Recitation 13, March 23, 2006

Fourier Series: Introduction

1. What is the general solution to $\ddot{x} + \omega_n^2 x = 0$? [Quick!]

2. Discuss why

$$\ddot{x} + \omega_n^2 x = a\cos(\omega t) \quad \text{has solution} \quad x_p = a\,\frac{\cos(\omega t)}{\omega_n^2 - \omega^2}$$

$$\ddot{x} + \omega_n^2 x = b\sin(\omega t) \quad \text{has solution} \quad x_p = b\,\frac{\sin(\omega t)}{\omega_n^2 - \omega^2}$$

3. For what values of ω_n is there a sinusoidal solution to $\ddot{x} + \omega_n^2 x = \sin(t)$? What is it when it exists? What is the general solution when no sinusoidal solution exists? Sketch the graph of one solution in that case.

4. What is the period of $f(t) = \sin(t) + (1/2)\sin(2t)$?

[A function is *periodic* if there is a number $P > 0$ such that $f(t+P) = f(t)$ for all t. Such a number P is then a "period" of $f(t)$. If $f(t)$ is a periodic function which is continuous and not constant, then there is a smallest period, often called *the* period.]

5. For what values of ω is there a periodic solution to $\ddot{x} + \omega_n^2 x = \sin(t) + (1/2)\sin(2t)$? What is it when such exists? What is the general solution for the values of ω when no periodic solution exists?