

Recitation 8, March 7, 2006

Homogeneous second order linear equations

1. Find two independent real solutions of $\ddot{x} + 4\dot{x} + 3x = 0$.
2. Find two independent real solutions of $\ddot{x} + 4\dot{x} + 5x = 0$. Check that they are solutions.
3. Find two independent real solutions of $\ddot{x} + 4\dot{x} + 4x = 0$.
4. In each case, find the solution with $x(0) = 0$, $\dot{x}(0) = 2$.
5. Explain why $e^{r_1 t}$ and $e^{r_2 t}$ are “linearly independent” functions whenever $r_1 \neq r_2$. This means: neither one is a constant multiple of the other. [Argue by “contradiction”: suppose one was a constant multiple of the other. What would follow?]
6. Explain why a nonzero solution to the equation in **(1)** can have at most one critical point (i.e. there’s at most one value of t for which $\dot{x}(t) = 0$). Ditto with solutions to **(3)**.