Recitation 16, April 11, 2006

Step and delta functions, and step and delta responses

1. Graph the functions

$$f(t) = 1 + \lfloor t \rfloor - t$$

(where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t) and

$$g(t) = 3(u(t-a) - u(t-b))$$

(where a < b). Then find their generalized derivatives and graph them, using harpoons to denote the delta functions that occur.

2. Find the unit step and unit impulse responses to the operator $mD^2 - kI$, for m > 0, and graph them.

3. Suppose $q(t) = 2u(t+1) + \delta(t) - 2u(t-1)$. Sketch a graph of this generalized function. Tell stories which might result in each of the equations $\dot{x} + kx = q(t)$ (your choice of k, it might be negative) and $2\ddot{x} + 4\dot{x} + 18x = q(t)$.

4. Find the unit step and unit impulse responses for $2D^2 + 4D + 20I$. Why is one the derivative of the other?