18.03 Class 17, March 17, 2006

Application of second order frequency response to AM radio reception with guest appearance by EECS Professor Jeff Lang.

[1] The AM radio frequency spectrum is divided into narrow segments which individual stations are required to broadcast in. The challenge of a receiver is to filter out the signals at frequences other than the target frequency.
This can be done using a very simple RLC system which has a sharp peak in the gain curve at a given frequency (which can be tuned by changing the strength of one of the components of the system).

Electronic circuits have mechanical analogues. Mechanical engineers often think of their systems in electronic terms, and vice versa.
The mechanical system analogous to the radio receiver is this:

```
|         b                  m          k        |/
|   |-----------          ______                 |/
|---|     |============|______|----/\/\/\------|/
|   |-----------             |                   |/
|                            |                   |/
|------->                    |-------->
   y(t)                         x(t)
```

We are driving the system by motion of the far end of the dashpot, while keeping the far end of the spring constant.

[2] Let's see the equation of motion for this system. Arrange the position parameter x so that $x = 0$ when the spring is relaxed.

The spring exerts a force $-kx$.

The dashpot exerts a force proportional to the speed at which the piston is moving through the cylinder.
This speed is $(y-x)'$. When $y' > x'$, the force is positive, so the dashpot force is $b(y-x)'$

$$mx'' = -kx + b(y-x)'$$

Putting the system terms on the left,

$$mx'' + bx' + kx = by'$$

If y is constant, we have a homogeneous equation; its solutions are transients.
A transient for the system was displayed; it oscillated, so we are in the underdamped situation.
We could measure the damped circular frequency ω_d and the "decrement," the ratio of the height of one peak to the preceding one; and using them we could determine b/m and $k/m = \omega_n^2$.

[3] Now let's drive the system with a sinusoidal signal. The radio waves are built up from them.
So set y = B cos(omega t) . The equation is then

mx" + bx' + kx = - b omega B sin(omega t)

We know that there will be a sinusoidal system response; and that that is
the response we'll see very quickly, since the transients damp out.
We also know that we should try to express the sinusoidal system response in terms of a gain and a phase lag with respect to the physical input signal. Despite the appearance of the right hand side of the equation, it's clear that we should take as physical input signal the function
y = B cos(omega t), so we look to find gain and phase lag phi such that

x_p = gain . B cos(omega t - phi)

We also know that gain and phi are computed by finding the "complex gain" W(i omega),
and then writing it out in polar terms:

W(i omega) = gain . e^{- i phi}

so that gain = |W(i omega)|

and - phi = Arg(W(i omega)) .

The handout "Driving through the dashpot" computes that

W(i omega) = b i omega / p(i omega)

= b i omega / (m(omega_n^2 - omega^2) + b i omega)

[4] The system was subjected to several input frequencies. One odd thing appeared: for small frequency,the system response is *ahead* of the system input:
< 0 in that case.
Also maximal gain seems to happen when the phase lag is zero.

We can analyze this mathematically, by dividing numerator and denominator in the complex gain by b i omega:

W(i omega) = 1 / (1 - (im/b) ((omega_n^2 - omega^2) / omega^2))

As omega varies, this sweeps out a curve in the complex plane. To see what
that curve is, look first at the denominator. Its real part is always 1 .
When omega = omega_n there is no imaginary part: W(i omega_n) = 1 .
When omega < omega_n , the imaginary part is negative,
when omega > omega_n , it is positive: the direction of movement is upward.

Now if z is on this line, then its reciprocal is on the circle of radius 1/2 and center 1/2 .
The angle gets reversed. So W(i omega) moves clockwise along that

circle.

Let's read off the gain and phase lag curves:

The gain starts small, grows to a maximal value of 1 at omega = omega_n , and then falls as omega --> infty.

The angle, which is - phi , starts at pi/2, falls through 0 at omega = omega_n , and then comes to rest near - pi/2 as omega ---> infty.

So we find mathematically that   phi < 0 for small omega.

[5] These curves were then reproduced experimentally by subjecting the RLC circuit to a series of different frequencies.

Then we tried to input a signal from an antenna, and we watched the system respond to some and not to others: 1030 khz was quite loud.