

### I. Value of Labor and Leisure Hours

In class, Professor Gruber suggested that the value of an hour of your time at work must be equal to the value of an hour of your time at leisure. Here's why:

Let  $x$ =consumption goods,  $l$ =leisure hours,  $h$ =work hours,  $p$ =price of consumption goods,  $w$ = nominal wage

Maximize your utility  $u(x,l)$  subject to the budget constraint  $px=wh$ .  
Note that  $l=24-h$ ,  $h=24-l$ .

Max  $u(x,l)$  s.t  $px-wh = 0$

Lagrangian:  $L=u(x,l)-\lambda(px-wh)=u(x,24-h)-\lambda(px-wh)$

FOC:  $\partial u/\partial x - \lambda p = 0$   
 $-\partial u/\partial h + \lambda w = 0$

$$\rightarrow \frac{\partial u/\partial h}{\partial u/\partial x} = \frac{w}{p} = \text{real wage}$$

Alternatively,  $L=u(x,l)-\lambda(px-wh)=u(x,l)-\lambda(px-w(24-l))$

FOC:  $\partial u/\partial x - \lambda p = 0$   
 $\partial u/\partial l - \lambda w = 0$

$$\rightarrow \frac{\partial u/\partial l}{\partial u/\partial x} = \frac{w}{p} = \text{real wage} = \text{MRS}(L \text{ for } X)$$

### II. Cost-Benefit Analysis

#### A. Present Value

- What is money today worth tomorrow?
  - Deposit money in a bank; the interest rate,  $r$ , is 5% per year.
  - Put in \$100, in one year you have  $\$100 * (1+.05) = \$105$
  - In two years, you have  $\$100 * (1+.05) * (1+.05) = \$100 * (1+.05)^2 = \$110.25$   
Why not just \$110? Compounding: in 2<sup>nd</sup> year, earn interest on interest from 1<sup>st</sup> year.
  - In general, if invest \$R for T years, at the end of T years you have  $\$R * (1+r)^T$ .
- What is money tomorrow worth today?

- Say your best friend offers to pay you \$100 in one year. What would you be willing to pay him or her for a contract guaranteeing you that? (assume no risk of default)
- The opportunity cost of letting him or her use the money for a year is the interest rate you could have earned on it: 5% (or more generally,  $r$ ).
- What would you have to put in the bank today to have \$100 in one year?  $\$100 / (1+.05) = \$95.24$ . We can see this because  $(\$100 / (1+.05)) * (1+.05) = \$100$ .
- The present value of a future amount of money is the maximum you would be willing to pay today for the right to receive that money in the future.
- In general, the present value of  $\$R$  dollars received in  $T$  years is  $\$R / (1+r)^T$ .
- In fact,  $r$  is called the discount rate because money in the future is discounted by  $r$ .

### 3. Examples

- What is the value of receiving \$100/year for five years (first payment today)?  

$$PV = 100 + 100/(1+.05) + 100/(1+.05)^2 + 100/(1+.05)^3 + 100/(1+.05)^4$$

$$PV = 100 + 95.24 + 90.70 + 86.38 + 82.27$$

$$PV = 454.59 \text{ (much less than \$500)}$$

- What is the value of receiving \$100/year forever?  

$$PV = 100 + 100/(1+.05) + 100/(1+.05)^2 + 100/(1+.05)^3 + \dots$$

$$PV/(1+.05) = 100/(1+.05) + 100/(1+.05)^2 + 100/(1+.05)^3 + \dots$$

$$PV - PV/(1+.05) = 100$$

$$(.05/(1+.05)) * PV = 100$$

$$PV = 100 * ((1+.05)/.05)$$

$$PV = 2100$$

More generally, the PV of  $\$R$  dollars received forever is  $\$R * ((1+r)/r)$

## B. Evaluating a project

### 1. Calculating Present Value

- Say we are considering some project, for example, the Big Dig.
- Our project has some costs and benefits each year:  $B_0, B_1, B_2, \dots, B_T; C_0, C_1, C_2, \dots, C_T$
- Present Value:  $PV = B_0 + B_1/(1+r) + B_2/(1+r)^2 + B_3/(1+r)^3 + \dots + B_T/(1+r)^T$   
 $- (C_0 + C_1/(1+r) + C_2/(1+r)^2 + C_3/(1+r)^3 + \dots + C_T/(1+r)^T)$

### 2. When To Do a Project

- If our project were something continuous (how many flowers to plant), we'd want to choose the level where  $MSB = MSC$ .
- However, most projects (like the Big Dig) require a yes-no decision.
- Our general decision rule is: do the project if the  $PV > 0$ .

- Note: If we are considering several mutually exclusive projects, we should do the one with the highest present value.

### C. Choosing a Discount Rate

#### 1. Discount Rate for Private Projects

- Say you are a firm trying to decide whether to undertake an investment. What discount rate do you use?
- Use the firm's opportunity cost of capital. For example, if the firm could have paid off some debt on which it pays 10% interest, then the firm should use a 10% discount rate.

#### 2. Discount Rate for Government Projects: Government or Private Sector Rate of Return

- Option 1: use the government's rate of return (treasury bond rate). This is the rate at which the government borrows.
- Option 2: discount rate = private pre-tax rate of return. Say the money for a government project comes from a firm that would have invested it. Then opportunity cost of the project is the pre-tax rate of return the firm would have earned in its investment (the pre-tax return measures the total value generated for society).
- Option 3: discount rate = weighted average of private pre-tax and after-tax rate of return. In practice, some government revenue comes from individuals who would otherwise have consumed it. The consumer's opportunity cost of giving up a dollar of consumption is after-tax rate of return, because if you don't consume it you would put it in a bank and earn  $r$ , then the government takes  $rT$  ( $T$ =tax rate), so you would get  $r*(1-T)$ . This option doesn't get used much in practice because it's hard to know the weights.

#### 3. Discount Rate for Government Projects: Social Discount Rate

- Social discount rate = rate at which society is willing to trade off between consumption today and consumption tomorrow. Why is this not equal to government or private rate of return?
- Future Generations: the private sector is only concerned with its own welfare, but it is the role of government to protect welfare of future generations. This leads us to choose a lower discount rate (because we value the future more highly). But do individuals care about future generations? I care about my own kids, grandkids, etc. – this is called intergenerational altruism. What about everyone else's kids? One argument is that my kids marry your kids, so my descendants are your descendants, so I care about your kids too (this may work for a small ethnic group that intermarries, though it seems unrealistic for the whole US).
- Paternalism: people are too short-sighted and don't value the future highly enough. This leads us to choose a lower discount rate.
- Externalities: if project has positive spillovers (for example, the government invests in some research and development at a university that creates know-how which benefits firms), then we want to use a lower discount rate.

#### 4. Conclusions

- Choice of a discount rate matters (you can construct examples where with one discount rate, the PV of a project is positive, and with another discount rate, the PV is negative).
- May want to do analysis with a range of discount rates to test the sensitivity of the results to your choice of a discount rate.

#### **D. Valuing Costs and Benefits**

1. Market prices: this is best if there are no market imperfections (monopoly, externalities, unemployment, etc.). However, market prices don't exist for everything (i.e., human life).
2. Shadow price: use this if there is a market imperfection. The shadow price is the marginal social cost of a good. Example: for unemployed labor, shadow cost is value of leisure, not the market wage. The reason is that employing them does not lower output elsewhere in the economy, so their wage is not an opportunity cost. (Note that shadow price is really a more general term that could include all the categories.)
3. Inferences from Economic Behavior (Revealed Preference): use this if there is no market price and there is some relevant economic choice. Example: value of time is the foregone income from one additional hour of work (for people who can control how much they work and have no benefits other than their hourly wage).
4. Contingent Valuation: use this if there is no market price and no possible revealed preference method. The idea is to ask people how much they value things. One problem is that people give inconsistent answers (value 1 bird at \$5, value 100,000 birds at \$5).

#### **E. Common Mistakes in C/B Analysis**

1. Counting Secondary Benefits. If build a road, one might count the additional commerce along the road as a benefit. Problem: the new road may be taking away from commercial activity elsewhere, so the net gain to society may be small or zero. People forget to count the lost benefits elsewhere.
2. Counting Labor as a Benefit. In arguing for "pork barrel" projects, politicians often talk about the jobs created by the project as a benefit. But wages are part of the cost of the project, not the benefits. (As mentioned above, if there is unemployment, the cost of labor is less than the wage.)
3. Double Counting Benefits. Example: in considering the value of an irrigation project, you count the increase in the value of the land and the PV of the increase in income from farming as benefits. Should only count one, because you either sell the land or keep it and get the gains as a stream of income.

#### **F. Complications**

1. Distributional Concerns.
  - Costs and benefits of a project don't necessarily accrue to the same people (think of expanding a road – commuters benefit, people living next to road lose).
  - Theoretically, if the PV of the project is  $> 0$ , it is possible to collect money from those who benefit and redistribute it to those who lose and make everyone better off.

- But in practice: 1) redistribution often doesn't happen; 2) redistribution is costly; 3) there are heavy information requirements.
- Another option is to use weights on costs and benefits depending on who is paying/receiving them. Example: weight \$1 of burden to a poor person as \$1.50.
- Problem: how do we pick the weights? Can make any project look good depending on weights you pick.

## 2. Uncertainty.

- Problem: future costs and benefits are not always known with certainty. How do we value uncertain costs and benefits?
- One strategy: expected value. If you take a gamble that pays \$100 with probability 50% and \$0 with probability 50%, the expected value of the gamble =  $100 \cdot .5 + 0 \cdot .5 = 50$ .
- This assumes people are risk-neutral. In simple terms, this means that a person cares only about the expected value of the gamble and not about any risk he or she may be taking. So in our example, a person is equally happy with getting \$50 with certainty and with getting \$100 with probability .5 and \$0 with probability .5.
- In fact, most people are risk-averse: they dislike risk and would be willing to trade away some of the expected value of the gamble to have a certain outcome. The certainty equivalent is the amount of certain income that would make an individual as happy as the gamble.
- If we know something about how risk-averse people are, we can calculate the certainty equivalent and use that instead of the expected benefit. [In a problem, if you are not told how risk-averse people are, assume they are risk-neutral, but make your assumption clear.]

## G. Example

### 1. Background

- CAFE (Corporate Average Fuel Economy) standards require that the average fuel economy of all the cars that a car manufacturer sells must meet a certain target (say, 30 MPG) or the manufacturer must pay a penalty.
- CAFE standards were imposed after the oil crises in the 1970s to reduce US dependence of foreign oil. The imposition of CAFE standards has been accompanied by a rise in average fuel economy of cars.
- More fuel efficient cars give less protection in car accidents because they are made of less strong, lightweight materials. However, they reduce gas usage and pollution.
- Question: Should we raise CAFE standards to 35mpg?

### 2. Costs and Benefits

- Costs:
  1. one-time R&D cost of \$500M

- 2. new cars cost \$100 more (10M cars sold per year) = \$1B/yr
- 3. 100 more fatalities per year in traffic accidents (value of life is \$3M) = \$300M/yr
- Benefits:
  - 1. individuals use \$10 less in gas/year (100M cars in US; to be totally accurate, only 10M cars have gas savings the first year, 20M the second year, but to keep the problem easy, assume all cars get the savings every year) = \$1B/yr
  - 2. less pollution in environment (valued at \$225M/yr)
  - 3. lower military budget because US takes less action in Persian Gulf (\$100M/yr).

### 3. Present Value Calculation

$$PV = (\$1.325B * (1+r)/r) - (\$500M + \$1.3B * (1+r)/r)$$

$$r = .10: PV = - \$225,000,000 \rightarrow \text{don't do project}$$

$$r = .05 \quad PV = \$25,000,000 \rightarrow \text{do project}$$

In this example, choice of r matters, but this is not always true (for reasonable values of r).