18.03 Problem Set 6

Due by 1:00 P.M., Friday, April 14, 2006.

I encourage collaboration in this course. However, if you do your homework in a group, be sure it works to your advantage rather than against you. Good grades for homework you have not thought through will translate to poor grades on exams. **You must turn in your own writeups of all problems, and, if you do collaborate, you must write on the front of your solution sheet the names of the students you worked with.**

Because the solutions will be available immediately after the problem sets are due, **no extensions will be possible**.

III. Fourier series

R13	Th 23 Mar	Fourier series: Introduction.
L20	F 24 Mar	Fourier series: EP 8.1.
L21	M 3 Apr	Operations on Fourier series: EP 8.2, 8.3.
R14	T 4 Apr	ditto
L22	W 5 Apr	Periodic solutions; resonance: EP 8.3, 8.4.
R15	Th 6 Apr	ditto

IV. The Laplace transform

L23	F 7 Apr	Ramp, step, and delta: SN 17.
L24	M 10 Apr	Step response, impulse response, convolution: SN 18, 19; Notes IR.
R16	T 11 Apr	ditto
L25	W 12 Apr	Laplace transform: basic properties: EP 4.1.

Part I.

22. (W 5 Apr) Notes 7C-1, 7C-2.

23. (F 7 Apr) Recitation 16 problems: **(a)** Graph the functions
$f(t) = 1 + \lfloor t \rfloor - t$ (where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t) and
$g(t) = 3(u(t-a) - u(t-b))$ (where $a < b$).
Then find their generalized derivatives and graph them, using harpoons to denote the delta functions that occur.

(b) $h(t) = (t-2)^2$ for $t > 2$ and $h(t) = 2 - t$ for $t < 2$; $k(t) = \sin(t)$ for $\pi < t < 2\pi$ and $k(t) = 0$ otherwise. Graph each function and express each as a sum using the step function.

24. (M 10 Apr) Notes 2H-1 [though the problem should be posed as: find the unit impulse response of the operator $D^2 - k^2 I$; the input signal $f(t)$ has nothing to

do with it]; and Recitation 16 problem 2: Find the unit step response for the same operator [k^2 is what is called w^2 in solutions to recitation 16 problem 2].

Part II.

22. (W 5 Apr) [Periodic solutions] **(a)** What sum of rational numbers is evaluated by considering the Fourier series of sq(t) at $t = \pi/4$ (and what is the sum)?

(b) (i) Find a periodic solution to $\ddot{x} + \omega_n^2 x = f(t)$, where $f(t)$ is the function which is periodic of period 4 and given by $|t| - 1$ for $-2 < t < 2$ (when one exists)? (ii) For what values of ω_n are there no periodic solution? (iii) For ω_n just less than the smallest such value, what is the solution like, approximately? (iv) For what values of ω_n are there more than one periodic solution? (v) If there's more than one periodic solution for a given value of ω_n, are there any NON-periodic solutions for that value?

23. (F 7 Apr) [Ramp, step, and delta] **(a)** Sketch graphs of the following functions:

(i) $f(t) = t + 2u(t+1) - 3u(t-1)$ (where $u(t)$ is the standard step function).

(ii) $g(t) = \begin{cases} 0 & \text{for} \quad t < 1 \\ t-1 & \text{for} \quad 1 < t < 2 \\ t+2 & \text{for} \quad 2 < t < 3 \\ 5 & \text{for} \quad 3 < t \end{cases}$.

(iii) $h(t) = \lfloor t \rfloor$, the greatest integer less than or equal to t.

(b) For each of the three functions in **(a)**, express the generalized derivative in terms of constant functions, unit step functions, and delta functions, and sketch it (using "harpoons" to denote delta functions).

24. (M 10 Apr) [Step and impulse response] For each of the operators $2D + kI$, $D^2 + 2D + 5I$, and D^3, find (i) the unit impulse response, and (ii) the unit step response, and graph them all.