14.03 Fall 2004
Problem Set 3 Solutions

Professor: David Autor

October 26, 2004

1 Sugarnomics

Comment on the following quotes from articles in the reading list about the US sugar quota system.

1. "In terms of minutes of work required to make enough money to buy one pound of sugar, the United States is the third-lowest in the entire world."

   VanDriesche's quotation implies that sugar in the US is relatively inexpensive. Are there any alternative explanations for this fact?

   The measure presented by VanDriessche is clearly a consequence of the high productivity of american workers. This does not imply that sugar is inexpensive in the US as the author seems to suggest and does not justify leaving the sugar quota system in place.

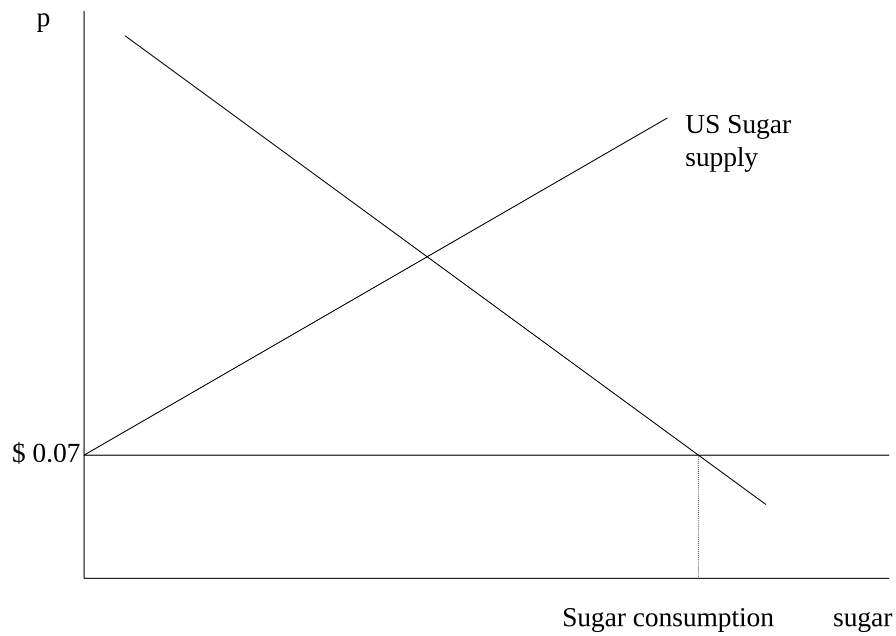
2. VanDriessche points out that, "these low, stable sugar prices have been achieved at no cost to taxpayers since 1985 and no payments to sugar growers whatsoever."

   (a) This statement is technically correct. Explain why it is misleading.

   First of all sugar price is not low and stability is not a very important issue when the domestic price of sugar is 3 times the international price. Second, although there has been no disbursement of money to sugar growers, the taxpayers have born the cost of a highly distorted price sugar through reduced consumption, higher price levels and consumption of sugar substitutes like HFCS.

Now consider an alternative sugar policy that guaranteed farmers the same surplus as does the U.S. sugar program, but achieved this through direct taxation and transfers, i.e., sugar would be sold at the world price of $0.068 per pound and farmers would be paid out of general revenue.

(b) Please draw a diagram similar to that used in class showing the likely demand and supply conditions under this alternative program.



(c) How much sugar would be consumed in the U.S. How much sugar would be grown in the U.S.? How much HFCS would be produced? What would be the deadweight loss associated with this policy? [You may want to assume that there is some deadweight loss associated taxation – perhaps $0.25 on the dollar.]

The quantity of sugar consumed would be 41.2 million pounds. The US sugar producers would not produce any sugar. The US producers are guaranteed 1billion in surplus (the same as under the previous program). To raise 1billion in tax revenues, the DWL is $0.25 * 1 = 0.25$ billion.

(d) Can you think of any political economy reasons why farmers might oppose this alternative program?

As we discussed in the case of the food stamps, cash programs, although more efficient, are less viable from a political economy point of view. A program that simply redistributes cash to farmers might not be well accepted by tax payers.

2 True/False questions

For each of the following statements say whether it's true/false/uncertain and give a short explanation.

1. A farmer that owns 100 acres of land and grows sugar earns $3000 per year when the price of sugar is $0.22. He claims that if the price of sugar is allowed to drop to its world level $0.07 he will earn only $1000. Therefore, he concludes, the value of his land will fall to a third of its initial value.

   UNCERTAIN This statement would be true if the next best employment for this kind of land earned less than $1000. If the next best employment of this land earns for instance $2000 the value of the land declines from $3000 to just $2000.

2. Consider an economy where only two goods are produced: cars and motorcycles. If one unit of capital can produce more cars than motorcycles then it is efficient to employ all the available capital in the production of cars.

   FALSE Allocative efficiency depends also on the demand side of the economy. For instance if there is a large demand for motorcycles, the price of motorcycles tends to be high and therefore capital allocated to the production of motorcycles has a high marginal revenue product.

3. An economy is populated by individuals who have the same preferences. We can conclude that there is no welfare gain from allowing them to trade.

   FALSE Gains from trade arise whenever individuals differ by preferences AND endowments.

3 Taxation

A consumer has the following indirect utility function:

$$V(p_x, p_y, I) = -\frac{p_x + 2\sqrt{p_x p_y}}{I} - \frac{p_y + \frac{1}{2}\sqrt{p_x p_y}}{I}$$

1. Find the consumer's expenditure function

$$V\left(p_x, p_y, E\left(p_x, p_y, U\right)\right) = -\frac{p_x + 2\sqrt{p_x p_y}}{E\left(p_x, p_y, U\right)} - \frac{p_y + \frac{1}{2}\sqrt{p_x p_y}}{E\left(p_x, p_y, U\right)} = U$$

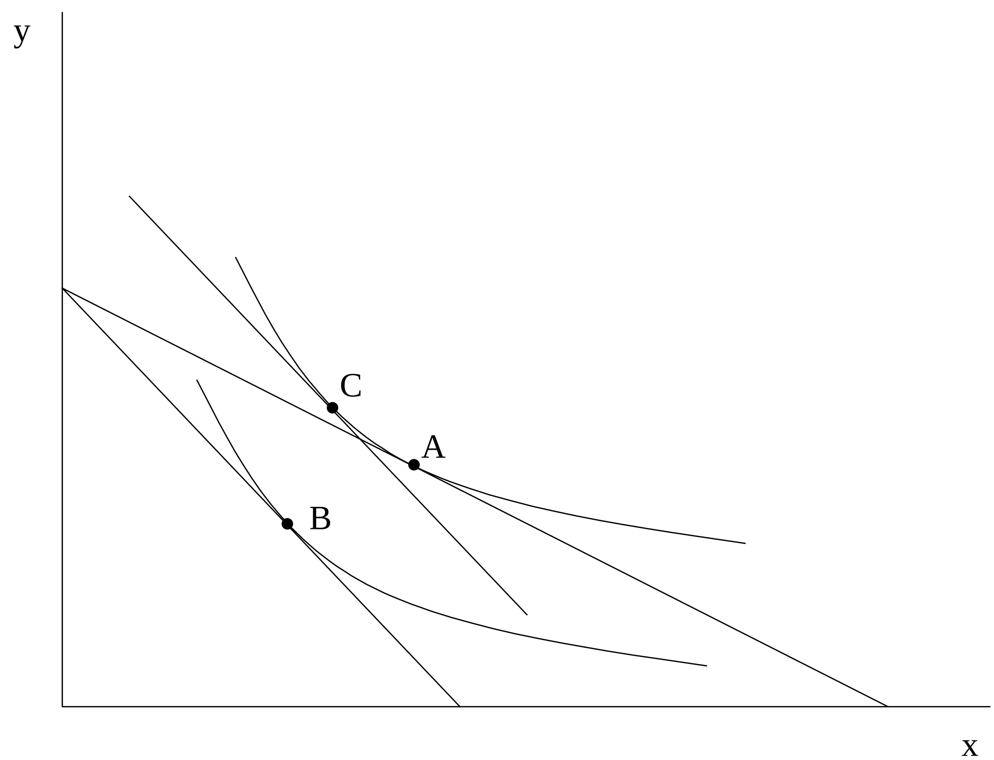
$$E\left(p_x, p_y, U\right) = -\frac{p_x + 2\sqrt{p_x p_y}}{U} - \frac{p_y + \frac{1}{2}\sqrt{p_x p_y}}{U}$$

2. Suppose that income is fixed at $I = 100$. Suppose that prices were $p_x = 4$ and $p_y = 9$. Suppose that the government puts in a tax of 5 on good x , but rebates enough to the consumer so that the consumer is as well off as he was before the tax. Assume that the consumer chooses his bundle without considering the rebate he is about to receive. Illustrate this scenario with a carefully labeled graph. Your graph should show:

   (a) the consumer's original budget set, indifference curve, and consumption bundle;

   (b) the 'taxed' budget set

   (c) the 'rebated' budget set that leaves the consumer as well off as the pre-tax budget. (Note: don't worry about drawing the price ratios exactly to scale.) The three above points are indicated as points A, B and C in the graph.



3. Calculate the deadweight loss of taxation in this example. That is, what is the difference between the revenue collected by the government and the amount of the rebate?

   First we have to calculate the utility level reached at the initial prices and income.

$$U^* = -\frac{4+2\sqrt{36}}{100} - \frac{9+\frac{1}{2}\sqrt{36}}{100} = -\frac{29}{100} = -0.28$$

   The necessary rebate after the tax is the following:

$$\begin{aligned} R &= E(9,9,-0.28) - 100 = \\ &= 144.64 - 100 = 44.64 \end{aligned}$$

   The tax revenue is calculated using the uncompensated demand and initial income plus the rebate::

$$\begin{aligned} d_x &= -\frac{\frac{\partial V}{\partial P_x}}{\frac{\partial V}{\partial I}} \\ d_x &= \frac{\left[1+1.25\left(\frac{p_y}{p_x}\right)^{0.5}\right] I}{p_x + p_y + 2.5\left(p_x p_y\right)^{0.5}} \\ d_x\left(9,9,100+44.64\right) &= 8.04 \end{aligned}$$

   Tax revenue:

$$T = 8.04 * 5 = 40.20$$

   So the DWL is:

$$DWL = R - T = 44.64 - 40.20 = 4.44$$

4. How would your answer to 1. change if the government also taxed good y by 11.25, keeping the tax on x at 5? [Hint: think before you start calculating.]

   With this tax the relative price of the two goods is unchanged therefore there is no distortion and no DWL.

4 Policy analysis

In the 1980's the US government imposed quotas on the importation of cars without imposing restrictions on the contruction of foreign-owned car factories in the US. This question is intended for you to analyze the welfare impact of this policy.

The demand for cars in the US has the following form:

$$C^D = 9P^{-1}$$

where C is million of cars.

1. What is the price elasticity of demand of this function?

   The price elasticity of demand is:

$$\frac{dC}{dP}\frac{P}{C} = -9\frac{1}{P^2}\frac{P}{9P^{-1}} = -1$$

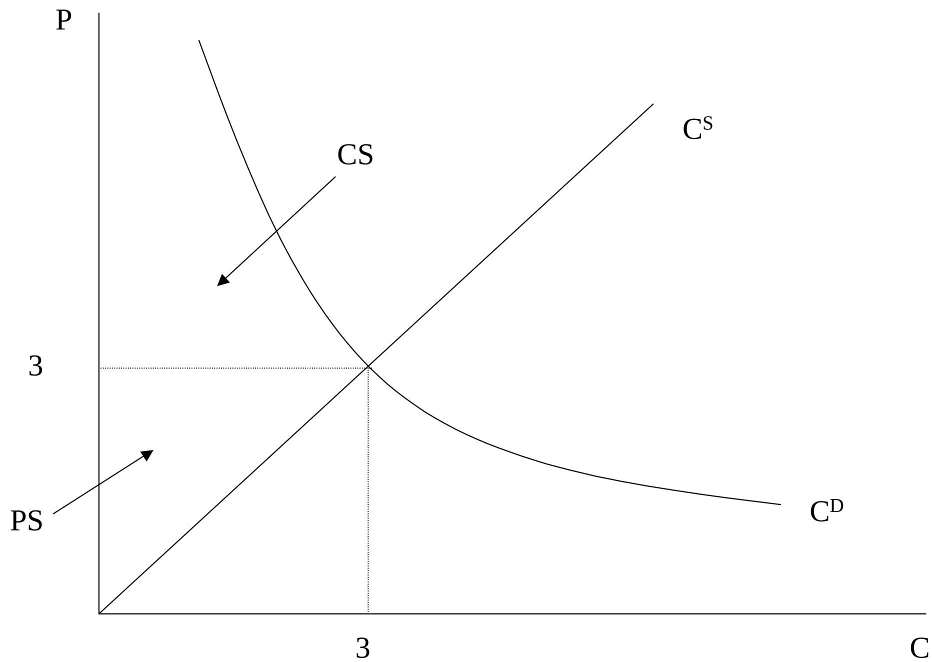
2. The supply of cars in the US takes the following form:

$$C^S = P$$

   Imagine the economy is closed to imports. What is the price of cars in a closed economy? What is the quantity of cars sold at this price? In a demand/supply graph indicate the equilibrium price and quantity and show the area representing consumer and producer surplus.

   In a closed economy the equilibrium price is the one equating demand and supply:

$$\begin{aligned} P &= 9P^{-1} \\ P^A &= 3 \\ C^A &= 3 \end{aligned}$$



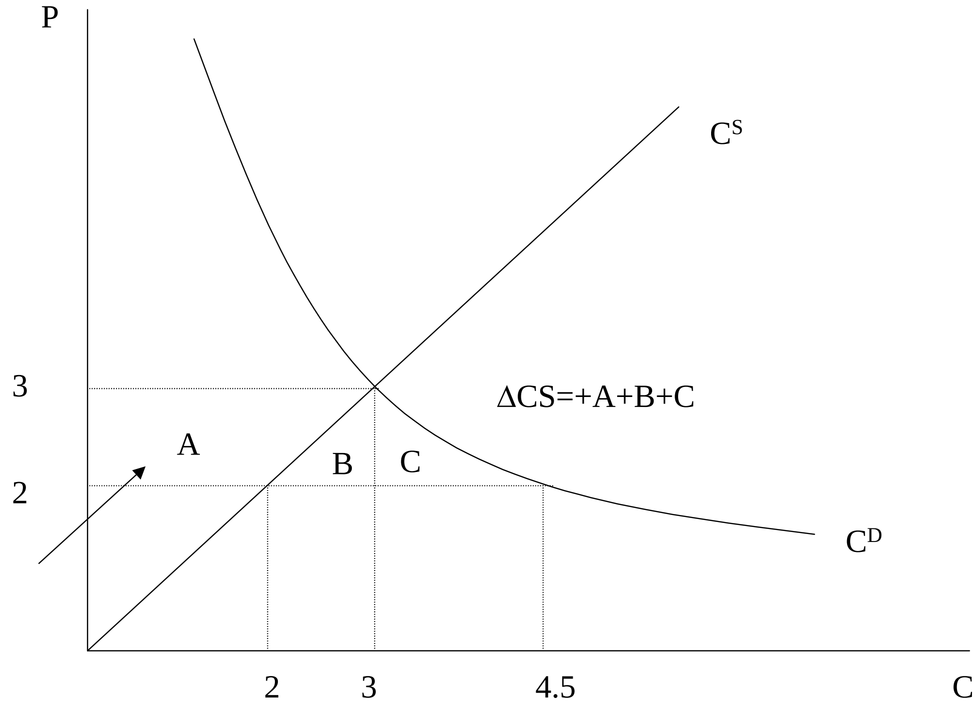
3. Now imagine that the international price of cars is $P^* = 2$. If there were no restrictions how many cars would be imported, how many would be produced? How many would be consumed? What would be the change in consumer surplus with respect to 2.? What would be the change in producer surplus with respect to 2.? (give quantitative answers)

   At $P^* = 2$:

$$\begin{aligned} C^D &= 4.5 \\ C^S &= 2 \end{aligned}$$

   The US would import 2.5 million cars. The change in consumer surplus would be:

$$\begin{aligned} \Delta CS &= \int_2^3 \frac{9}{P} dP = [9 \ln P]_2^3 = 9 \ln 1.5 = 3.6 \\ \Delta PS &= 2 - 4.5 = -2.5 \end{aligned}$$



4. The US imposes a quota on imports of 1 million cars from abroad. What is the price paid by consumers to buy a car in the US following the introduction of the quota? Assume that the quota rights are assigned to Japanese car producers. What is the change producer surplus in this market with respect to 3.? What is the change in consumer surplus in this market with respect to 3.? What is the dead-weight loss caused by the introduction of the quota (compared to the open economy case)?

   The equilibrium domestic price is the one that equates demand minus supply to 1 million:

$$\begin{aligned} \frac{9}{P} - P &= 1 \\ P^Q &\cong 2.5 \\ C^S &= 2.5 \\ C^D &= 3.5 \end{aligned}$$

The changes in welfare:

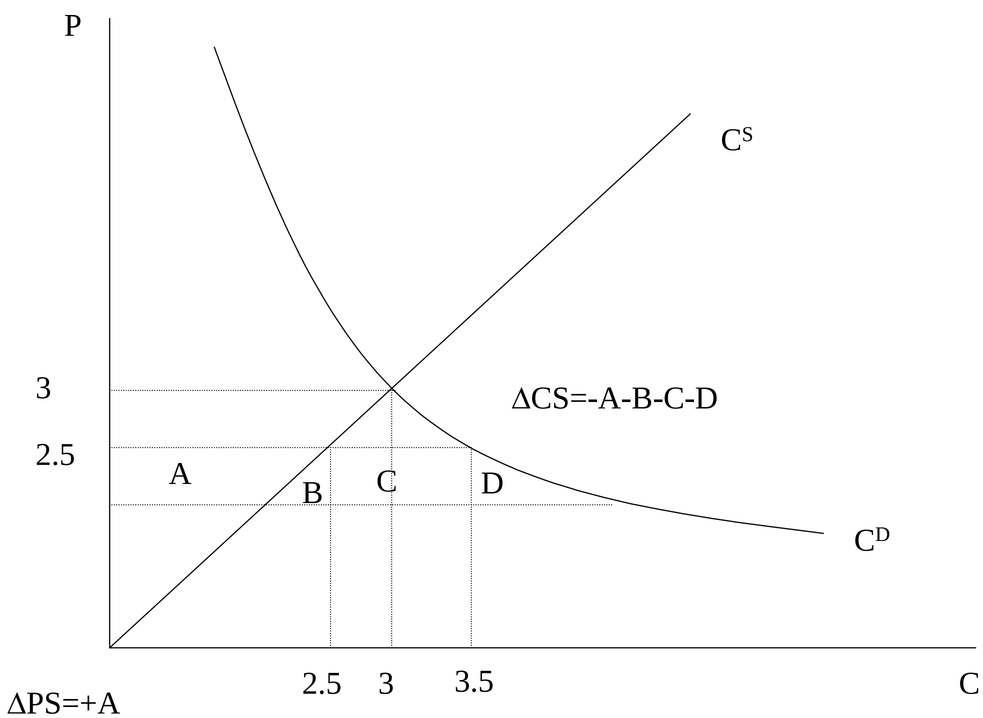
$$\Delta CS = -\int_{2}^{2.5} \frac{9}{P} dP = -\left[9 \ln P\right]_2^{2.5} = -9 \ln 1.25 = -2$$

$$\Delta PS = \frac{(2.5)^2}{2} - \frac{2^2}{2} = 1.125$$

$$DWL = -0.875$$

The DWL corresponds to the sum of areas $B + C + D$ in the graph.

Notice that the DWL could be reduced if quota rights were assigned to the US producers, since they would earn 0.5 million in quota rents, but these are handed to Japanese.



5. Imagine that due to the high US price for cars some Japanese car producers open plants in the US. So now the supply of cars in the US is the sum of the US producers supply and the Japanese producers supply. Imagine for simplicity that the Japanese manufacturers that start producing in the US supply a total of 0.5 million cars and that their cost of production is 2 per car. What is the price of cars in the US following the entry of Japanese producers? Draw the supply function in

this market and show the equilibrium in this market. Indicate in this graph the US producer surplus in this market. Qualitatively, what is the change in consumer surplus in this market with respect to 4. does it increase or decrease? What is Japanese producer surplus in this economy?

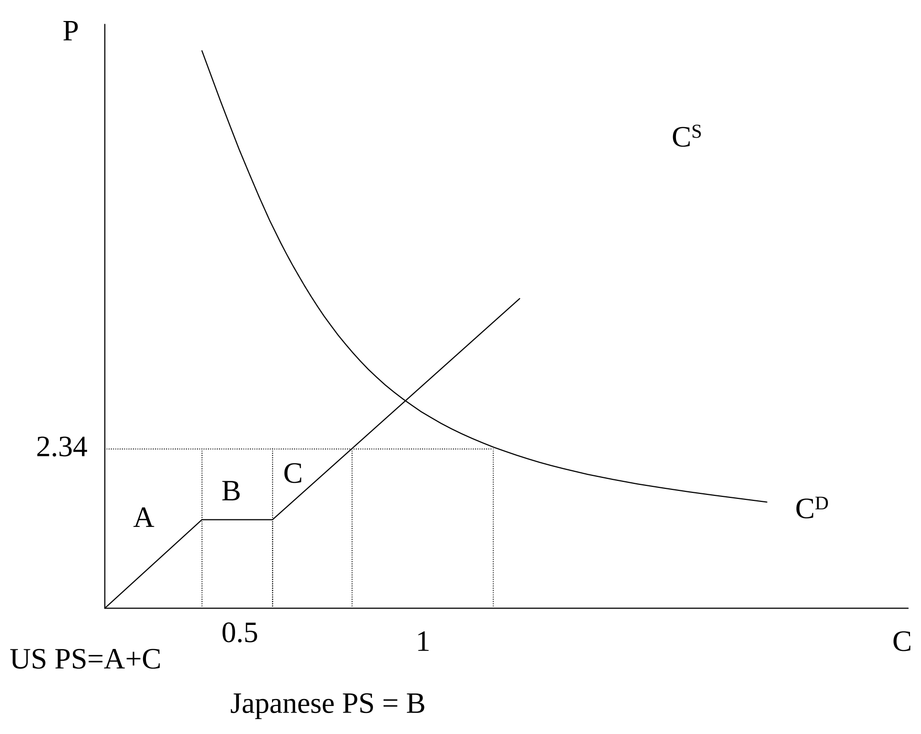
The supply in the US market has the shape indicated in the figure below. The domestic price is again the one that equates the difference between demand and supply to 1 million:

$$
\begin{aligned}
\frac{9}{P} - (P + 0.5) &= 1 \\
P^{Q1} &= 2.34 \\
C^{US} &= 2.34 \\
C^{J} &= 0.5 \\
C^{D} &= 3.84
\end{aligned}
$$

Japanese producer surplus is:

$$PS^J = (2.34 - 2)\, 0.5 = 0.17$$

Consumer surplus increases in this market relative to the case where no Japanese producers enter the market, because the price of cars declines.



5 General equilibrium in a pure exchange economy

Imagine a small country, CanLand, with two individuals, Ann and Bob, and two goods, canned corn (C) and canned ham (H). Ann and Bob's utility functions are given by:

$$\begin{aligned} U^A &= .2\ln C + .8\ln H \\ U^B &= .8\ln C + .2\ln H \end{aligned}$$

The economy as a whole is endowed with 100 units of canned corn and 50 units of canned ham. Assume for simplicity's sake that the price of canned corn is normalized to $P_c = \$1$. So, you can define the price ratio as $P \equiv \frac{P_H}{P_C} = P_H$.[HINT Remember that for Cobb-Douglas utility functions like the one above $U = x^\alpha y^{1-\alpha}$ expenditure shares are constant that is $p_x x = \alpha I$ and $p_y y = (1-\alpha) I$]

1. Solve for the marginal rates of substitution between canned corn and canned ham for Ann and Bob, and show the condition that represents allocative efficiency in C and H. [Recall that $\partial \ln X = \frac{dX}{X}$.]

   We set up the following Lagrangians for Ann and Bob and find the derivative with respect to C and H.

$$\begin{aligned} L^A &= .2\ln C + .8\ln H + \lambda(I^A - C - pH) \\ L^B &= .8\ln C + .2\ln H + \lambda(I^B - C - pH) \end{aligned}$$

   In equilibrium, Ann's MRS is the ratio of these derivatives. The inverse of this ratio equals the price of C (since $p_c = 1$)

$$\begin{aligned} \partial L^A/\partial C &= .2/C - \lambda = 0 \\ \partial L^A/\partial H &= .8/H - p\lambda = 0 \\ &\Rightarrow \frac{4C}{H} = p \end{aligned}$$

   Likewise for Bob,

$$\begin{aligned} \partial L^B/\partial C &= .8/C - \lambda = 0 \\ \partial L^B/\partial H &= .2/H - p\lambda = 0 \\ &\Rightarrow \frac{C}{4H} = p \end{aligned}$$

Allocative efficiency requires that Ann and Bob have the same marginal rate of substitution:

$$\frac{4C^A}{H^A} = \frac{C^B}{4H^B}$$

where superscripts indicate the consumer ($A = \text{Ann}, B = \text{Bob}$)

2. Now calculate Ann and Bob's demands (uncompensated) for C and H as a function of Prices P and Income I.

   We now rearrange this equality to isolate C (H) and plug into the budget constraint to solve for the optimal H (C). For Ann:

$$\begin{aligned} \frac{pH}{4} + pH &= I^A \\ \frac{5}{4}pH &= I^A \\ H^A &= \frac{4}{5p}I^A = \frac{4}{5p}(\alpha + \beta p) \\ &\Rightarrow \ C^A = \frac{1}{5}I^A = \frac{1}{5}(\alpha + \beta p) \end{aligned}$$

   Likewise for Bob:

$$\begin{aligned} 4pH + pH &= I^B \\ 5pH &= I^B \\ H^B &= \frac{1}{5p}I^B = \frac{1}{5p}[(100 - \alpha) + (50 - \beta)p] \\ &\Rightarrow \ C^B = \frac{4}{5}I^B = \frac{4}{5}[(100 - \alpha) + (50 - \beta)p] \end{aligned}$$

   where we have already replaced values of endowments α, β, etc.

3. Assume that Ann has an endowment of C equal to α and an endowment of H equal to β. Using Ann and Bob's respective MRS between the two goods, and the given societal endowment, solve for the equilibrium price ratio in terms of α and β. This equilibrium price ratio will clear the market for both goods given the prices and endowments. [Note that Bob's endowment of C is equal to $100 - \alpha$ and his endowment of H is equal to $50 - \beta$. Income I for each individual is equal to their endowment of each good multiplied by its price.]

   To solve for p, we use the identity that in equilibrium, there are no unconsumed goods; $C^{A*} + C^{B*} = 100$ or $H^{A*} + H^{B*} = 50$. So

$$\frac{1}{5}(\alpha + \beta p) + \frac{4}{5}[(100 - \alpha) + (50 - \beta)p] = 100$$

Collecting terms and simplifying, we get:

$$p = \frac{100 + 3\alpha}{200 - 3\beta}$$

4. Now assume Ann has 80 units of canned corn and 10 units of canned ham. Using your answer from Part 3, calculate the quantities consumed by each individual in equilibrium. Draw the equilibrium in an Edgeworth Box diagram, making sure to label the equilibrium point and price vector. [If you were not able to solve Part 3, explain the properties of the solution to this problem, refering to the Edgeworth box.]

   At endowments $E_C^A = 80, E_H^A = 10, E_C^B = 20$, and $E_H^B = 40$, we get $p = 2$. At this price, we get

$$\begin{aligned} C^{A*} &= 20 \\ H^{A*} &= 40 \\ C^{B*} &= 80 \\ H^{B*} &= 10 \end{aligned}$$

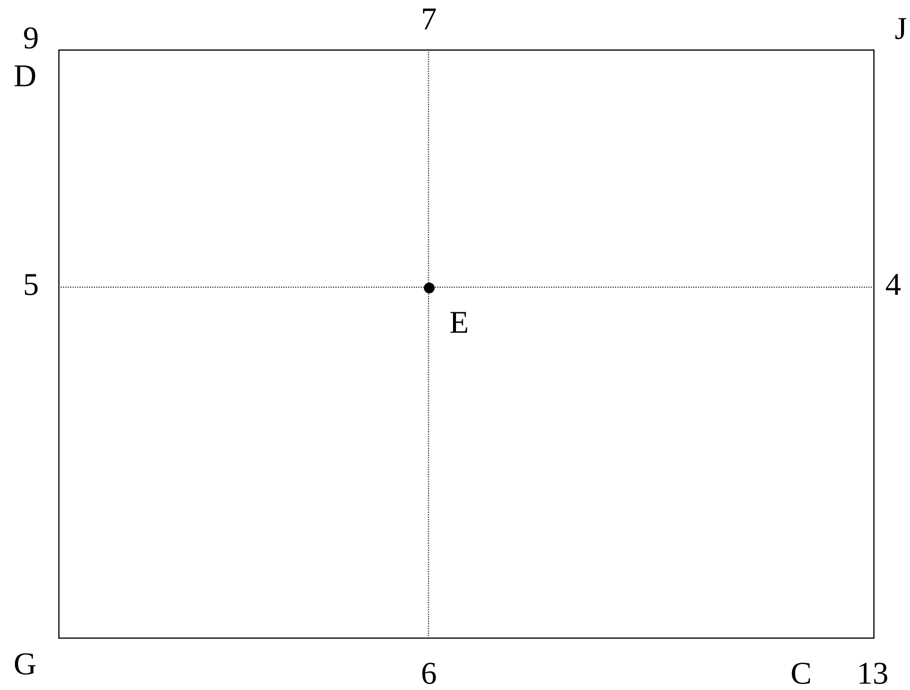
6 Linear utility and general equilibrium

George and John have the following utility functions over capable campaign managers (C) and political "attack dogs" (D).

$$\begin{aligned} U_G &= C + 3D \\ U_J &= C + D \end{aligned}$$

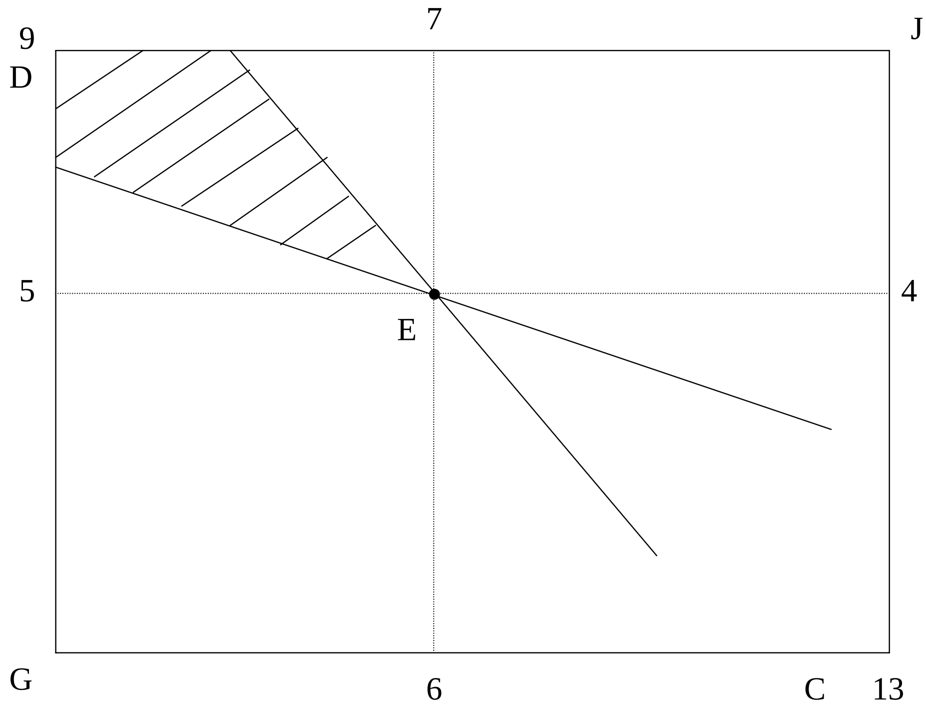
George is endowed by his party with 6 campaign managers and 5 political "attack dogs". John is endowed by his party with 7 campaign managers and 4 political "attack dogs". Imagine George and John are allowed to exchange campaign managers and attack dogs.

1. Draw the Edgeworth box for this economy clearly indicating where the initial allocation lies and what the sides of the box measure.



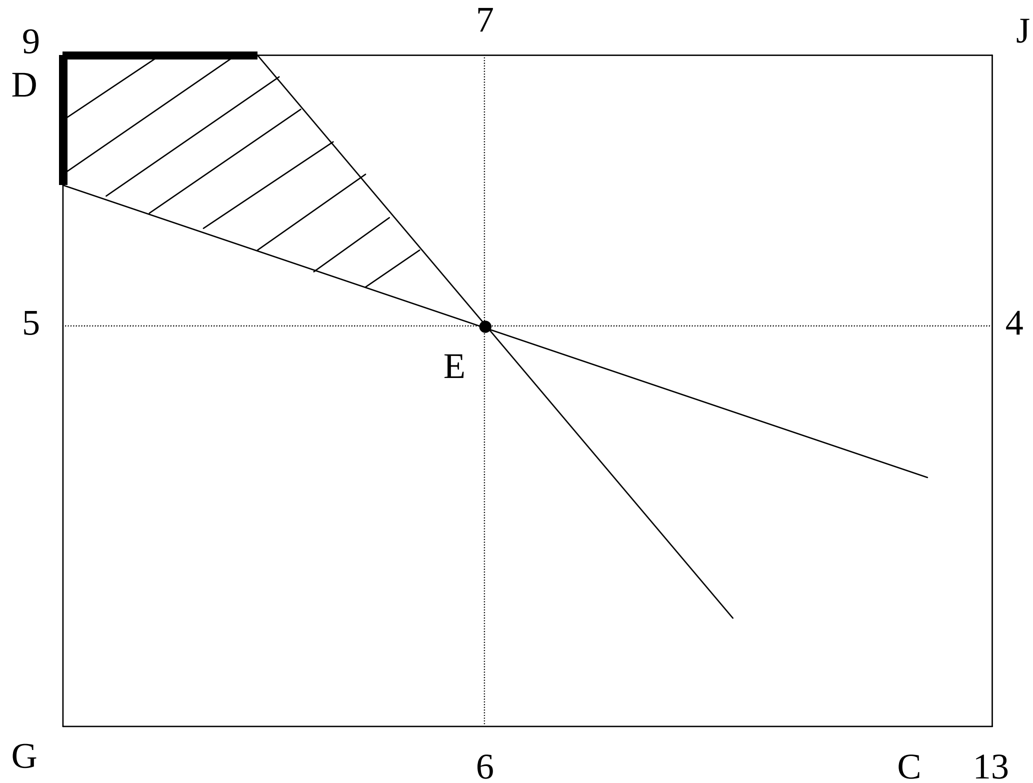
2. Indicate the set of points in the Edgeworth box that are a Pareto improvement with respect to the endowment point.

   The set of allocations that are pareto superior to the initial endowment is the shaded area below.



3. In the Edgeworth box indicate the possible final allocations of C and K after trade has taken place.

   The nature of the utility functions in this problem implies that the final allocation will involve a corner solution. If you consider any other point in the interior of the core you will realize that further trade would increase utility of both George and John, so it cannot be an equilibrium.


9
7
J
D
5
E
4
G
6
C
13