# Pricing Apartment Attributes: A Hedonic Analysis of the Dallas/Fort Worth Multifamily Rental Housing Market 

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B.S.B.A. Finance, 1986

University of Florida
SUBMITTED TO THE DEPARTMENT OF URBAN STUDIES AND PLANNING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN REAL ESTATE DEVELOPMENT AT THE

## MASSACHUSSETS INSTITUTE OF TECHNOLOGY

SEPTEMBER, 1999
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Submitted to the Department of Urban Studies and Planning on August 2, 1999 in Partial Fulfillment of the Requirements for the Degree of Master of Science in Real Estate Development


#### Abstract

A hedonic regression analysis is performed using data collected from 1007 multifamily properties within the Dallas/Fort Worth, Texas metropolitan area. A Model is estimated that is capable of a) predicting rent given certain inputs regarding the attributes of a property and b) pricing and determining the relative impact on rent of certain attributes and groups of attributes.

The analysis produced significant results with important implications for valuation, design, development and acquisition/development of multifamily projects. The Model can be utilized by multifamily developers and investors to assist in optimizing configuration and investment decisions in the Dallas/Fort Worth market. The Model may also provide conceptual insight into tenant preferences applicable to other similar multifamily markets.


Thesis Supervisor: William C. Wheaton
Title: Professor of Economics

This study would not have been possible without the generous and expert assistance of Wayne Williams and ALN Systems, Inc of Dallas, TX.

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## CHAPTER ONE: INTRODUCTION

The development or acquisition of a real estate asset is a complex undertaking. The degree of complexity will vary significantly depending on the scale and type of project, its location and a host of other factors. However, the crucial consideration for any project is "matching" its bundle of characteristics or attributes with the preferences of the market and the financial structure of the development or acquisition.

There are many components or "inputs" to this matching process but there is only one important "output" - Rent. Rent is the primary determinant of land cost, loan value, cash flow and terminal value. Attributes determine Rent. Thus, in order to maximize value in the development or acquisition decision, it is crucial for the developer/investor to thoroughly understand the relationship between the attributes of a given product type and its value or Rent.

An obstacle, however, to this understanding is the fact that the prices of these attributes are not directly observable. They are "implicit" prices. It can be observed that a tenant contracts to pay $\$ 840$ per month for an apartment unit. But this figure is simply an aggregate representation of the value of all of the separate attributes (location, physical characteristics and amenities) contained by the property. What value is the tenant placing on the appliances? Does the economic value tenants place on a pool justify its initial construction cost? These are questions with important implications for project design, property management and acquisition and development strategies.

In many metropolitan markets, apartments account for a significant share of new construction. Apartments are among the largest and most economically significant sectors of the U.S. real estate markets. In the multifamily arena, increased land costs,
increased institutionalization of development and ownership, a more competitive marketplace and an increasingly discerning consumer have combined to create a management-intensive, lower-margin operating environment. Furthermore, income is capitalized to derive loan and sale values at historically low rates.

These factors add to the importance of the developer/investor optimizing the multifamily cost/rent equation. In optimal development, a developer does not pay for an attribute whose contribution to rent or "implicit value" is not sufficient to justify its cost. This requires a precise awareness of the economic value a tenant places on a given attribute when renting an apartment unit. When this economic value is capitalized and compared to the estimated construction cost associated with the attribute, a more informed development decision can be made. In an acquisition scenario, a property under consideration could be lacking an attribute that the informed investor knows has a strong correlation with higher rent in the target market. If the projected contribution to Rent and thus value is greater than the cost to construct or install (and operate) the attribute upon acquisition, a value-added opportunity has been identified.

To date, the primary means of gathering information regarding consumer preferences includes surveys of consumers, reviewing small samples (nearby comparable projects) and anecdotal evidence. These are valuable tools and have been used effectively by savvy developers and operators. However, a large national developer headquartered in Dallas, complains of "shooting in the dark" when deciding on the inclusion or exclusion of many attributes or features. A statistical analysis across a very large sample of units will improve upon the information gathered by these methods,
providing a more accurate picture of the correlation between a given feature and higher rent.

Since the prices of individual attributes or characteristics are not in most cases directly observable, they must be estimated. A frequent means of estimating these prices is through hedonic regression analysis. A hedonic regression model specifies a dwelling's rent (or value) as a function of the structural, neighborhood and other attributes it contains.

Since the mid-1970's, much research has been conducted using hedonic regression - primarily with respect to single family residential property values. However, a relatively small amount of research has been initiated to estimate hedonic -rent equations for multifamily housing, particularly in light of the economic significance of the sector.

Schenkel (1) developed a multiple regression model to estimate the market value of apartment projects. He used data from forty-seven apartment complexes and analyzed sixty-nine property characteristics.

Londerville (2) used data on 809 apartment building sales in Vancouver, Canada from 1971-1985 to estimate a hedonic price equation for this market. However, her primary purpose was to derive a trading model and a limited number of explanatory variables - age, building area and suite area - were tested. The results of her study confirm the results of this analysis with respect to those variables. Age is negatively correlated as expected. Size of a project is positively correlated with unit rent. Area of the unit or suite is negatively correlated with rent per square foot.

Benjamin, Lusht and Shilling (3) performed a hedonic regression analysis on the Washington, DC and State College, Pennsylvania multifamily rental housing markets. They used data collected from 81 apartment properties, consisting of 253 unit types or observations in Washington, DC and 423 individual units in State College. Although they tested a variety of physical characteristics and location variables, their primary purpose was to explore the relationship between up-front security deposits and rental rates. Results for the Washington D.C. analysis differed with respect to several key variables from the results reported herein. They found a positive correlation between project age and rent and a negative correlation between number of units in a project and rent. The results of this analysis indicate just the opposite.

Guntermann and Norrbin (4) used regression analysis to analyze rent variations in a sample of apartment data from the Phoenix, Arizona metropolitan area. They collected data on 104 apartment properties, consisting of 291 different unit types. Their primary purpose as is that of this report was to "...relate variations in rent to various physical characteristics and amenities of projects as well as to their location". The results of their study were significant and established a solid methodological model that could be used to conduct further research. It is this study that is most similar in methodology and purpose to this report.

Interestingly, the Guntermann and Norrbin results with respect to several significant attributes were very different than the results obtained in the study reported herein. A compelling conclusion of this report is that configuration as defined by number of bedrooms is insignificant with respect to rent per square foot. Guntermann and Norrbin reached the conclusion that "given that there is sufficient size to accommodate
division into extra bedrooms...there is a substantial rent increase as a result of the additional bedroom". Their results included a positive coefficient for a swimming pool where results of this analysis indicate a negative correlation to rent for this feature. A fireplace is implicitly priced in their study at a level almost three times that of the results reported herein. They found age to be only moderately related to rent. In this study, it is the most significant factor.

With the exception of Londerville, all of the above analyses use monthly rent as the dependent variable. This study uses rent per square foot as the dependent variable primarily in order to better isolate the pricing effects of configuration features like number of bedrooms and number of baths.

These differences underscore the importance of updating existing research and further study of new markets. This study seeks to add to the body of existing research relevant to the subject by:

- Focusing exclusively on the relationship of attributes to multifamily rent
- Using rent per square foot as the dependent variable as opposed to monthly rent in order to more effectively isolate the pricing effects of configuration
- Focusing on an important but heretofore empirically unexamined multifamily housing market - the Dallas/Fort Worth, Texas MSA
- Utilizing a database many times larger and more comprehensive than previously analyzed
- Updating existing research - an important consideration in light of the dynamic nature of the multifamily operating and ownership environment within the last decade.

Specifically, this study seeks to determine the relationship between specific multifamily attributes and rent in the Dallas/Fort Worth MSA. This relationship will be expressed in the form of an economic model, which can be used to estimate the implicit value or price of individual characteristics and predict rents given input regarding these characteristics. This model can be manipulated to estimate an optimal mix of these characteristics and features within a specified location, resulting in maximum rent.

The following chapters will describe in detail the data and methodology used to conduct the analysis, discuss and interpret the results of the analysis and conclude with a Model and recommendations based upon these results. These recommendations and the Model should contribute toward optimization of multifamily development and acquisition within the Dallas/ Fort Worth multifamily market and provide conceptual insight applicable to other markets as well.

## CHAPTER TWO: THE DATA

## Description of Database:

As the basis for analysis, data was collected on 1007 properties located in the Dallas/Fort Worth area known as the "Metroplex". This sample represents 339,401 apartment units and $280,648,782$ square feet of residential space. As of June, 1999, it represents in excess of $75 \%$ of all existing units and $100 \%$ of existing units located in properties containing 200 or more total units in the Dallas/Fort Worth MSA. Thirty-five separate cities, thirty-two Independent School Districts (ISDs) and five counties are represented in the database.

The raw data concerning asset features and characteristics was compiled by ALN Systems, Inc., a Dallas-based information services firm specializing exclusively in providing apartment data to apartment locator services and real estate professionals in the Metroplex. This data was compiled by direct phone contact with each individual property and updated monthly by phone or facsimile. Since this data is used by virtually all apartment locator services in the area, it is in the best interest of these services as well as that of the individual properties that the latest information be reflected in the data. In addition, this data does not include subjective or quality-based data such as condition of property, or exterior appearance etc., which could lead to biased data in the interest of marketing. For these reasons, the level of accuracy of the data that has been compiled is considered to be quite high.

The overwhelmingly predominant property type represented in the database is "suburban garden". This constitutes an estimated $99 \%$ of the observations. This is primarily a function of the decentralized, suburban character of the Dallas/Fort Worth

MSA. It should certainly be considered when interpreting or applying the results of the report. However, the 200+ unit, suburban garden prototype is by far the most prevalent type of institutional multifamily holding in the United States. And the decentralized, multi-nodal suburban city model is correspondingly prevalent among U.S. cities. So the fact that the database is heavily weighted in this sector should not detract from its usefulness.

The data was configured so that one observation equaled a single unit type in a single property. For example, Property A contains 200 total units comprised of 50 one bedroom/one bath units, 75 two bedroom/two bath and 75 three bedroom/two bath units. Property A would constitute a total of three observations - Property A-1/1, Property A $2 / 2$ and Property A-3/2. The database totals 7,885 observations, (meaning that the average property of 200 units or more in the Dallas/Fort Worth Metroplex offers 7885/1007 or 7.8 different floor-plans).

## Description of variables:

Information concerning thirty-eight attributes was collected for each of the $\mathbf{7 , 8 8 5}$ observations. They include the following:

## Variables - Asset Features and Characteristics

- Number of Bedrooms: total number of bedrooms (not including dens) in unit;
- Number of Bathrooms: total number of bathrooms in unit; The fractions, .3, . 5 and .8 are used to indicate sink-only, sink and shower only and commode and shower only respectively.
- Square Feet: total area of the unit in square feet;
- Effective Age: age in years of the property in which the unit is located;

Calculated as " 99 minus year-built". In the case of a substantial renovation, the most recent age is used. For example if a property was built in 1972 but renovated in 1994, effective age equals 5 years. Many large properties were constructed in phases. In this case, the first and last year of the construction period are averaged, and rounded to the nearest whole number if necessary. For example, if a property were built in three phases from 1990-1995, the effective age of its units is calculated as $99-(\{90+95\} / 2)=6$ years.

- Total Units: total number of units contained in the property in which the unit is located;
- Parking Facilities: Database rents do not reflect additional rent for optional parking upgrade available to units at selected properties. Therefore, the parking score reflects only the type of parking available to all units without additional charge. Including optional parking facilities without the additional charge (rent) would skew the results of this portion of the data. In addition, although there are seven parking categories in the database, there are only four differentiated scores. For example, underground parking is a result of a highrise configuration and not applicable to the mostly suburban garden database. The four categories are: score 1 if Open; score 2 if Covered (there is no differentiation between Covered and Covered/Assigned.); score 3 if Detached Garage (Parking Garage and Underground Parking are counted as Detached Garage.); Score 4 if Attached Garage;
- Number of Pools: total number of pools located on the property in the which unit is located;
- Number of Tennis Courts: total number of tennis courts located on the property in which unit is located;
- Water Volleyball: presence of water volleyball facilities on the property in which unit is located;
- Volleyball: presence of community volleyball facilities;
- Basketball: presence of community basketball facilities;
- Racquetball: presence of community racquetball facilities;
- Jacuzzi: presence of community Jacuzzi;
- Sauna: presence of community Sauna;
- Jogging Trail: presence of community jogging trail;
- Playground: presence of community playground;
- Barbecue Grills: presence of community Barbecue grills;
- Clubhouse: presence of community clubhouse;
- Fitness Center: presence of community fitness center;
- Social Activities: presence of regular management-organized community social activities;
- Washer/Dryer: four categories - Stackable Connections provided (in unit), score 1, Full-size Connections provided, score 2, Stackable W/D provided, score 3, Full-size W/D provided, score 4;
- Self-Cleaning Oven;
- Continuous Cleaning Oven;
- Electric Garbage Disposal;
- Frost-Free Refrigerator;
- Microwave Oven;
- Electric Dishwasher;
- Security Patrol: presence of regular courtesy or security patrol of property in which unit is located;
- Private Alarms: presence of security alarm installed in unit;
- Controlled Access: presence of controlled access fencing and gating on property in which unit is located;
- Patio/Balcony;
- Fireplace: presence of fireplace in unit;
- Extra Storage Space: presence of "extra storage space" defined as non-closet space that is intended exclusively for storage (usually located under stairs or adjacent to patio or balcony);
- Vaulted Ceilings: presence of vaulted ceilings in unit;
- Walk-In Closet: presence of at least one walk-in closet in unit;
- Den: presence of "den" not included in the bedroom total;


## Variables - Location

Each individual parcel of land in the world is unique. There is no other one exactly like it anywhere in the world. Many factors affect the desirability of a specific site from a real estate perspective - ingress and egress, topography, soil, surrounding uses, and zoning among others.

Consider a site with tremendous ingress and egress, a beautiful topography, perfect soil and liberal zoning. But it is located in an area in which general socioeconomic factors are not aligned with the economics of the proposed development. In this case, the aforementioned positives are meaningless. The project has very little, if any chance of success. Conversely, consider a site that does not possess the specific positive characteristics listed above. However, the intended use is perfectly suited to the socioeconomic characteristics of the general area. There are quite possibly actions that can be undertaken to mitigate the specific negatives of the site and still develop a successful project. It may have a chance for success - a good chance. With respect to location, a "top-down" approach to evaluation beginning with the socioeconomic characteristics of the general area in which a property is to be built or exists is required. And these socioeconomic characteristics have the greatest impact on the eventual success of a project. Therefore, they are the most important element in the location decision.

Two location attributes are used in this study as a proxy for the socioeconomic characteristics of the general area in which the units are located:

Average Home Price: The average single family residential sales price for the year ending 12/31/98 for the city in which the property is located. This information was provided by Dallas-based North Texas Real Estate Information Systems, Inc.

A priori expectations were that there would be a significant positive relationship between the price of an average single family home in a city and the multifamily rents within that city.

Mean SAT: The mean Scholastic Aptitude Test score of high school seniors in the Independent School District in which the property is located. This information was provided by the Texas Education Agency, Division of Performance Reporting.

There have been numerous studies exploring the relationship between schools and housing values - Edel and Selar (5), Kain and Quigley (6) and Jud (7), among others. Most have found that the quality of local schools has a significant, positive impact on housing values. The a priori expectation concerning this variable was that there would be a positive correlation between the quality of a school district (as measured by mean SAT score) and multifamily rent within that school district.

## CHAPTER THREE: METHODOLOGY

## Multicollinearity:

Given the large number of variables, the issue of possible multicollinearity needed to be addressed. Multicollinearity is the existence of one or more predictor variables that are very highly correlated with each other and thus have a very similar linear relationship with the dependent variable. A very simple example of collinearity would be if separate variables were assigned in a regression to the presence of a billiards table and the presence of billiard balls. The presence of one would generally indicate the presence of another. The two would be highly collinear. Guntermann and Norbin note that "A model with a large number of variables, particularly if they measure similar attributes or features, is likely to have a high degree of multicollinearity. ...The result would be an estimated equation that is misleading in terms of which features are important. In addition, the regression coefficients might not provide an accurate estimate of the market value of the various features..."

## Consolidation of Variables into Indexes:

To address the multicollinearity issue, preliminary correlation tests to establish degrees of collinearity were performed on the data - primarily the physical characteristics and features attributes represented by binary variables. Significant cross-correlation (over $50 \%$ ) was exhibited by certain variables of similar type. For example, the presence of dishwasher and disposal exhibited a correlation of .826 . In order to mitigate the negative effects of multicollinearity and estimate a more reliable equation, twenty-two variables were grouped into four distinct categories or indexes. NOTE: Some of the variables included in the categories below did not exhibit significant cross-correlation
and hence are included individually in the regression in addition to being included in the appropriate index. For example, the presence of a fireplace is a significant amenity and yet it is also a component of the Interior Amenity Index. The four categories and methods of scoring are:

- Community/Recreational Amenity Score: This consists of recreational "quality-oflife" features that are located not in individual units but in common areas for the use of all residents. The features and characteristics contributing to this score are: \# of Pools, \# of Tennis Courts, Water Volleyball, Volleyball, Basketball, Racquetball, Jacuzzi, Sauna, Clubhouse, Barbecue Grills, Playground, Fitness Center, Jogging Trails and Social Activities. Each feature, with the exception of Number of Pools and Number of Tennis Courts is valued using a binary variable - a value of one if the feature is present, a value of zero if it is not. \# Pools and \# of Tennis Courts are continuous variables and are scored at one point per facility - two pools equals two points etc.).
- Security Score: consists of three security features, each of which is valued using a binary variable - a value of one point if the feature is present, zero if it is not: Security Patrol, Controlled Access and Private Alarms;
- Appliance Score: consists of the following appliance-related features: Self-Cleaning Oven, Continuous Cleaning Oven, Double Oven, Microwave Oven, Frost-Free Refrigerator, Icemaker, Disposal, Electric Dishwasher, Washer/Dryer. Each feature with the exception of Washer/Dryer is valued using a binary variable - a value of one if present, zero if not. Washer/Dryer is valued at zero to four points as detailed in the above "Variables" section.
- Interior Amenity Score: consists of unit specific aesthetic or quality of life features: Ceiling Fans, Patio/Balcony, Extra Storage, Vaulted Ceilings and Walk-In Closet; NOTE: Wall-Wall Carpet was considered as a variable. However, it was eliminated for two reasons: 1) it is ubiquitous in the predominantly suburban garden Metroplex market 2) in the absence of Wall-Wall Carpet, a unit normally has hardwood floors which are considered by many to be a superior surface. If presence of carpet had value one and zero for none, this could result in a unit's interior amenity score being reduced for having hardwood floors. In the alternative, it would be inaccurate to value Carpet at zero and No Carpet at one given \#1.

Ranking Individual Attributes within the Indexes:
Although the collective effects of the various attributes are of primary importance, it is also important to determine some relative impact or significance of the individual components of the indexes. If a developer or investor believes that a property's collective score in one or more of the indexes should be enhanced in an effort to maximize rent, understanding the correlation of individual attributes to rent per square foot is necessary. Once the priority of attributes by virtue of correlation to rent has been established, a cost benefit analysis can be performed to determine the most cost effective way to increase the Index score and positively impact rent.

A simple correlation test was performed to determine the relationship between each individual attribute and rent per square foot. Although few of the individual attributes will by themselves have a significant correlation, the higher the correlation, the more relevant the individual attribute is in the relationship between the Index score and rent. Included in the "Interpretation of Results" section are the individual components of
each Index ranked in order of correlation to the dependent variable - rent per square foot from highest to lowest: (Fireplace, WD and \# POOL are not included in these rankings as they are variables in the regression equation.)

## Regression Analysis:

With monthly rent as the dependent variable and with the data consolidated into the above independent variables, a number of reduced form regression equations were estimated using a standard Microsoft Excel package in an effort to determine the most reliable model. Detailed discussion of the results will focus on the one equation selected as the optimal model. However, the results of all six regression analyses are significant and will be presented in the "Results" chapter.

There were several considerations in determining the best form for the final hedonic equation:

Form of Rent to be used as the Dependent Variable: One aspect of rent variability to be analyzed is preferred configurations of apartments by households (\#BED, \#BATH, DEN and SQFT variables). A priori expectations were that monthly rent would be positively impacted by these variables irrespective of preferred configuration and thus would render inconclusive results. The value of rent per square foot, however, is not necessarily positive with respect to the configuration variables. An equation with rent per square foot as the dependent variable should have a higher probability of yielding conclusive results regarding household apartment configuration preferences and thus was selected as the dependent variable in the final form regression equation.

Linear or Non-Linear Model: The basic regression equation is a linear equation of the form:
1.

$$
y=a x+b
$$

where $y$ is the dependent variable, $a$ is the coefficient of the predictor or independent variable $x$ and $b$ is the $y$-intercept. In many cases, however, a linear expression of the relationship between a dependent variable and one or more independent variables does not result in the most reliable model. One reason for this is the economic principal of Diminishing Marginal Utility. An example would be an apartment with nine bedrooms. A consumer may pay more for a one bedroom unit than a two, and perhaps more for a three bedroom unit than a four. However, as the number of bedrooms increases, the utility of each and thus the additional rent the consumer is willing to pay for each decreases and at some point becomes negative. A variable adhering to this principal would exhibit a non-linear or curvilinear relationship best approximated by a non-linear equation.

There are various methods of accounting for non-linear relationships in a regression analysis. One is to perform the regression using the natural logarithm (ln) of the variables in the equation expected to exhibit non-linear characteristics. If only the dependent variable is logged, the equation takes the form:
2.

$$
\ln (y)=a x+b
$$

If both the dependent and independent variables in the equation are logged, the equation takes the form:
3.

$$
\ln (y)=a[\ln (x)]+b .
$$

If the dependent variable and selected independent variables in the equation are logged, the equation takes the form:
4.

$$
\ln (y)=a\left[\ln \left(x_{1}\right)\right]+b x_{2}+c
$$

Since only positive integers can be logged, variables for which properties have zero values are not logged. Hence, binary variables such as Fireplace and Den as well as \#Bed (efficiencies $=0$ BED) are not logged. The various indexes cannot be logged as well since properties exist with index scores of zero. However, these are variables with a narrow, finite range of possible values so there is little value in logging them. Variables that could exhibit significant non-linear relationship with the dependent variable and were logged in the log/log form regressions are SQFT, UNITS, AGE, HOME\$ and SAT. The dependent variable (rent) was also logged for log form regressions as it was expected to exhibit a non-linear relationship with the independent variables.

The above considerations necessitated performing six regressions in order to select an optimal model. These six consisted of a linear, log-linear and log-log equation using both rent and rent per square foot as the dependent variable.

## Selection of Optimal Model:

The results indicated increased explanatory capability using the log-log model. It resulted in a higher R-Square using both rent and rent per square foot as dependent variables. In addition, unlike the other models, it resulted in virtually identical coefficients irrespective of the form of the dependent variable. For these reasons, the final equation takes the form of Equation 4, log-log form, with rent per square foot as the dependent variable.

TABLE I: SUMMARY OF FINAL REGRESSION INDEPENDENT VARIABLES:

| Regression Variable/Index | If Index - Variables Included |
| :---: | :---: |
| \#BED | - |
| \#BATH | - |
| LnSOFT | - |
| LnUNITS | - |
| LnAGE | - |
| PARK | - |
| \#POOL | - |
| Recreational/ <br> Community <br> Amenity Index RCA | \#POOL, \# tennis courts, water volleyball, volleyball, basketball, racquetball, Jacuzzi, sauna, clubhouse, BBQ grills, playground, fitness center, jogging trail, social activities |
| Security Package - SEC | Controlled access, courtesy patrol, private alarms |
| Washer/Dryer WD | - |
| Appliance Index - APP | Microwave oven, electric disposal, icemaker, frost-free refrigerator, double oven, self cleaning oven, continuous cleaning oven, dishwasher, washer/dryer |
| Fireplace - FP | - |
| DEN | - |
| Interior Amenity Index INT | Walk-in closet, vaulted ceiling, extra storage, patio/balcony, ceiling fans, fireplace, den |
| LnHomeS | - |
| LnSAT | - |

## CHAPTER FOUR: RESULTS OF ANALYSIS

Results of the analyses are presented below:
TABLE II: MEAN AND STANDARD DEVIATION OF REGRESSION
VARIABLES

| Variable |  | Mean |
| :---: | :---: | :---: |
| \#BED |  | Standard Deviation |
| \#BATH | 1.54 | 0.71 |
| SQFT | 1.44 | 0.51 |
| UNITS | 886 | 257.7 |
| AGE | 336 | 143.9 |
| PARK | 12.9 | 9.1 |
| POOL | 1.246 | 0.53 |
| RCA | 1.979 | 1.376 |
| SEC | 6.51 | 3.17 |
| WD | 1.78 | 0.89 |
| APP | 1.58 | 1.15 |
| FP | 5.89 | 2.03 |
| DEN | 0.63 | N/A |
| INT | 0.08 | N/A |
| Home $\$$ | 5.66 | 1.39 |
| SAT | 131126 | 31889 |
|  | 1002 | 85.5 |
| Rent |  |  |
| Rent per SF | 717 | 275.6 |
|  | 0.816 | 0.19 |

## TABLE III: LINEAR EQUATION: MONTHLY RENT AS DEPENDENT

## VARIABLE

## SUMMARY OUTPUT

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.8697963 |
| R Square | 0.7565455 |
| Adjusted R Square | 0.7560504 |
| Standard Error | 136.129 |
| Observations | 7885 |

ANOVA

|  | df | $S S$ | $M S$ | $F$ | Significance $F$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Regression | 16 | 453088397.3 | 28318025 | 1528.135 | 0 |
| Residual | 7868 | 145802723 | 18531.1 |  |  |
| Total | 7884 | 598891120.3 |  |  |  |


|  | Coefficients |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper <br> $95 \%$ |
| Intercept | -199.30207 | 23.62247514 | -8.43697 | $3.84 \mathrm{E}-17$ | -245.6084042 | -152.996 |
| \#BED | -40.721737 | 4.58257493 | -8.88621 | $7.74 \mathrm{E}-19$ | -49.70480184 | -31.7387 |
| \#BATH | 0.2067312 | 5.437179225 | 0.038022 | 0.969671 | -10.45158508 | 10.86505 |
| SQFT | 0.7928435 | 0.013194821 | 60.08748 | 0 | 0.766978114 | 0.818709 |
| UNITS | 0.1840087 | 0.013566363 | 13.5636 | $1.92 \mathrm{E}-41$ | 0.157415036 | 0.210602 |
| AGE | -6.4614544 | 0.211888162 | -30.4946 | $3.8 \mathrm{E}-193$ | -6.876811555 | -6.0461 |
| PARK | 56.835601 | 3.017757233 | 18.83372 | $1.98 \mathrm{E}-77$ | 50.91999519 | 62.75121 |
| \#POOL | -32.262041 | 1.542050516 | -20.9215 | $1.24 \mathrm{E}-94$ | -35.28487031 | -29.2392 |
| RCA | 4.5182994 | 0.676843504 | 6.675545 | $2.63 \mathrm{E}-11$ | 3.191506209 | 5.845093 |
| SEC | 23.681575 | 2.004386688 | 11.81487 | $6.08 \mathrm{E}-32$ | 19.75244428 | 27.61071 |
| WD | -13.585682 | 2.404251345 | -5.65069 | $1.65 \mathrm{E}-08$ | -18.29865312 | -8.87271 |
| APP | 22.436161 | 1.556517387 | 14.41433 | $1.64 \mathrm{E}-46$ | 19.38497319 | 25.48735 |
| FP | -0.0909496 | 4.181982909 | -0.02175 | 0.98265 | -8.28874756 | 8.106848 |
| DEN | -18.380107 | 6.538746159 | -2.81095 | 0.004952 | -31.19778694 | -5.56243 |
| INT | 0.8775079 | 1.691042148 | 0.518915 | 0.603834 | -2.437384172 | 4.1924 |
| HOME\$ | 0.0011811 | $4.88212 \mathrm{E}-05$ | 24.19237 | $8.9 \mathrm{E}-125$ | 0.001085397 | 0.001277 |
| SAT | -0.0497652 | 0.019441192 | -2.55978 | 0.010492 | -0.087875074 | -0.01166 |

TABLE IV: LINEAR EQUATION: MONTHLY RENT PER SQUARE FOOT AS

## DEPENDENT VARIABLE

SUMMARY OUTPUT

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.7121658 |
| R Square | 0.5071802 |
| Adjusted R Square | 0.506178 |
| Standard Error | 0.1343445 |
| Observations | 7885 |

ANOVA

|  | df | SS | $M S$ | $F$ | Significance $F$ |
| :--- | ---: | ---: | ---: | :---: | ---: |
| Regression | 16 | 146.1430772 | 9.133942 | 506.0792 | 0 |
| Residual | 7868 | 142.0051658 | 0.018048 |  |  |
| Total | 7884 | 288.148243 |  |  |  |


|  | Coefficients Standard Error |  | Stat | P-value | Lower 95\% | Upper 95\% |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 0.6074309 | 0.023312812 | 26.05567 | $1.3 \mathrm{E}-143$ | 0.561731592 | 0.6531302 |
| \#BED | -0.043667 | 0.004522503 | -9.65547 | $6.17 \mathrm{E}-22$ | -0.052532182 | -0.034802 |
| \#BATH | 0.0115947 | 0.005365904 | 2.160804 | 0.030741 | 0.001076068 | 0.0221133 |
| SQFT | $-8.7 \mathrm{E}-05$ | $1.30219 \mathrm{E}-05$ | -6.67954 | $2.56 \mathrm{E}-11$ | -0.000112506 | $-6.15 \mathrm{E}-05$ |
| UNITS | 0.0001986 | $1.33885 \mathrm{E}-05$ | 14.83092 | $4.24 \mathrm{E}-49$ | 0.000172319 | 0.0002248 |
| AGE | -0.006189 | 0.000209111 | -29.5947 | $1.3 \mathrm{E}-182$ | -0.006598484 | -0.005779 |
| PARK | 0.062701 | 0.002978198 | 21.05333 | $9 \mathrm{E}-96$ | 0.05686291 | 0.068539 |
| \#POOL | -0.032195 | 0.001521836 | -21.1553 | $1.17 \mathrm{E}-96$ | -0.035178091 | -0.029212 |
| RCA | 0.0050048 | 0.000667971 | 7.492497 | $7.49 \mathrm{E}-14$ | 0.003695369 | 0.0063142 |
| SEC | 0.0224309 | 0.001978111 | 11.33954 | $1.42 \mathrm{E}-29$ | 0.018553244 | 0.0263085 |
| WD | -0.007984 | 0.002372734 | -3.36502 | 0.000769 | -0.012635494 | -0.003333 |
| APP | 0.0232591 | 0.001536113 | 15.1415 | $4.5 \mathrm{E}-51$ | 0.02024787 | 0.0262703 |
| FP | -0.010533 | 0.004127162 | -2.55206 | 0.010728 | -0.018623082 | -0.002442 |
| DEN | -0.014799 | 0.006453031 | -2.29331 | 0.021856 | -0.027448473 | -0.002149 |
| INT | 0.0018519 | 0.001668875 | 1.109643 | 0.267187 | -0.001419583 | 0.0051233 |
| HOME\$ | $1.268 \mathrm{E}-06$ | $4.81812 \mathrm{E}-08$ | 26.32231 | $2 \mathrm{E}-146$ | $1.17379 \mathrm{E}-06$ | $1.363 \mathrm{E}-06$ |
| SAT | $-3.16 \mathrm{E}-05$ | $1.91863 \mathrm{E}-05$ | -1.64712 | 0.099574 | $-6.92125 \mathrm{E}-05$ | $6.008 \mathrm{E}-06$ |

TABLE V: LOG-LINEAR EQUATION: MONTHLY RENT AS DEPENDENT

## VARIABLE

SUMMARY OUTPUT

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.89491396 |
| R Square | 0.800871 |
| Adjusted R Square | 0.80046606 |
| Standard Error | 0.15096622 |
| Observations | 7885 |

ANOVA

|  | Df | SS | MS | $F$ | Significance $F$ |
| :--- | ---: | :---: | :---: | :---: | ---: |
| Regression | 16 | 721.1937842 | 45.07461 | 1977.755 | 0 |
| Residual | 7868 | 179.3180116 | 0.022791 |  |  |
| Total | 7884 | 900.5117958 |  |  |  |


|  | Coefficients | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper <br> $95 \%$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 5.20509956 | 0.02619718 | 198.6893 | 0 | 5.153746125 | 5.256453 |
| \#BED | -0.0087357 | 0.005082047 | -1.71893 | 0.085667 | -0.018697834 | 0.001226 |
| \#BATH | 0.00534562 | 0.006029798 | 0.886534 | 0.375357 | -0.006474388 | 0.017166 |
| SQFT | 0.00086131 | $1.4633 \mathrm{E}-05$ | 58.86091 | 0 | 0.000832626 | 0.00089 |
| UNITS | 0.00023486 | $1.5045 \mathrm{E}-05$ | 15.61033 | $3.99 \mathrm{E}-54$ | 0.000205365 | 0.000264 |
| AGE | -0.0072092 | 0.000234983 | -30.6797 | $2.4 \mathrm{E}-195$ | -0.007669827 | -0.00675 |
| PARK | 0.06269066 | 0.003346674 | 18.73223 | $1.23 \mathrm{E}-76$ | 0.056130292 | 0.069251 |
| \#POOL | -0.0351274 | 0.001710125 | -20.5409 | $2.25 \mathrm{E}-91$ | -0.038479727 | -0.03178 |
| RCA | 0.00628139 | 0.000750615 | 8.368319 | $6.85 \mathrm{E}-17$ | 0.004809983 | 0.007753 |
| SEC | 0.02592705 | 0.002222853 | 11.66387 | $3.52 \mathrm{E}-31$ | 0.021569672 | 0.030284 |
| WD | 0.00625796 | 0.0026663 | 2.347057 | 0.018947 | 0.001031303 | 0.011485 |
| APP | 0.02557937 | 0.001726168 | 14.81858 | $5.07 \mathrm{E}-49$ | 0.022195622 | 0.028963 |
| FP | 0.01593015 | 0.004637793 | 3.434855 | 0.000596 | 0.006838841 | 0.025021 |
| DEN | 0.00931229 | 0.007251429 | 1.284201 | 0.199109 | -0.004902436 | 0.023527 |
| INT | 0.00360483 | 0.001875355 | 1.92221 | 0.054615 | $-7.13686 \mathrm{E}-05$ | 0.007281 |
| HOME $\$$ | $1.5627 \mathrm{E}-06$ | $5.41424 \mathrm{E}-08$ | 28.86306 | $3.3 \mathrm{E}-174$ | $1.45658 \mathrm{E}-06$ | $1.67 \mathrm{E}-06$ |
| SAT | $7.344 \mathrm{E}-05$ | $2.15602 \mathrm{E}-05$ | 3.406272 | 0.000662 | $3.11761 \mathrm{E}-05$ | 0.000116 |

TABLE VI: LOG-LINEAR EQUATION: MONTHLY RENT PER SQUARE

## FOOT AS DEPENDENT VARIABLE

## SUMMARY OUTPUT

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.72757882 |
| R Square | 0.52937094 |
| Adjusted R Square | 0.52841389 |
| Standard Error | 0.15726138 |
| Observations | 7885 |

ANOVA

|  | df | SS | MS | $F$ | Significance $F$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Regression | 16 | 218.8718326 | 13.67949 | 553.1281 | 0 |
| Residual | 7868 | 194.58462 | 0.024731 |  |  |
| Total | 7884 | 413.4564525 |  |  |  |


|  | Coefficients | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper <br> $95 \%$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | -0.5769518 | 0.027289579 | -21.1418 | $1.53 \mathrm{E}-96$ | -0.63044663 | -0.523457 |
| \#BED | -0.0466855 | 0.005293964 | -8.81863 | $1.41 \mathrm{E}-18$ | -0.05706308 | -0.036308 |
| \#BATH | 0.02052369 | 0.006281236 | 3.267461 | 0.00109 | 0.008210802 | 0.032837 |
| SQFT | -0.0001647 | $1.52432 \mathrm{E}-05$ | -10.8074 | $4.91 \mathrm{E}-27$ | -0.00019462 | -0.000135 |
| UNITS | 0.00024721 | $1.56724 \mathrm{E}-05$ | 15.7737 | $3.29 \mathrm{E}-55$ | 0.000216489 | 0.000278 |
| AGE | -0.0069672 | 0.000244781 | -28.4631 | $1.1 \mathrm{E}-169$ | -0.00744708 | -0.006487 |
| PARK | 0.0672252 | 0.003486228 | 19.28308 | $5.4 \mathrm{E}-81$ | 0.060391264 | 0.074059 |
| \#POOL | -0.0381018 | 0.001781435 | -21.3883 | $1.07 \mathrm{E}-98$ | -0.04159392 | -0.03461 |
| RCA | 0.00672578 | 0.000781915 | 8.601674 | $9.38 \mathrm{E}-18$ | 0.005193018 | 0.008259 |
| SEC | 0.02662707 | 0.002315544 | 11.49927 | $2.32 \mathrm{E}-30$ | 0.022087985 | 0.031166 |
| WD | -0.005292 | 0.002777482 | -1.9053 | 0.056777 | -0.01073655 | 0.000153 |
| APP | 0.02815752 | 0.001798148 | 15.65918 | $1.9 \mathrm{E}-54$ | 0.024632672 | 0.031682 |
| FP | -0.0044849 | 0.004831185 | -0.92833 | 0.353265 | -0.01395534 | 0.004985 |
| DEN | -0.015209 | 0.007553808 | -2.01342 | 0.044104 | -0.03001647 | -0.000402 |
| INT | 0.00361355 | 0.001953556 | 1.849732 | 0.06439 | -0.00021593 | 0.007443 |
| HOME \$ | $1.6016 \mathrm{E}-06$ | $5.64001 \mathrm{E}-08$ | 28.39647 | $5.9 \mathrm{E}-169$ | $1.491 \mathrm{E}-06$ | $1.71 \mathrm{E}-06$ |
| SAT | $5.7968 \mathrm{E}-05$ | $2.24592 \mathrm{E}-05$ | 2.581029 | 0.009869 | $1.39419 \mathrm{E}-05$ | 0.000102 |

## TABLE VII: LOG-LOG EQUATION: MONTHLY RENT AS DEPENDENT

VARIABLE

SUMMARY OUTPUT

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.90301517 |
| R Square | 0.81543639 |
| Adjusted R Square | 0.81506107 |
| Standard Error | 0.14534014 |
| Observations | 7885 |

ANOVA

|  | df |  |  |  |  |  | SS | MS | F | Significance $F$ |
| :--- | ---: | :---: | :---: | :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| Regression | 16 | 734.3100876 | 45.89438 | 2172.643 | 0 |  |  |  |  |  |
| Residual | 7868 | 166.2017081 | 0.021124 |  |  |  |  |  |  |  |
| Total | 7884 | 900.5117958 |  |  |  |  |  |  |  |  |


|  | Coefficients |  | Standard Error | $t$ Stat | P-value | Lower 95\% |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | Upper 95\%

## TABLE VIII: LOG-LOG EQUATION: MONTHLY RENT PER SQUARE FOOT

## AS DEPENDENT VARIABLE

SUMMARY OUTPUT

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.77331677 |
| R Square | 0.59801883 |
| Adjusted R Square | 0.59720138 |
| Standard Error | 0.14534014 |
| Obsenations | 7885 |

ANOVA

|  | Df | SS | MS | $F$ | Significance $F$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Regression | 16 | 247.2547444 | 15.45342 | 731.566 | 0 |
| Residual | 7868 | 166.2017081 | 0.021124 |  |  |
| Total | 7884 | 413.4564525 |  |  |  |


|  | Coefficients | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper <br> $95 \%$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | -1.03444132 | 0.185993747 | -5.5617 | $2.76 \mathrm{E}-08$ | -1.39903849 | -0.66984 |
| \#BED | -0.00074564 | 0.005000467 | -0.14911 | 0.881467 | -0.010547887 | 0.009057 |
| \#BATH | 0.04840519 | 0.00568539 | 8.513961 | $1.99 \mathrm{E}-17$ | 0.037260314 | 0.05955 |
| LnSQFT | -0.35929128 | 0.012580391 | -28.5596 | $8.8 \mathrm{E}-171$ | -0.383952192 | -0.33463 |
| LnUNITS | 0.10016476 | 0.00589708 | 16.98548 | $1.42 \mathrm{E}-63$ | 0.088604914 | 0.111725 |
| LnAGE | -0.08636778 | 0.001980354 | -43.6123 | 0 | -0.090249801 | -0.08249 |
| PARK | 0.06174813 | 0.003220675 | 19.17242 | $4.14 \mathrm{E}-80$ | 0.055434752 | 0.068062 |
| \#POOL | -0.03125263 | 0.001604026 | -19.4839 | $1.3 \mathrm{E}-82$ | -0.034396947 | -0.02811 |
| RCA | 0.00692321 | 0.000722555 | 9.581565 | $1.25 \mathrm{E}-21$ | 0.00550681 | 0.00834 |
| SEC | 0.01656457 | 0.002163233 | 7.657319 | $2.12 \mathrm{E}-14$ | 0.012324054 | 0.020805 |
| WD | 0.00820591 | 0.002585347 | 3.174008 | 0.001509 | 0.003137945 | 0.013274 |
| APP | 0.0207039 | 0.001679165 | 12.32988 | $1.3 \mathrm{E}-34$ | 0.017412288 | 0.023996 |
| FP | 0.01967075 | 0.004519803 | 4.352125 | $1.37 \mathrm{E}-05$ | 0.010810732 | 0.028531 |
| DEN | 0.02470463 | 0.007001469 | 3.528492 | 0.00042 | 0.01097989 | 0.038429 |
| INT | 0.00771936 | 0.001802955 | 4.281505 | $1.88 \mathrm{E}-05$ | 0.004185089 | 0.011254 |
| LnHome\$ | 0.1892209 | 0.006583878 | 28.74004 | $8.1 \mathrm{E}-173$ | 0.176314753 | 0.202127 |
| LnSAT | 0.03817842 | 0.020060563 | 1.903158 | 0.057056 | -0.00114561 | 0.077502 |

## Discussion of Optimal Model:

The Log-Log Model presented in Table VIII has excellent explanatory capability with an R-Square of .82 with rent as the dependent variable and .60 with rent per square foot as the dependent variable. These are very high R-Square measurements considering the large number (7885) of observations. The R-Square represents the proportion of the variance in rent that is attributable to the variance in the independent variables tested. Therefore, eighty-two percent of the variance in rent for Dallas/Fort Worth multifamily properties of 200 units or more can be explained by the attributes included in the analysis. And sixty percent of the variance in rent per square foot can be explained by the attributes included in the analysis.

## Analysis of Variance:

The $F$-value is used to determine whether or not the observed relationship occurs by chance. This value must exceed a certain value (the F-critical value) for the model to be considered useful. The F-critical value can be calculated using statistical tables. To read the tables, the Degrees of Freedom $d f$, must be calculated. Degree of Freedom $k$, usually referred to as $v l$, is the number of variables in the regression analysis. Degree of freedom $n$, referred to as $v 2$, is equal to the number of observations less $(k+1)$. There are different F-critical values for different Alphas. Alpha represents the possibility of erroneously concluding that there is a relationship between the variables. For example an Alpha of .05 represents a probability of .05 that there is no relationship between the variables i.e. that the observed relationship is occurring by chance. The Confidence level is equal to 1 minus Alpha. The lowest Alpha included in statistical tables is generally .001. Thus the highest Confidence Level is .999 .

As illustrated in Table VIII, the F-value for the Model was 731.56. The F-critical value using an Alpha of .001 (Confidence Level $=.999$ ) is calculated at 2.46. The observed $F$-value is substantially greater than the $F$-critical value at a .999 confidence level. Therefore, the probability of the observed relationship occurring by chance is infinitesimally small or zero. Thus, the Model is statistically useful in predicting multifamily rents as a function of the attributes tested.

A Least Squares regression analysis involves choosing a line that best fits a given set of data points. The line that is chosen, among all possible lines, is that which results in the smallest sum of squared deviations of the data points from the line. In Table VIII, the heading SS in the ANOVA section represents the Sum of the Squares. It is a measurement of the total variance of the all of the data points from the line that the model equation represents. The MS heading represents the Mean Square of the data points from the line. It is a measurement of the mean squared deviation of the data points from the line that the model equation represents. Smaller values for SS and MS mean less variance in the data points from the model and thus a more accurate model ceteris paribus. The SS and MS values in Table VIII (rent per square foot as dependent variable) of 247.25 and 15.45 respectively are significantly lower than the SS and MS values of 734.31 and 45.89 in Table VII (monthly rent as dependent variable). So, while the R-Square is lower in the rent per square foot model, this is outweighed by the fact that there is much less variance (by a factor of $2 / 3$ ) that the model must explain. Thus, the rent per square foot model is a more precise predictive equation than the rent model.

## Standard Error:

The Standard Error of a regression equation is the standard deviation of the observed value of the dependent variable about its predicted value. It is not a standard error in the conventional use of the term, as a measure of the standard deviation of the sampling distribution of a statistic. Rather, it is an estimate of the standard deviation of the predicted value of the dependent variable about the true regression line. It is equal to the square root of the (sum of the differences between the observed and predicted values for y) squared, and divided by (number of observations minus two).

The Standard Error of the model, as illustrated in Table VII is equal to .145340137. This means that the standard deviation of rent per square foot predicted by the model is $14.53 \%$. This equates to a standard variance of $2.11 \%$ with respect to predicted rent about the true regression line.

## The t-statistic:

Just as the F-statistic is used to determine the usefulness of the regression equation as a whole, the $t$ Statistic is used to determine the usefulness or significance of individual independent variables within the regression equation. It is calculated by dividing the value of a coefficient by its standard error. T distribution tables list the critical value of $t$ given inputs of degrees of freedom and Alpha. If the $t$-statistic for a coefficient is greater than the $t$ critical value, the corresponding variable is useful in estimating the value of the dependent variable - i.e. it is significant.

Fifteen of the sixteen variables included in the model were significant ( $t$ statistic>1.75) assuming a one-tailed test with sixteen degrees of freedom and Alpha $=$
.05. Fourteen of the sixteen variables were significant $(\mathbf{t}$-statistic $>2.92)$ at Alpha $=$ .005.

## The Model:

The Dallas/Fort Worth Hedonic Rent Model derived from the optimal equation is:

$$
\begin{aligned}
& \text { LnRent/SF }=-1.034444132-.00074564(\# B E D)+.048405189(\# B A T H)- \\
& .35929128(L n S Q F T)+.100164758(\text { LnUNITS })-.08636778(L n A G E)+ \\
& .061748131(P A R K)-.03125263(\# P O O L)+.00692321(R C A)+.016564566(S E C)+ \\
& .008205913(W D)+.020703898(A P P)+.019670747(F P)+.02470463(D E N)+ \\
& .007719359(I N T)+.189220903(\text { LnHome } \$)+.038178424(S A T)
\end{aligned}
$$

Using this Model, rent per square foot can be predicted given inputs of the attributes of a given property and an estimate of the pricing or contribution to overall rent of the various attributes represented by the independent variables can be calculated.

## Testing the Model:

A random observation (\#32 of 7885) can be used to test the model's predictive ability and to illustrate the implicit pricing of the various attributes of the property. Observation \#32 in the database is a one bedroom, one bath unit. It has an area of 600 square feet. It is contained in a property containing 252 total units. It has an effective age of 16 years. It has open parking and contains one pool. It has a Recreational/Community Amenity (RCA) Index score of 4 (presence of Jacuzzi, pool, clubhouse and organized social activities). It has a Security (SEC) Index score of 2 (presence of courtesy patrol, and private alarms in selected units). It has an Appliance (APP) Index score of 6 (presence of frost-free refrigerator, dishwasher, disposal, and stackable washer/dryer units). It has a fireplace and no den. It has an Interior Index
score of 5 (presence of fireplace, patio or balcony, extra storage space, vaulted ceilings and walk-in closets). It is located in Arlington; a city in which the sales price for a single family home in 1998 averaged $\$ 112,768$. It is located in a school district, the Arlington ISD, in which the mean SAT score for graduating seniors in 1998 was 1042.

## TABLE IX: HEDONIC RENT MODEL CALUCLATOR



Referring to Table VIII above, the data concerning the attribute profile of observation \#32 are entered in the "Input" column of the model. The "Input" column contains the input values for the attributes. The "Values" column converts these inputs to their natural log where necessary. These values are then multiplied by the corresponding coefficient in the Hedonic Equation and the values are summed at the bottom of the "Attribute Price" column. Since the dependent variable - rent per square foot - is in logged form, this sum is converted to rent per square foot in dollars below the sum. The
result is multiplied by the area in square feet of the unit type to obtain the predicted monthly rent.

The predicted rent per square foot for observation \#32 is calculated at $\$ .814$ per square foot. Monthly rent is predicted at \$488. Actual average monthly rent for observation 32 is $\$ .792$ per square foot or $\$ 475$ monthly. However, the range of rent for this observation was $\$ 470-\$ 480$. If an attribute is available in selected units, it is counted as present in all units. The rationale for this is an assumption that the lower rent reflects the units that do not have the optional features and the higher rents reflects the units that do. The dependent variable rent was averaged in the case of a range for each unit type. So, this assumption equates to one-half of the units of that type not possessing the optional attribute(s) and renting at the low end of the range and half of the units of that type possessing the optional attribute(s) and renting at the high end of the indicated range.

As a result, in the case of an observation with a range of rents for the same unit type, the proper comparison between the predicted and the actual rent is to compare the predicted with the high end of the range for that unit type.

If the actual figures of $\$ .80$ per square foot and $\$ 480$ monthly rent are compared to the predicted figures of $\$ .814$ and $\$ 488$ monthly results, the variance between the predicted and actual dependent variable is calculated at $.0175(1.75 \%)$. This is slightly better than the "standard" variance between the predicted and actual $x$ values of $2.11 \%$. The Model demonstrates quite accurate explanatory and predictive power. Later in the report, it will be utilized to analyze the implicit pricing of the various attributes represented by the independent variables. The weights or percentage contribution of each
attribute or group of attributes will be determined and a sensitivity analysis can be performed to ascertain the effect of changes in the "package" of attributes on rent for the selected unit type.

Statistical Results for Individual Variables (complete results in tabular form can be found in TABLE VIII for regression and TABLE II for mean and standard deviation.)
\#BED is not statistically significant ( t -statistic -.149). The model indicates that adding or deleting bedrooms has little to no effect on multifamily rent per square foot. The mean number of bedrooms contained by units in the database is 1.54 with a standard deviation of 71 .
\#BATH: The number of bathrooms contained in a unit as measured by the variable \#BATH is statistically significant (t-statistic 8.51 ) and is positively correlated to rent. The coefficient indicates that each additional bath generates $4.84 \%$ in additional rent per square foot. A half-bath generates half this percentage increase or $2.42 \%$. The mean number of bathrooms contained by units in the database is 1.44 with a standard deviation of .51 .

SQFT: The floor area in square feet of a unit as measured by the variable SQFT is statistically very significant ( t -statistic -28.56 ) and has a negative coefficient. According to the model, as the area of a unit increases, the rent per square foot tends to decrease. As the floor area of a unit doubles, predicted rent per square foot decreases by approximately $22 \%$. The mean square footage of units in the database is 886 with a standard deviation of 257.7.

UNITS - the number of units contained in the property in which the unit is located as measured by the variable UNITS is a surprisingly significant variable (t-
statistic 16.99). It is positively correlated with rent. The relationship between UNITS and rent per square foot is non-linear so there is not a constant effect across ranges of units. As an example, the predicted rent for a unit contained in a property consisting of 400 units would be approximately $7.2 \%$ higher than a unit with the same attribute profile located in a property consisting of 200 units. The mean number of units contained in properties represented by the database is 336 with a standard deviation of 143.9.

AGE: Effective age as measured by the variable AGE is the most significant variable ( $\mathbf{t}$-statistic -43.61) in the Model. There is a notable negative correlation between the effective age of a property and its rent. The relationship between AGE and rent per square foot is non-linear, so there is not a constant effect over time. As an example, the predicted rent of a given unit type that has an effective age of 10 years would be approximately $18.1 \%$ less than a unit with the same attribute profile having an effective age of 1 year. The mean effective age of units in the database is 12.9 years with a standard deviation of 9.1 years.

PARK: The level of parking facilities as measured by the variable PARK is statistically very significant (t-statistic 19.17) and is positively correlated with rent per square foot. The range of possible values for PARK is 1 through 4. According to the model, as the score representing level of parking facilities increases by 1 (for example, from open parking to covered parking), rent per square foot increases by approximately $6 \%$. The mean parking score (on a scale of 1-4) of properties in the database is 1.25 with a standard deviation of .53
\#POOL: The number of pools contained in the property in which the unit is located as measured by the variable \#POOL is a significant variable ( $\mathbf{t}$-statistic -19.48).

Surprisingly, the coefficient has a negative sign indicating that \#POOL is negatively correlated with rent per square foot. For each pool present on a property, there is an approximate $3 \%$ decrease in predicted rent for unit types contained in that property. The mean number of pools for properties in the database is 1.979 with a standard deviation of 1.376 .

RCA: The level of recreational/community amenities as measured by the RCA Index variable is statistically significant (t-statistic 9.58 ) and is positively correlated with rent per square foot. The index is composed of eleven individual attributes. According to the model, an increase of 1 point in the RCA Index score results in predicted rent per square foot increasing approximately 70 basis points ( 7 of one percent). The mean RCA index score of properties in the database is 6.51 with a standard deviation of 3.17.

SEC: The level of security-related features as measured by the SEC Index variable is statistically significant ( $\mathbf{t}$-statistic 7.66 ) and is positively correlated with rent per square foot. The index is composed of three individual attributes. According to the model, an increase of one point in the SEC Index score results in predicted rent per square foot increasing approximately $1.6 \%$. The mean SEC index score of units in the database is 1.78 with a standard deviation of .89 .

WD: The presence and type of washer/dryer facilities as measured by the variable WD is statistically significant (t-statistic 3.17) and is positively correlated with rent per square foot. WD is scored on a scale of 0-4 depending on the type of facilities provided. According to the model, an increase of one point in the WD score results in predicted rent per square foot increasing approximately 80 basis points ( .8 of one
percent). The mean WD score for properties in the database is 1.58 with a standard deviation of 1.15 .

APP: The level of appliances present in the unit as measured by the APP Index variable is the most statistically significant of the Indexes ( $\mathbf{t}$-statistic 12.33) and is positively correlated with rent per square foot. The Index is composed of nine individual attributes, each valued at one point toward the total Index score with the exception of washer/dryer, which is a component of the index and is valued at $0-4$ points. According to the model, an increase of one point in the APP Index score for a unit results in a predicted rent per square foot increase of approximately $2.1 \%$. The mean APP Index score of units in the database is 5.89 with a standard deviation of 2.03 .

FP: The presence of a fireplace as measured by the variable FP is statistically significant (t-statistic 4.35) and is positively correlated with rent per square foot. According to the model, the presence of a fireplace increases the predicted rent per square foot of a unit by $1.96 \%$. The mean FP score for units in the database is .63. (FP is a binary variable so standard deviation is not applicable).

DEN: The presence of a den as measured by the variable DEN is statistically significant (t-statistic 3.52) and is positively correlated with rent per square foot. According to the model, the presence of a den increases the predicted rent per square foot of a unit by $2.47 \%$. The mean DEN score for units in the database is .08 . (DEN is a binary variable so standard deviation is not applicable).

INT: The level of interior appointments as measured by the INT Index variable is statistically significant (t-statistic 4.28) and is positively correlated with rent per square foot. The Index is composed of seven individual attributes. Each is valued at one point
toward total Index score. According to the model, a one point increase in INT index score results in predicted rent per square foot increasing by approximately 80 basis points (. 8 of one percent). The mean INT Index score of units in the database is 5.66 with a standard deviation of 1.39 .

Homes: The average home price within the city in which the unit is located as measured by the location variable Home\$ is statistically very significant (t-statistic 28.74) and is positively correlated with rent per square foot. According to the model, as Home\$ doubles, rent per square foot increases approximately $14 \%$. The mean Home\$ score for the units in the database is $\$ 131,126$ with a standard deviation of $\$ 31,889$.

SAT: Contrary to a priori expectations, quality of the school district in which the unit is located, as measured by the variable SAT, is not significantly correlated with rent per square foot (t-statistic 1.90). According to the model, a 100 -point decrease in mean SAT score decreases the predicted rent per square foot of a unit by only 50 basis points ( .5 of one percent). The mean SAT score for the school districts in which the units are located is 1002 with a standard deviation of 85.5 .

## CHAPTER FIVE: INTERPRETATION OF RESULTS

## Implications of Independent Variable Results:

\#BED: The fact that this variable is not significant is compelling. By using rent per square foot as the dependent variable as opposed to total rent, the relative pricing of additional bedrooms can be determined. If observation \#32 is changed to an efficiency by inputting 0 for \#BED rather than 1 , the effect is an insignificant increase in rent per square foot from $\$ .814$ to $\$ .815$.

It appears that, aside from a desire to have their basic needs met, tenants do not place any economic value on bedrooms. Therefore, developers should make decisions regarding the configuration or unit mix of a project simply from a total size and demographic perspective without consideration for the number of bedrooms as a determinant of rent.
\#BATH: Intuitively, one would think that if tenants do not place any economic value on additional rooms to sleep in, they would not place any value on bathrooms either. However, the configuration of a unit with respect to number of baths is significant. The model indicates a value per bathroom of $\$ 23$ per month or $4.77 \%$ of monthly rent. This is probably more applicable for the addition of a second bathroom, however. The value of one bathroom versus zero is certainly greater than this. If observation \#32 is manipulated by adding a half-bath, the model suggests an increase of $\$ .02$ per square foot per month or $\$ 12$ per month in additional rent (\$144 per year). Using this information, a developer could compare the Net Present Value (NPV) of these projected cash flows with the NPV of the incremental construction and operating costs of
the half bath. A rational economic configuration decision could then be made as a result of this analysis.

Perhaps extra baths are a proxy for higher quality as developers believe they are not a necessary item and not economically justified at lower rent levels. The number of bedrooms in a unit is economically irrelevant. However, developers should consider the cost/benefit ratio of adding a half or full bath to a unit depending on the number of bedrooms as this does have a positive impact on rent per square foot.

SQFT: Like other goods and services, apartment units are subject to "volume discounts" and the principal of diminishing marginal utility. Tenants would seem to prefer to live in a larger space as opposed to a smaller one and yet it appears that they are not only unwilling to pay more (per square foot) to do so but in fact demand a discount.

There are additional factors to consider when interpreting this information such as the construction cost of incremental square feet as compared to the additional rent (in absolute terms) received. For example, changing the floor area of observation \#32 to 900 square feet would result in a reduction in rent per square foot of $\$ .11 / \mathrm{ft}$ to $\$ .704$ but an increase in monthly rent of $\$ 145$ to $\$ 633$. Configuration is thus an optimization exercise, as the developer must balance the incremental costs of constructing the additional space with the increase in total rent and relative decrease in rents per square foot.

On a per square foot basis, in the subject market, a developer is not rewarded with higher rent for providing tenants with additional floor area. A developer must anticipate a decrease of approximately $22 \%$ in rent per square foot as space doubles and calculate optimal space based on the additional total rent versus the incremental costs of constructing and operating the additional space.

UNITS: The fact that this has a positive correlation to rent is a rather surprising result. One explanation could be that the database includes only properties containing 200 or more units. The "boutique" urban lofts and walk-ups are omitted from this database. These typically have high rent per square foot and, if included, could have an impact on this variable.

The fact remains, however, that for Dallas/Fort Worth suburban garden properties containing 200 or more units, there is a significant positive correlation between total units and rent. For example, if observation \#32 were contained in a property with 350 total units rather than 252 , predicted rent would be $\$ .841 /$ sf and $\$ 505$ per month -an increase of $3.4 \%$.

Another explanation could be that there are a limited number of developers in the market with the resources to develop the larger properties. These developers with superior resources choose perhaps to concentrate on the higher quality properties with correspondingly higher rents.

Another explanation could have to do with the AGE variable. The AGE variable is significantly positively correlated with rent. The increased availability of debt and equity funding since 1994 and the ensuing building boom has caused larger and larger projects to be recently constructed. And these newer, larger projects have correspondingly higher rent in large part because of a lower effective age.

This is positive information for the developer/investor. Intuitively, one would surmise that as the number of units increased, relative rent per square foot would decrease as the dwelling experience is less personalized. But this does not appear to be the case. It appears that a developer/investor can enjoy the benefits that may accrue from
construction/acquisition and operating economies of scale and still capture top-of-the market rent.

AGE: If the effective age of observation \#32 is reduced from 16 years to 1 , the predicted rent per square foot becomes $\$ 1.034$ from $\$ .814$, an increase of $27 \%$. Conversely, as effective age decreases from 1 to 16 years, a decrease in rent of $21.2 \%$ occurs. This illustrates the very significant impact of the effective age of a unit on its rent.

Separating Age into Two Components: Clapp and Giaccotto (8) used 8024 single family residential properties that sold twice between 1981 and 1991 in Fairfax County, VA to develop a model demonstrating two components of the age coefficient in a standard hedonic model - a pure cross-sectional depreciation component and a demand side component. They argue that traditional views of the age coefficient focus solely on the depreciation aspect i.e. older properties are worth less because they are less productive and more costly to maintain. By separating the age coefficient into a growth (decline) component in addition to the depreciation component, they demonstrate that the age coefficient can vary depending on expected returns and features (like high ceilings) associated with homes of a certain vintage as compared to newer homes.

It is not clear whether a similar dynamic exists in the multifamily housing rental market. Certainly, tenants of multifamily rental housing have no growth or return expectations. But it is possible that apartment buildings of a certain vintage could have a demand component related to construction, features or location that "dominates" the depreciation component of the age coefficient from time to time. However, this would likely apply to smaller, urban properties, a profile not represented in the database upon which this model is based.

The Age-Rent Depreciation Effect: With respect to the suburban garden class of apartment assets, the relationship between age and rent is clearly negative, with very little exception. This fact may have important implications - particularly with respect to acquisition and disposition strategies. Revisiting observation \#32, the difference in rent between this unit type having an effective age of one year compared to that of the identical unit type having an effective age of five years would be $-12.96 \%$. This of course does not mean that the unit's rent per square foot will decrease by $12.96 \%$ over this time period, it simply represents the unit's decrease in relative rent i.e. its rent as compared to newer units having an identical attribute profile. Demand-induced increases in rent levels would offset this. Referring again to the example, this age depreciation effect is considerably less pronounced for years $6-10$ at $-4.28 \%$. For years $11-15$, it is just -2.20\%.

An Application to Acquisition and Disposition Strategy: As the effective age of a property increases, operating costs generally increase as additional repair and replacement is required. The depreciation in rent and the increase in costs both have a relationship with effective age and the combination of the two has a negative compound effect on Net Operating Income. However, there is a potential opportunity to use the relationship between effective age and rent illustrated in the model to develop an optimal acquisition and disposition strategy. For example, if it assumed that the age-induced increase in operating costs begins slowly and accelerates as the effective age increases, then there is an opportunity to mitigate the overall negative effect of the rent depreciation and operating expense increase on NOI given the age-rent depreciation pattern illustrated by the model.

If the sum of the percentage rent depreciation and operating expense increase are viewed as an overall age effect, this age effect will be less in the years when rent depreciation has leveled off but before operating expense increases have begun to accelerate. If it assumed that operating expense increases are greatest in years 16-25 and rent depreciation is greatest in years $1-5$, then the overall age effect is least negative in years 6-15. Although a quantified, detailed examination of this is beyond the scope of this report, quantifying a general age rent depreciation effect is the first step in identifying the relationship between these two effects and incorporating it into an acquisition and disposition strategy.

According to the model, a developer/investor should anticipate relative rent per square foot (that is, the rent per square foot of the subject property as compared to a newer unit with the same attribute profile) to decrease by approximately $2 \frac{1}{2} \%$ as the effective age of a property increases by one year during the first five years, just under one percent per year during years 6-10 and .4-. $5 \frac{1}{2} \%$ per year during years 11-25.

PARK: Parking facilities are indeed a very significant factor of rent. This is an attribute for which the predicted rent generated by the model can assist in identifying a value-added opportunity. An approximate $6 \%$ increase in predicted rent results from each one point improvement in PARK.

Applying the model to observation \#32, the current parking facilities are valued at $\$ 29$ per month or $5.94 \%$ of total rent. If the facilities were upgraded from open parking (score 1) to detached garage (score 3), the predicted rent per square foot increases from $\$ .814$ to $.922-\mathrm{a} 13 \%$ increase. This indicates that tenants would be willing to pay an additional $\$ 64.80$ in monthly rent for detached garage parking facilities.

If there are 40 of these unit types contained in the property, the result is additional Gross Potential Income of $\$ 2592$ per month or $\$ 31,104$ per year. Deducting a vacancy allowance and comparing the Present Value of this incremental cash flow to the Discounted Cost of constructing and operating the detached garages would inform the developer/investor as to whether or not the addition of detached parking garages would add value to the project.
\#POOL: The fact that a strong negative correlation exists between number of pools and rent is a surprising result. Intuitively, one would think that a pool represents an enjoyable amenity and an attractive view asset in most cases. There are several possible explanations for this relationship.

Although there is a negative correlation between number of pools and rent, this does not necessarily mean that a property should not have a pool in order to maximize rent. The negative correlation could exist primarily because properties with lower rent tend to have more pools. A property that is oriented toward families with children may tend to have more pools. Properties that are oriented toward families with children typically have lower rents in the subject market than properties oriented toward young professionals and "lifestyle renters".

The most plausible explanation is related to the AGE variable. Recent construction seems to indicate a trend in Dallas/Fort Worth to decrease both the size and number of pools relative to the total units of a project. Perhaps increased skin cancer concerns related to sunbathing, increased personal injury liability exposure on the part of apartment owners and a decrease in leisure time have contributed toward reduced demand for and hence a reduced supply of apartment pools. If this were indeed a trend, it would
explain the negative correlation. Newer properties have higher predicted rent than older properties. If newer properties indeed have fewer pools on average than older properties, this would contribute toward a negative correlation between number of pools and rent.

This model does not present a definite answer to this issue. What is clear however, is that in the subject market, the inclusion of multiple pools does not increase rent per square foot and in fact results in decreased predicted rent per square foot. The installation and operating expenses associated with a pool are substantial. Developers have the opportunity to value-engineer by incorporating fewer, smaller pools as a means of reducing construction and operating costs with little risk of decreasing rent in the process.

RCA: The level of recreational/community amenities is collectively correlated with rent. An increase of one point in the Index score results in a relatively small increase in predicted rent per square foot ( 7 of one percent). Applying the Model to observation \#32 (which has a below average RCA score of 4), a 3-point upgrade in this Index results in a monthly rent increase of $\$ 11$ ( $(\$ 132$ per year). Collectively, the unit's recreational/community amenities are priced at only $\$ 13$ per month or $2.66 \%$ of total rent according to the model.

However, there is leverage in the sense that the cost of constructing or installing one community attribute affects the predicted rent per square foot on all of the units. So although the effect on a per unit and per-square foot basis is modest, there may be valueadded opportunities within this category for the developer/investor.

Notable among components of the Index is the relatively high correlation to rent of inexpensive amenities such as barbecue grills, jogging trail and social activities.

Ranking of Individual Attributes: Recreational/Community Amenities (RCA)
Attribute

1) Fitness Center ..... 487
2) Clubhouse ..... 298
3) Jacuzzi ..... 290
4) Barbecue Grills ..... 262
5) Social Activities .....  258
6) Jogging Trail ..... 195
7) Water Volleyball ..... 135
8) Racquetball ..... 101
9) Volleyball ..... 050
10) Basketball .....  040
11) Tennis Courts ..... -. 071
12) Playground ..... -. 194

SEC: The mean database score for the Security Index (SEC) is 1.78 which indicates that a large percentage of communities possess more than one of the attributes that comprise this index. Apartment security concerns have heightened over the past decade because of increasing crime rates and liability exposure on the part of landlords. Strictly from a liability standpoint, many consider it a positive step to invest in the latest security technology.

Applying the Model to observation \#32, an upgrade of one point in the Security Index score results in an increase of \$9 in monthly rent (\$108 per year). Collectively, the Model prices the unit's security features at $\$ 16$ per month or $3.28 \%$ of total rent.

Notable among the components of the Index is courtesy patrol, which is negatively correlated with rent.

The fact that security features collectively have a significant positive correlation to rent as well as the potential to mitigate liability should encourage developers/investors to strongly consider controlled access and private alarms when looking for value enhancement opportunities.

Ranking of Individual Attributes: Security Index (SEC):

## Attribute Correlation

1) Controlled Access . 373
2) Private Alarms . 356
3) Courtesy Patrol $-.093$

WD: Washer/Dryer facilities as represented by the WD variable are significant and positively correlated with rent. Applying the Model to observation \#32, which currently has stackable washer/dryer units provided (3 points out of a possible 4), an upgrade to full-size washer/dryers would add only $\$ 4$ per month to rent or $\$ 48$ per year. The current washer/dryer facilities are valued by the model at $\$ 11$ per month or $2.25 \%$ of total rent.

Washer/dryers is an amenity for which a cost/benefit analysis should be performed by the developer/investor as it is not clearly a value-added item. If, for example, observation \#32 provided only full-size connections instead of stackable
washer/dryer units, rent would decrease only $\$ 4$ per month and yet the cost of the stackable washer/dryers would be eliminated.

APP: Applying the Model to observation \#32, the implicit pricing of the collective appliance package is $\$ 57$ per month or $11.68 \%$ of total monthly rent. An upgrade from 6 to 7 in this score would increase predicted rent by $\$ 11$ per month.

Notable among components of the Index is the clear priority of microwave and icemaker in terms of correlation. It is also apparent that type of oven is fairly irrelevant to a unit's rent. And the low correlation of the ubiquitous frost-free refrigerator, dishwasher and disposal simply indicates that low, medium and high rent units all have these attributes. The relatively high median score for APP of 5.89 suggests that a unit must have a fairly complete appliance package just to be average in this category. Ranking of Attributes: Appliance Index (APP)

## Attribute

1) Ice-maker . 497
2) Microwave469
3) Self Cleaning Oven ..... 117
4) Dishwasher ..... 051
5) Frost-Free Refrigerator ..... 048
6) Continuous Clean Oven .....  028
7) Disposal ..... 023
8) Double Oven ..... 007

FP: Interestingly, the mean fireplace score, as it is a binary variable, represents the percentage of units in which this attribute is present $-63 \%$. This is a relatively high figure and yet the presence of a fireplace in a unit increases predicted rent per square foot by only $1.97 \%$. For observation \#32, this amounts to $\$ 9.61$ in monthly rent or $\$ 115.32$. This is an attribute for which a cost benefit analysis should be performed as it may add value for some projects and detract from it for others.

DEN: The percentage of units in the database with a den is $8 \%$. The presence of a den in a unit increases predicted rent for a unit by $2.47 \%$. Observation \#32 does not have this attribute. If it did, predicted monthly rent would increase by $\$ 12.05$ per month or $\$ 144$. This incremental income generated by presence of a den should be weighed against the costs of construction and any additional operating/maintenance costs to determine if it enhances value or detracts from it.

INT: Collectively, the sample \#32 interior amenities (score -5 ) are priced by the model at $\$ 18$ per month. The mean INT Index score for the database is 5.66 out of a possible 7. This combined with the relatively low correlation of some of the individual attributes suggests that, like the appliance package, the presence of most of these attributes is required in order to simply "keep pace" with the market.

Notable among components of the index is the low ranking of Patio/Balcony, which is surprising. This may be related to AGE, as fewer and smaller patio/balconies seem to be a trend in recent project configuration.

Attribute Ranking: Interior Index (INT) Correlation

1) Vaulted Ceilings . 300
2) Extra Storage
3) Ceiling Fan . 170
4) Patio/Balcony . 078
5) Walk-In Closet . 000

Homes: The relative location of a project within which a unit is located has a tremendous impact on the rent per square foot of the unit. Location is defined by the Home\$ variable as the average sales price (1998) for the city in which the property is located. There are 35 cities represented in the observations. Observation \#32 is located in Arlington, a city with an average 1998 single family residential sales price of $\$ 112,768$. If a unit identical in every other respect were located in Addison, the city with the highest average home price in the metroplex $(\$ 204,659)$, predicted rent would increase from $\$ .814$ to $\$ .911$ per square foot, an increase of $12 \%$.

This is a strong positive correlation. However, this relationship equates to a $100 \%$ increase in average home price correlating with only a $14 \%$ increase in rent. While significant, there is a notable discrepancy between the increase in home price and the corresponding increase in rent as the unit is "relocated" to Addison. This discrepancy indicates a disconnect in the relationship between these two factors. For whatever reason, renters place a lesser value on the services or attributes related to location than do homeowners.

The average home price data used in the regression analysis is not qualitycontrolled. It represents average sale price of a home in the various cities comprising the Dallas/Fort Worth metropolitan area. From year to year, the relative quality of homes sold can vary. Assume the average sale price for a home in a given city in year x is $\$ 100,000$. If the relative quality of homes sold in that year is high relative to the overall
quality of homes in the city, then the $\$ 100,000$ figure would overstate the price of the average home in that city. If this is the case with the data used in this analysis, then the disconnected nature of the relationship between Home\$ and rent could be exaggerated. Of course, there is the possibility that the relative quality of homes sold is relatively low. In this case, the indicated disconnect would be understated.

The home price data used in the analysis represents a total of 49,075 transactions in 35 different cities. Despite the lack of statistical controls for quality, it seems likely that with this number of observations and distinct markets, the results of the analysis in the aggregate represent fairly the nature of the relationship between home price and rent in the subject market.

## Relative Costs of Homeownership and Renting: A general relationship between

 rent and Home\$ can be established using the price of the average home and monthly rent per square foot. However, a more accurate measure of the cost of homeownership as compared to rent must compare the annual cost of homeownership versus the annual cost of renting. Wheaton and DiPasquale (9) define the annual cost of homeownership as the purchase price multiplied by the user cost of capital. The user cost of capital is a function of the user's after-tax mortgage rate, the opportunity cost of the equity/down payment and expected house appreciation. The relative costs of homeownership versus renting vary over time and from market to market.Home price data used in the analysis was for calendar 1998 - a year during which mortgage rates reached historic lows. At the same time, prices soared in the Dallas/Fort Worth residential market, with growth expectations at extremely high levels. This combination has resulted in a very low user cost of capital and correspondingly low
relative annual homeownership costs in this market. So, in historic terms, a higher priced home could be bought in 1998 than could be bought say, in 1994 for the same annual cost. Simultaneously, in the rental housing market, tremendous new supply has been delivered in the last five years. Beginning in 1998, this has caused a certain degree of concessions and "softness" to arise in rental rates.

So, a possible explanation of the disconnect between increases in home prices compared to rental rates is the combination of a very low user cost of capital. This enables a prospective homeowner to afford a higher priced house relative to annual cost. Factors contributing to this low user cost of capital (mortgage rates, growth expectations, opportunity cost) do not apply to the annual cost of renting. So renters are not willing to pay more for an apartment as a result of these factors. The huge supply of new projects keeps rents increasing only at moderate levels. Therefore, there is a divergence between the increase in Home\$ and the increase in rent.

Another possible explanation has to do with the fact that the Dallas/Fort Worth market has relatively low apartment rent compared to other major metropolitan markets. Markets like San Francisco, Seattle, Boston, New York, Chicago, and others have significantly higher rents - particularly in the high-end luxury niche. Dallas/Fort Worth has undergone tremendous economic growth and perhaps the apartment market has lagged behind the housing market in adjusting to this growth.

Application: From an apartment development perspective, if a developer expects this divergence to continue, sites within cities having relatively low housing prices would produce maximum profits assuming land pricing is commensurate with Home\$. However, if a developer believes that this discrepancy will be reduced as a result of
higher top-line rents, then the opposite approach should be taken, focusing on in-fill locations within the highest Home\$ cities and areas in the market.
'SAT: A priori expectations were that this would be a significant location variable. This was not the case. The most likely explanation relates to the Dallas Independent School District. This is the largest ISD in the metroplex and encompasses the metropolitan Dallas area. Over $40 \%$ of the 7885 observations used in the analysis were located in this ISD. Mean SAT score for this ISD is the lowest of observed ISD's at 872. Although this ISD represents some lower-income areas, it also represents some extremely affluent "pockets". These pockets of affluence affected the relationship between SAT and the location variable Home\$. These two were expected to be fairly collinear but were not. Similarly, these pockets of affluence caused the relationship between rent per square foot and SAT to be inconclusive as well. Although not tested, it is hypothesized that for the remaining $60 \%$ of observations (suburban), the relationship between SAT and rent per square foot would follow the expected relationship.

## SUMMARY AND CONCLUSION

## Summary of Study:

In order to maximize value in the development or acquisition decision, it is crucial for the developer/investor to thoroughly understand the relationship between the attributes of a given product type and its value or rent. Since the prices of individual attributes or characteristics are not in most cases, directly observable, they must be estimated.

One means of estimating these prices is through a hedonic regression. A relatively small amount of research has been initiated to estimate hedonic rent equations for multifamily housing. There are significant differences in the results of the studies that have been conducted. These differences underscore the importance of updating existing research and further study of new markets.

As the basis for the hedonic analysis conducted for this report, data was collected on 1007 properties located in the Dallas/Fort Worth area known as the "Metroplex". Information concerning thirty-eight attributes was collected. Twenty-two variables were grouped into four distinct categories or indexes in order to avoid multicollinearity and estimate a more reliable equation.

Using both monthly rent and rent per square foot as dependent variables and with the data consolidated into sixteen independent variables, a number of regression equations were estimated. A Model in which the dependent variable and certain independent variables are logged was selected as the optimal form for the final regression equation. This Model has excellent explanatory capability with an R-Square of .82 with rent as the dependent variable and .60 with rent per square foot as the dependent
variable. Rent per square foot was chosen as the dependent variable in the final regression equation because it more effectively isolated configuration preferences. Thus, the final Model was of the log-log form with rent per square foot as the dependent variable.

Fourteen of the sixteen independent variables used in the hedonic analysis were found to be significant in explaining variations in the dependent variable. Using the Model, rent per square foot can be predicted given inputs of the attributes of a given property and an estimate of the pricing or contribution to overall rent of the various attributes represented by the independent variables can be calculated. Several of the relationships between independent variables and rent per square foot may have important implications for multifamily developers and investors in the subject market:

- The number of pools is clearly negatively correlated with rent per square foot in the study.
- A fairly dramatic decrease in predicted rent per square foot occurs as the floor area of a unit increases.
- Age is the most significant factor with respect to rent. This age-rent depreciation effect is greatest during the first five years after development and decrease as age progresses.
- Parking facilities appear to offer a significant value-added opportunity, as do controlled access, fitness centers and microwaves.
- Other attributes such tennis courts, playgrounds, courtesy patrol, and walk-in closets are clearly not correlated with higher rent.
- Although the relationship between a unit's rent and the city in which it is located is very strong, there appears to be a "disconnect" in this relationship. As the average home price of a city doubles, multifamily rent rises only $14 \%$. This discrepancy may provide an opportunity for excess profits in the market.


## Qualifications and Suggestions for Further Research:

The database used in this analysis is heavily weighted in one segment of the multifamily market - suburban garden. As a result, the conclusions reached may not be applicable to smaller, urban properties. A study that includes data on properties under 200 units would involve more urban, high-rise, mid-rise and walk-ups and could provide insight as to the differences in tenant preferences between the suburban and urban prototypes.

Similarly, the conclusions contained in this report may not be applicable to other types of markets. A comprehensive study of the multifamily market located in city conforming to the traditional urban, monocentric model would provide insight into the differences between this type of market and the decentralized, polycentric market analyzed in this study.

Questions without empirical answers were generated by this study concerning the negative correlation of pools and the disconnect in home price versus rent. A more detailed study of the pool question and a more detailed quality-controlled study of the relationship between home prices and rent in the subject market could yield interesting and potentially valuable answers to these questions.

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North Texas Real Estate Information Systems, Inc., Dallas, TX. Special thanks to Jan Blanchard of Home Partners, Dallas, TX for compilation of this data.

