MAS.963 Special Topics: Computational Camera and Photography
Fall 2008

# Fourier Optics

Se Baek Oh and Prof. Ramesh Raskar October 24th, 2008
Notes by anonymous MIT student Lecture 8

## Introduction to Fourier Optics

This section of the lecture was given by Se Baek Oh.

Fourier Optics are an alternative method to model and understand phenomenon occuring in classic optics. It uses the Fourier transform technique to tackle problems including:

- Diffraction (and also briefly touching wave optics)
- Spatial Frequency Domain
- Imaging
- Phase in wave optics is analogous to the direction of rays in a light field
- Point spread function

There are many other topics that Fourier Optics touches upon. However, in this presentation, we don't consider things like scalar diffraction theory or the polarization of light. Please use the related references to learn more[1] [2].

Fourier Transofrmations are the basic premise of looking at Fourier-based optics. In order to introduce this transformation to those who have not seen it before, it is important to review:

1. Fourier Transform relationships

   Basic Insight: A particular domain s may be mapped to another domain w via a mathematical transform.

   This transform allows many phenomenon in lenses to be analyzed more thoroughly.'

2. Linear Time Invariant System analysis

   Basic Insight: An input $x(t)$ may be passed into a system to create $y(t)$.

   The system may be modelled as the transfer function $h(t)$ which is subject to analysis.

[1] Introduction to Fourier Optics by Joseph Goodman
[2] 2.71/710 Optics lecture notes by Professor George Barbastathis

To give some basic intution for the relationship between the classic modelling and Fourier space modelling, we can look at a few basic transformations (in the Lecture slides, there are images accompanying these basic descriptions):

1. $\delta(x) -> 1$

   a single impulse maps to a constant value in the Fourier domain

2. $\cos(\alpha x) -> \pi(\delta(v - \alpha) + \delta(v + \alpha))$

   a cosine or sine wave maps to two impulses in the Fourier domain

3. $rect(\alpha x) -> \frac{1}{\alpha} sinc(\frac{v}{2\pi\alpha})$

   a box function maps to a special sinc function in the Fourier domain which is periodic and resemblant of a wave

Remark: Since we can model optical phenomenon using basic principles from Fourier Analysis and LTI systems, one way to look at the optical system as the parallel of the audio amplifier of an audio system. Many of the transformations that may be done within audio systems may also be done within optical systems. Additionally, the tools used to understand audio systems may also be used to understand optical systems.

## Light as an EM wave

Now, if we think about light as an Electromagnetic wave with a corresponding time-varying field as described by Maxwell's equation, we can show that this is analagous to thinking about the direction of the "ray" in the light field.

Remark: Since we are now thinking about light as a wave front, one question people have gotten confused about is what happens when a laser pointer is pointed in the ewater. From looking at the wave equations, people have asked: If the water has a different refraction index than the air, how come the color of the laser beam does not change as it bounces off the water? The answer is that color depends on frequency.

The key to understanding the behavior of a EM wave in an optical system is to understand the wavefront. We can describe this wave front with a 3-dimensional wave equation.

The wave equation dictates how we waves behave in this open space:

$$f(x, y, z, t) = A\cos(\frac{2\pi}{\lambda}(x\cos(\alpha) + y\cos(\beta) + z\cos(\gamma)) - \omega t - \phi_0)$$

which may be rewritten in many different formats that are equivalent:

$$f(x, y, z, t) = A\cos(k_x x + k_y y + k_z z - \omega t - \phi_0) \quad (1)$$

where $k_x = \frac{2\pi}{\lambda}\cos(\alpha)$

$$f(x,y,z,t) = Ae^{i(k_x x + k_y y + k_z z - \omega t - \phi_0)} \quad (2)$$

which is a complex representation and the general format is known to be composed of an amplitude and a phasor. This representation is particulary important since it allows us to define the wavefront which is when the exponential portion $\phi(x,y,z) = k_x x + k_y y + k_z z - \omega t - \phi_0$ is a constant.

Almost all the understanding of how waves behave when thinking about them in this way is derived from this wave equation. There are many practical insights from looking at the wave equation

- From the wave equation, phase implies the direction of propagation and also indicates the direction of the rays. Holography is study of how to control the directions of these rays.
- From the wave equation, we can see that light of different wavelengths (colors) will bend differently. E.g. red light will bend differently from blue light

  Diffraction gratings are very thin and can demonstrate this property

  A prism achieves a very similar effect
- From the wave equation, we see that the bending of light may be achieve by both amplitude gratings as well as phase gratings

Insight: Consider a waterhose which does not have a very narrow opening. The water will flow steadily out of the house. However, as the obstruction of the hose becomes very small, the water in that hose will begin to spirt outward with it's own properties. This analogy helps describe the phenomenon which we see with camera phones where the aperture size of the camera begins to get smaller and smaller. The resulting capture of light begins behaving in a different way.

Insight: Consider a car driving along a curb with two wheels on the curb and two wheels on the ground. As the car drives at constant, the difference in the height between the ground and the curb will cause the car to curve. This analogy helps describe what we can achieve with an amplitude grating or a phase grating.

The concept of coherence is important when we inspect the wave equation.

- Temporal coherence

  Temporal coherence results in a synchronized and visible interference pattern
- Spatial coherence

  Spatial coherence is not synchronized so there is no visible interference pattern

There are many applications using the concept of coherence. Optical coherence tomography is a method which uses the concept of coherence to do 3D imaging and display.

## Point spread function (PSF)

The point spread function may be explained from the wave equations, fourier equations. The point spread function may also be represented by the optical transfer function (OTF) by using the Fourier relationship discussed. I.e. the Fourier transform of the PSF is the OTF. Essentially, the point spread function describes the transfer function of light from a point source through a particular system.

Generally, the point spread function is modelled using integrals over two dimensions with various parameters which ultimately determine the behavior. From here, many properties may be observed using LTI and Fourier theory. Often modelling things using Fourier dimension is easier. Furthermore, some properties such as shift-invariance in LTI theory are easier to see when the point spread function is modelled in this way. E.g. an example of a point spread function might look like:

$$O(x_0, y_0) = \int \int O(u, v)\delta(x_o - u, y_o - v)\partial u \partial v$$

In many cases, the PSF is a Fourier Transform of the function which describes the aperture. Intensity is basically the magnitude squared.

Remark: In many cases, the PSF is the Fourier transform of the aperture function. Here, there are many instances where only the lens itself describes the point spread function. In these cases, the transfer function or $H(f_x)$ or the PSF cannot look better than a triangle.

The PSF may be used for many things including the modelling of what blur should look like.

## Fourier Optics summary

With this taste of Fourier Optics modelling, we see that there is more than one way to think about optics. We see that in order to think of light as a EM wave and see the same results as ray optics, we must think about wavefronts.

Remark: It is necessary to think about how closely physical phenomenon follow these models. There was much discussion on this point in class. In general, a LSI system is fairly true in applications such as paraxial apps. The use of wave optics is fairly useful in the understanding of the wavefront, diffraction and coherence. The optical transfer functions (OTF) and point spread functions (PSF) are often used to describe phase change. Also, these methods are particularly useful in understanding how broadband light behaves.

Remark: One question that might arise is to ask when to think of light as a EM wavefront for a system and when to think of light as ray optics. A good rule of thumb is to say that whenever there are features on the order of wavelengths and you are interested in interference features, then it is viable to model with Fourier optics. If these features do not exist, then it suffices to use ray optics.

With this knowledge in mind, let's switch gears and think about projects.

## Good Projects and Why They Are Good

Many of the following projects may be found in Ramesh's project pages [3]. The main point of this part of lecture is to give you insight about what to choose to do for your own projects.

There are many interesting projects emerging which are related to cameras in the human computer interaction field. It is good to go through those and get a sense of which ideas are interesting and relevant and which are not.

Some interesting projects related to HCI:

- Cameras looking at people i.e. facial tracking, emotion tracking, segmentation
- Cameras looking at fingers i.e. multi-touch, finger tracking, optical mouse, smart pens
- Multiflash
- Projects questioning complexity of optical transmission
- Motion capture

    2D, 1D, 0D

    Motion detector

    Wii

Some not so interesting projects related to HCI:

- location tracking in unrealistic and unbuildable (not everyday) scenes
- triangulation of a LED with two webcams
- segmentation that then requires you put on a hat or a glove or something else unnatural
- artistically interactive display: blobs, waving, motion etc.

The idea is to create something with technology, physic and processing which appears to be magic. Lets go through and talk about some recent project which serve as good examples of interesting work.

[3] Ramesh Raskar at *http://web.media.mit.edu/~raskar/*

### Frustrated Total Internal Reflection (TIR)

Low-Cost Multi-Touch Sensing through Frustrated Total Internal Reflection. In Proceedings of the 18th Annual ACM Symposium on User Interface Software and Technology. The goal of this work is to achieve the multitouch effect.

- What can you do when you use light beyond the critical angle
- By using total internal reflection, the surface acts as a waveguide and is able to recognize the final output in a controlled fashion at the every end
- Achieved effect is a multi-touch screen
-

### Hacking the Wii-Mote

What can you do with something that is fairly cheap and has an amazing number of inputs and outputs with reasonable specs? Johnny Lee figured out that with the wii-mote and corresponding sensor bar:

- Resolution of 1,024 768 pixels
- more than 4 bits of dot size or light intensity
- 100 Hz refresh rate
- 45 degree horizontal field of view.

you can achieve many interesting applications such as:

- Desktop virtual reality

  *http://www.youtube.com/watch?v=5s5EvhHy7eQ*
- *Display with changing viewpoint dependent on angles*

  *http://www.youtube.com/watch?v=Jd3-eiid-Uw*
- *Finger tracking*

  *http://www.youtube.com/watch?v=0awjPUkBXOU*

### Multiflash and Colored Multiflash

What kind of phenomenon does multiflash and colored multiflash create. Work done by Raskar et. al and Feris [4]

- canny edges
- depth edges
- more information embedded in a single flash.

### Smart Pens [5] and Optical Mice

With the specifications that are available on a traditional optical mice, there are many projects that might result. Consider specs that are available on a standard optical mouse camera. And imagine all the things you could do!

- 800 counts per inch (CPI) resolution
- a pixel size of 30.4 $\mu$m and an array size of 20x20 pixels
- frame rate 9600 frames per second (super high rate!)
- high speed motion detector of up to 40 ips

### Gaze Tracking

Many works in the area of gaze tracking and following where one's eyes go.

### Optical information

- Using lense to achieve a long distance information transmission
- Tradeoff in complexity and information conveyed
- Active RFID with handheld projector
- Labelling space withi projectors

[4] *http://ilab.cs.ucsb.edu/publications/FerisSIB06.pdf*
[5] *http://www.acreo.se/upload/Publications/Proceedings/OE00/00-KAURANEN.pdf*

### Video motion capture

Key point here is that this video motion capture work everywhere and not just in a green screen area. Also, the use of demodulating cameras to capture things in one go. This is also important. Many interesting ideas in this space which is relavent to human computer interaction. Also, an important constraint here is cost. Can you do something complex with something cheap and not obtrusive?

- High speed motion capture
- Multiplexing cameras
- Demodulating cameras
- Cameras which work in any environment

## Summary

We introduced some of the theories which are useful for modelling physical phenomenon and we give some pointers of when it is useful to look at what. Then we went over a vast space of project ideas which hopefully can give others ideas of things to try out in their final projects.