Recitation 6, February 28, 2006

Using the complex exponential; Autonomous equations

1. Find the sinusoidal solution of $\dot{x}+2x=\cos(2t)$ in polar form, $A\cos(\omega t-\phi)$, in the following way: First find the exponential solution of the corresponding equation with complex exponential right hand term; it is $z_p = \dfrac{1}{2i+2}e^{2it}$. Then find A and ϕ such that $\dfrac{1}{2i+2} = Ae^{-i\phi}$, and use this to re-express $z_p = Ae^{(2t-\phi)i}$. Now take the real part.

2. The growth-rate of the population of Ivory-billed Woodpeckers falls to zero as the population density does, because there are just so few of them that it's hard to find a mate. On the other hand, they require many acres of forest and compete with other IBWs when the population grows. This can be modeled using a growth-rate of $k(y) = 4k_0\,\dfrac{y}{p}\left(1-\dfrac{y}{p}\right)$, where k_0 and p are constants.

Show that the maximal growth-rate is k_0; this explains the factor $4k_0$ in the front.

What is the autonomous equation for $\dot{y}$? (NB: it's not $\dot{y} = k(y)$.)

Sketch the phase line and some solutions. Classify the critical points: stable, unstable, semi-stable.

Pressure from bird watchers reduces the population growth by a constant rate a. How high can that rate go before the IBWs face certain extinction? (First find the IBW population where that maximum "harvest rate" occurs.)