18.03 Recitation 19, April 25, 2006

Hour exam review

Suppose you have an LTI system (what does that term mean?) which is modeled by a differential operator $p(D)$. You don't know the coefficients of the operator, but you investigate the system by delivering a blow to it—a unit impulse input signal—and recording the system response, $w(t)$. (Of course, it was at rest beforehand.)

Review what we know from this:

(1) How do we determine the characteristic polynomial (and hence the coefficients) of the operator?

(2) How can we write down (in terms of $w(t)$) the solution (with rest initial conditions) to $p(D)x = q(t)$ for some arbitrary input signal?

(3) How can we determine the multiplier $W(r)$ such that $x_p = W(r)e^{rt}$ is a system response to the exponential input signal e^{rt} (for r constant)?

(4) How can we determine the frequency response of the system?—that is, A and ϕ (both functions of ω) such that $p(D)x = \cos(\omega t)$ has sinusoidal solution $A\cos(\omega t - \phi)$?

A unifying visual image is the graph of $|W(s)|$, which is largely controlled by the poles of $W(s)$.

Work these out in case $w(t) = e^{-t}\sin(3t)$.