Recitation 7, March 2, 2006

Solutions to second order ODEs

1. What does the inequality $f''(a) > 0$ mean about the graph of the function $f(x)$ at $x = a$? How about the inequality $f''(a) < 0$?

What does the equality $f''(x) = 0$ say about the function $f(x)$? That is: what are the solutions $y = f(x)$ of the equation $y'' = 0$?

Which of these functions satisfy $y(0) = 0$? Does the "initial value" determine the solution of a second order ODE?

2. Find all the solutions of $y'' = x^3$. Suppose I add the "initial condition" $y(0) = 1$, $y'(0) = 2$?

3. Find all functions of the form $x = at^2 + bt + c$ which are solutions of $\ddot{x} + \dot{x} + 2x = 4t^2$.

4. Show that $\sin(5t)$ and $\cos(5t)$ are both solutions of the "harmonic oscillator $\ddot{x} + 25x = 0$.

Can you think of solutions to $\ddot{x} - 25x = 0$?