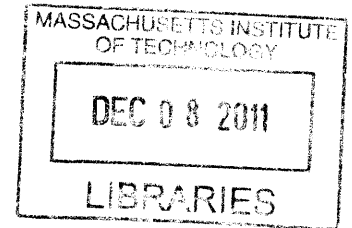


The Market Economy of Trips

By
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Master of Science in Architecture Studies
Massachusetts Institute of Technology, 2008

Professional Diploma in Architectural Engineering
National Technical University of Athens, 2004



ARCHIVES

Submitted to the Program in Media Arts and Sciences, School of Architecture and Planning in
Partial fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE IN MEDIA ARTS AND SCIENCES
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ABSTRACT

Mobility on Demand (MOD) systems allow users to pick-up and drop-off vehicles (bikes, automobiles) ubiquitously in a network of parking stations. Asymmetric demand patterns cause unbalanced fleet allocation decreasing level of service. Current redistribution policies are complex to plan and typically cost more than the usage revenues of the system. The Market Economy of Trips (MET) explores a new operation model based on a double auction market where cost-minimizing users are both buyers and sellers of trip rights while profit-maximizing stations are competing auctioneers that trade them. Trip rights are priced relatively to the inventory needs of origin and destination stations. A theory, a game, and a model are presented to explore equilibrium and limits of efficiency of MET under different demand patterns and income distribution.

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1 INTRODUCTION

1.1 Research area

The high cost of private vehicle ownership compared to their low utilization rates, and the increasing parking requirements compared to the decreasing available urban land, make private automobiles an unsustainable solution for the future of dense urban environments. In the US the average household has nearly 2 vehicles, which spend around 90% of their time parked while require three to five times their footprint in urban land to travel from an origin to a destination (National Household Travel Survey, 2001). Furthermore, capacity limitations in transportation roadways cause level of service to follow decreasing marginal gains as quantity of flowing vehicles increases. For example, more cars in the streets transport more commuters but throughput speed decreases as streets get more congested. In fact, urban economists know well that there is no such thing as sufficient capacity for an infrastructure system: the larger it gets, the more its demand grows such that a new saturation level occurs inhibiting its further growth. For example, constructing wider streets increases urban development attracting even more vehicles until a new traffic congestion level comes that eventually restrains further development (Duranton and Turner, 2009; Sterman 2000).

1.1.1 Sharing and Mobility on Demand systems

Sharing is an effective and popular method for sharing the cost, while increasing utilization of a resource allocation network when demand is greater than its supply capacity. A sharing policy provides shareholders with fractional ownership rights over the allocated resources allowing them to deposit or withdraw anywhere in the network. Banking systems with bank accounts, freight rental service networks with trucks, and airports with luggage carts, are few of the numerous examples. Recently, vehicle sharing has entered public transit systems as a complementary way to provide customized personal mobility (figure 1). One-way vehicle sharing systems, also known as Mobility on Demand (MOD) systems, utilize a decentralized network of parking stations and a fleet of shared vehicles that allow users to pick up a vehicle from any station and drop it off to any other station (one-way trips). If users don't find an available vehicle at the origin they switch to their best alternative option; if they don't

find an available parking place at their destination, they drive to the nearest available station.



Figure 1. A bike sharing system (left), and a car sharing system (right)

1.1.2 Operation problems of MOD Systems

Despite their convenience MOD systems have drawbacks too. Due to uncorrelated departure and arrival patterns, soon some stations end up having no vehicles while other stations end up having no parking spaces (Kaltenbrunner et al., 2008). This inventory imbalance not only decreases throughput, but it furthermore increases trip time as drivers search for parking spaces. To maintain level of service a MOD system needs to constantly feed origin with vehicles while drain destinations from occupied parking spaces. A redistribution process reallocates vehicles from stations with a surplus to stations with a deficit. To have a balanced system the vehicle redistribution flow should on average equal the net vehicle flow between sources and sink areas.

1.2 Area background

1.2.1 Manual redistribution methods

Existing bike-sharing companies rebalance the system by manually redistributing bikes among the stations using trucks, which is an operationally challenging and complex task (figure 2). Not only it is difficult to plan but it is also expensive: either the system requires large capacity or frequent redistributions¹. Moreover, redistributions keep vehicles unutilized reducing further service capacity. In addition, limited parking space

¹ For example Bixi, one of the most successful bike sharing service providers today reports they need 1 truck with 2 employees of 16 hours continuous operation per day for each 1000 bikes of their fleet in Montreal, Canada (From discussion with Xavier Barrera, from Public Bike System (Bixi)).

near stations increase average stopover time of trucks compared to the effective redistribution time. Furthermore, carrying capacity of redistributing means limits further efficiency: while bikes can be carried away with trucks, this is not an easy solution for cars, in which cases a service vehicle with two employees, one to drive the vehicles from full stations back to empty ones and another one to transport the first one to the next empty station, is required. As a consequence, existing bike-sharing systems end up spending higher costs for sustaining their performance than the revenues from the service they provide (EACI, 2011).



Figure 2. Truck carrying bicycles during redistribution, in Velib Paris

1.2.2 Incentive-based policies

Many bike-sharing experts today believe that the next generation of MOD systems will rely significantly on information technology and dynamic pricing to mitigate its operation costs (DeMaio, 2009; Mitchell, 2010). Various forms of incentive mechanisms have been implemented in the past in resource allocation networks for demand pattern regulation: congestion pricing zones, peak pricing in electricity grids, electronic markets/auctions such as eBay or Amazon, carbon trading programs, and recently, water banking (Karimov et al., 2010), are some of the current examples. The Changing Places group of the MIT Media Lab has been similarly exploring the potential of using qualitative incentives such as persuasive mechanisms or recommender systems to smooth demand imbalances in MOD systems. While qualitative incentives may have an impact on human behavior, quantitative incentives such as market mechanisms and dynamic pricing are particularly attractive because they translate directly into quantifiable payoffs that can be studied by the tools and methods of economics and

game theory. In transportation networks, incentive mechanisms focus on two goals: to improve throughput performance at links, and/or to prevent stock overflow at nodes. However, as information technology comes at a cost, the resulting savings from a policy must justify the costs and complexity of its implementation. The following cases illustrate examples that have used such mechanisms to improve efficiency.

1.2.3 Variable pricing in one-way truck rentals

Interstate one-way truck rental companies use variable pricing depending on the pick-up and drop-off locations. For example, U-Haul charges \$1154 the trip from Detroit, MI, to Houston, TX while only \$679 the other way around¹, a price difference of about 170%. While U-Haul mentions that their rates are also based on demand and supply of a fleet that is always on the move, major state tax differences between states such as Texas and Michigan have a significant impact on pricing too. U-Haul does not provide further information neither on how it calculates rates nor on what the estimated benefits are.

1.2.4 Congestion pricing systems

London was the first European city to employ congestion pricing as a means to regulate traffic and improve transport efficiency. The revenues from pricing were used to directly fund transport improvements fulfilling two goals: decreasing car traffic while increasing public transportation performance. The London program was based on the economic principle that consumers should pay directly for the costs they impose as an incentive to use resources efficiently. Nevertheless it used a fixed pricing structure that did not reflect individual utilization, raising questions about the fairness of the price levels. Despite its criticism the system is considered successful by increasing by 37% the average traffic speed and decreasing significantly social commuting costs (Litman, 2006).

Bangalore launched a similar pilot project named INSTANT to explore congestion reduction by awarding drivers monetary credits depending on how far from the peak hour their trip arrival times were (Merugu et al., 2009). Despite its behavioral impact on travel patterns the project did not explain whether the benefits from congestion reduction justified the costs of the incentives.

¹ Quote for a 10' truck taken from U-Haul website on 8/15/2011

1.2.5 Dynamically-priced parking systems

In summer 2011, San Francisco launched SFpark, the first pilot program in the US on dynamically priced public parking (figure 3). In its pricing policy documentation SFpark specifies that while currently pricing is still a complex and empirical process done with the help of staff from the San Francisco Municipal Transportation Agency (SFMTA), the end goal of the system is to eventually establish a demand-responsive pricing mechanism (SFpark, 2011). Currently, pricing of each location is organized in three time zones (morning, noon, evening) while the pricing rates of each zone are being readjusted each month based on occupancy data collected from the installed smart meters. Price information of parking spaces is available on line on a Google-maps application through SFpark's website. As of now, the system is still under calibration and there are no scientific studies on its behavioral impact.



Figure 3. The SFpark system

1.2.6 Bike sharing incentive systems

Velib, a bike sharing company in Paris, follows a similar policy to SFpark but by rewarding 15 minutes of bonus time of service time to the users who ride their bikes up to the 100 elevated stations at hills of Paris with the sign V+¹ (figure 4). While Velib's incentives had a positive impact on demand patterns there have been limited results on reducing overall system's redistribution costs and furthermore there is no justification on how incentives are calculated.

¹ "Velib, Frequently Asked Questions: Service & usage", accessed August 5, 2011, <http://en.velib.paris.fr/How-it-works/FAQ2>

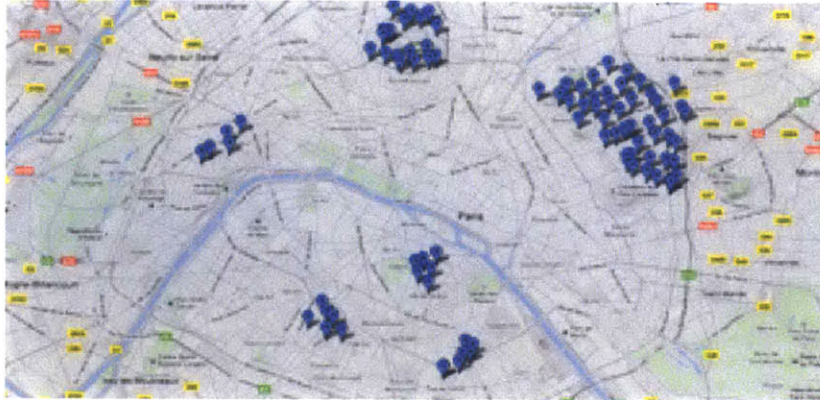


Figure 4. Velib V+ bonus stations located at hills in Paris

Experience shows that pricing trips based on their drop-off locations can significantly affect user behavior in shared systems. However, no MOD system to date has managed to decrease its operation costs in a sustainable way (EASI, 2011).

Can incentive-based mechanisms create sustainable MOD systems?

1.3 Research goals

This thesis has the following goals:

To explore the extent to which market price incentives can create autonomous, sustainable, scalable, and self-organizing MOD systems that eliminate, or minimize, the need for manual redistribution.

To design a simple, scalable and easily implementable pricing model that does not rely on expensive and complex technological infrastructure dealing with vehicle tracking.

To explain decision-making in dynamically priced MOD systems and explore the conditions under which an equilibrium state may exist.

To develop an educational toolset for designers and non-expert policy makers to study behavior and stability of dynamically priced MOD systems.

1.4 Literature review

MOD systems are a relatively new domain in the transportation literature. Most of current work focuses on analysis of datasets that existing bike sharing programs release to understand human travel patterns (Jensen et al. 2010; Kaltenbrunner et al. 2008). Other work focuses on modeling system performance given a demand pattern (Wang et al. 2010; Barth and Todd, 1999) as well as fleet management redistribution planning (Nair and Miller-Hooks, 2010) using stochastic modeling. In logistics, a similar problem known as backhauling is mostly pertinent to the freight transportation industry where the goal is to decrease empty trips of trucks between server and client nodes of centralized supply chain networks by routing optimization.

The use of market and auction mechanisms as a means to manage resource allocation, scheduling, decision making, and energy consumption in distributed networks has been studied thoroughly in computer science, mechanism design and game theory with applications in sensor networks, computational grids and train networks (Parkes, 2001). For mechanism design the question is to find a pricing policy that can achieve a desired outcome while for game theory the question is to find a payoff equilibrium in a game. Market mechanisms and user interfaces have also been studied as ways to create *hidden markets* (systems where users participate without being aware of it) with applications in P2P backup systems, smart grids, and online display advertising (Seuken, 2010).

The effect of land prices on population migration patterns as well as the social costs of public parking has been thoroughly studied in urban economics (DiPasquale and Wheaton, 1995; Shoup, 2011). Such studies however typically focus on long-term urban transformation processes rather than potential application on MOD systems. Economists have also used hydraulic models to explain the concept of value, prices, and marginal utility (Fisher, 1892).

Performance and stability of resource allocation systems have been studied in system dynamics and agent based systems. System dynamics (SD) studies macroscopic behavior of complex feedback systems under different policy scenarios through causal loop analysis and determinist stock-flow models of differential equations (Forrester, 1968) providing abstraction, educational clarity, and computational robustness (figure 5). Agent Based (AB) modeling on the other hand simulates dynamics of complex

systems by replicating the microscopic behavior of discrete individual agents through behavioral rules, lacking therefore educational clarity as a decision making tool (Rahmandad and Sterman 2006). Several ecosystems have been modeled and studied in the past in the system dynamics literature including behavior of economies under different governmental policies (Wheat, 2007), performance of industrial systems (Forrester, 1961), and evolution of urban infrastructures (Forrester, 1969).

Diffusion processes are typically modeled in SD literature by cascading compartments that describe different states of the resource and simulating the diffusion rate between them. System dynamics model often include stochastic elements and decision rules. However, for simplicity, the model described here omits these; the trajectories of the variables should be thought of as the average or expected values of the variables. Application examples are the famous SEIR model for propagation of epidemics and the Bass model for diffusion of market innovation (McKendrick, 1925; Bass, 1969). To describe heterogeneity compartment models cascade many compartments, each one representing a different state, or density, of the flowing resource (Rahmandad and Sterman 2008).

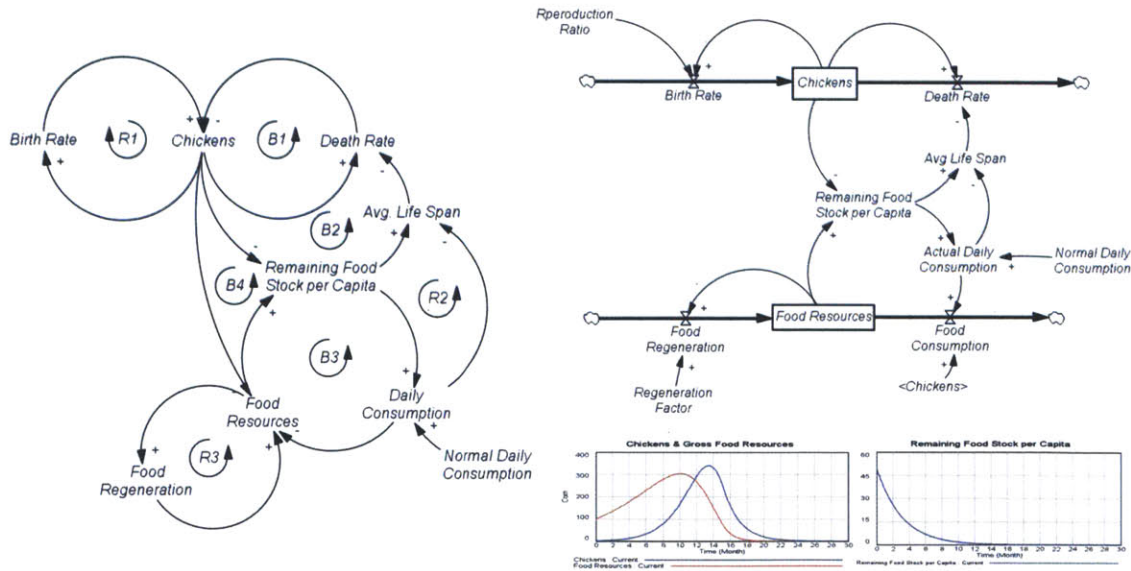


Figure 5. A system dynamics example: causal-loop analysis (left) and stock-flow model with simulation results (right) that describes the rise and fall of a population of chickens based on the consumption and regeneration of food resources

1.5 Contribution

This thesis makes the following contributions:

It defines a dynamically priced MOD system as a marketplace with users and stations as participants and trip rights as goods. The goal of this system is to maximize the circulation rate of vehicle resources.

It shows that a typical market model where stations are sellers and users are buyers of service cannot address sustainably the above goal.

It rather proposes a new operation model to a MOD system of a distributed double auction hidden market where cost-minimizing users are both buyers and sellers of trip rights while profit-maximizing stations are competing auctioneers that trade those trips.

It presents a solution using both a competitive and a flow equilibrium that takes into account the average price and speed of the substitutes and the distribution of cost of time of the participants.

It proposes a pricing model that uses information from station inventories. Such model decreases implementation complexity, scales up easily, and applies to any vehicle type. Furthermore, it can either be controlled by a computer, or provide a business model where individual station owners control pricing.

It presents the design of a game that captures the economic principles of the pricing model to empirically assess how far human behavior is from that suggested by theory.

Finally, It presents a simple system dynamics model to study behavior of MOD systems under different demand patterns.

There is no prior work I am aware of that has addressed these goals in the context of a MOD system.

1.6 Thesis organization

The rest of this thesis is organized as follows:

Chapter 2 describes the key requirements of a price mechanism and explains why current dynamically priced parking policies cannot meet a MOD system's goal. Next, it

presents the operation model of the Market Economy of Trips, analyzes it as a marketplace, and explains decision-making and equilibrium. Chapter 3 presents the design and rules of a board game experiment that follows the economic principles of MET and can be used to empirically explore decision-making and sustainability. Chapter 4 presents a system dynamics framework that explores performance of MOD systems under different demand patterns. Finally, chapter 5 concludes and discusses future work.

2 THEORY

2.1 The requirements of a policy

The goal of a MOD system is to sustainably increase the circulation rate of its resources to serve as many users as possible with its limited capacity. Given this goal, a pricing policy should address at least three issues:

First, it should help the system converge to the desired goal by rewarding actions based on the level of contribution of the users towards that goal. Pricing trips based on the demand and supply condition of their destinations cannot create a converging mechanism for a MOD system; such strategy aims at balancing station inventories, rather than increasing circulation rate. Consider a MOD system with a pattern of prices distributed at the stations. Now suppose that two users start at two different origins A, and C, and want to go to the same destination B. The drop-off prices of stations P_A , P_B , and P_C reflect the supply-demand trends at their inventories such that $P_A > P_B > P_C$ (figure 6). While each user departs from stations with different inventory trends, it is equally expensive for both of them to go to B. Clearly, it would be more beneficial for the system if the user from station A completed the trip instead of the user from station C since A has a greater need for parking spaces than C. However, by taking into account only the drop-off price of C it is impossible to distinguish which of the two actions will make better for the system. Since a trip requires both a vehicle at an origin and a parking space at a destination, a price mechanism should take into account the price condition of both origin and destination stations. In a parking system what is priced is a stock; in a MOD system what must be priced is a flow, defined as a relative difference between stocks.

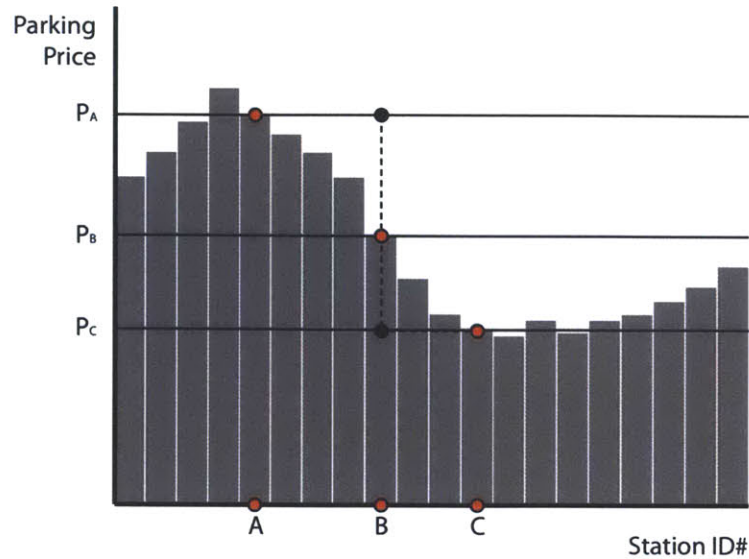


Figure 6. Drop-off price distribution: payoffs should be calculated as price differences

Second, it should be sustainable during convergence by generating the funds required for financing the rewards. While many obvious forms of incentives can change human behavior (who wouldn't drop-off his bike few blocks further for \$100?) not all can create sustainable ecosystems (where will these \$100 come from?). In a self-balancing MOD system some users are driving vehicles from stations with decreasing inventories (net sources) to stations with increasing inventories (net sinks), while other users are returning the vehicles back from the stations with increasing inventories to the stations with decreasing inventories¹. The pricing mechanism should explain that during price equilibrium while high-payers are seemingly paying the system higher prices for unbalancing trips in practice they should be paying the low payers to redistribute the vehicles. This is a form of double auction game where users are both sellers and buyers while stations are the auctioneers.

Third, it should ensure that prices would not change faster than the average time it takes for the system to converge (system's latency). Prices change based on the results from users' actions while users' actions in turn adapt to the price changes. The time it takes for the actions to have a result on the system's state depends on the

¹ The term *net source* describes a temporary state of a station during which the demand rate of departure requests is currently higher than the demand rate of arrival requests. Similarly, the term *net sink* describes a temporary state of a station during which the demand rate of departure requests is currently lower than the demand rate of arrival requests. A station can change from a net source to a net sink depending on its demand pattern.

average trip time: in a small MOD deployment, say a university campus, price changes would take minutes to reflect on inventory levels. In large MOD deployment, say a metropolitan district where people commute from suburbs to the center, price changes would take hours to reflect on inventory levels. If prices changed faster than the system's latency period, the system would be constantly out of control. The volatility of prices is a function of the gross capacity of the system given a demand: undersized systems would change prices faster, while oversized systems would change prices slower.

2.2 The operation model of the Market Economy of Trips (MET)

The Market Economy of Trips (MET) is an operation model for MOD systems consisting of a pricing mechanism, a financial system, and a graphic user interface that prices trips relatively to the inventory needs of the origin and destination stations (figure 7). Each station calculates a pick-up price and a drop-off reward based on its throughput rate, its inventory change rate and price information from its neighbor stations (figure 8). The price of a trip is paid during drop-off and is determined by the algebraic difference between the pick-up price of an origin at the time of departure and the drop-off reward of a destination station at the time of arrival. If the origin has higher pick-up price index than the destination's drop-off price index then the user gets the difference from the system as a reward; if the destination has a higher drop-off price index than the origin's pick-up price index then the user pays the system the difference; if the origin has the same pick-up price index as the destination's drop-off index then the user pays nothing. In a variation of this method, stations compute only one price index, and the price of each trip consist of the difference between the origin and destination price indexes plus a standard fare for the traveled distance. Thus the same drop-off location may be cheap for someone and expensive for someone else depending on where they come from. MET is a self-organizing system operated by its users for its users.

Users can access online price information at the stations or while they are driving, through handheld mobile devices. MET is not a recommender system; it does not suggest what a user should do, since this would imply that his cost of time is known. Instead it aims to help users perceive their payoffs to make better decisions. Users that consider a trip to be expensive may opt out choosing another option (e.g. walking, taxi, public transit, or private automobile). Such form of location-based price information

could be visually communicated to the users with various ways such as scalar fields, heat maps, vector fields, or contour maps (figure 9): isometric price curves describe areas with same price indexes. Like navigating through a price landscape, climbing from valleys up to hills is expensive, descending from hills down to valleys is rewarding, while traveling through flat areas is neutral. The price landscape may be continuous, with few hills and valleys, or discontinuous, with multiple segregated hills and valleys. Acceptable price continuity and volatility during trip time would depend on the calibration of the pricing mechanism and the latency of the system. The amount of information displayed to the users will depend on the cognitive burden that an average user may afford and is yet to be determined experimentally as a future step of this work.

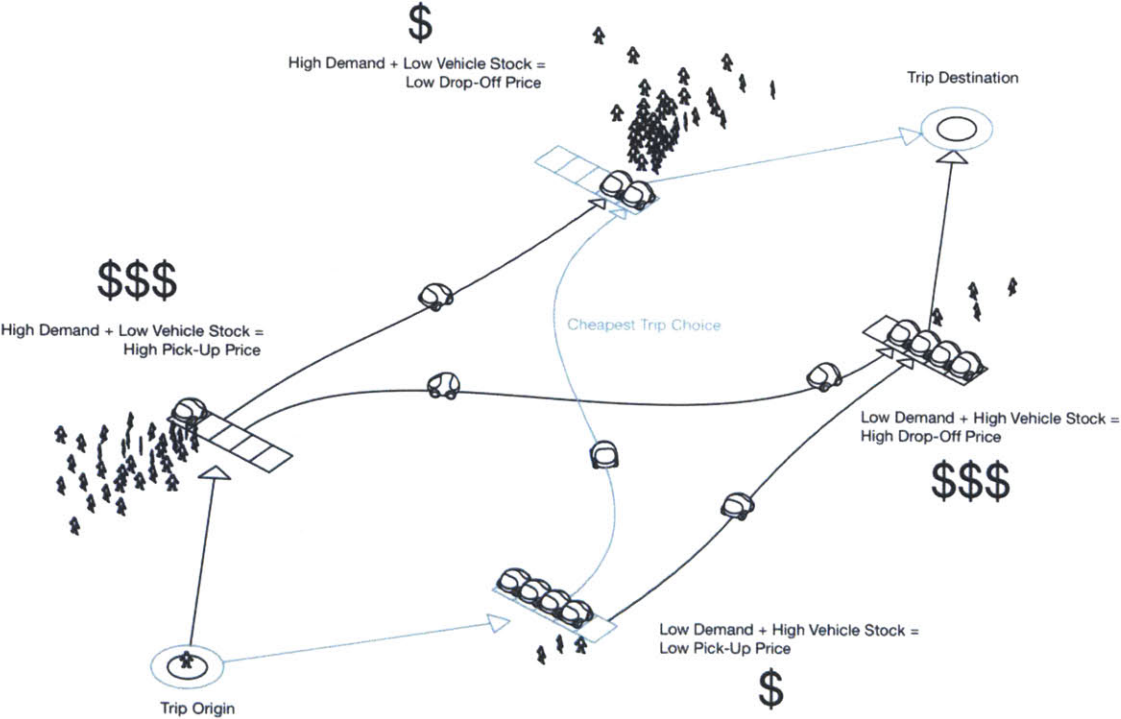


Figure 7. The origin-destination path (in blue) with highest payoffs in MET

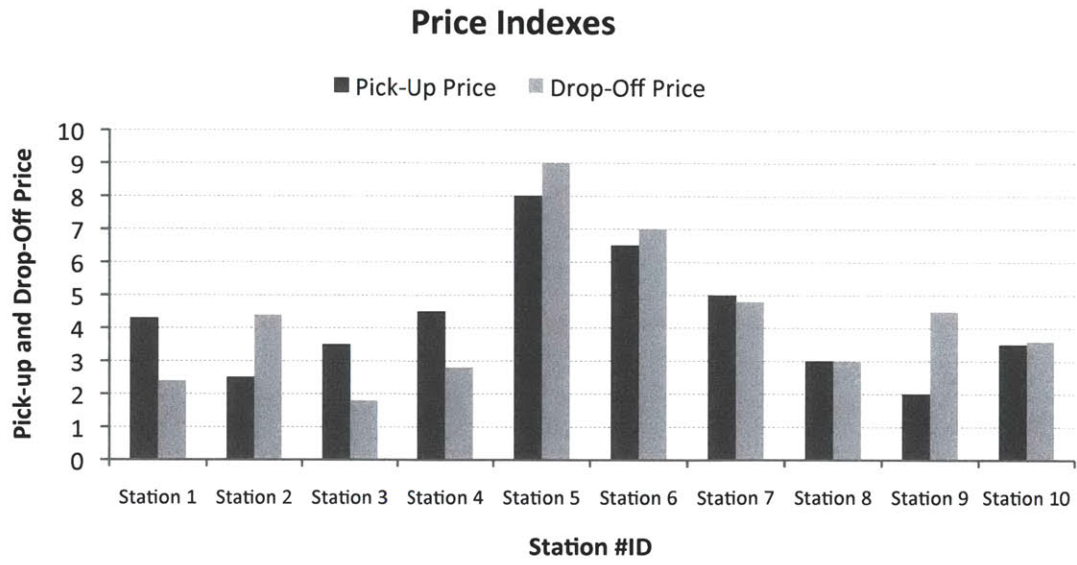


Figure 8. Price indexes of stations

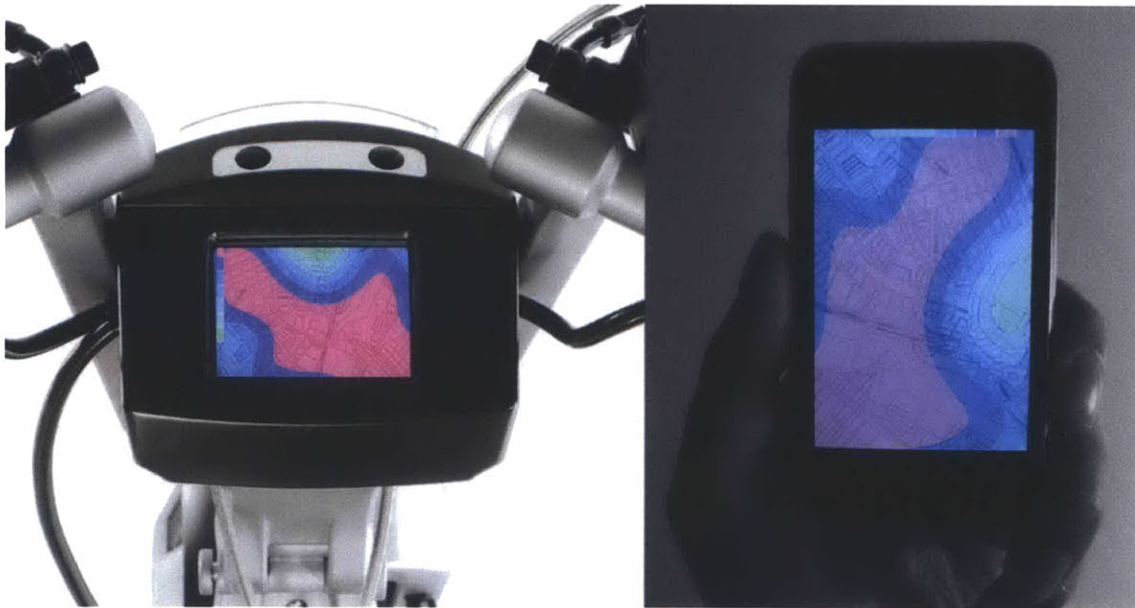


Figure 9. Contour-map GUI: a possible interface for communicating payoffs using isometric curves of price indexes

2.3 The cost of trips

Time is money. Urban mobility options are priced based on the level of service and commuting times they offer in relation to the cost of time of the commuters. Commuters whose cost of time is high will be willing to pay higher for faster options while commuters whose cost of time is low will be willing to pay lower prices for slower options. Urban trips however are combinations of multiple mobility options: you walk from your house down to a nearby bus station; you take the bus to go to the downtown center; you ride a bike inside the center; finally, you walk again to your final destination. The commuting cost of each compound trip is the sum of the prices that have been paid for each option plus the total cost of time that was spent while traveling. People select bundles of options that maximize their utility¹.

2.4 Analysis of MET as a marketplace

The Market Economy of Trips is a marketplace where price-taking users trade vehicle and parking rights (trip rights from now on) while price-making stations act as brokers that handle the transactions. Users buy trip rights from origin stations to pick-up a vehicle and sell them back to destination stations to drop the vehicle off. Pick-up prices are determined based on the demand and supply of vehicles at the origins while drop-off rewards are determined based on the demand and supply of parking spaces at the destinations. The Market Economy of Trips consists of the following participants: the MOD vehicles and the alternative transportation option; the high-paying and low-paying users; and the stations-brokers with a central bank. The following analysis assumes that people think rationally and can efficiently perceive their payoffs.

For the price-taking users the net result of the transaction should yield less commuting costs than using their best alternative option. For example, a user traveling from an origin O, and ends to a final destination D, will select any drop-off station Q if the price from O to Q with MOD plus the price from Q to D with the substitute (e.g. bus, taxi, walking, etc.), plus the total time costs (from O to Q with MOD, and from Q to D with substitute) are in sum less or equal than the original price from O to D with the substitute plus time cost with substitute (figure 10, figure 11). Users whose time cost is

¹ While utility can be defined by many factors such as comfort, convenience, reliability, etc., this thesis focuses only on the commuting cost as this is defined by the price of an option and the associated time cost of traveling.

high will therefore be willing to pay higher prices in MET to minimize their total commuting time, while users whose time cost is low would be willing to spend more time in commuting for a better price in MET. For the price-making stations, the transactions should balance revenues from pick-ups with costs from drop-offs and capital expenses such as vehicles and urban land occupation, and yield a marginal profit.

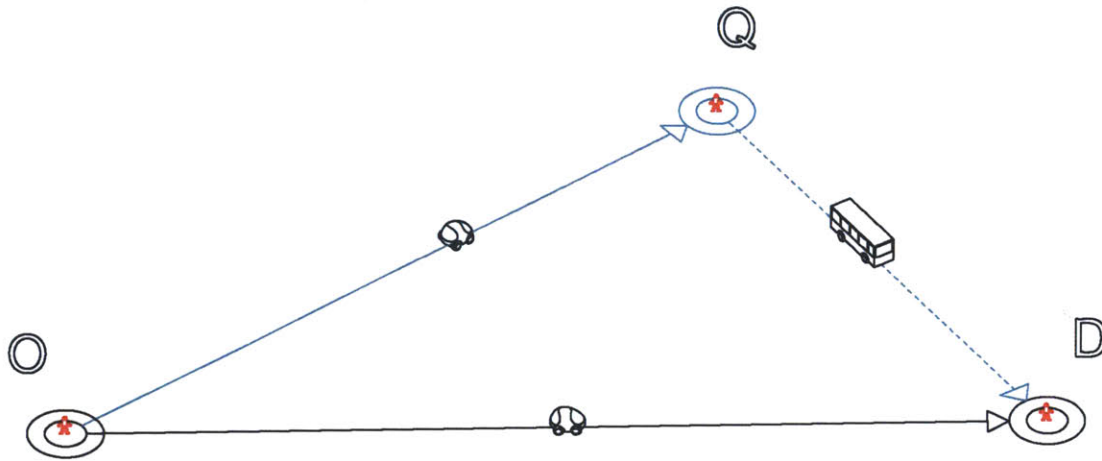


Figure 10. User decision making process. In equilibrium payoffs from OD should equal payoffs from OQ plus QD

The Market Economy of Trips is a form of a strategy game. Territorial decisions of users change the pricing of stations, which changes the payoff landscape affecting decision making of other users and vice versa. Urban Economic theory (DiPasquale and Wheaton, 1995) suggests that users with sufficient information would make decisions that minimize their time-adjusted commuting costs bringing eventually the system into a *competitive equilibrium* where no further action can be taken to increase anyone's payoffs. Such equilibrium would come when no high paying and no low paying users prefer any other resources at the prices, including not participating.

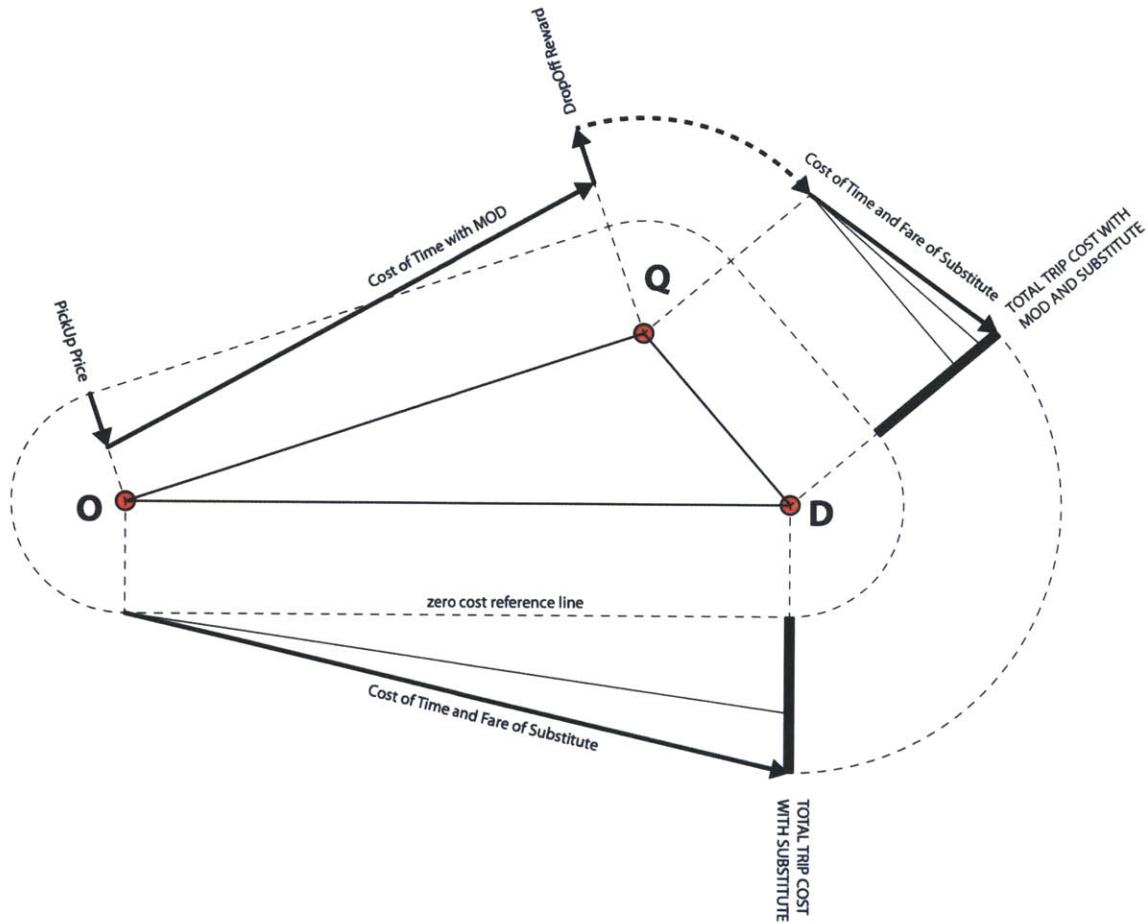


Figure 11. Price equilibrium graphic analysis of figure 10: The total trip cost with MOD from O to Q and with substitute from Q to D consist of: the downward pointing pick-up price vector at O; the up-warding inclined price vector from O to Q that depends on user's cost of time and MOD's traveling speed; the up-warding drop-off reward vector at Q; and the up-warding inclined vector from Q to D that depends on user's cost of time, substitute's traveling speed, and substitute's fare per distance traveled. The total cost is indicated by the thick line at D. Similarly, the total trip cost from O to D with the substitute consist of the up-warding inclined price vector from O to D that depends on user's cost of time, substitute's traveling speed, and substitute's fare per distance traveled.

2.5 Convergence of MET

Suppose the Market Economy of Trips starts in *maximal flow equilibrium*. In this ideal state, vehicle inflows and outflows are equal in each station and as a consequence all inventory levels are stable (flow equilibrium). Furthermore vehicle flows are maximal and as a consequence all inventory levels are the same (maximal flow equilibrium)¹. As

¹ The maximal flow equilibrium of the system is described in section 4.7.

demand pattern becomes unbalanced, net source stations increase pick-up prices and drop-off rewards to decrease departures and increase arrivals, while net sink stations decrease pick-up prices and drop-off rewards to increase departures and decrease arrivals. As the gap between price indexes increases, moving from net sources to net sinks gets more expensive, while moving from net sinks to net sources gets cheaper or profitable. Commuters with low cost of time will be willing to select low-cost pick-up stations further from their origins and drop-off stations further from their destinations as long as this is still cheaper for them than using the alternative option. Similarly, commuters with high cost of time will be willing to select expensive pick-up stations closer to the net sources and drop-off stations closer to the net sinks as long as the net result is still cheaper for them than using the alternative. In the most unbalanced demand pattern, the cost that each user is paying is marginally the same as the cost he would be otherwise paying with the alternative option. Any additional change from this point will cause some users to opt out, selecting the alternative option. As some users respond to prices while others opt out, the demand pattern asymmetry decreases causing the gap between prices at each station to decrease. This in turn increases throughput performance bringing eventually the system into a new equilibrium state.

2.6 Price hills and price valleys

All else being equal, a station that increases its drop-off reward relatively to the drop-off rewards of its neighbors is effectively increasing its drop-off area of service by paying both the commuting cost and the time of a user that would otherwise drop-off at a destination nearby to drop it off to the station and then commute back to his original destination with the alternative (figure 12). Similarly, all else being equal, a station that decreases its pick-up price relatively to nearby stations is effectively increasing its pick-up area of service by paying both the commuting cost and the time of a user that would otherwise pick-up from a station nearby to drop-off at the station and drive back to his original destination with the alternative.

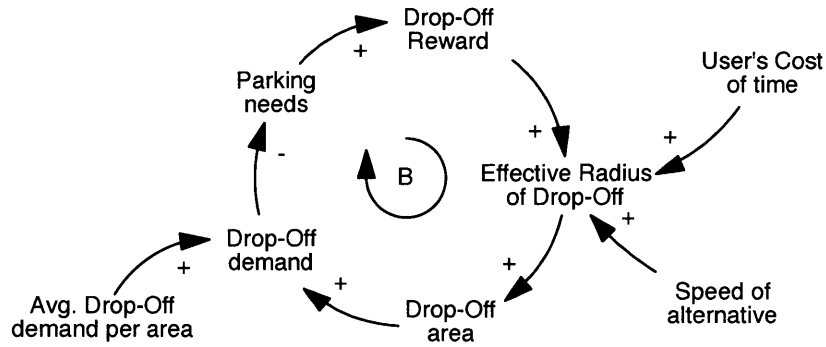


Figure 12. Change of effective drop-off area as rewards increase due to a decrease in drop-off demand

As a net sink station decreases its drop-off reward and its pick-up price, it increases the cost of the inflowing (inbound) trips, causing inbound flow to be directed to its relatively cheaper neighbors. This snowball effect causes gradually its neighbor stations to overflow too, redirecting inbound flow even further. Gradually the overflowed net sink stations create a local valley of low price indexes. The initial overflowed station that caused the snowball effect lies in center of the valley while the drop-off reward prices gradually increase towards the periphery. The price valley will continue lowering until the portion of the inbound demand flow with the lowest time costs cannot travel efficiently from the periphery to the center of the valley using the alternative option. Similarly, a local price hill will be formed when outbound demand drains inventories at net source stations. Each additional inbound demand causes all prices in a price hill to increase or a price valley to decrease expanding its boundary periphery outwards.

2.7 The financial system of MET

While the physical resource system is a watertight system (the vehicles can never escape the vehicle sharing system), the financial resource allocation system is an open one (money flows in as revenues from users and flows out as costs). The financial system of MET can be studied in two levels. At the microeconomic level, the revenues of each station are determined by the pick-up price multiplied by the departures rate, while its costs are determined by the drop-off reward multiplied by the arrivals rate, and the payments for capital expenditures. During convergence, stations are trying to balance revenues with costs by adjusting pick-up and drop-off prices to redirect revenues from departures to costs for rewarding arrivals. At the macroeconomic level,

the gross revenues of the system come from the flow of high-paying source-sink trips multiplied by the difference between the pick-up price at net sources and drop-off reward at net sinks, while the gross costs go on financing the flow of redistributing sink-source trips plus capital expenditures. During convergence, MET is trying to balance revenues with costs by adjusting trip prices to redirect revenues from expensive source-sink trips to costs for rewarding sink-source trips.

While each station has a local balance account for revenues and costs, during operation it may enter in a state of either financial deficit or financial surplus. There are two main reasons for this. First, there is a delay between the impact of pricing on users and the effect of their actions at the stations, which mainly depends on the average travel time. Second, during pricing the local financial resources of the station may not be sufficient to provide the desired rewards to incentivize users which could potentially bring the performance of the station down to a vicious cycle: less available funds, less rewards offered to low-payers, less redistribution from low-payers, less demand from high-payers, less profits, and finally even less available financial resources. To avoid such circumstances, stations with financial deficit may need to borrow from stations with financial surplus. This fund reallocation must be carried out by a central bank and a tax/subsidy policy. Stations make tax payments to the bank as a percentage of their profits while pulling subsidy funds from the bank to cover their needs for rewards. The central bank provides a buffer that can sustain the system during demand fluctuations.

The above observations are illustrated in the following two stock-flow diagrams. During maximal flow equilibrium (all inventories are the same and demand pattern is symmetric) no money goes to the public transit, all pick-up prices and all drop-off rewards are equal; both low and high payers pay the same price (figure 13, top). During financial disequilibrium financial resources are redirected from high-paying users to low-paying users through the stations and the bank, which in turn are redirected from low-paying users to the alternative option (figure 13, bottom).

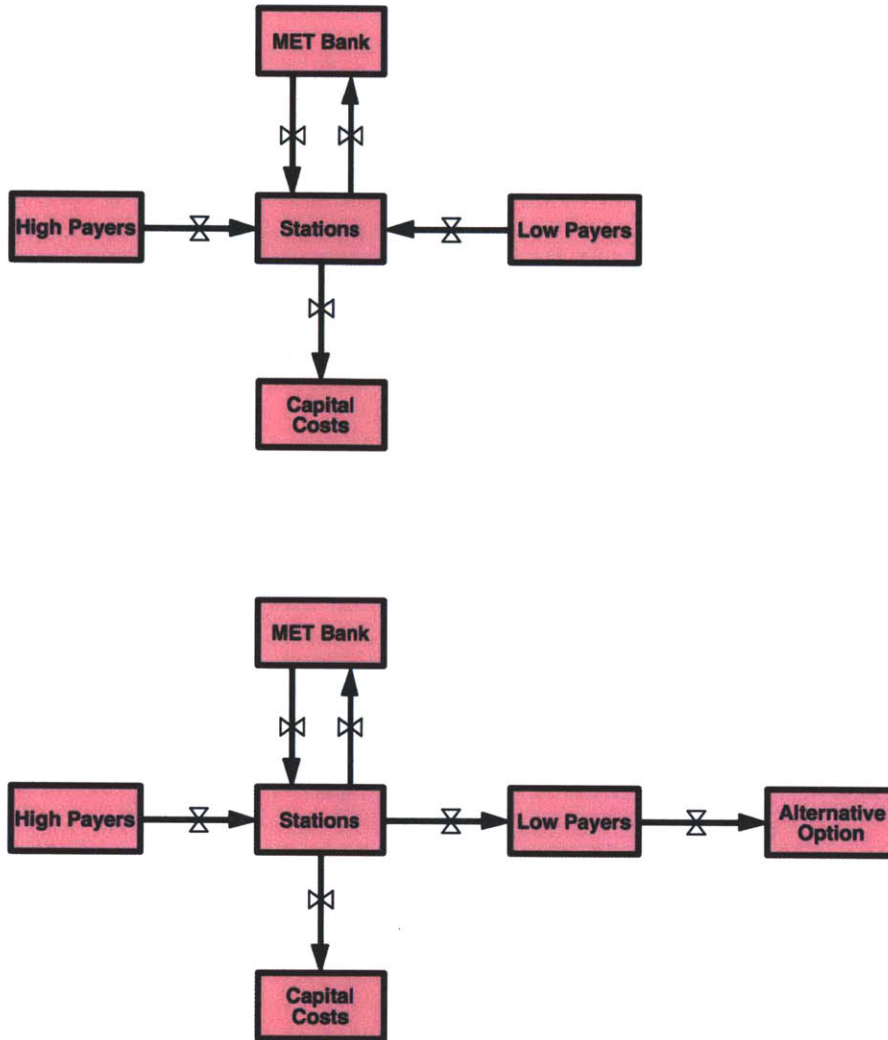


Figure 13. Cash flow system during financial disequilibrium and symmetric demand pattern (top), and asymmetric demand pattern (bottom)

2.8 Competitive equilibrium

During long-run stable price equilibrium the following observations should hold true for the system.

Vehicles flow from net sources to net sinks equals vehicles flow from net sinks to net sources and each flow equals the average throughput rate.

Both high-paying users and low-paying users make a portion of their trip using a MOD vehicle and another portion of their trip using the substitute. The distance that the high-

paying users cover with the substitute is on average less than the distance that the low-paying users cover with the substitute.

The total commuting costs that each of the two groups of payers pay consist of the price they pay for MOD as this is defined by the pick-up price and the drop-off reward, the price they pay for the substitute, the time cost they suffer by using a MOD vehicle and the time cost they suffer by using the substitute. Since the substitute is slower than the MOD vehicle its time costs are higher than those of the MOD vehicle.

The cash flows that high-paying users pay to MET equal the cash flows that MET pays to the underpaying users plus the cash flows that MET pays for capital costs.

The cash flows that high-paying users pay to MET equal the throughput rate times the difference between average pick-up prices at net source stations and average drop-off rewards at net sink stations.

The cash flows that MET pays to the underpaying users equal the throughput rate times the difference between average pick-up prices at net sink stations and average drop-off rewards at net source stations.

The average difference between pick-up price and drop-off reward at any station equals the tax payment that the average station pays to the bank to cover capital costs.

The cash flows that low-paying users pay to the substitute plus their total time costs (by both MOD and the substitute) minus the cash flows that they receive from MET are less or equal to the cash flows that they would otherwise have to pay to travel from sources to net sinks with the substitute plus their time costs with from the substitute

Similarly, the cash flows that the high-paying users pay to MET plus the cash flows they pay to the substitute plus their total time costs (by both MOD and the substitute) are less or equal to the cash flows that they would otherwise have to pay to travel from net sources to net sinks with the substitute plus their time costs with from the substitute (which are much higher than those of the low-payers because high-payers have high time costs).

Therefore the distribution of the personal cost of time on the population of commuters has a significant role on the performance of MET as it indicates the likelihood that some commuters would be willing to buy the time for other commuters. In a population

where everyone evaluates time in the same manner nobody would be willing to spend more to commute faster and consequently nobody could earn rewards to drive further.

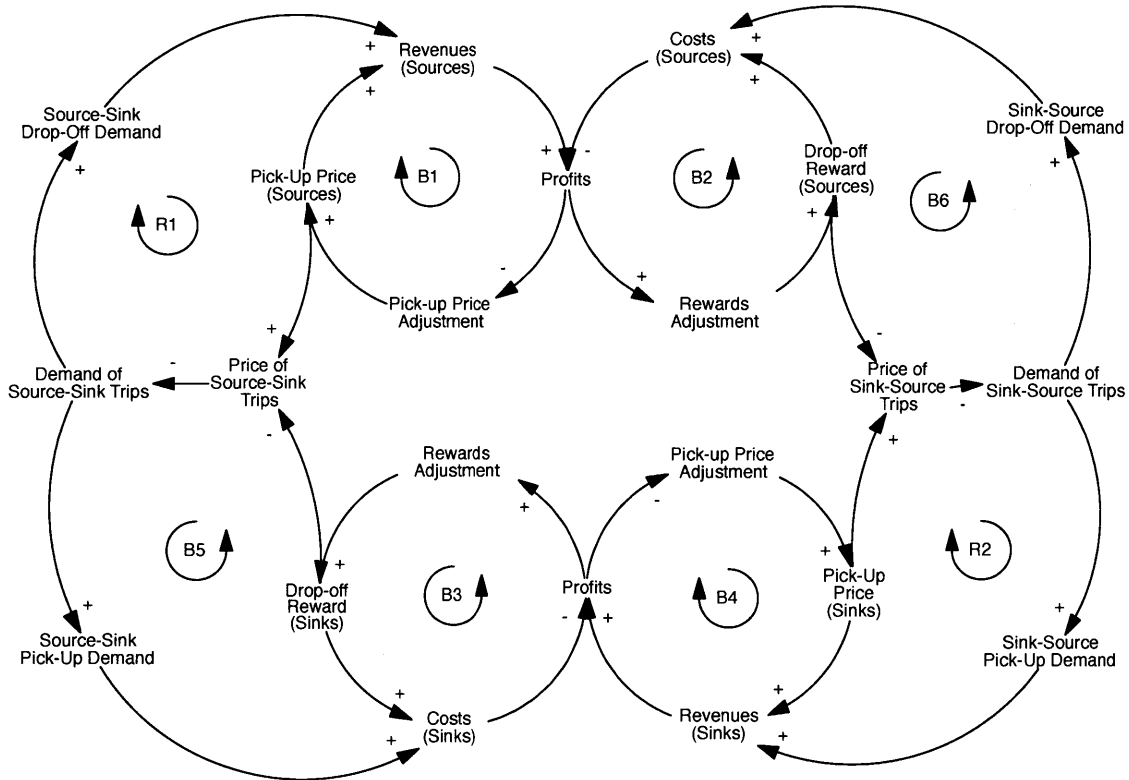


Figure 14. Causal-loop diagram of MET showing the stabilizing force of pricing to demand for both source-sink and sink-source trips. The cost of substitute, the total time costs, and the trip distances for group of users are not shown

2.8.1 Price Limits

The maximum payments that the remaining high-paying users are willing to pay will determine the maximum rewards that the remaining redistributing users can take so that the system is still breaking even. The cost of time of the low-paying users will in turn determine how far they are willing to drive for those rewards. Furthermore, the asymmetry of the demand pattern in combination with the maximum distance that the low-paying users are willing to drive will determine how much rebalancing can be made. Finally, feeding the net sources with vehicles will determine the reliability of the sharing system compared to the best alternative which in turn will determine the adoption rate and level of performance, closing this way the main balancing feedback loop of MET.

Generally, the caused fractional increase in the cash outflows for rewards should be no more than the expected fractional increase in the revenue cash inflows from payments after adding the capital expenses (vehicle and land capital expenditures). Marginally, they should be equal.

$$\frac{\Delta(\text{arrivals_rate}) * \Delta(\text{dropOff_reward})}{\Delta(\text{departures_rate}) * \Delta(\text{pickUp_price})} \leq 1$$

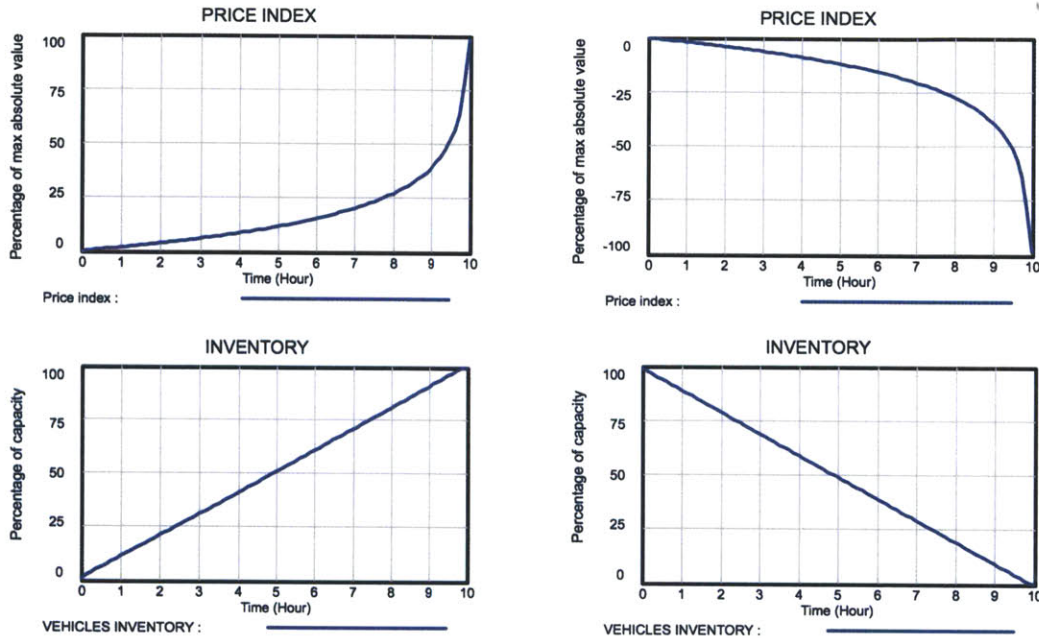


Figure 16. Relation of inventory level to price index level in time: A price index should increase non-linearly as inventory increases (left, parking spaces deplete) and similarly decrease non-linearly as inventory decreases (right, vehicles depletes). Graphs are illustrative indicating curve trend.

3 THE MET GAME

3.1 Purpose

Whatever theory suggests, it assumes that people can efficiently perceive their payoffs and they will think rationally, an assumption that is far from being realistic. The MET game is a board game that aims to empirically investigate how far from optimum can human intuition be in perceiving and evaluating payoffs, by replicating the essential economic principles of MET. A second purpose of MET game is to collect data that can be then used to calibrate the Vehicle Dynamics framework.

3.2 Description

The current design of the game is based on multiples of six, a physical constraint coming from the sides of the dice used to play the game. In a future version the MET game will be computer based and therefore the layout will be redesigned allowing multiple players to play simultaneously.

3.2.1 Layout

MET game consists of a one-dimensional array of 24 neighborhood blocks that contain commuters and MOD vehicles. Neighboring blocks are connected through a substitute commuting line. The layout of the array may be a straight line or a closed ring (figure 42). Neighborhood blocks have placeholders for placing commuter pawns of each player; a placeholder with limited capacity for placing vehicle pawns; and 2 special placeholders for bidding the drop-off reward and the pick-up price in the form of currency chips.

3.2.2 Players

The game is played with two or more players and one banker. Each player controls a team of 6 numbered commuter pawns and needs to accomplish a list of missions by reallocating his pawns between origin and destination blocks. On each round each player throws a dice to determine which pawn he will move. Assigning more than one pawns per player allows each pawn to spend random time in each neighborhood block. The banker controls the pricing of the MOD stations and the currency transactions with

the commuter players. In a future version of the game, once the pricing algorithm is finished, the computer will be controlling the banker and the prices.

3.2.3 Movements

There are two options to move a commuter pawn between two blocks: either using the substitute, or using a MOD vehicle, as long as there is an available vehicle pawn at an origin and an available parking space at the destination; in such case the vehicle must be moved too. Any combination of those two ways is possible so for example if a player needs to move his commuter pawn from block A to block F, he can use the substitute from A to C, then use a vehicle from C to G, and then use the substitute again to move from G to F (figure 18).



Figure 18. MET game block array

3.2.4 Cost and time

Since commuting consumes both time and money, players have two scarce resources to manage for accomplishing their missions: time chips and currency chips. Both time and currency chips have the same exchange value.

Each transportation option consumes different amount of time chips and money chips per trip. For example, a MOD vehicle, which is faster, takes 1 time chip per block, and as many currency chips as the algebraic difference between the pick-up price at the origin block and the drop-off reward at the destination block that the banker sets. The substitute on the other hand, which is slower, takes 2 time chips and 1 currency chip per block traveled. The total commuting cost of a player is the sum of all the currency and time chips he needs to spend to move from an origin to a destination.

Players begin with the same aggregate budget of time and currency chips; however, the percentage of each type of chip that a player gets is different. For example, player 1 may start with 30 time chips and 70 money chips while player 2 may start with 60 time chips and 30 money chips. The first player thus has less time but more money to spend, while the second player has more time and less money to spend. Players with many time chips and few currency chips should be willing to travel longer for better

prices while players with many money chips but few time chips should be willing to spend more money for minimizing their trip distance.

3.2.5 Game play

On each round, the banker can modify prices at each block by placing currency chips at the drop-off reward placeholder and writing down the requested amount at the pick-up price placeholder. If a player uses a MOD vehicle he will pay the banker the pick-up price at the origin and receive from the banker the drop-off reward at the destination. He will also spend the corresponding amount of time chips for the distance he covers, namely 1 time chip per traversed block. If instead the player uses the substitute, he will spend the corresponding amount of time and currency chips for the distance he covers, namely 1 currency chip and 2 time chips per each traversed block. After each origin-destination mission has been completed, the banker may readjust the prices at the blocks. Obviously, the banker cannot bid more currency chips than the ones he currently has in the bank. To prevent large price changes the banker has a limited amount of price changes he can make in each block. For example he might be able to adjust each of the pick-up and drop-off prices, if needed, by adding or subtracting only 1 or 2 currency chips. Since on each round a player may use both a MOD vehicle and the substitute to complete his mission, the resulting cost will be a combination of the total time and currency chips he spent in each option.

As the game continues, time chips drain out of the system while money chips first circulate several times among the players through the banker and then they drain out of the system too. For example, the player with more currency chips than time chips will be paying high prices to the banker to minimize his trip time and the banker will be then offering this money back to the player with less currency chips than time chips to redistribute vehicles. The player that receives those rewards will in turn be spending them to travel with the substitute. Any payment to the substitute drains out of the game.

3.2.6 Scoring

Each time a player completes an origin-destination mission he earns 1 point. Each time a player uses a MOD vehicle the banker earns 1 point. If players don't play strategically the currency will quickly drain away and eventually they will drop out of the game with the score of points they made so far. It may be possible as a variation to occasionally have players earn currency and time chips, after completing several missions for

example, in order to increase the game duration. While none of those additional chips guarantees that a player will win, they do provide him resources to accomplish more missions and earn points.

3.2.7 Winning

Winner will be the player that at the end of the game has the highest score in points. Therefore, each of the player needs to make the decisions that will maximize his points while making sure that the financial resources are being circulated between the players instead of draining out. For example, the banker has an incentive of bidding the prices in such a way that the users will prefer the MOD vehicles rather than the substitute. Therefore he must guess how the players will think. Similarly, the commuters try to take the decisions that will keep them as long as possible in the game given their resources, in order to score as high as possible. Therefore players will try to manage their available resources in order to earn as many points as possible: the commuters by wisely selecting combinations of options to complete their OD missions, while the banker by selecting those price schemes that will attract the players to use a MOD vehicle.

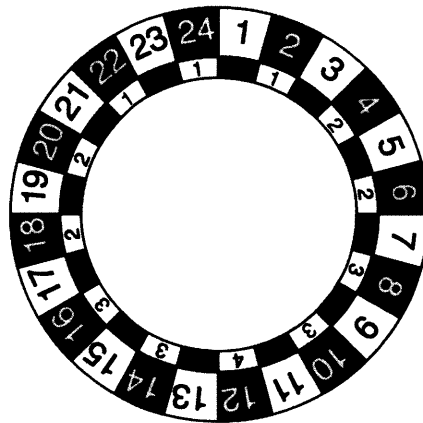
3.2.8 Creating the demand pattern

To study the behavior of MET we must find a way to create asymmetric demand patterns using the dice. I present the following two methods:

In the ring layout, commuter pawns can move only clockwise (figure 19 top). On each round, the player whose turn is to play throws first one six-sided dice to decide which of his 6 numbered commuter pawns to move. Next he throws a set of dice ranging from 1 to 4 to decide how many blocks away (clockwise) his final destination will be. Each neighborhood block has a number ranging from 1 to 4 indicating how many dice a player needs to throw if his pawn happens to be at that block. This method creates an asymmetry in the demand pattern since there will be clusters of neighborhood blocks from which outgoing trips will be statistically shorter and other blocks from which outgoing trips will be statistically longer.

In the straight-line layout, commuter pawns can move in both directions (figure 19 bottom). Each block is numbered incrementally starting from 1 to 24 and has a second number ranging from 1 to 4 that indicates how many dice a player needs to throw if his

pawn happens to be at that block. Similarly to the ring layout case, the player whose turn is to play first throws one six-sided dice to decide which of his 6 numbered commuter pawns to move. Next he throws as many dice as it is written in the block where his pawn is in order to decide the index number of the destination block for his pawn. This model has a more centralized layout, as the first 6 blocks have higher probability to become the next destination for a pawn.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4

Figure 19. The MET board game layout: each block has an incremental ID number ranging from 1 to 24 and a second number ranging from 1 to 4 indicating the number of dice that must be thrown to determine the next destination of a pawn starting from that block

4 THE VEHICLE DYNAMICS FRAMEWORK

4.1 Purpose

Vehicle Dynamics is a new system dynamics framework that models behavior of resource allocation in MOD systems with limited capacity. The framework's purpose is study throughput capacity under different demand pattern scenarios, with potential applications in other domains.

4.2 Understanding the problem

Throughput performance of a MOD system decreases either because departure rate drops due to unavailability of vehicles at origins or because trip time increases due to unavailability of parking spaces at destinations. The following causal-loop diagram, consisting of one reinforcing loop (R1) and four balancing loops (B1, B2, B3, B4), illustrates this concept (figure 20). A positive sign (+) of an arrowhead means that the effect is positively related to the cause; a negative sign (–) of an arrowhead means that the effect is negatively related to the cause. For any distribution of the demand pattern (depended on the Upstream and Downstream demand), throughput rate increases with recycling of resources (R1) and decreases with depletion of vehicles (B1, B3) and parking spaces (B2, B4).

In equilibrium, all inventory stocks remain stable; during disequilibrium however, vehicles move from areas with higher density in users than vehicles (net sources) to areas with lower density in users than vehicles (net sinks). This is a form of spatial diffusion.

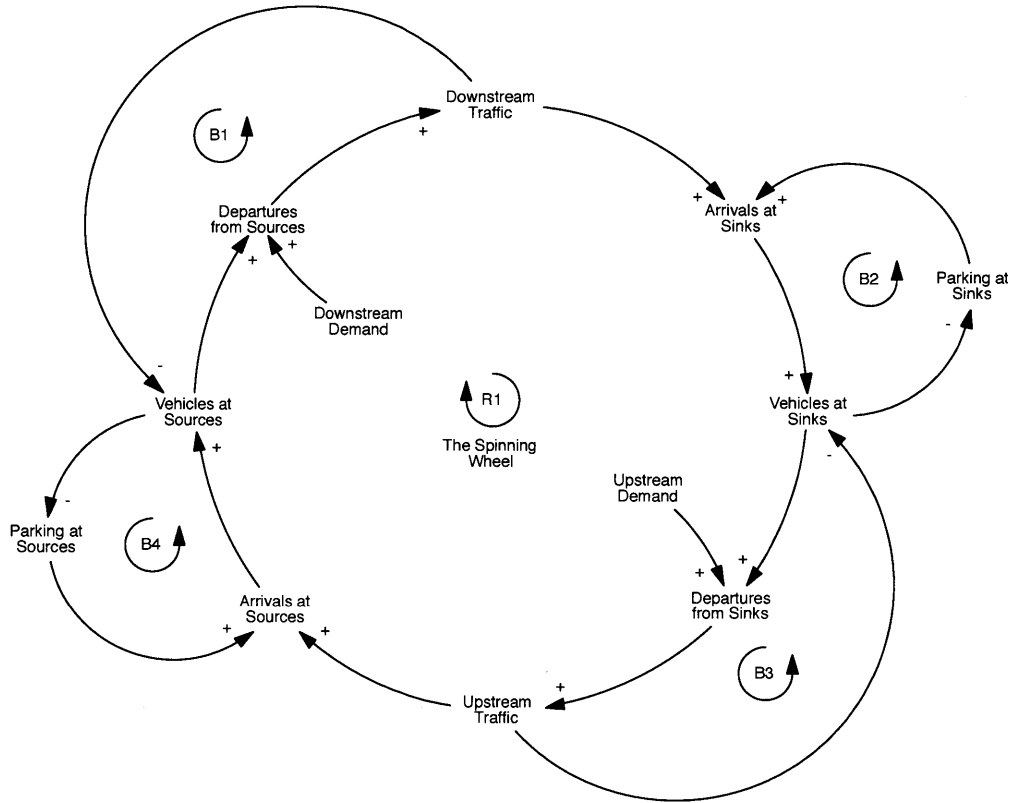


Figure 20. MOD system causal-loop diagram

4.3 Modeling spatial diffusion in system dynamics

Diffusion models describe the dissipation of a resource from areas of high density to areas of low density. Diffusion rate is relative to the distribution of the density of the resource in space and therefore ceases when density is homogeneously distributed. As an example consider the dissipation of a highly pressured gas in a void room. As the pressure of gas is higher than that of the air, the gas will continue dissipating in the room. Eventually dissipation stops when gas and air have the same pressure (figure 21).

In a spatial diffusion like our case, the diffusion rate between each compartment depends on the spatial correlation of the distributions of users and vehicles at the stations, and moreover on how many total inventory capacity net source and net sink areas include: since a trip needs both a vehicle and a user to coexist in a station, diffusion ceases when there are no more vehicles at the stations where users wait, or alternatively when there are no more users at the stations where vehicles are parked.

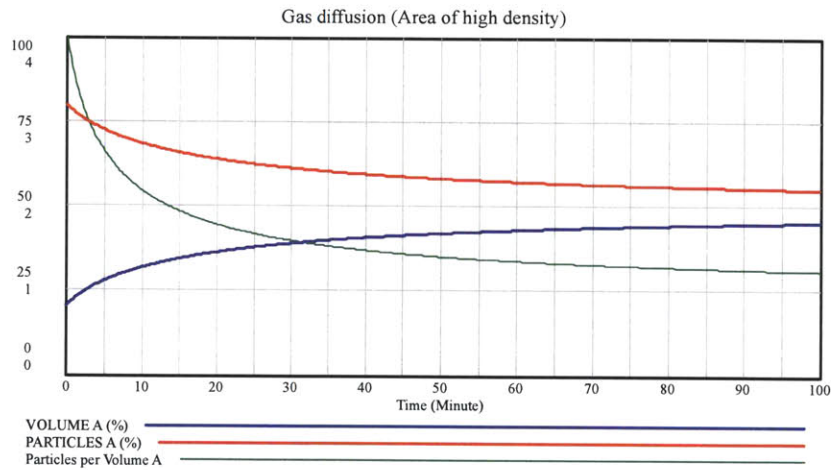
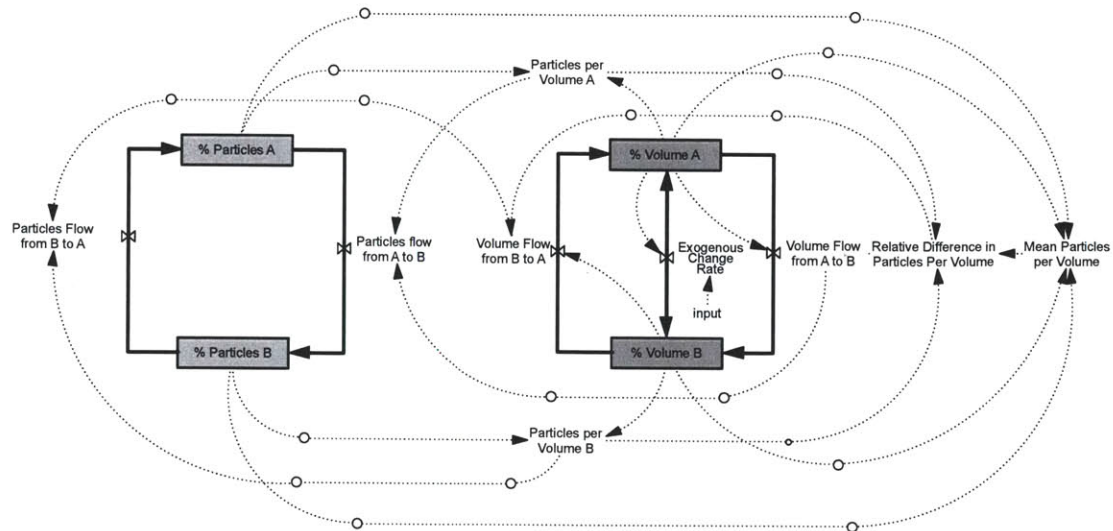


Figure 21. A simple system dynamics model of a diffusion process of a gas modeled as pressure exchange between areas of high (A) and low (B) density. The blue curve indicates the high-density volume of the gas as a percentage of the total volume, while the red curve indicates the particles contained in the high-density volume as a percentage of the total number of particles. The green curve indicates the particles per volume.

4.4 Describing a heterogeneous distribution as two homogenous groups

In practice, people describe distributions with phrases such as “81% of drop-off demand concentrates on 22% of the stations”. Our model should both capture intuition accurately and provide control to simulate different scenarios by describing the threshold boundary between the high and low density and how this boundary changes

in time. Statistically, distributions are described using derivatives of standardized moments (e.g. the averaged sum of the nth power of the difference of each sample from the mean); the more the standardized moments are, the better the description of the distribution is. In practice, most descriptions use only the first three standardized moments (e.g. the mean, variance, and skewness). This is the same as approximating the discrete distribution of n samples with a continuous distribution of only two values above (Y1) and below (Y2) the mean such that the mean, variance, and skew are the same. Practically we are looking for X1, X2, Y1, and Y2 if we know the mean, variance, and skewness of a distribution (Figure 22).

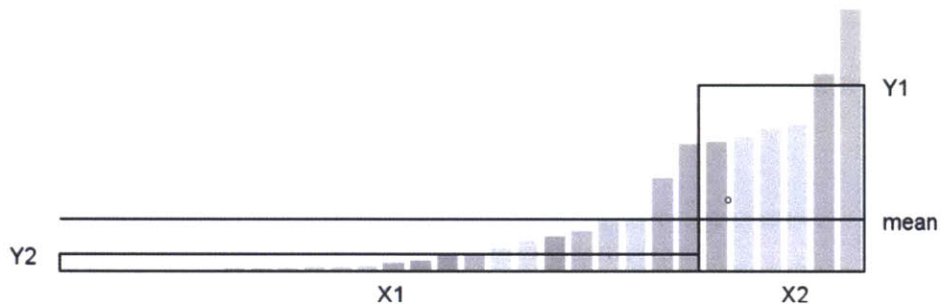


Figure 22. Simplification of a distribution

From the first three standardized moments we get the following four equations:

$$\mu_1(y) = \frac{(y_1 \cdot x_1 + y_2 \cdot x_2)}{x_1 + x_2} \quad (1)$$

$$\mu_2(y) = \sqrt{\frac{(y_1^2 \cdot x_1 + y_2^2 \cdot x_2)}{x_1 + x_2}} \quad (2)$$

$$\mu_3(y) = \sqrt[3]{\frac{(y_1^3 \cdot x_1 + y_2^3 \cdot x_2)}{x_1 + x_2}} \quad (3)$$

$$x_1 + x_2 = \text{sample population} \quad (4)$$

Then, solving the system of equations we get the following for x1, x2, y1, y2:

$$Y_1 = \frac{1}{2} \frac{1}{\mu_2(y)} \left((x_1 + x_2) \mu_3(y)^3 \pm \sqrt{(x_1(y) + x_2(y))^4 \mu_3(y)(y)^6 + 4(x_1(y) + x_2(y)) \mu_2(y)^3} \right) + \mu_1(y)$$

$$Y_2 = \frac{\mu_3^3(y) \cdot (x_1 + x_2)}{\mu_2(y)} - Y_1 + 2 \cdot \mu_1(y)$$

$$x_2 = \frac{\mu_2(y)}{(Y_2 - Y_1) \cdot (Y_2 - \mu_1(y))}$$

$x_1 = \text{sample population} - x_2$

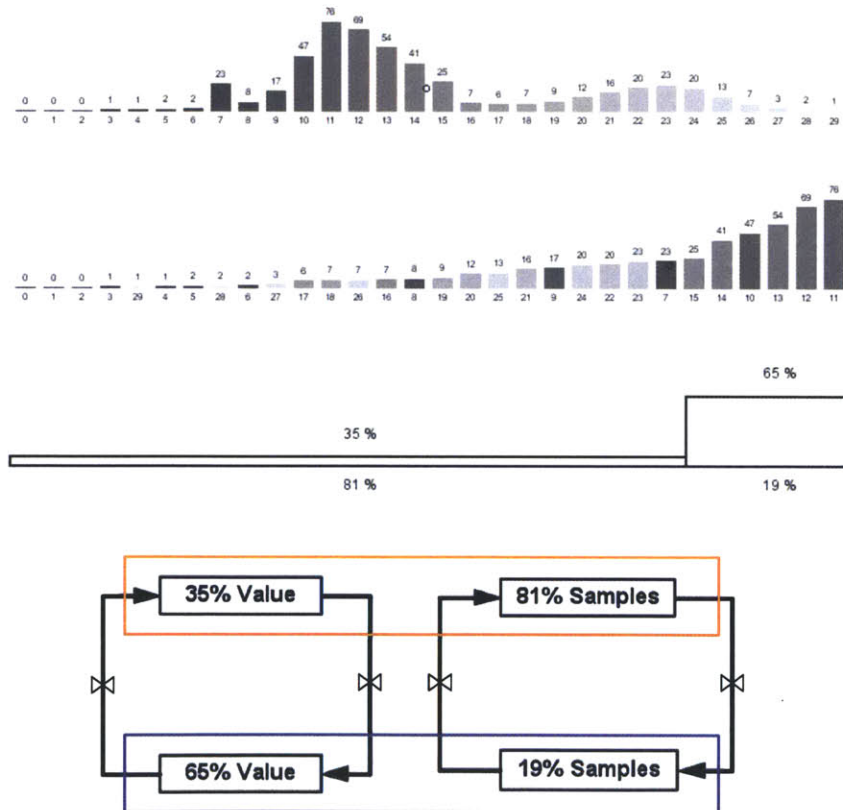


Figure 23. A distribution approximated into two groups (top) and a stock-flow interpretation (bottom)

Figure 23 top shows a distribution of samples with their average value, the same distribution sorted from highest to lowest, and its simplified description: 65% of value

concentrates on 19% of samples while the rest 35% of value concentrates on 81% of samples. Figure 23 bottom shows the same description as a stock-flow model: the circuit on the left represents the segregation of aggregate value into the two groups of samples while the circuit on the right represents the segregation of the samples.

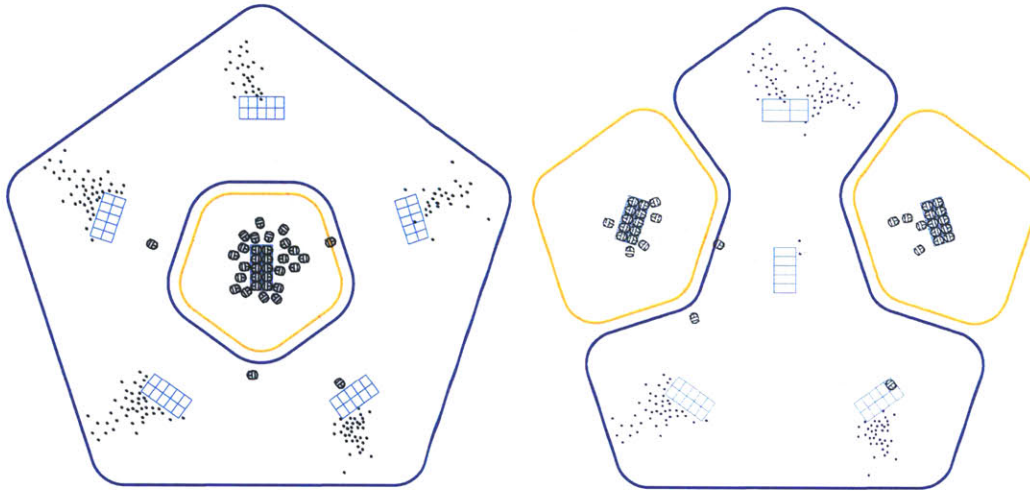


Figure 24. Illustrative spatial representation of net source (blue) and net sink (yellow) areas and how these change in time

4.5 Description of the stock-flow model architecture

Vehicle throughput rate depends on the correlation of vehicle and user distributions at the stations (figure 24). To describe this correlation I have built a model in Vensim, a popular software package for system dynamics modeling, consisting of three stock-flow subsystems: Stations, Users, and Vehicles, each segregated into net source areas (top) and net sink areas (bottom). The Stations model (figure 25, bottom-middle) describes the urban area that each of the two groups occupies (measured in numbers of stations, assuming that stations equally distributed in land). The Users model (figure 25, bottom-left) describes the user levels and the arrivals and departures flows into and out from each group. The Vehicles model (figure 25, bottom-right) describes the vehicle levels and the arrivals and departures flows into and out from each group as well as the stock that contains the vehicles in transit. The vehicle departures rate from each group (net sources and net sinks) depends on the minimum of the user and vehicle levels in that group. The vehicle arrivals rate to each group depends on the minimum of the portion of the vehicles in transit that currently travels towards that group (as this is defined by the input demand pattern), and the parking capacity that the group currently

has (as this is determined by the number of stations, the number of parking spaces, and the number of vehicles in that group).

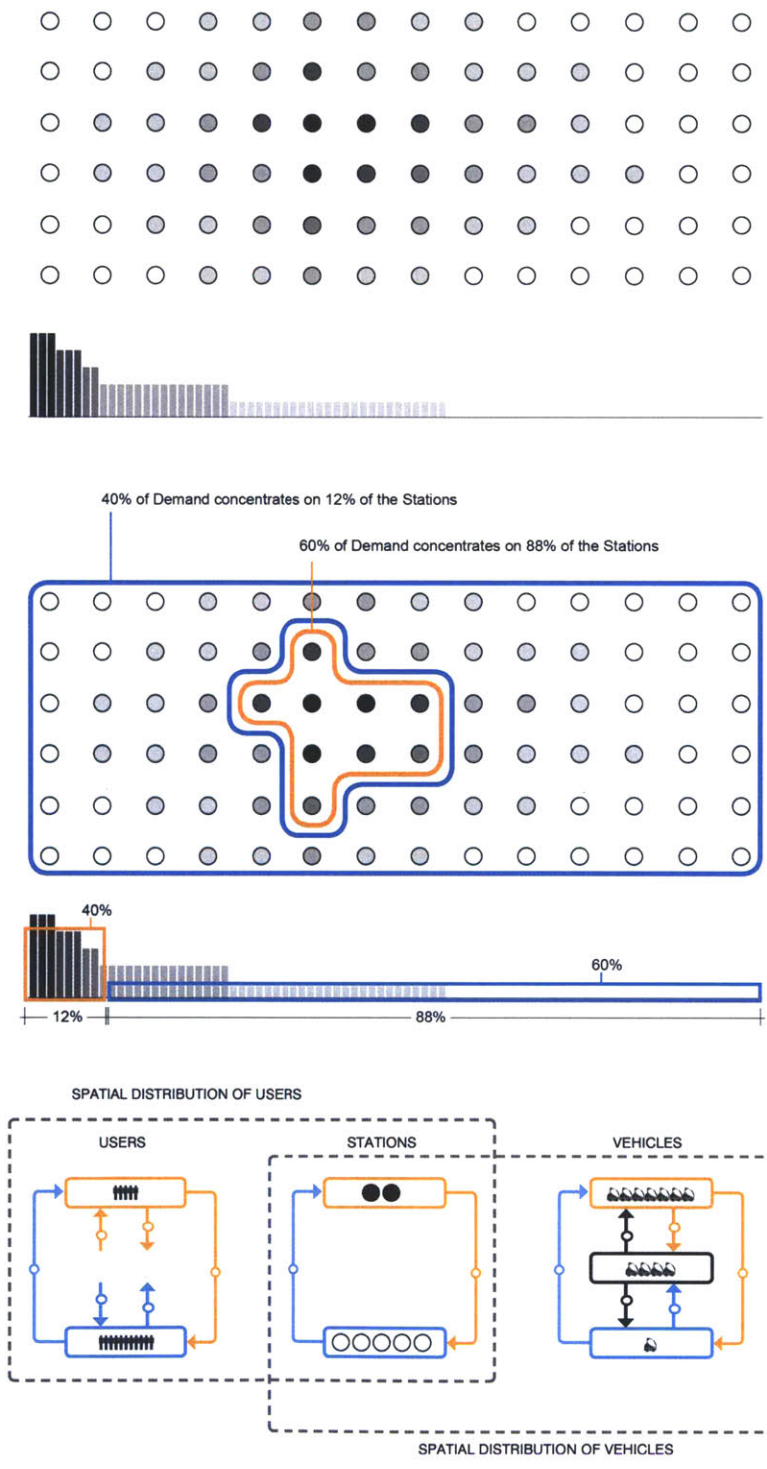


Figure 25. The spatial interpretation of the 3 stock-flow subsystems of the Vehicle Dynamics framework. The numbers used in the example are illustrative

4.6 Illustrative model behavior

4.6.1 Demand pattern

A demand pattern is a pair of time-based distributions of pick-up and drop-off requests at the stations. Demand patterns allocate vehicle and user resources in the system depending on the correlation of these two distributions and they therefore describe the rate of change of the distribution of both vehicles and users in the system. For example, in a highly correlated demand pattern for each station for each pick-up there is one drop-off. On the other hand, in a highly uncorrelated demand pattern massive drop-offs or pick-ups are persistently concentrated on few stations causing unbalanced fleet transfers, delays, and overflows.

4.6.2 Initial equilibrium state

Suppose that a MOD system starts in maximal flow equilibrium: all inventory levels are equal and the demand pattern is such that in every station, for each outgoing trip there is an incoming trip, and as a consequence inventory levels remain stable (figure 26).

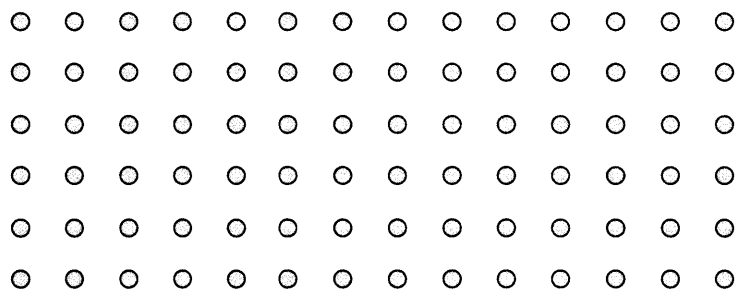


Figure 26. The system during maximal flow equilibrium: all inventories (dots) are stable and equal. Inflows and outflows are equal for each station

4.6.3 Disequilibrium

At some point in time suppose that the demand pattern changes in a way that now some stations have persistently higher vehicle arrivals than user arrivals, while some other stations have persistently lower vehicle arrivals than user arrivals. Obviously, inventories in the first group will be increasing and user queues will be decreasing, while inventories in the second group will be decreasing and user queues will be

increasing. Following the methodology described in section 4.4 let us call the first group as net sinks and the second group as net sources (yellow and blue areas in figure 27). A mutual resource reallocation occurs: vehicles are moving from net sources to net sinks, while users are moving from net sinks to net sources. Furthermore, since departures rate depends to the minimum of users and vehicles waiting at the stations (there needs to be at least a vehicle and a user to have a departure) throughput performance drops in both groups.

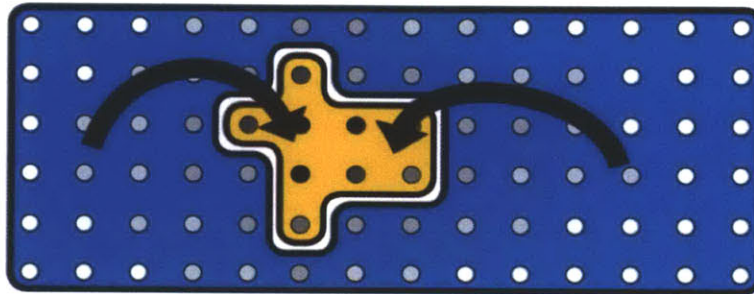


Figure 27. The system during disequilibrium. Stations are segregated into net sources (blue) and net sinks (yellow). Vehicles are reallocated from net sources to net sinks

4.6.4 Shift of disequilibrium

Suppose now that the demand pattern changes again such that net sink areas are now changing in shape, number, and location (figure 28, top). A mutual exchange of stations between them occurs (figure 28, bottom-left); if net source area expands spatially, then stations that were previously inside the net sink area are now entering the net source area (figure 28, bottom-middle); if instead the net source area contracts, then stations from it will flow towards the net sink area; if however it simply moves or changes shape without changing in size then there is a mutual inflow and outflow of stations between net source and net sink areas as their boundary changes (figure 28, bottom-right). It should be obvious that as stations enter and exit each group the aggregate levels of both users and vehicles in that group change too. For example, if a station changes from net sinks to net sources, then the total inventory and user stock in net sinks will decrease, while in net sources it will increase as defined by the average levels on each group.

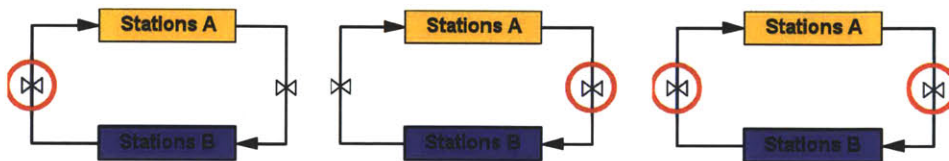
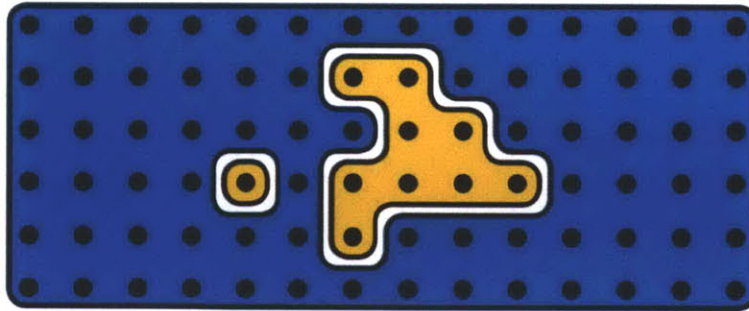


Figure 28. The system during transformations of net source and net sink areas as demand pattern changes. Vehicles continue flowing from net sources to net sinks.

4.6.5 Overflow of sinks

If demand pattern continues unchanged, soon net sources will start depleting from vehicles, while net sinks will start depleting from parking spaces. Arriving vehicles to net sinks that find no parking will drive to the closest available station towards net sources. As a consequence, vehicle arrivals in net sources will tend to increase *relative* to the departures, while the vehicle arrivals in net sinks will tend to decrease *relative* to the departures. This behavior will increase the average trip time of trips from net sources to net sinks, increase the vehicles in transit, and shift the boundary between the two groups: net sinks will tend to expand while net sources will tend to contract. The increase in time depends on the length of the border between the two groups and the density according which stations are scattered in space (figure 29). For example, if net sinks required X amount of additional stations from net sources to accommodate the overflowing demand, then the average increase in trip distance would depend on the ratio of the required acquired urban area from net sources divided by the aggregate length of the boundary of the two areas: dispersed clustered patterns with long boundaries would have less increase in trip time than centralized clustered patterns with short boundaries.



Figure 29. Overflow of net sink areas as inventories deplete. Additional vehicle flow from net sources to net sinks is pushed outwards. As a result the area of net sinks increases while the area of net sources decreases (red area)

4.6.6 New equilibrium state

Eventually the system reaches a new equilibrium state of less throughput rate, higher average trip time, and lower fleet in transit. Depending on the demand pattern, the size of the two groups and the amount of resources contained in them change continuously. The time that it takes the system to move from one equilibrium state to another depends on the average trip time and the stock in transit. The larger they are, the longer it takes.

4.7 Maximal flow equilibrium of the system

The *carrying capacity rate* is the maximum service rate that the system can provide during equilibrium and it depends on the distribution of the stocks in each of the three subsystems. Any user demand rate beyond carrying capacity cannot be served. Modeling how carrying capacity is affected by the demand pattern provides an objective benchmark not only for evaluating the ability of a vehicle sharing system to serve a demand pattern, but also for evaluating the potential redistribution cost. Multiple flow equilibrium states exist based on how the inventory levels are distributed among stations, ranging from an even distribution to an uneven distribution. Since the service rate from each of the two groups depends on the correlation of the users and vehicles stocks in that group but also on the available parking capacity, the system should reach its maximum capacity rate when both user and vehicle levels are equal in each group. During maximal flow equilibrium, the capacity rate depends on the average trip time, pick-up time, and fleet size. For example, if average trip time is 13min and the average time to pick up a vehicle is 2min, then each vehicle can serve on maximum

4 users per hour. For a fleet of 100 vehicles, the average maximum service rate would be 400 trips per hour and would segregated the fleet into 86.6% vehicles in transit and 13.4% vehicles in stations. The maximum capacity rate can be defined by the following equilibrium equations¹:

$$\text{Departures Rate} = \text{Arrivals Rate} \Rightarrow \frac{\text{Vehicles in Stations}}{\text{Avg. Pick-Up Time}} = \frac{\text{Vehicles in Transit}}{\text{Avg. Trip Time}}$$

$$\text{Fleet Size} = \text{Vehicles in Stations} + \text{Vehicles in Transit}$$

$$\text{Vehicles in Stations} = (\text{Fleet Size} - \text{Vehicles in Stations}) * \frac{\text{Avg. Pick-Up Time}}{\text{Avg. Trip Time}}$$

$$\text{Vehicles in Stations} = \text{Fleet Size} * \frac{\text{Avg. Pick-Up Time}}{\text{Avg. Pick-Up Time} + \text{Avg. Trip Time}}$$

$$\text{Max. Throughput Rate} = \frac{\text{Fleet Size}}{\text{Avg. Pick-Up Time} + \text{Avg. Trip Time}}$$

4.8 Throughput performance of a Station

A station's inventory can be increasing, decreasing, or being stable depending on the ratio of inflow to outflow rates. Stations serve drop-off requests by providing enough parking spaces and pick-up requests by providing enough vehicles. Arrivals rate depends on the minimum of the accumulated demand and the stock of available vehicles and the average pick-up time. Departures rate depends on the minimum of arrivals demand and parking spaces and the average drop-off time. Throughput rate (or average circulation rate) measures the rate according which a station serves both pick-up and drop-off requests. It is the average of pick-up and drop-off rated over a period, typically the operation cycle of the system. Depleting from either vehicles or parking spaces decreases throughput rate. The inventory level of a station that maximizes throughput rate during equilibrium depends on the inventory capacity, and the pick-up and drop-off times. To maintain a high throughput rate stations should aim to adjust accordingly arrival and departure rates so that their inventory level approaches the maximal flow equilibrium level (figure 30).

For each station it should be:

¹ These equations are "in expectation" and refer to the long-run average equilibrium. Due to stochastic effects, the actual values would vary.

$$\text{Station Departures Rate} = \frac{\min(\text{Users}, \text{Vehicles})}{\text{Pick-Up Time}}$$

$$\text{Station Arrivals Rate} = \min\left(\text{Requested Arrivals}, \frac{\text{Parking Spaces}}{\text{Drop-Off Time}}\right)$$

$$\text{Station Throughput Rate} = \frac{\text{Station Departures Rate} + \text{Station Arrivals Rate}}{2}$$

During maximal flow equilibrium inflow (arrivals) and outflow (departures) rates are equal and maximum:

Max Inflow = Max Outflow

$$\frac{\text{"Inventory Capacity"} - \text{"Inventory"}}{\text{"Drop-Off Time"}} = \frac{\text{"Inventory"}}{\text{"Pick-Up Time"}}$$

$$\text{"Inventory"} = \frac{\text{"Pick-Up Time"} * \text{"Inventory Capacity"}}{\text{"Pick-Up Time"} + \text{"Drop-Off Time"}}$$

$$\text{Maximum Throughput Rate} = \frac{\text{"Inventory Capacity"}}{\text{"Pick-Up Time"} + \text{"Drop-Off Time"}}$$

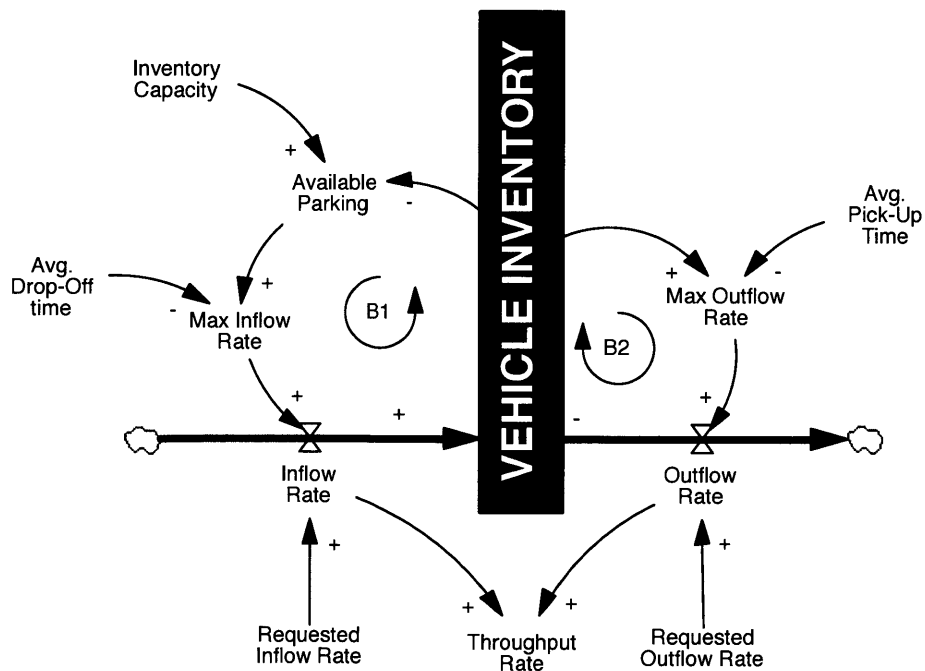


Figure 30. Stock-flow model of a station's throughput rate

4.9 Vensim model and early simulation results

Figure 31 (and figures 34, 35, and 36 in the appendix) show the Vensim stock-flow model, which contains around 120 equations. The model receives a constant gross demand rate equal to the maximum carrying capacity of the system and allows the distribution of the demand pattern to be modified. The controlling parameters are the following: Portion of gross user demand rate to net sinks (and net sources); portion of gross vehicle arrivals rate to net sinks (and net sources); circulation rate of stations from net source areas into net sink areas and vice versa; Average Trip Time; Average Pick-up Time. All other parameters are computed endogenously in the model.

To focus on the impact of demand pattern distribution characteristics, we start the model in equilibrium using the maximum capacity rate (net source and net sink areas are each 50% of the total area). The simulation horizon is 24 time units. At $t=5$ we introduce the following pulse change in the demand pattern for two time units after which it returns to the normal state: sink areas decrease from 50% to 20% and source areas increase from 50% to 80%; departure requests at sink areas increase from 50% to 60% while departure requests at source areas decrease from 50% to 40% of the gross demand rate; arrival requests at sink areas decrease from 50% to 40% while arrival requests at source areas increase from 50% to 60% of the gross arrivals rate. Figure 32 shows the throughput performance of net sources (blue), the increase in trip time (red) of the trips towards net sinks, as well as the vehicle inventories and available parking at net sink areas. Although the model needs calibration and improvement, early simulation results show that it models increase in trip time as demand pattern becomes asymmetric.

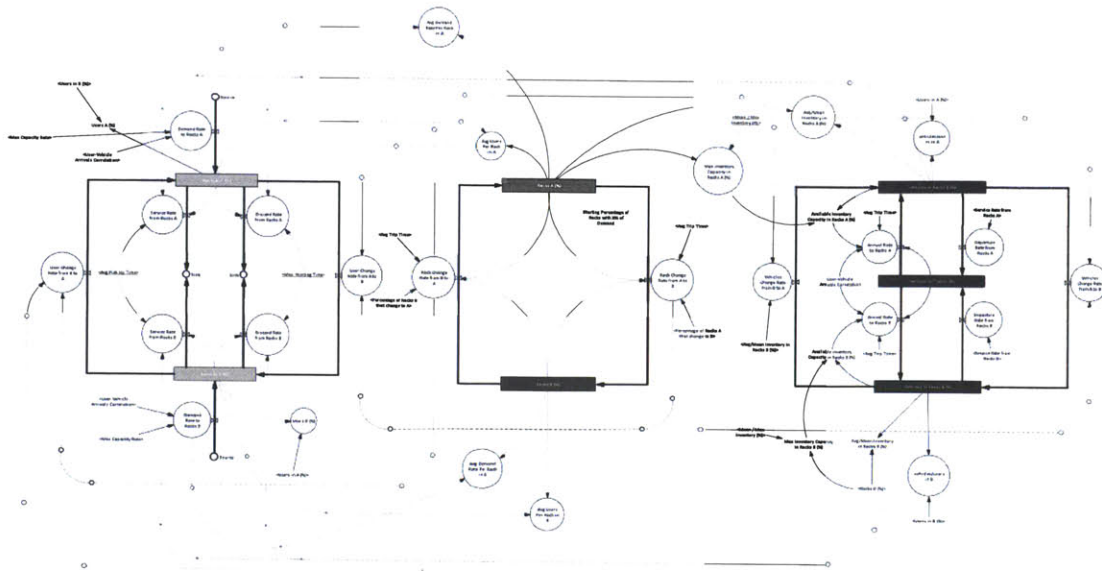


Figure 31. System dynamics stock-flow model built in Vensim software

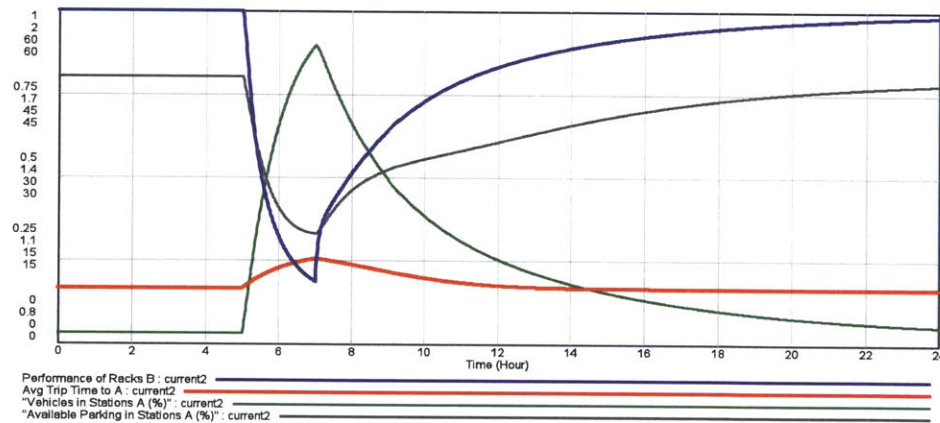


Figure 32. Early simulation results showing increase of trip time (red)

4.10 Validation methods

I consider three validation methods. As a first validation method I have programmed a second simulation model in Java based on a combination of system dynamics and Agent Based systems. The model in Java uses the same equations as the SD model in Vensim but is expanded to model each station individually; it takes as input two controllable discrete distributions, one representing inflow of users and the other

representing the vehicle arrivals over the stations, and simulates the throughput rate of the system by correlating the distribution of users and vehicles stocks. Vehicles are departing individually from each station, aggregate in the stock in transit, and return back to their destinations after having spent the average time in the stock in transit (figure 33). A second validation method is to build an Agent Based model of the system and compare the result with the lumped model. One possible approach for this direction is to first run a simulation in the AB model recording both input parameters and output results and use them as inputs in the lumped model to compare the results. A third validation approach is to conduct a series of experiments with the MET board game, record the results, and run a simulation with the same parameters, comparing at the end results from both sides.

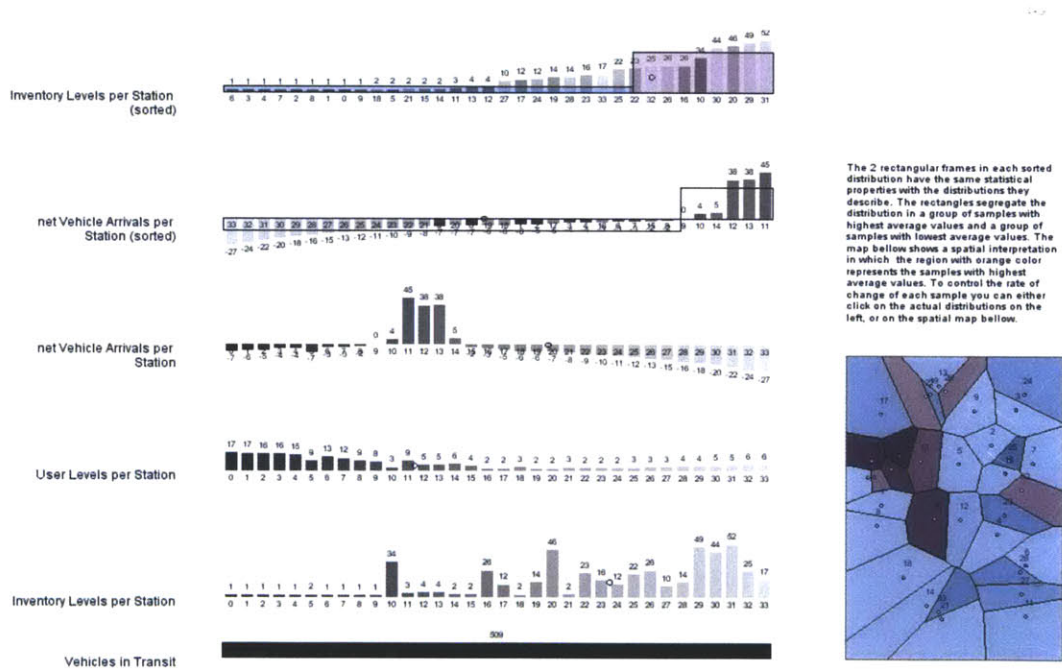


Figure 33. Programmed Vehicle Dynamics simulation model in Processing (Java). The input parameters are dummy data similar to the ones used in the lumped model of section 4.9

4.11 Limitations

Vehicle Dynamics framework has several limitations. First of all, the model currently segregates the stations into net sinks and net sources exogenously by controlling the circulation rate of stations between the two groups. The model must be modified so that the circulation rate is controlled endogenously following the description of section 4.4. A second limitation, inherent to all system dynamics stock-flow models, is the

simplification of behavior due to the mixing that occurs when aggregating resources in the stocks, which tends to drift the results as simulation time advances. Furthermore, the model assumes all trips having the same average trip time, pick-up time, and maximum waiting time which microscopically might make little sense, however macroscopically may be a reasonable assumption. A calibration must be done to take into account these effects.

5 CONCLUSION

5.1 Discussion

This thesis described the current state of the Market Economy of Trips, a new operation model for a MOD system that uses market price incentives to bring itself into balance. What is new in this work is the perspective that the user becomes both service provider and service recipient without being aware of it. This thesis made the following contributions:

First of all, it proposed a new market-based operation model to address redistribution problems in MOD systems and formulated MET as a game theoretic problem by describing the participants, their interests and their decision-making processes. It showed that the global goal of the system is to incentivize circulation throughput by inversely pricing both pick-up and drop off points. This can be done through the form of a double auction game where profit-maximizing auctioneer stations have a direct incentive to increase throughput flow.

It presented a pricing model that reflects an important requirement for sustainability: the flow of the funds that the overpaying users pay the system should balance the flow of rewards that the system pays the underpaying users. The underpaying users in turn pay the alternative option as they redistribute the vehicles. This means that a dynamically priced MOD system must exist synergistically with existing public transit infrastructure.

It proposed and designed the rules, layout, and game play of a board game that captures the economic principles of MET as these are described in this work, and may be used to empirically study decision-making, as well as provide valuable data for further future analysis.

Finally, this thesis developed a simple system dynamics framework that may be used as an educational tool for studying the behavior of MOD systems and explore the impact of pricing policies on throughput capacity.

5.2 Future work

Future steps of this work are the following:

To develop the pricing algorithm of MET following the directions that are specified in this thesis.

To incorporate the pricing effect in the Vehicle Dynamics framework in order to explore throughput performance under different pricing policies and demand patterns.

To develop further the economic analysis and compare the social costs of a dynamically priced MOD system with those of a manually redistributed MOD system.

To build a prototype for the MET board game, conduct a series of experiments and collect data. As a next step, the MET game can be implemented as an online computer game, or as an interactive physical computing application in which stations will be controlled by microcontrollers.

To develop a working prototype of the information system of MET including a sensor network deployment to collect data, a server to upload the data, and the contour map graphic user interface to display location based information to the users.

To run a pilot experiment using either bikes or cars and test the efficiency of the system including the pricing mechanism, the sensor network, the server to collect the data and publish the prices, and a graphic user interface to visualize prices on a map.

Finally, to compile and analyze the data results from the board game and the pilot experiment following the methodology described in section 4.4 and use them as input parameters to the Vehicle Dynamics framework for calibration. The results from the model, the game and the pilot experiment will then be compared to explore discrepancy of actual behavior from theoretical limits of efficiency.

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8 APPENDIX

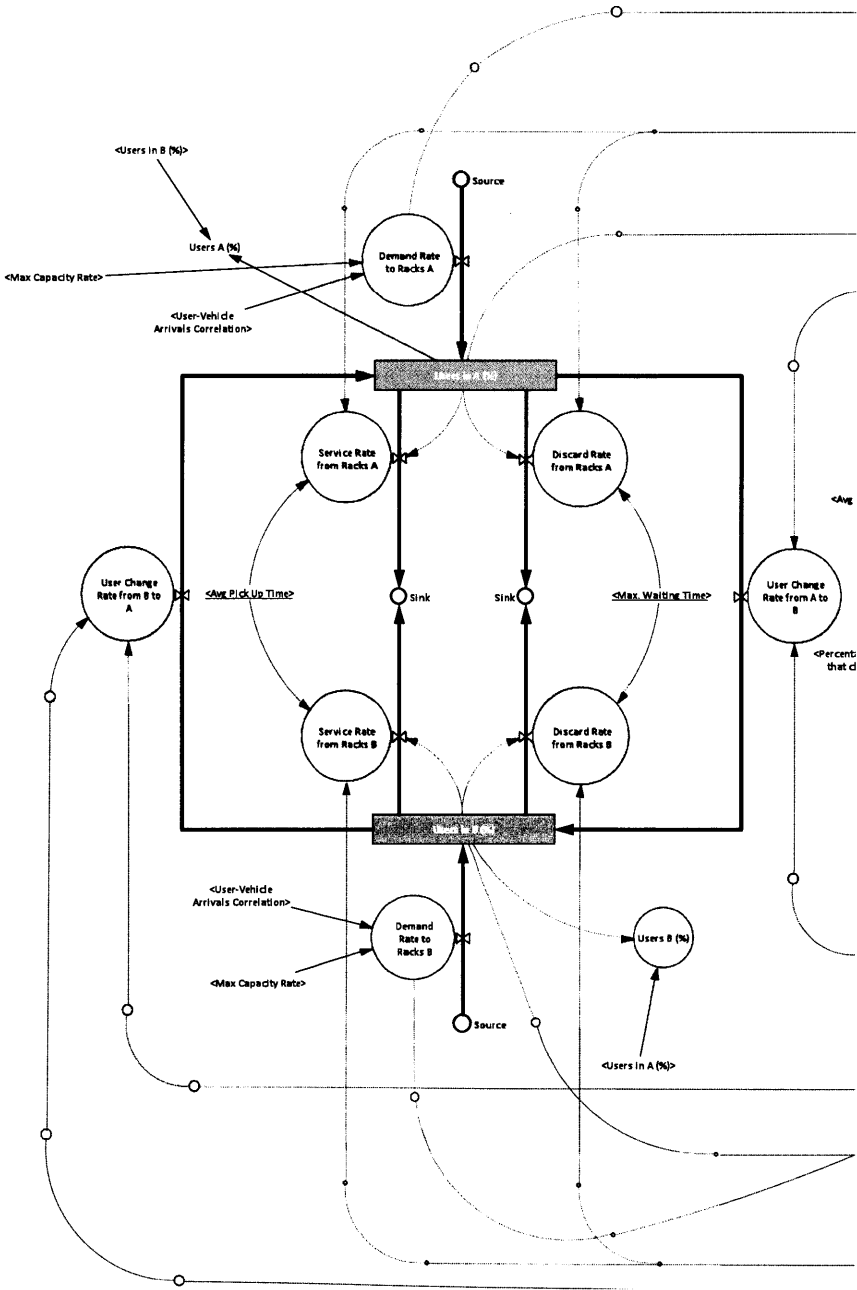


Figure 34. Users sub-model (in Vensim)

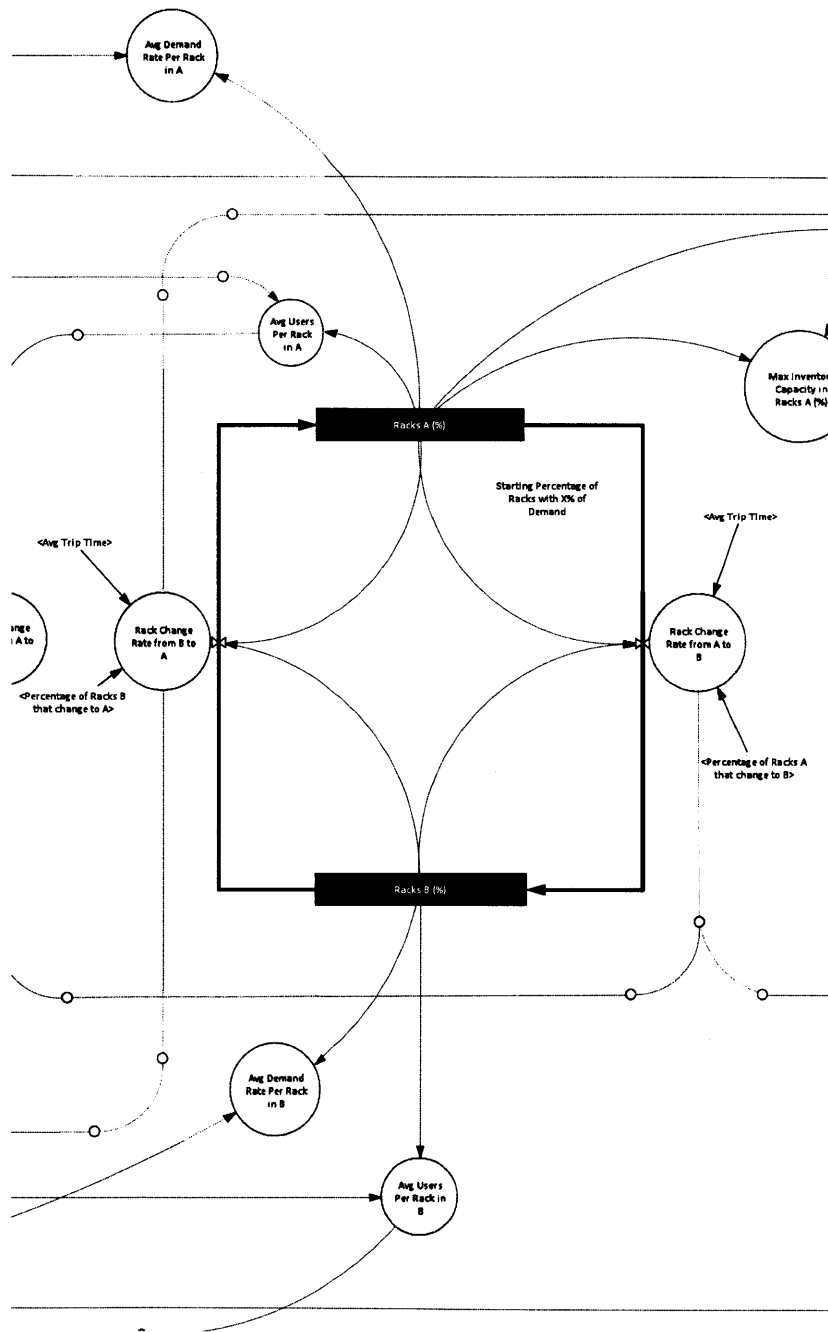


Figure 35. Stations sub-model (in Vensim)

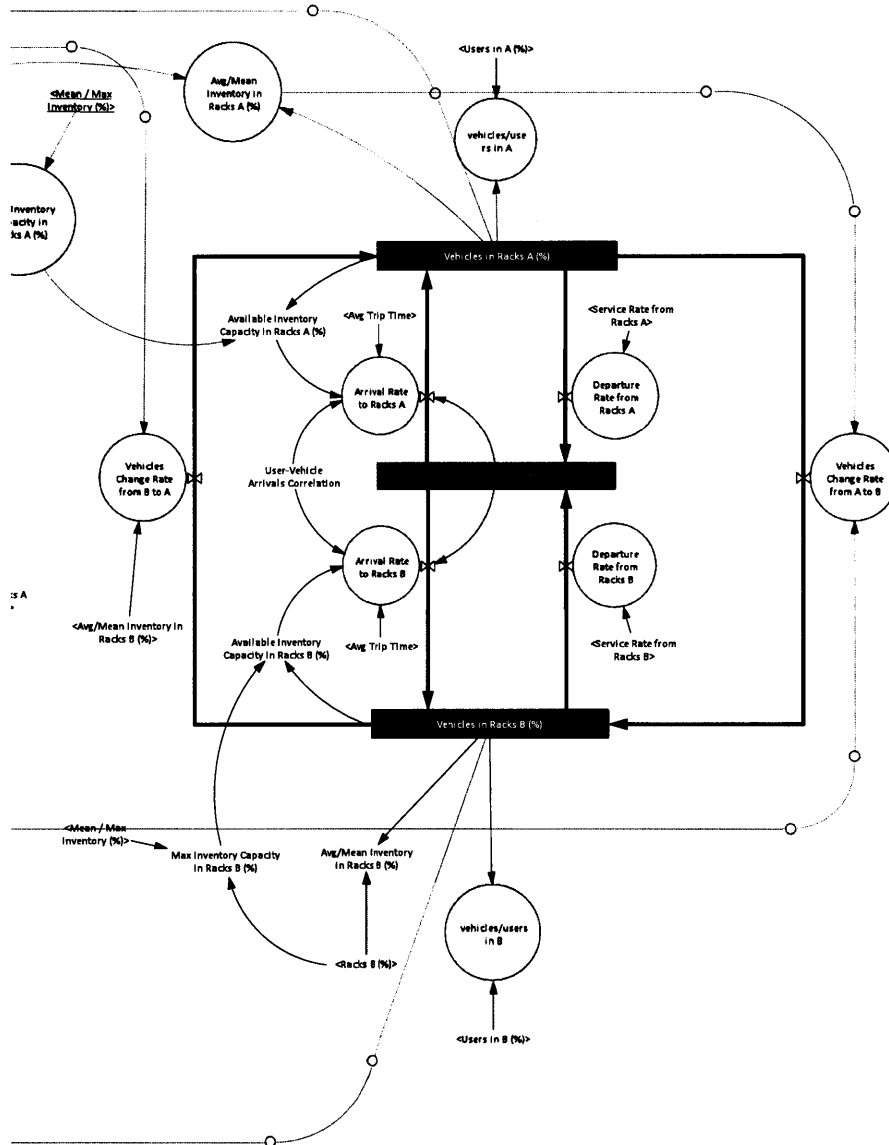


Figure 36. Vehicles sub-model (in Vensim)