Recitation 10, March 14, 2006

Operators, Exponential Response, Exponential shift, Undetermined Coefficients

1. What is the general solution of $\dfrac{d^4 x}{dt^4} - x = 0$?

2. What is an exponential solution of $\dfrac{d^4 x}{dt^4} - x = e^{-2t}$?

3. What is a sinusoidal solution of $\dfrac{d^4 x}{dt^4} - x = \cos(2t)$?

4. What is a polynomial solution of $\dfrac{d^2 x}{dt^2} - x = t^2 + t + 1$?

5. Compute $p(D)\cos(2t)$ if $p(s) = s^4 - 1$. How does this relate to **(3)**?

6. Compute $D^2(e^{2t}\cos(t))$ in three ways: (1) directly; (2) by writing the function as the real part of an exponential; and (3) using the exponential shift law.

7. What is a solution of $\ddot{x} + 9x = \cos(3t)$? Do this by replacing the signal with a complex exponential signal and using the "resonant exponential response formula."

Exponential response formula. $Ae^{rt}/p(r)$ is a solution of $p(D)x = Ae^{rt}$ provided $p(r) \neq 0$.

Exponential shift law. $p(D)(e^{rt}u) = e^{rt}p(D + rI)u$.

Resonant ERF. If $p(r) = 0$, $Ate^{rt}/p'(r)$ is a solution to $p(D)x = Ae^{rt}$ provided $p'(r) \neq 0$.