Recitation 9, March 9, 2006

Second order linear equations

1. Show that $\ddot{x} + 4x = 10$ has a constant solution.

What is the general real solution?

2. Find A so that $A \sin t$ is a solution of $\ddot{x} + 4x = 3\sin(t)$.

3. What is the general solution of $\ddot{x} + 4x = 3\sin t + 10$?

4. What is the solution of $\ddot{x} + 2\dot{x} + 65x = 0$ with $x(0) = 1$ and $\dot{x}(0) = 7$?

Sketch the graph of this function. Begin by sketching the graph of the exponential factor.

At what times does this function peak? (For this it may be easier to use the "rectangular" form of the solutions, using a linear combination of cosine and sine rather than an amplitude and phase shift.)

5. For what non-negative values of b does $\ddot{x} + b\dot{x} + 4x = 0$ exhibit solutions which are damped sinusoids?

What is the circular pseudofrequency of these solutions, as a function of the damping constant b? Sketch the graph of this function. Sketch the the graph of the pseudoperiod as a function of b. Can you envision the graphs of solutions of these equations, and how they vary as b varies?