanation on the basis of rational economic decision making on the part of the family and the capitalist farms.

#### 8. Generalizations

It is possible to generalize our basic analytical model in two directions, by relaxing (a) the simplifying assumptions that were made about the agricultural sector itself and (b) the ones made about the relationship between the agricultural and the industrial sector. We shall mention some of these possibilities very briefly.

1. The landless laborers were not included in the description of the initial state of the model. This, however, is not a restrictive assumption, because it has been shown that the model, starting with an initial state involving the family and the capitalist farms, will itself evolve in a way that the family farms, through a process of immiserization, will be found converted into the landless laborers. In other words, the landless laborer can always be accommodated into our discussion as representing a particular stage in the evolution of the model. However, if we also want to explain why some of the family farms may become landless laborers faster than others, so that at some point of time in history, there can exist the family farm, the capitalist farm and also the landless laborer, we will have to allow for some variation among the family farms themselves, say, in their landownership, in the description of the initial state.

2. To simplify calculations, the parameter  $\mu$  was assumed to be constant. This assumption, however, can be relaxed and the implications of a variation in  $\mu$  analyzed. We have seen in Section 3 that  $\mu$  can be considered as depending on  $L_2$  and  $\bar{L}_2$ :  $\mu = \mu(L_2, \bar{L}_2)$  with  $\partial\mu/\partial L_2 > 0$  (opportunity cost) and  $\partial\mu/\partial \bar{L}_2 < 0$  (economics of scale). Since  $d\bar{L}_2/dt > 0$  and it is expected that  $dL_2/dt > 0$ , the sign of  $\mu$  can go in either direction. If  $d\mu/dt < 0$ , then there will be yet another force to depress the value of  $y_1$  and as a result the tendency toward stagnation will only be strengthened. On the other hand, if  $d\mu/dt > 0$  and if  $w$  also increases corresponding to that, the effect

will be similar to the one that followed from the family farms collectively bargaining for wages. However, given the characterization of the initial state in terms of a slow rate of capital accumulation, it is unlikely that the effect of an increase in  $L_2$  will be strong enough to overcome the corresponding effect of an increase in  $\bar{L}_2$ , so that the initial behavior of  $\mu$  is more likely to be as in the first situation. And, then, with its negative feedback on capital accumulation, it is also possible that the downward tendency of  $\mu$  and of capital accumulation may start reinforcing each other without the second possibility,  $d\mu/dt > 0$ , ever getting materialized.

3. Technological progress was assumed in our model to be exogenous and the reason was again essentially to simplify algebra. It is possible to endogenize technological progress by adding another input, to represent, say, the category of biochemical inputs, into the production function and then regarding that input as the vehicle of technological progress. The allocation decision with respect to this input will be essentially similar to that of capital, only the number of equations and variables will increase.

4. It may be recalled that in the description of the initial state of the model, the labor market, unlike the credit market, was not assumed to be segmented. However, as a consequence of imperfection in the credit market and the possibility of nonrepayment of loan, it is possible for this initial state in the labor market to be replaced by localized monopsony requiring that the family farms supply their labor

to the local capitalist farm. In the face of nonrepayment of loan, imperfection of credit market may also give rise to monopsony in the commodity market in the sense that the family farms may have to sell their output to the capitalist farm at a price lower than the competitive market price. This kind of monopsonization, for one thing, may represent additional institutional means through which the process of immiserization of the family farm will go on. For another, by reducing the number of sellers in the commodity market, it may also give rise to some form of regionally localized monopolistic competition in the commodity market, in which case a part of the burden may also be shifted to the consumers outside the agricultural sector.

5. It is possible to include some other institutional forms of agriculture within the basic structure of our analysis. Inclusion of sharecropping, for example, will alter some of the allocation rules, but it can be shown, and here some of the results of Bhaduri's model can be profitably used, that the basic tendencies of the agricultural sector will not change in their qualitative properties. In the same vein, a more interesting generalization can be made if the money lenders and the capitalist farmers are considered as two separate classes. In a sense, it is somewhat difficult to visualize this situation, because it is not clear why, given an unequal distribution of income and the assured profitability of an imperfect credit market, a capitalist farmer will not consider money lending as another source of income. But if, because of reasons of uncertainty or some other non-economic consideration, e.g., the influence of the caste system, such a separation really exists,



then it will have a significant effect in eliminating the basic source of conflict responsible for the agricultural stagnation.

6. Finally, the results obtained exclusively within the agricultural sector can be generalized to accommodate the interactions with the industrial sector. The basic characteristics of this industrial sector have been outlined in Section 2. It is known to be partitioned into a private sector producing consumer goods and a government sector producing capital goods. The product as well as the credit market of this sector will have characteristics of imperfection. The imperfection of the product market will be implied in the properties of the relevant average revenue curve, whereas the credit market imperfection will be reflected by the dependence of the terms of borrowing on the wealth of the borrower. Because of the existence of these two kinds of imperfection at the same time, it will be found that a problem of conflict will again arise in the decision making about industrial expansion, and this conflict will have some similarity with the one faced by the capitalist farm in agriculture.

Given this structure of the industrial sector, its two most important links with agriculture will be through the commodity market and the labor market. The labor market link can be characterized by a Harris-Todar type of migration rule, and the product market by an expression of terms of trade involving the price and the income elasticities of the sectoral demand and supply functions. The interaction through labor market will have the effect of making the rate of growth of labor

supply,  $g$ , dependent on the effects of the industrial sector, whereas the impact of product market interaction will be felt in terms of the variation of  $\bar{P}$ . With these variations in  $g$  and  $\bar{P}$  precisely characterized, it will be possible to generalize the allocation rules of our basic model, which were initially derived with constant  $g$  and  $\bar{P}$ , to accommodate these variations and, through them, the interacting effects of the industrial sector.

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