18.03 Muddy Card responses, April 3, 2006

1. A number of people were confused by my derivation of the Fourier coefficients of the function $f(t)$, even, periodic, period 2π, with $f(t) = 4$ for $0 < t < \pi/2$ and $f(t) = 0$ for $\pi/2 < t < \pi$. I think the process of expressing this in terms of the standard square wave, $f(t) = 2 + 2\text{sq}(t + (\pi/2))$, was pretty clear. Note that the gap in the square wave is 2, while the gap in the graph of $f(t)$ is 4: hence the factor of 2. Then you get

$$f(t) = 2 + \frac{8}{\pi}\left(\sin(t + (\pi/2)) + \frac{\sin(3(t + (\pi/2)))}{3} + \cdots\right)$$

so we have to understand $\sin(n(t + (\pi/2))) = \sin(nt + (n\pi/2))$. I did not explain that well. Think of the nt as a single unit, so we want to understand $\sin(\theta + (n\pi/2))$, especially for n odd. When $n = 1$, this is $\sin(\theta + (\pi/2))$, which is $\cos(\theta)$—look at the graphs! When $n = 3$, this is $\sin(\theta + (3\pi/2) = -\cos(\theta)$—look at the graphs again! When $n = 5$, $\sin(\theta + (5\pi/2)) = \sin(\theta + (\pi/2))$, and we are back in the $n = 1$ case again. The values thus alternate between $\cos(\theta)$ and $-\cos(\theta)$. Therefore

$$f(t) = 2 + \frac{8}{\pi}\left(\cos(t) - \frac{\cos(3t)}{3} + \frac{\cos(5t)}{5} - \cdots\right).$$

2. When I was computing the Fourier coefficients for $\text{sq}(t)$, I wanted to evaluate $\cos(n\pi)$ for various values of n. I started with $n = 0$, despite the fact that b_0 does not occur as a Fourier coefficient. This was because I knew that these coefficient would repeat after a while, so by starting early I would see the repetition quicker.

3. There were some questions about Fourier series for more general functions—by which I guess you meant non-periodic functions. This is possible, but you have to use $\sin(\omega t)$ for *all* values of ω, not just values which are multiplies of some fundamental circular frequency. This is the "Fourier transform," and it is closely related to the Laplace transform.

4. Some people wanted to hear more about the Gibbs effect: please see the Supplementary Notes, §16.