

14.41 Fall 2004  
Problem Set #2  
Due Friday, Oct. 22, at 1PM

1) In the town of Springfield, residents work at home and receive \$100,000 of income each year *unless* their house burns down, in which case they receive \$0. Half of the residents are descended from the Flanders clan, and are by nature careful; their probability of fire is 0.2% each year. Half of the residents are descended from the Simpson clan, and are not as cautious. Their probability of fire is 1.0% per year.

The residents also have different utility functions. The descendants of Flanders have utility:

$$U_F=(X+5)^{1/4}$$

Where X is their income. The descendants of Simpson are from two lines—half descended from Bart and the other half from Maggie. Bart Simpson's descendants have utility:

$$U_{BS}=(X+5)^{1/2}$$

While Maggie's have utility:

$$U_{MS}=X$$

- a) Write down each type's expected utility in a world with no insurance.
- b) The town's sole insurer, appropriately named Burns Insurance Co., cannot distinguish between the residents. If it offers actuarially-fair insurance based on the town's average rate of fires, what will be the equilibrium price? Who will buy insurance?
- c) A new safety device is invented that can cut the risk of fires in half. It costs \$5 per year. Assume that Burns has no ability to monitor device use. Assume also that those who are indifferent between buying and not buying the new device *don't* buy it. Under these assumptions, who will buy the device? What will be the new equilibrium price of insurance? Who will buy insurance?
- d) Compute the change in welfare for each group and society as a whole. Who is better off? Who is worse off? Why?
- e) Explain how moral hazard and adverse selection affect the equilibrium choice of insurance and safety device use in (c).
- f) Now assume Burns can require the use of the device among all the insured. What will be the new equilibrium price of insurance? Who will buy insurance? What is the level of social welfare?
- g) Explain the changes in part (f) from part (c).

2) An economy produces two goods - ice cream and hot chocolate. Each of these goods is produced by a separate person. Individual 1 produces ice cream and receives a profit of \$64 if it is hot, \$0 if it is cold. Individual 2 produces hot chocolate and receives a profit of \$0 if it is hot, \$64 if it is cold.

Individual 1 and 2 maximize expected utility:

$$E[U_1] = p * U(Y_{1H}) + (1-p) * U(Y_{1C})$$

$$E[U_2] = p * U(Y_{2H}) + (1-p) * U(Y_{2C})$$

where  $Y_H$  and  $Y_C$  are income in state H (hot) and state C (cold),  $U(Y)$  is the utility function, and  $p$  is the probability of state H. There is a social welfare function of the form:

$$\text{Total Welfare} = E[U_1] + E[U_2]$$

(a) Suppose that  $p=1/2$ , and that the form of the utility function is:

$$U(X) = X^{1/2}$$

(i) What is the initial expected utility of each individual?

(ii) Suppose that the two individuals are considering entering into an arrangement today which insures their income against this uncertain weather outcome tomorrow (there is no weather forecasting in this society). That is, depending on the weather outcome (state H or C), each person will either receive some of the other person's income, or give some of their income to the other person. Can such an insurance arrangement be struck that makes both parties better off? What arrangement will maximize social welfare? How does social welfare compare to the level in (i)? Is this arrangement acceptable to both parties?

(iii) Now, suppose a perfectly accurate weather prediction system is invented, and it is announced that it will be hot tomorrow. How much insurance will be bought and sold now? What expected utility does this give each person? Has this increased or decreased social welfare, relative to (ii)? Why?

(iv) Using what you've learned from this problem, discuss the implication of economic theory for the use of genetic screening by insurance companies.

(b) Now, suppose that the weather prediction system turns out not to work after all, so  $p=1/2$  again. But now assume that the form of  $U$  is  $U(X) = 1/2 X$ . Answer (i)-(iii) above. Is your answer to (iii) different from (a)? Why or why not?

3) Consider an economy that is composed of identical individuals who live for two periods; in each period, half the workers are in their first period of life and half are in their second. These individuals have log utility over consumption in each period.

a) Write down their lifetime utility function.

b) They receive an income of 100 in period 1 and no income in period 2. They can save as much of their income as they like in bank accounts, earning an interest rate of  $r$  per period. They want their kids to work hard and become self-made, so they do not want to leave bequests. Write down their lifetime budget constraint.

c) Individuals choose consumption in each period by maximizing lifetime utility subject to this lifetime budget constraint. What is the individual's optimal consumption in each period? How much savings does he or she do in the first period?

d) Now, the government decides to set up a social security system. This system will take \$50 from each individual in the first period, put it in the bank, and transfer it to them with interest in the second period.

i) What is the name for this type of social security system?

ii) How does the system affect the amount of private savings? How does the system affect national savings (total savings in society)?

iii) What is the effect on social welfare, if we assume that the government interest rate is the same as the private interest rate? How would your answer change if we believed people were myopic?

iv) What is the effect on social welfare, if we assume that due to economies of scale the government interest rate is higher than the private interest rate?

e) Due to an economic disaster, the savings of those who are about to enter the second period are wiped out. The government decides to modify the social security system: instead of putting the \$50 in the bank, and transferring it back to the individual who paid it with interest in the second period, it will transfer the \$50 to a current retiree, and in the next period will "pay back" the individual who paid the \$50 by transferring \$50 to him or her from a current worker.

i) What is the name for this type of social security system?

ii) What is the effect on national savings of this change?

iii) What is the effect on total social welfare in society, relative to maintaining the old system in the face of this disaster? Who are the "winners" and "losers"?

(Answer intuitively--there is no need to show math.)

- 4) You have been asked to make an assessment of several potential reforms to Social Security using your skills as a public economist.
- (a) For each of the reforms listed below, briefly discuss the pros and cons of the reform, paying attention in particular to efficiency implications (through potential behavioral responses to the change) and equity implications (who wins and who loses). [Note that all reforms are intended to save the system money, so you do not need to list this as a benefit.]
- (i) Increase the number of years used to calculate benefits from 35 to 40.
  - (ii) Reduce benefits for beneficiaries with high asset levels (wealth)
  - (iii) Add new state and local government workers to the pool of covered workers (i.e., they pay payroll taxes now and receive benefits when they are old).
  - (iv) Gradually increase the normal retirement age from 65 to 70 (under current laws, the NRA will gradually rise to 67 by 2022; the proposal is to speed up this process so the NRA will be 70 by 2022).
- (b) One proposal for fixing Social Security is to move from the current pay-as-you-go system to a fully funded system with individual accounts. In one version of this new system, everyone would be required to contribute 10% percent of his or her income into an individual account (like an IRA) that he/she could choose to invest in a limited number of investment vehicles regulated by the government. Write a brief summary of the arguments in favor of this proposal. Then, write a brief summary of the arguments against this proposal.