18.03 Differential Equations, Spring 2006
Transcript – Lecture 8

I usually like to start a lecture with something new, but this time I'm going to make an exception, and start with the finishing up on Friday because it involves a little more practice with complex numbers. I think that's what a large number of you are still fairly weak in. So, to briefly remind you, it will be sort of self-contained, but still, it will use complex numbers. And, I think it's a good way to start today.

So, remember, the basic problem was to solve something with, where the input was sinusoidal in particular. The k was on both sides, and the input looked like cos(omega t). And, the plan of the solution consisted of transporting the problem to the complex domain. So, you look for a complex solution, and you complexify the right hand side of the equation, as well. So, cos(omega t) becomes the real part of this complex function. The reason for doing that, remember, was because it's easier to handle when you solve linear equations. It's much easier to handle exponentials on the right-hand side than it is to handle sines and cosines because exponentials are so easy to integrate when you multiply them by other exponentials. So, the result was, after doing that, y tilda turned out to be one, after I scale the coefficient, $y\sim = 1 / (1 + omega/k)$.

And then, the rest was $e^{(i (omega t - phi)}$, where phi had a certain meaning. It was the arc tangent of a, it was a phase lag. And, this was then, I had to take the real part of this to get the final answer, which came out to be something like $1 / sqrt(1 + (omega/k)^2) * cos (omega t - phi)$. It's easier to see that that part is the real part of this; the problem is, of course you have to convert this.

Sorry, this should be i omega t, in which case you don't need the parentheses, either. So, the problem was to use the polar representation of this complex number to convert it into something whose amplitude was this, and whose angle was minus phi. Now, that's what we call the polar method, going polar. I'd like, now, for the first few minutes of the period, to talk about the other method, the Cartesian method. I think for a long while, many of you will be more comfortable with that anyway. Although, one of the objects of the course should be to get you equally comfortable with the polar representation of complex numbers. So, if we try to do the same thing going Cartesian, what's going to happen? Well, I guess the same point here.

So, the starting point is still $y\sim = 1 / (1 + i(omega/k)) * e^{(i omega t)}$. But now, what you're going to do is turn this into its Cartesian, turn both of these into their Cartesian representations as a + bi. So, if you do that Cartesianly, of course, what you have to do is the standard thing about dividing complex numbers or taking the reciprocals that I told you at the very beginning of complex numbers. You multiply the top and bottom by the complex conjugate of this in order to make the bottom real. So, what does this become? This becomes $(1 - i(omega/k)) / (1 + (omega/k)^2)$.

So, I've now converted this to the a + bi form. I have also to convert the right-hand side to the a plus bi form. So, it will look like cos(omega t) + i sin(omega t). Having done that, I take the last step, which is to take the real part of that. Remember, the reason I want the real part is because this input was the real part of the complex input.

So, once you've got the complex solution, you have to take its real part to go back into the domain you started with, of real numbers, from the domain of complex numbers. So, I want the real part is going to be, the real part of that is, first of all, there's a factor out in front. That's entirely real. Let's put that out in front, so doesn't bother us particularly. And now, I need the product of this complex number and that complex number. But, I only want the real part of it. So, I'm not going to multiply it out and get four terms. I'm just going to look at the two terms that I do want. I don't want the others. All right, the real part is cos(omega t), from the product of this and that.

And, the rest of the real part will be the product of the two i terms. But, it's i times negative i, which makes one. And therefore, it's (omega / k) * sin(omega t). Now, that's the answer. And that's the answer, too; they must be equal, unless there's a contradiction in mathematics. But, it's extremely important. And that's the other reason why I'm giving you this, that you learn in this course to be able to convert quickly and automatically things that look like this into things that look like that. And, that's done by means of a basic formula, which occurs at the end of the notes for reference, as I optimistically say, although I think for a lot of you will not be referenced, stuff in the category of, yeah, I think I've vaguely seen that somewhere.

But, well, we never used it for anything. Okay, you're going to use it all term. So, the formula is, the famous trigonometric identity, which is, so, the problem is to convert this into the other guy. And, the thing which is going to do that, enable one to combine the sine and the cosine terms, is the famous formula that a times the cosine, I'm going to use theta to make it as neutral as possible, --

-- so, theta you can think of as being replaced by omega t in this particular application of the formula. But, I'll just use a general angle theta, which doesn't suggest anything in particular. So, the problem is, you have something which is a combination with real coefficients of cosine and sine, and the important thing is that these numbers be the same. In practice, that means that the omega t, you're not allowed to have omega1 t here, and some other frequency, omega2 t here. That would correspond to using theta one here, and theta two here. And, though there is a formula for combining that, nobody remembers it, and it's, in general, less universally useful than the first.

If you're going to memorize a formula, and learn this one, it's best to start with the ones where the two are equal. That's the basic formula. The others are variations of it, but there is a sizable variations. All right, so the answer is that this is equal to some other constant, real constant, times the cos(theta - phi). Of course, most people remember this vaguely. What they don't remember is what the c and the phi are, how to calculate them. I don't suggest you memorize the formulas for them. You can if you wish. Instead, memorize the picture, which is much easier. Memorize that a and b are the two sides of a right triangle. Phi is the angle opposite the b side, and c is the length of the hypotenuse.

Okay, that's worth putting up. I think that's a pink formula. It's even worth two of those, but I will thrift. Now, let's apply it to this case to see that it gives the right answer. So, to use this formula, how I use it? Well, I should take, I will reproduce the left-hand side. So that part, I just copy. And, how about the right? Well, the amplitude, it's combined into a single cosine term whose amplitude is, well, the two sides of the right triangle are one, and omega/k.

The hypotenuse in that case is going to be, well, why don't we write it here? So, we have one, and omega/k. And, here's phi. So, the hypotenuse is going to be the sqrt(1 + (omega/k)^2). And, that's going to be multiplied by the cos(omega t - phi). You can write, if you wish, phi equals the arc tangent, but you are not learning a lot by that. phi = arctan(omega / k). That's okay, but it's true. But, notice there's cancellation now. This over that is equal to what? Well, it's equal to this.

And, so when we get in this way, by combining these two factors, one gets exactly the same formula that we got before. So, as you can see, in some sense, there's not, if you can remember this trigonometric identity, there's not a lot of difference between the two methods except that this one requires this extra step. The answer will come out in this form, and you then, to see what it really looks like, really have to convert it to this form, the form in which you can see what the phase lag and the amplitude is.

It's amazing how many people who should know, this includes working mathematicians, theoretical mathematicians, includes even possibly the authors of your textbooks. I'm not sure, but I've caught them in this, too, who in this form, everybody remembers that it's something like that. Unfortunately, when it occurs as the answer in an answer book, the numbers are some colossal mess here plus some colossal mess here.

And theta is, again, a real mess, involving roots and some cube roots, and whatnot. The only thing is, these two are the same real mess. That amounts to just another pure oscillation with the same frequency as the old guy, and with the amplitude changed, and with a phase shift, move to the right or left. So, this is no more general than that. Notice they both have two parameters in them, these two coefficients. This one has the two parameters in an altered form. Okay, well, I wanted, because of the importance of this formula, I wanted to take a couple of minutes out for a proof of the formula, --

-- just to give you chance to stare at it a little more now. There are three proofs I know. I'm sure there are 27. The Pythagorean theorem now has several hundred. But, there are three basic proofs. There is the one I will not give you, I'll call the high school proof, which is the only one one normally finds in books, physics textbooks or other textbooks. The high school proof takes the right-hand side, applies the formula for the cosine of the difference of two angles, which it assumes you had in trigonometry, and then converts it into this. It shows you that once you've done that, that a turns out to be c cos(phi) and b, the number b is c sin(phi), and therefore it identifies the two sides.

Now, the thing that's of course correct and it's the simplest possible argument, the thing that's no good about it is that the direction at which it goes is from here to here. Well, everybody knew that. If I gave you this and told you, write it out in terms of cosine and sine, I would assume it dearly hope that practically all of you can do that. Unfortunately, when you want to use the formula, it's this way you want to use

it in the opposite direction. You are starting with this, and want to convert it to that. Now, the proof, therefore, will not be of much help. It requires you to go in the backwards direction, and match up coefficients. It's much better to go forwards. Now, there are two proofs that go forwards. There's the 18.02 proof. Since I didn't teach most of you 18.02, I can't be sure you had it.

So, I'll spend one minute giving it to you. What is the 18.02 proof? It is the following picture. I think this requires deep colored chalk. This is going to be pretty heavy. All right, first of all, the a and the b are the given. So, I'm going to put in that vector. So, there is the vector whose sides are, whose components are a and b. I'll write it without the i and j. I hope you had from Jerison that form for the vector, if you don't like that, write ai + bj, okay? Now, there's another vector lurking around. It's the unit vector whose, I'll write it this way, u because it's a unit vector, and theta to indicate that it's angle is theta.

Now, the reason for doing that is because you see that the left-hand side is a dot product of two vectors. The left-hand side of the identity is the dot product of the vector with the vector whose components are cosine theta and sine theta. That's what I'm calling this unit vector. It's a unit vector because (cos^2(theta) + sin^2(theta) = 1.

Now, all this formula is, is saying that scalar product, the dot product of those two vectors, can be evaluated if you know their components by the left-hand side of the formula. And, if you don't know their components, it can be evaluated in another way, the geometric evaluation, which goes, what is it? It's a magnitude of one, times the magnitude of the other, times the cosine of the included angle.

Now, what's the included angle? Well, theta is this angle from the horizontal to that unit vector. The angle phi is this angle, from this picture here. And therefore, the included angle between (u)theta and my pink vector is theta - phi. That's the formula. It comes from two ways of calculating the scalar product of the vector whose coefficients are , and the unit vector whose components are cosine theta and sine theta . All right, well, you should, that was 18.02.

There must be an 18.03 proof also. Yes. What's the 18.03 proof? The 18.03 proof uses complex numbers. It says, look, take the left side. Instead of viewing it as the dot product of two vectors, there's another way. You can think of it as the part of the products of two complex numbers. So, the 18.03 argument, really, the complex number argument says, look, multiply together a - bi and the complex number cos(theta) + i sin(theta). There are different ways of explaining why I want to put the minus i there instead of i. But, the simplest is because I want, when I take the real part, to get the left-hand side.

I will. If I take the real part of this, I'm going to get a cos(theta) + b sin(theta) because of -i * i = 1, multiplied together. All right, that's the left-hand side. And now, the right-hand side, I'm going to use polar representation instead. What's the polar representation of this guy? Well, if has the angle theta, then a - b, a - bi goes down below. It has the angle minus phi. So, this is, has magnitude. It is polar representation. Its magnitude is a^2 + b^2, and its angle is negative phi, not positive phi because this a - bi goes below the axis if a and b are positive.

So, it's e^(-i phi). That's the first guy. And, how about the second guy? Well, the second guy is e^(i theta). So, what's the product? It is sqrt(a^2 + b^2) e^(i (theta

- phi)). And now, what do I want? The real part of this, and I want the real part of this.

So, let's just say take the real parts of both sides. If I take the real part of the left-hand side, I get a cos(theta) + b sin(theta). If I take a real part of this side, I get sqrt(a^2 + b^2) cos(theta - phi), which is just what it's supposed to be.

Well, with three different arguments, I'm really pounding the table on this formula. But, I think there's something to be learned from at least two of them. And, you know, I'm still, for awhile, I will never miss an opportunity to bang complex numbers into your head because, in some sense, you have to reproduce the experience of the race. As I mentioned in the notes, it took mathematicians 300 or 400 years to get used to complex numbers. So, if it takes you three or four weeks, that's not too bad.

Now, for the rest of the period I'd like to go back to the linear equations, and try to put into perspective and summarize, and tell you a couple of things which I had to leave out, but which are, in my view, extremely important. And, up to now, I don't want to leave you with any misapprehensions. So, I'm going to summarize this way, whereas last lecture I went from the most general equation to the most special.

I'd like to just write them down in the reverse order, now. So, we are talking about basic linear equations. First order, of course, we haven't moved as a second order yet. So, the most special one, and the one we talked about essentially all of the previous two times, or last Friday, anyway, was the equation where the k, the coefficient of y, is constant, and where you also get it on the right-hand side quite providentially.

So, this is the most special form, and it's the one which governed what I will call the temperature-concentration model, or if you want to be grown up, the conduction-diffusion model, conduction-diffusion which describes the processes, which the equation is modeling, whereas these simply described the variables of things, which you usually are trying to calculate when you use the equation. Now, there are a class of things where the thing is constant, but where the k does not appear naturally on the right hand side. And, you're going to encounter them pretty quickly in physics, for one place. So, I better not try to sweep those under the rug.

Let's just call that q of t. And finally, there is the most general case, where you allow k to be non-constant. That's the one we began, when we talked about the linear equation. And you know how to solve it in general by a definite or an indefinite integral. Now, there's one other thing, which I want to talk about. I will do all these in a certain order. But, from the beginning, you should keep in mind that there's another between the first two cases. Between the first two cases, there's another extremely important distinction, and that is as to whether k is positive or not. Up to now, we've always had k>0. So, I'm going to put that here. So, it's understood when I write these, that k is positive.

I want to talk about that, too. But, first things first. The first thing I wanted to do was to show you that this, the first case, the most special case, does not just apply to this. It applies to other things, too. Let me give you a mixing problem. The typical mixing problem gives another example. You've already done in recitation, and you did one for the problem set, the problem of the two rooms filled with smoke.

But, let me do it just using letters, so that the ideas stand out a little more clearly, and you are not preoccupied with the numbers, and calculating with the numbers, and trying to get numerical examples. So, it's as simple as k sub mixing. It looks like this. You have a tank, a room, I don't know, where everything's getting mixed in. It has a certain volume, which I will call v. Something is flowing in, a gas or a liquid. And, r will be the flow rate, in some units. Now, since it can't pile up inside this sealed container, which I'm sure is full, the flow rate out must also be r. And, what we're interested in is the amount of salt. So, x, let's suppose these are fluid flows, and the dissolved substance that I'm talking about is not carbon monoxide, it's salt, any dissolved substance, some pollutant or whatever the problem calls for.

Let's use salt. So, it's the amount of salt in the tank at time t. I'm interested in knowing how that varies with time. Now, there's nothing to be said about how it flows out. What flows out, of course, is what happens to be in the tank. But, I do have to say what flows in. Now, the only convenient way to describe the in-flow is in terms of its concentration. The salt will be dissolved in the in-flowing water, and so there will be a certain concentration. And, as you will see, for a secret reason, I'm going to give that the subscript e.

So, e is the concentration of the incoming salt, in other words, in the fluid, how many grams are there per liter in the incoming fluid. That's the data. So, this is the data. r is part of the data. r is the flow rate. v is the volume. I think I won't bother writing that down, and the problem is to determine what happens to x of t. Now, I strongly recommend you not attempt to work directly with the concentrations unless you feel you have a really good physical feeling for concentrations. I strongly recommend you work with a variable that you are given, namely, the dependent variable, which is the amount of salt, grams. Well, because it's something you can physically think about. It's coming in, it's getting mixed up, and some of it is going out.

So, the basic equation is going to be that the rate of change of salt in the tank is the rate of salt inflow, let me write salt inflow, minus the rate of salt outflow. Okay, at what rate is salt flowing in? Well, the flow rate is the flow rate of the liquid. I multiply the flow rate, 1 L per minute times the concentration, 3 g per liter. That means 3 g per minute. It's going to be, therefore, the product of the flow rate and the concentration, incoming concentration.

How about the rate of the salt outflow? Well, again, the rate of the liquid outflow is r. And, what is the concentration of salt in the outflow? I must use, when you talk flow rates, the other factor must be the concentration, not the amount. So, what is the concentration in the outflow? Well, it's the amount of salt in the tank divided by its volume. So, the analog, the concentration here is x divided by v. Now, here's a typical messy equation, dx / dt, let's write it in the standard, linear form, plus r times x over v equals r times the given concentration, which is a function of time. Now, this is going to be some given function, and there will be no reason whatsoever why you can't solve it in that form.

And, that's normally what you will do. Nonetheless, in trying to understand how it fits into this paradigm, which kind of equation is it? Well, clearly there's an awkwardness in that on the right-hand side, we have concentration, and on the left-hand side, we seem to have amounts. Now, the way to understand the equation as opposed to the way to solve it, well, it's a step on the way to solving it. But, I emphasize, you can and normally will solve it in exactly that form.

But, to understand what's happening, it's better to express it in terms of concentration entirely, and that's why it's called the concentration, or the diffusion, concentration-diffusion equation. So, I'm going to convert this to concentrations. Now, there's no problem here. x over v is the concentration in the tank. And now, immediately, you see, hey, it looks like it's going to come out just in the first form.

But, wait a minute. How about the x? How do I convert that? Well, what's the relation between x? So, if the concentration in the tank is equal to x over v, so the tank concentration, then x is equal to C times the constant, V, and dx / dt, therefore, will be c times dC / dt. You see that? Now, that's not in standard form. Let's put it in standard form. To put it in standard form, I see, now, that it's not r that's the critical quantity. It's r divided by v.

So, it's dC/dt + (r/v)C = (r/v)Ce. That's the equation expressed in a form where the concentration is the dependent variable, rather than the amount of salt itself. And, you can see it falls exactly in this category. That means that I can talk about it. The natural way to talk about this equation is in terms of, the same way we talked about the temperature equation.

I said concentration. I mean, that concentration has nothing to do with this concentration. This is the diffusion model, where salt solution outside, cell in the middle, salt diffusing through a semi-permeable membrane into that, uses Newton's law of diffusion, except he didn't do a law of diffusion. But, he is sticky. His name is attached to everything. So, that's this concentration model. It's the one entirely analogous to the temperature. And the physical setup is the same. This one is entirely different. Mixing in this form of this problem has really nothing to do with this model whatsoever. But, nor does that concentration had anything to do with this concentration, which refers to the result of the mixing in the tank.

But, what happens is the differential equation is the same. The language of input and response that we talked about is also available here. So, everything is the same. And, the most interesting thing is that it shows that the analog of the conductivity, the k, the analog of conductivity and diffusivity is this quantity. I should not be considering r and v by themselves. I should be considering as the basic quantity, the ratio of those two. Now, why is that, is the basic parameter. What is this? Well, r is the rate of outflow, and the rate of inflow, what's r over v? r over v is the fractional rate of outflow. In other words, if r over v is one tenth, it means that 1/10 of the tank will be emptied in a minute, say.

In other words, we lumped these two constants into a single k, and at the same time have simplified the units. What are the units? This is volume per minute. This is volume. So, it's simply reciprocal minutes, reciprocal time, which was the same units of that diffusivity and conductivity had, reciprocal time. The space variables have entirely disappeared. So, it that way, it's simplified. It's simplified conceptually, and now, you can answer the same type of questions we asked before about this.

I think it would be better for us to move on, though. Well, just an example, one really simple thing, so, suppose since we spent so much time worrying about what was happening with sinusoid inputs, I mean, when could Ce be sinusoidal, for example? Well, roughly sinusoidal if, for example, some factory were polluting. If this were a lake, and some factory were polluting it, but in the beginning, at the beginning of the day, they produced a lot of the pollutant, and by the end of the day

when it wound down, it might well happen that the concentration of pollutants in the incoming stream would vary sinusoidally with a 24 hour cycle.

And then, we would be asking, so, suppose this varies sinusoidally. In other words, it's like cos(omega t). I'm asking, how closely does the concentration in the tank follow Ce? Now, what would that depend upon? Think about it. Well, the answer, suppose k is large. Closely, let's just analyze one case, if k is big. Now, what would make k big? We know that from the temperature thing. If the conductivity is high, then the inner temperature will follow the outer temperature closely, and the same thing with the diffusion model. But we, of course, therefore, since the equation is the same, we must get the same result here. Now, what would make k big? If r is big, if the flow rate is very fast, we will expect the concentration of the inside of that tank to match fairly closely the concentration of the pollutant, of the incoming salt solution, or, if the tank is very small.

For fixed flow rates, if the tank is very small, well, then it gets emptied quickly. So, both of these are, I think, intuitive results. And, of course, as before, we got them from that, by trying to analyze the final form of the solution. In other words, we got them by looking at that form of the solution up there, and seeing if k is big. As k increases, what happens to the amplitude, and what happens to the phase lag? But, that summarizes the two. So, this means, closely means, that the phase lag is, big or small? The lag is small. And, the amplitude is, well, the amplitude, the biggest the amplitude could ever be is one because that's the amplitude of this.

So, the amplitude is near one, one because that's the amplitude of the incoming signal, input, whatever you want to call it. Okay, now, I'd like to spend the rest of the time talking about the failures of number one, and when you have to use number two, and when even number two is no good. So, let me end first-order equations by putting my worst foot forward. Well, I'm just trying to avoid disappointment at misapprehensions from you. I'll watch you leave this room and say, well, he said that, okay. So, the first one you're going to encounter very shortly where one is not satisfied, but two is, so on some examples where you need two, well, it's going to happen right here.

Somebody, sooner or later, it's going to draw on that loathsome orange chalk, which is unerasable, something which looks like that. Remember, you saw it here first. r, yeah, we had that. Okay, see, I had it in high school too. That's the capacitance. This is the resistance. That's the electromotive force: battery, or a source of alternating current, something like that. Now, of course, what you're interested in is how the current flows in the circuit. Since current across the capacitance doesn't make sense, you have to talk about the charge on the capacitance. So, q, it's customary in a circle this simple to use as the variable not current, but the charge on the capacitance.

And then Kirchhoff's, you are also supposed to know that the derivative, that the time derivative of q is what's called the current in the circuit. That sort of intuitive. But, i in a physics class, j in an electrical engineering class, and why, not the letter Y, but why is that? That's because of electrical engineers use lots of lots of complex numbers and therefore, you have to call current j, I guess. I think they do j in physics, too, now. No, no they don't. I don't know. So, i is ambiguous if you are in that particular subject. And it's customary to use, I don't know.

Now it's completely safe. Okay, where are we? Well, the law is, the basic differential equation is Kirchhoff's voltage law, but the sum of the voltage drops across these three has to be zero. So, it's R times i, which is dq / dt. That's Ohm's law. That's the voltage drop across resistance. The voltage drop across the capacitance is Coulomb's law, one form of Coulomb's law. It's q divided by C. And, that has to be the voltage drop. And then, there is some sign convention.

So, this is either plus or minus, depending on your sign conventions, but it's E of t. Now, if I put that in standard form, in standard form I probably should say q' + q/RC = E/R. And, this is what would appear in the equation. But, it's not the natural thing. The k = 1 / RC. And, that's the reciprocal. The RC constant is what everybody knows is important when you talk about a little circuit of that form. On the other hand, the right-hand side, it's quite unnatural to try to stick in the right-hand side that same RC. Call this EC over RC. People don't do that, and therefore, it doesn't really fall into the paradigm of that first equation. It's the second equation that really falls into the category. Another simple example of this is chained to k, radioactively changed to k.

Well, let's say the radioactive substance, A, decays into, let's say, one atom of this produces one atom of that for simplicity. So, it decays into B, which then still is radioactive and decays further. Okay, what's the differential equation, which is going to be, it's going to govern this situation? What I want to know is how much B there is at any given time. So, I want a differential equation for the quantity of the radioactive product at any given time. Well, what's it going to look like? Well, it's the amount coming in minus the amount going out, so to speak. The rate of inflow minus the rate of outflow, except it's not the same type of physical flow we had before. How fast is it coming in? Well, A is decaying at a certain rate, and so the rate at which A decays is by the basic radioactive law.

It's k1, it's constant, decay constant, times the amount of A present . If I used the differential equation with A here, I'd have to put a negative sign because it's the rate at which that stuff is leaving A. But, I'm interested in the rate at which it's coming in to B. So, it has a positive sign. And then, the rate at which B is decaying, and therefore the quantity of good B is gone, --

-- that will have some other constant, B. So, that will be the equation, and to avoid having two dependent variables in there, we know how A is decaying. So, it's k1, some constant times A, sorry, A will be e to the negative, you know, the decay law, so, times the initial amount that was there, A0 e^(-k1t). That's how much A there is at any given time. It's decaying by the radioactive decay law, -k2 B. Okay, so how does the differential equation look like? It looks like B' + k2B = k1 A0 e^(-k1 t). But, there's no reason to expect that that constant really has anything to do with k2. It's unnatural to put it in that form, which is the correct one.

Now, in the last two minutes, I wish to alienate half the class by pointing out that if k is less than zero, none of the terminology of transient, steady-state input response applies. The technique of solving the equation is identical. But, you cannot interpret. So, the technique is the same, and therefore it's worth learning. The technique is the same. In other words, the solution will be still e^(-kt) integral(q(t) e^(kt) dt) plus a constant times e to the k, oh, this is terrible, no.

dy / dt, let's give an example. The equation I'm going to look at is something that looks like this: y equals q of t, let's say, okay, a constant, but the constant a is positive. So, the constant here is negative. Then, when I solve, my k, in other words,

is now properly written as negative a. And therefore, this formula should now become not this, but the $-k = at$. And, here it's negative a t. And, here it is positive a t. Now, why is it, if this is going to be the solution, why are all those things totally irrelevant? This is not a transient any longer because if a is positive, this goes to infinity. Or, if I go to minus infinity, then C is negative.

So, it's not transient. It's not going to zero, and it depends heavily on the initial conditions. That means that of these two functions, this is the important guy. This is just fixed, some fixed function. Everything, in other words, is going to depend upon the initial conditions, whereas in the other cases we have been studying, the initial conditions after a while don't matter anymore. Now, why did I say I would alienate half of you? Well, because in what subjects will a be positive? In what subjects will k be negative? k is typically negative in biology, economics, Sloan. In other words, the simple thing is think of it in biology. What's the simplest equation for population growth? Well, it is dP / dt equals some, if the population is growing, a times P, and a is a positive number.

That means P prime minus a P is zero. So, the thing I want to leave you with is this. If life is involved, k is likely to be negative. k is positive when inanimate things are involved; I won't say dead, inanimate.