18.03 Recitation 17, April 13, 2006

Convolution

$$f(t) * g(t) = \int_0^t f(t-\tau)g(\tau)\,d\tau$$

1. (a) What is the differential operator $p(D)$ whose weight function (i.e. unit impulse response) is the unit step function $u(t)$?

Verify that $u(t)*q(t)$ is the solution to $p(D)x = q(t)$ with rest initial conditions. (Since we are always interested only in $t > 0$, we could write $1 * q(t)$ instead of $u(t) * q(t)$.)

(b) What is the differential operator $p(D)$ whose weight function is $u(t)t$?

Verify that $t * t^n$ is the solution, with rest initial conditions, to $p(D)x = t^n$.

2. (a) Suppose $a \geq 0$. Figure out what $w(t) * \delta(t-a)$ is by using the fact that it is the solution to the equation $p(D)x = \delta(t-a)$ with rest initial conditions.

(b) Then figure out what $w(t) * \delta(t-a)$ is using the definition.

3. Compute the convolution product $e^{-t} * (1 + \cos(t))$ by using the integral. (Solution: $1 + \frac{1}{\sqrt{2}}\cos\left(t - \frac{\pi}{4}\right) - \frac{3}{2}e^{-t}$.)