14.03 Exam 3 Fall 2004

Please do not open this exam until directed

December 13, 2004

- There are 60 points on this exam and you have 120 minutes to complete it.
- Answer 3 of the 4 problems.
- Each question is worth 20 points total. Associated points for each sub-part are listed below.
- Even very hard questions give abundant points for the initial (less difficult) steps – so please do not be discouraged.
- Answers without explanation will *not* receive credit.
- You will *not* receive extra credit for answering more than three questions.
- **No calculators or other reference materials are permitted (or needed)**

1 RA and moral hazard

A professor hires a research assistant (RA) to perform some regressions. The RA is highly risk averse, so he does not like variation in his income. His utility depends on the wage he receives from the professor (w) and the effort he puts in running regressions (e):

$$U(w, e) = w^{\frac{1}{2}} - e$$

The number of useful regression results, r, can vary. Let r_g be the good outcome, and r_b be the bad outcome. This uncertainty cannot be completely eliminated. But the probabilities of these outcomes are affected by the RA's effort. If the RA exerts no effort ($e = 0$), there is 50% probability that the good outcome would occur. If the RA exerts an effort that costs him $e = 1$ then there is a 75% probability that the good outcome will occur. The professor cannot monitor effort directly, so he must provide the right effort incentives to the RA. He knows that his RA is risk averse, so it is costly for him to have uncertainty in his future income. The professor's objective is to maximize the number of regression results net of the wage paid to his RA:

$$\Pi = r - w$$

Suppose that the RA could get a job at the campus cafeteria that paid a wage of 4 and required no effort ($e = 0$).

1. (4 POINTS) If the professor COULD observe the level of effort of his RA, what wage would he pay him to exert high effort? (Hint: the professor must convince the RA to work for him rather than taking the job at the campus cafeteria)

2. (4 POINTS) Now assume the professor cannot observe the effort exerted. The only thing he observes is the outcome in terms of regression results (r_g, r_b). Why will the professor have to pay his RA two different wage levels according to the output r (call the two wage levels w_g and w_b)?

3. (4 POINTS) Write down the incentive compatibility constraint, i.e. the condition under which, given the two wage levels w_g and w_b, the RA will choose to exert high effort.

4. (4 POINTS) Write down the participation constraint i.e. the condition under which, given the two wage levels w_g and w_b, the RA will choose to stay and work for the professor instead of the campus cafeteria.

5. (4 POINTS) Now combine the constraints derived in part 3 and 4 to find the optimal wages w_g and w_b that the professor will offer to his RA.

2 Safety and regulation

Consider a firm that produces an allergy medication. The marginal cost of producing the drug is \$60. The medication may cause drowsiness (sleepiness). Two types of consumers buy the drug, truck drivers and grandmothers. Both are risk neutral. Truck drivers and grandmothers enjoy a utility of \$100 from having their allergy symptoms relieved by the drug. Since truck drivers spend most of their time on the road, the probability that a truck driver has an accident after taking the drug is 25% (0.25). Grandmothers stay home most of the time. Their probability of having a car accident if they take the drug is only 0.5% (0.005). The damages from an accident for either truck drivers or grandmothers is \$200. There are no other monetary, psychic or utility cost of an accident. Define λ as the proportion of potential consumers who are grandmothers.

1. (4 POINTS) Is it socially efficient for grandmothers to consume the medication? How about truck drivers? Explain.

2. (4 POINTS) Suppose we are under a *caveat emptor* (buyer responsible, seller not liable) regime [assume the market is competitive]. What would be the selling price of the medication? Would grandmothers buy it? Would truck drivers? Is this outcome efficient? Explain.

3. (4 POINTS) Now assume we are under a *strict liability* regime (seller responsible for damages) and that the seller cannot distinguish the two types of consumers.

   (a) If $\lambda = \frac{1}{3}$, what would be the price of the medication? Would grandmothers buy the drug? Would truck drivers? Is this outcome efficient? Explain.

   (b) If $\lambda = \frac{1}{5}$ does the strict liability regime produce the efficient outcome? Explain.

   Now, suppose that a new discovery allows the firm to make a non-drowsy version for \$20 more per pill. The company can decide either to produce both versions (drowsy and non-drowsy), only the standard drowsy version, or only the non-drowsy version.

4. (4 POINTS) Under a strict liability regime and $\lambda = \frac{3}{5}$, what would be the company's choice? Is this choice socially efficient? Explain.

5. (4 POINTS) Now assume the company can sell the standard version under a *caveat emptor* regime (the buyer signs a contract agreeing not to sue for damages) and the non-drowsy version under strict liability (company pays if consumer is harmed). What version(s) of the medication would the company produce? What would be the price(s)? Is the socially efficient outcome reached in this case? Explain.

3 Signaling and romance

Every night, men and women go out on blind dates. Men differ only according to their sensitivity (η), which is uniformly distributed on $[0, 1]$. They all want a second date, which gives them utility $\frac{3}{4}$. Women like the sensitive types. Women get utility $8\eta - 4$ if they agree to go on a second date with a man of sensitivity η and 0 if they refuse to go on a second date. Men know how sensitive they are (that is, they know their own η_i). Women cannot tell how sensitive a man is, but they do know the distribution of η. On the first date, women can ask men to recite a romantic poem from memory. No one enjoys poetry (neither men nor women). However, men and women know that that it costs $(1 - \eta)$ for a man of sensitivity η to memorize a poem.

1. (14 POINTS) Find a separating equilibrium of this model – that is, an equilibrium where some men memorize a poem and others do not. Some of the steps you will have to take to solve this problem are:

   (a) Determine which men would memorize a poem if only men who memorize a poem get a second date.

   (b) Determine whether a woman will agree to a second date as a function of her beliefs about a man's η.

   (c) Confirm that if the men act according to (a), then women will indeed agree to a second date if and only if a man has memorized a poem.

2. (6 POINTS) The company Romance Technologies, Inc. develops an ingenious test that reveals men's exact sensitivity. A man taking a test watches a short film of a sunrise, and the weight of his tears (in ounces) is an exact measure of η. The company sells the test for p. (Assume that all utilities are measured in money, so buying the test also costs p units of utility.)

   (a) Calculate a woman's expected utility if she purchases the test before going on a date. (Hint: You'll need to use the fact that $E[\eta|\eta \geq q] = \frac{1+q}{2}$.) Compare this with expected utility from (1), where she doesn't purchase the test and demands that her date recite a poem. What is the most that women would pay for the test?

   (b) Suppose now that the test were freely available to everyone. Would any men still memorize poems? Would your answer change if the test were only freely available to men?

4 Externalities and military service

The country of Freeland is at war and is reconsidering how it enlists soldiers. There are 100 Freelanders, each of whom gets utility $\frac{1}{200}M$ when a number $M \in [0, 100]$ of Freelanders are enrolled in the military. Citizens pay a utility cost c_i if they join the military, with $c_i = \frac{i}{100}$ for citizen number i (so c_i is distributed uniformly on the $[0, 1]$ interval). Freelanders also get utility T_i when they receive a payment worth T_i from the government or their fellow citizens (note that T_i could be negative). Hence, the utility of a Freelander is: $U_i = \frac{1}{200}M - c_i + T_i$.

1. (3 POINTS) If each citizen decides individually whether to join the military and there is no payment ($T = 0$), who will join the military voluntarily? [Notice that an individual's cost is c_i and her benefit is $\frac{1}{200}M + T_i$]

2. (4 POINTS) If Freeland's government knew each person's c_i and wanted to maximize social welfare (here, the sum of Freelanders' utilities), who would they enroll in the military? [Notice that utility of every Freelander is increasing in M, regardless of whether or not the citizen is in the military.] Why are the answers to (1) and (2) different?

3. Now suppose that the government cannot observe each person's c_i but still wants to maximize social welfare (i.e., the sum of utilities). Consider three schemes for enlisting soldiers:

   (a) (3 POINTS) Sergeant Rumsfeld proposes a draft. Each citizen is forced into the military with probability 1/2.

   (b) (3 POINTS) Sergeant Pigou proposes a volunteer military. The government pays $T = 1/2$ to anyone who enrolls in the military; the cost of these salaries is raised by lump sum taxes on the general population.

   (c) (3 POINTS) Sergeant Coase proposes a tradable draft. Each citizen is drafted into the military with probability 1/2, but those drafted are allowed to pay someone else to go in their place as long as both parties agree.

   For each scheme, answer the following questions:

   i. Which people (i.e., with what values of c_i) end up in the military?

   ii. What is the expected sum of Freelanders' utilities?

4. (2 POINTS) Compare the social efficiency of the three schemes. Which scheme(s) are more efficient than others and why? (The explanation is important.)

5. (2 POINTS) Sergeant Rumsfeld complains that the Pigou and Coase proposals have the same efficiency properties, hence he does not see any difference between them. Explain why these two schemes have different distributional consequences.