18.03 Recitation 21, May 2, 2006

Eigenvalues and Eigenvectors

We'll solve the system of equations $\left\{ \begin{array}{rcr} \dot{x} & = & -5x - 3y \\ \dot{y} & = & 6x + 4y \end{array} \right.$

1. Write down the matrix of coefficients, A, so that we are solving $\dot{\mathbf{u}} = A\mathbf{u}$. What is its trace? Its determinant? Its characteristic polynomial $p_A(\lambda) = \det(A - \lambda I)$? Relate the trace and determinant to the coefficients of $p_A(\lambda)$.

2. Find the eigenvalues and then for each eigenvalue find a nonzero eigenvector.

3. Draw the eigenlines and discuss the solutions whose trajectories live on each. Explain why each eigenline is made up of three distinct non-intersecting trajectories. Begin to construct a phase portrait by indicating the direction of time on portions of the eigenlines. Pick a nonzero point on an eigenline and write down all the solutions to $\dot{\mathbf{u}} = A\mathbf{u}$ whose trajectories pass through that point.

4. Now study the solution $\mathbf{u}(t)$ such that $\mathbf{u}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Write $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as a linear combination of a vector from the first eigenline and a vector from the second eigenline. Use this decomposition to express the solution, and sketch its trajectory. Fill out the phase portrait.

5. Same sequence of steps for $\left\{ \begin{array}{rcr} \dot{x} & = & 4x + 3y \\ \dot{y} & = & -6x - 5y \end{array} \right.$