MAS.963 Special Topics: Computational Camera and Photography
Fall 2008

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Scientific Imaging, Range sensing camera and Compressive Sensing

Douglas Lanman, Ashok Veeraraghavan and Prof. Ramesh Raskar November 3rd, 2008
Notes by Jaewon Kim Lecture 9

Computational Imaging: A Survey of Medical and Scientific Applications

This section of the lecture was given by Douglas Lanman.

1. Medical Imaging

Mostly, this field is related with reconstruction of internal structures of living organisms. By such feature, this kind of imaging should be non-invasive and various indirect method to view inner shape have been used like radiology, endoscopy, thermal imaging, microscopy, etc.

- **What is Tomography?**

Tomography is one of popular methods to view inner shape of an object using X-ray. Of course, this method is not perfectly safe because X-ray is harmful to human body. However, in natural environment, we are living with certain amount of X-ray daily and very small amount of X-ray is hardly harmful. In this method, all cross sections of an object are generated and reconstruct 3D shape of it by assembling all cross sections. Generally, much X-ray projection data is needed to generate a single cross section and thus, huge amount of date is used to reconstruct 3D shape.

No

There are many methods in computational tomography and they are classified into two groups in macro vision. One is back projection method and the other is numerical iterative method. Usually, back projection method is used when we can get X-ray projection spaced uniformly from 0 degree to 360 degree. Numerical iterative method is used when only X-ray projection at limited angle is available. ART (Algebraic Reconstruction Technique) is representative method in the group.

In back projection method, there are classic two ways, parallel beam projection and fan beam projection method. Generally, parallel beam projection method is faster than fan beam method in the aspect of computational time. But fan beam projection is possible to make simple and fast scan mechanism of X-ray source. By the reason, most CT machines have adopted the fan beam method.

- Parallel beam projection method

$$p_{\phi}(x') = \int\int_{-\infty}^{\infty} f(x, y)\delta(x\cos\phi + y\sin\phi - x')\,dxdy$$

$$= \int_{-\infty}^{\infty} f(x'\cos\phi - y'\sin\phi, x'\sin\phi + y'\cos\phi)\,dy'$$

$$P_{\phi}(\omega) = \int_{-\infty}^{\infty} p_{\phi}(x')\exp(-i\omega x')\,dx'$$

$$= \int\int_{-\infty}^{\infty} f(x'\cos\phi - y'\sin\phi, x'\sin\phi + y'\cos\phi)\exp(-i\omega x')\,dx'dy'$$

$$P_{\phi}(\omega) = \int\int_{-\infty}^{\infty} f(x, y)\exp[-i\omega(x\cos\phi + y\sin\phi)]\,dxdy$$

Coordinate translation from x', y' to x, y

$$= F(\omega\cos\phi, \omega\sin\phi)$$

$$= F(\omega, \phi)$$

$$f(x, y) = \int_{0}^{\pi}\int_{0}^{\infty} F(\omega, \phi)\exp[i\omega(x\cos\phi + y\sin\phi)]|f|\,d\omega d\phi$$

f(x,y)= iFFT of $F(\omega_x, \omega_y)$
$F(\omega,\phi)$: polar coordinate transform of $F(\omega_x, \omega_y)$

$$f(x, y) = \int_{0}^{\pi}\int_{-\infty}^{\infty} P_{\phi}(\omega)|\omega|\exp[i\omega(x\cos\phi + y\sin\phi)]\,d\omega d\phi$$

$$= \int_{0}^{\pi}\int_{-\infty}^{\infty} P_{\phi}(\omega)|\omega|\exp(i\omega x')\,d\omega d\phi$$

$$= \int_{0}^{\pi} p^{*}_{\phi}(x')\,d\phi$$

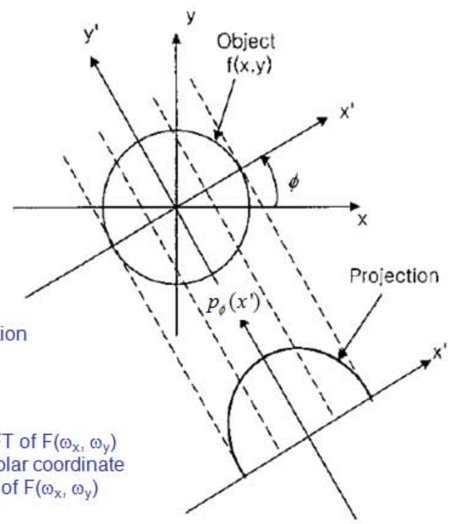

Diagram of Parallel-Beam projection

Source: Lee, S.C, M.H. Cho, and S.Y. Lee. "Performance Comparison of Reconstruction Algorithms for Fan-Beam Computerized Tomography." *J. Biomed Eng. Res.* 22, no. 3 (2001): 223-229.

• Fan beam projection method

$$f(x,y)=\frac{1}{2\pi}\int_0^{2\pi}d\phi\int_{-\infty}^{\infty}dx'\,p_\phi(x')h(x\cos\phi+y\sin\phi-x')$$

$$x'=R_d\sin\beta$$
$$\phi=\alpha+\beta$$

$$f(x,y)=$$
$$\frac{1}{2\pi}\int_0^{2\pi}d\alpha\int_{-\beta_m}^{\beta_m}d\beta\,p_\alpha(\beta)h\{x\cos(\alpha+\beta)+y\sin(\alpha+\beta)-R_d\sin\beta\}|J|$$
$$(|J|=R_d\cos\beta)$$

$$f(x,y)=\frac{1}{2\pi}\int_0^{2\pi}d\alpha\int_{-\beta_m}^{\beta_m}d\beta\,p_\alpha(\beta)h\{v\sin(\beta'-\beta)\}|J|$$
$$v=\sqrt{(x\cos\alpha+y\sin\alpha)^2+(x\sin\alpha-y\cos\alpha+R_d)^2}$$
$$\beta'=\tan^{-1}[\frac{x\cos\alpha+y\sin\alpha}{x\sin\alpha-y\cos\alpha+R_d}]$$

$$h\{v\sin(\beta'-\beta)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega|\omega|\exp[i\omega v\sin(\beta'-\beta)]$$ From h(x)= iFFT of |ω|
$$\omega'=\omega\frac{v\sin(\beta'-\beta)}{\beta'-\beta},$$
combining
$$h\{v\sin(\beta'-\beta)\}=\frac{1}{v^2}[\frac{\beta'-\beta}{\sin(\beta'-\beta)}]^2h(\beta'-\beta)$$

$$f(x,y)=\int_0^{2\pi}d\alpha W_2\int_{-\beta_m}^{\beta_m}d\beta[W_1p_\alpha(\beta)]g(\beta'-\beta)$$
$$W_1=|J|=R_d\cos\beta,$$
$$W_2=\frac{1}{2\pi}\frac{1}{v^2}$$
$$g(\beta)=[\frac{\beta}{\sin(\beta)}]^2h(\beta)$$

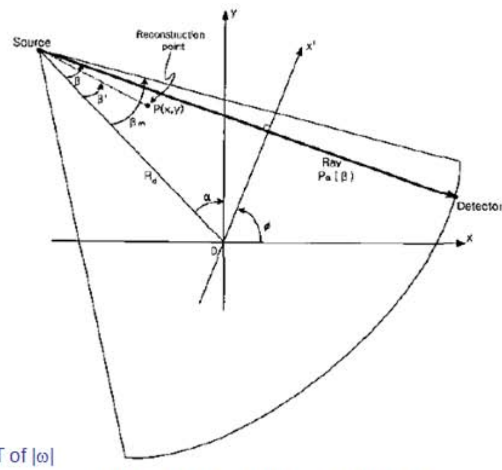

Diagram of Fan-Beam projection

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• Reconstruction: Backprojection[1]

Below figures explain easily how to reconstruct a cross section of an object by back-projection. In Fig 1, 1D projection data give us 1D volume information which is shown as 1D line arrays. In Fig 2, when two projection data is available, we can know that the volume of object is placed in crossing region between two 1D line arrays. By similar way, we can confine the volume data of an object more and more with multiple projection data as Fig 3.

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1 http://engineering.dartmouth.edu/courses/engs167/12%20Image%20reconstruction.pdf

2 R. G. Baraniuk, "Compressive Sensing," Lecture Notes in IEEE Signal Processing Magazine, Vol. 24,

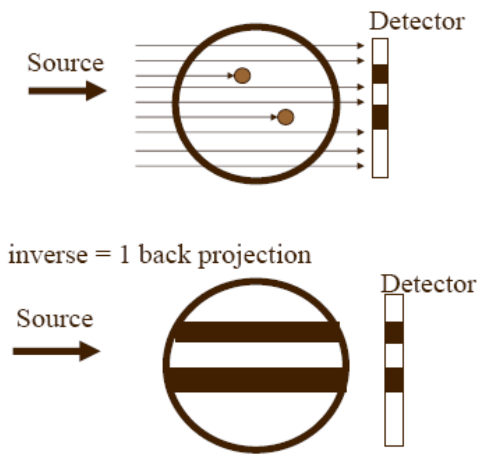


Fig 1. Linear Single Backprojection

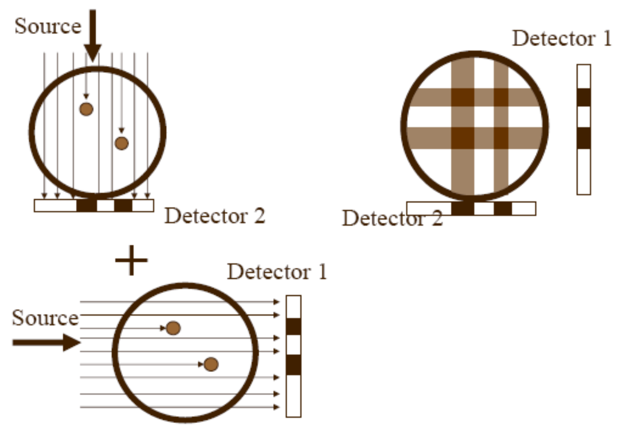


Fig 2. Linear Two Backprojection

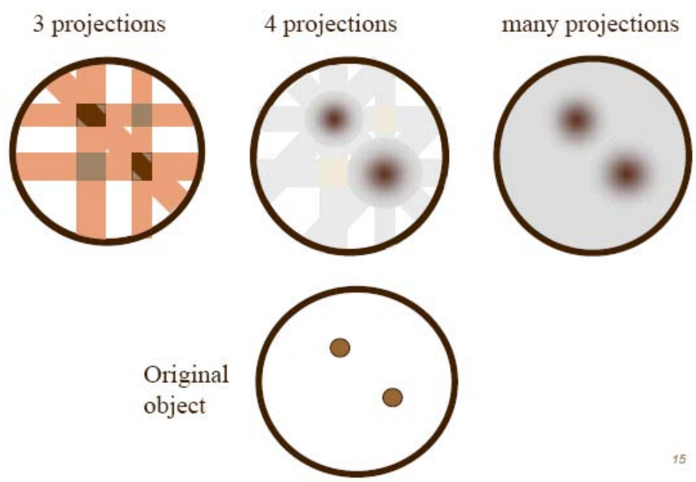


Fig 3. Linear Multipul Backprojection

• Sampling Requirements and Limitations

When the project data is sampled around the center of an object, the value at the origin in frequency domain become too much larger than other frequency values. To correct this, Ram-Lak filter is used. The reconstruction quality depends significantly on the spacing of rotating angle, θ. Even in 5 degree spacing, many artifacts can be happened. In this case, to decrease the artifacts, filtered backprojection method or ART can be considered.

• How to Eliminate Moving Parts in CT?

Prior CT is time-sequential because the projection images should be captured according to rotation of X-ray source. If an attenuating mask is used for CT like shield field imaging, simultaneous projection is possible. But it can be very challenging with the problem of projecting X-rays at different view angles simultaneously.

• Biology: 3D Deconvolution
In this method, object is assumed to be semi-transparent, hence scattering is minimal.

object * PSF → focal stack
F{object} × F{PSF} → F{focal stack}
F{focal stack} ÷ F{PSF} → F{object}

Range Sensing

This section of the lecture was given by Prof. Ramesh Raska.

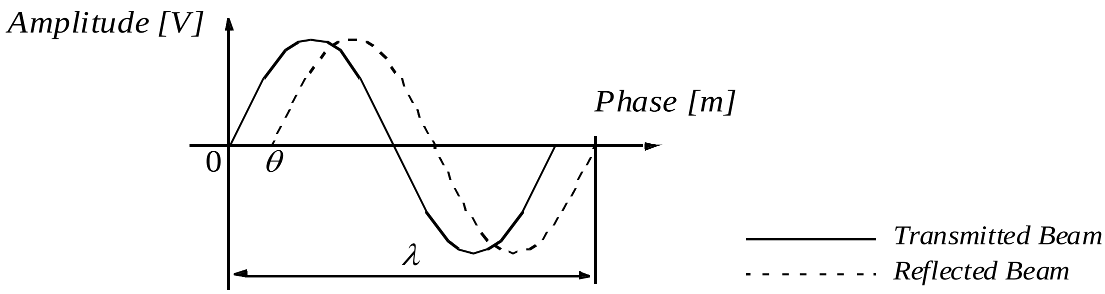
• 3D Cameras : Time of Flight, Phase decoding of modulated illumination, and structured light. Range(Depth) camera which can get depth information in every pixel might have big impact on photography and medical imaging in near future. It might also make refocusing issue trivial. Thus, the technique to involve range sensing with camera deserves to get attention from now.

The technology to recognize talking of people in the house from outside was discussed. The sound inside of room can be detected from outside by measuring the vibration of a window. Human’s voice is discernable from vibration of window by natural movement of air because it has high frequency.

• Time of Flight

Basic idea : pulse of light(usually laser) is sent out and time how long it takes to return is measured. Very fast measuring system which can measure signal in a few nano-seconds is need. The simple way of it is using sinusoidal signal. Then the phase difference(θ) between sending and receiving signals is measured. When the wavelength of the signal is λ, distance is calculate by the below. In this method, getting good accuracy is challenging and the accuracy is at best 5mm.

$$D = \frac{\lambda}{4\pi}\theta$$



• Depth Cameras

Depth cameras are 2D array of time-of-flight sensors. Because jitter is too big on single measurement, the average value of many measurements is used to get depth image. To get the accurate depth image, the range sensing data should be normalized by normal camera image because the intensity of received signal depends of the color of an object.

• Mid-Term Exam

-. 11/7 13:30 ~ 15:30 (Someone can take the exam early if he or she need more time.)
-. The exam is open-book test but copying all class slides is prohibited.
-. The topics on “mid-term” slide will be covered in the exam including all materials.
-. The exam questions will be emphasized on testing concepts like drawing a diagram to explain X, very simple calculations, comparisons, open ended questions (essay), Homework insights, and so on.

Compressive Sensing

This section of the lecture was given by Ashok Veeraraghavan.

We learned Shannon sampling theorem which states that if we sample densely enough at the Nyquist rate, we can perfectly reconstruct the original analog data. But sampling at the Nyquist rate often asks us too much data. This method pursues capture and represent compressible signals at a rate significantly below the Nyquist rate. This method employs nonadaptive linear projections that preserve the structure of the signal; the signal is then reconstructed from these projections using an optimization process. This method might be very useful in many cases dealing huge data like holography and CT. In the case of CT, the amount of projection data is very critical to the quality of reconstruction. Ashok mentioned that this method will improve the reconstruction result when small number of data is available.

[2]Let's consider a real-valued, finite-length, one-dimensional, discrete-time signal x, which can be viewed as an $N \times 1$ column vector in R^N with elements x[n], n = 1, 2, . . . , N. Any signal in RN can be epresented in terms of a basis of $N \times 1$ vectors ψ_i. Using the NxN basis matrix of ψ_i, a signal x can be expressed as

$$x = \psi s \quad (1)$$

where s is the $N \times 1$ column vector of weighting coefficients. Clearly, x and s are equivalent epresentations of the signal, with x in the time or space domain and s in the ψ domain. The signal x is K-sparse if it is a linear combination of only K basis vectors; that is, only K of the s_i coefficients in (1) are nonzero and $(N - K)$ are zero. The case of interest is when $K << N$. The signal x is compressible if the representation (1) has just a few large coefficients and many small coefficients.

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[2] R. G. Baraniuk, "Compressive Sensing," Lecture Notes in IEEE Signal Processing Magazine, Vol. 24, No. 4, pp. 118-120, July 2007.

References

[1] A.C. Kak and M. Slaney, Principles of Computerized Tomographic Imaging, 1988

[2] Douglas Lanman, Ramesh Raskar, Amit Agrawal, and Gabriel Taubin,Paper, Modeling and Capturing 3D Occluders, SIGGRAPH Asia 2008

[3] BAYINDIR Mehmet, ABOURADDY Ayman F, FABIEN, SORIN, JOANNOPOULOS John D. , FINK Yoel, Fiber photodetectors codrawn from conducting, semiconducting and insulating materials, 2004

[4] Emmanuel Candès and Terence Tao, Near optimal signal recovery from random projections: Universal encoding strategies (IEEE Trans. on Information Theory, 52(12), pp. 5406 - 5425, December 2006)

[5] David Donoho, Compressed sensing. (IEEE Trans. On Information Theory, 52(4), pp. 1289 - 1306, April 2006)

[6] R. G. Baraniuk, "Compressive Sensing," Lecture Notes in IEEE Signal Processing Magazine, Vol. 24, No. 4, pp. 118-120, July 2007.