

Microeconomics Review

I. Consumers and Utility Maximization

A. Basics

- Utility function :

Preferences over two goods can be represented by a utility function: $U(x_1, x_2)$.

Example: $U(x_1, x_2) = a \log x_1 + b \log x_2$

The most common case is where the two goods are consumption and leisure.

Common properties: $U'_i > 0$

Utility is increasing in consumption of good x_i

$U''_i < 0$

Marginal utility is decreasing in consumption of good x_i

- Indifference curves: $U(x_1, x_2) = \text{constant}$

The IC represents all possible combinations of the two goods that give the consumer the same level of satisfaction.

Higher IC represent higher levels of happiness: $U^A > U^B > U^C$.

(More is better. Also, there is free disposal: you can always throw something away if you don't want it).

- Budget constraint: $p_1x_1 + p_2x_2 \leq I$, where p_i = price of good i , I = income

Slope = $- p_1 / p_2$

Price change: rotation in or out of BC (one endpoint fixed)

Income change: parallel shift in or out of BC

B. Utility Maximization: Graphical

- Goal: given your income and preferences, make yourself as happy as possible.
- How? You clearly want to spend all your income, so the BC is binding.
- Because you have convex preferences, there will be a tangency between the IC and the BC at the optimum.

C. Utility Maximization: Calculus

- Set-up: $\max U(x_1, x_2)$ s.t. $p_1x_1 + p_2x_2 \leq I$
- Use Lagrange's method: $L = U(x_1, x_2) + \lambda (I - p_1x_1 + p_2x_2)$
- First-order condition: $\partial L / \partial x_i = 0 \rightarrow \partial U / \partial x_i = \lambda p_i, i=1,2$

- At the optimum:
$$\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = \frac{p_1}{p_2}$$

marginal rate of substitution (MRS) = negative of slope of BC
= ratio of prices

This is the tangency condition from the graph

(rate at which you are willing to trade between goods at the margin

= rate at which you can trade between goods in the market).

- Example: $U(x_1, x_2) = a \log x_1 + b \log x_2$
 $L = a \log x_1 + b \log x_2 + \lambda (I - p_1x_1 + p_2x_2)$
FOCs: (1) $\partial L / \partial x_1: a / x_1 = \lambda p_1$
(2) $\partial L / \partial x_2: b / x_2 = \lambda p_2$
(3) $\partial L / \partial \lambda: I - p_1x_1 + p_2x_2$

(1) and (2) yield : (4) $a x_2 / b x_1 = p_1 / p_2$

D. Extensions

- Demand curves: Derived by varying the price of one good and reoptimizing.
Example: $x_1 = x_1(p_1, p_2, I)$
- Income expansion path: Derived by varying income and reoptimizing.

- To solve for the demand curves and income expansion path, use algebra:

Example: Set: $a + b = 1$
Then solve FOC for x_i :

(4) and (3) yield : (5) $x_1 = a I / p_1$
(6) $x_2 = b I / p_2$

These are the demand curves for x_1 and x_2 .

- Income and Substitution Effects

When a price changes, there are two different effects to consider:

1. Substitution effect: the effect of a change in the price ratio. When the price of a good rises, the substitution effect leads agents to consume less of the good.
2. Income effect: the effect of a change in purchasing power. When the price of a normal good rises, the agent will move to a lower indifference curve and consume *less* of the good. (If the good is inferior, the decrease in purchasing power caused by the price increase may cause the agent to purchase *more* of the good.)

- Elasticities:

$$\eta_p = \partial x / \partial p * p / x$$

$$\eta_I = \partial x / \partial I * I / x$$

Think of it as the % change in demand to a 1% change in prices or income.

II. Perfectly Competitive Equilibrium

A. Competitive equilibrium

- Aggregate demand: $D^x(p_x) = \sum_i x_i(p_x)$, where x=good, i=individual
Aggregate supply: $S^x(p_x) = \sum_f x_f(p_x)$, where x=good, f=firm

- Competitive equilibrium occurs at the intersection of aggregate supply and demand

- The competitive equilibrium is pareto optimal since any other quantity of x results in a lower social surplus. A pareto optimal outcome is called efficient.

- Consumer surplus: $CS = \int_p^\infty D(p) dp$
Producer surplus: $PS = \int_0^p S(p) dp$
Social surplus: $SS = CS + PS$

B. Efficiency conditions and competitive equilibrium

- Exchange efficiency
 - Condition: $MRS_i = MRS_j$, where i, j = individuals
 - Idea: Rate at which individuals are willing to trade goods is equal across all individuals.
 - Met in perfect competition (PC)? Yes, since each person sets $MRS_i = p_1/p_2$

- Production efficiency
 - Definitions: K =capital, L =labor, $f(K,L)$ =production function
 - $MP_K = \partial f / \partial K$ = marginal product of capital
 - $MP_L = \partial f / \partial L$ = marginal product of labor
 - $RTS_i = (MP_K/MP_L)_i$
 - = rate of technical substitution between K and L in producing output i
 - $G(x_1, x_2)$ = production possibility frontier (PPF)
 - = efficient level of output given inputs
 - $RPT_i = (\partial x_1 / \partial x_2)_i$
 - = rate of product transformation in producing outputs x_1 and x_2 by firm I

 - Condition 1: $RTS_i = RTS_j$, where i, j = outputs
 - Condition 2: $RPT_i = RPT_j$, where I, j = firms
 - Idea: allocate resources among firms and among outputs within firms such that no further reallocation would permit more of one good to be produced without necessarily reducing the output of some other.
 - Met in PC? Yes, when solve for FOC for firms (not done here).

- Product mix efficiency
 - Condition: $MC_1 / MC_2 = MU_1 / MU_2$
 - Idea: choose mix of products such that marginal cost of production equals marginal benefit. Ties individual preferences to production possibilities.
 - Met in PC? Yes, since firms set $MC_x = p_x$ and consumers set $MU_x = p_x$

- First Welfare Theorem: A competitive equilibrium is pareto-efficient.

- Second Welfare Theorem: Any pareto-efficient allocation may be implemented as a competitive equilibrium for the appropriate set of endowments.
Where you start matters for where you end up; efficiency and equity are different issues.