18.03 Recitation 23, May 9, 2006

Qualitative analysis of linear systems

The matrices I want you to study all have the form $A = \begin{bmatrix} a & 2 \\ -2 & -1 \end{bmatrix}$.

1. Compute the trace, determinant, characteristic polynomial, and eigenvalues, in terms of a.

2. For these matrices, express the determinant as a function of the trace. Sketch the $(\operatorname{tr} A, \det A)$ plane, along with the critical parabola $\det A = (\operatorname{tr} A)^2/4$, and plot the curve representing the relationship you found for this family of matrices. On this curve, plot the points corresponding to the following values of a: $a = -6, -5, -2, 1, 2, 3, 4, 5$.

3. Make a table showing for each a in this list (1) the eigenvalues; (2) information about the phase portrait derived from the eigenvalues (real and distinct; real and repeated; non-real) and the stability type (stable if all real parts are negative; unstable if at least one real part is positive; undesignated if neither); (3) further information beyond what the eigenvalues alone tell you: if a spiral, the direction (clockwise or counterclockwise) of motion; if the eigenvalues are repeated, whether the matrix is defective or complete.

In each case, make a small sketch of the phase portrait which conveys this information, but does not try to get the eigendirections right.