Multi-echelon Inventory Systems

- *Why?*
- *Issues and decisions?*
- *Models*

Why have a multi-echelon distribution system?

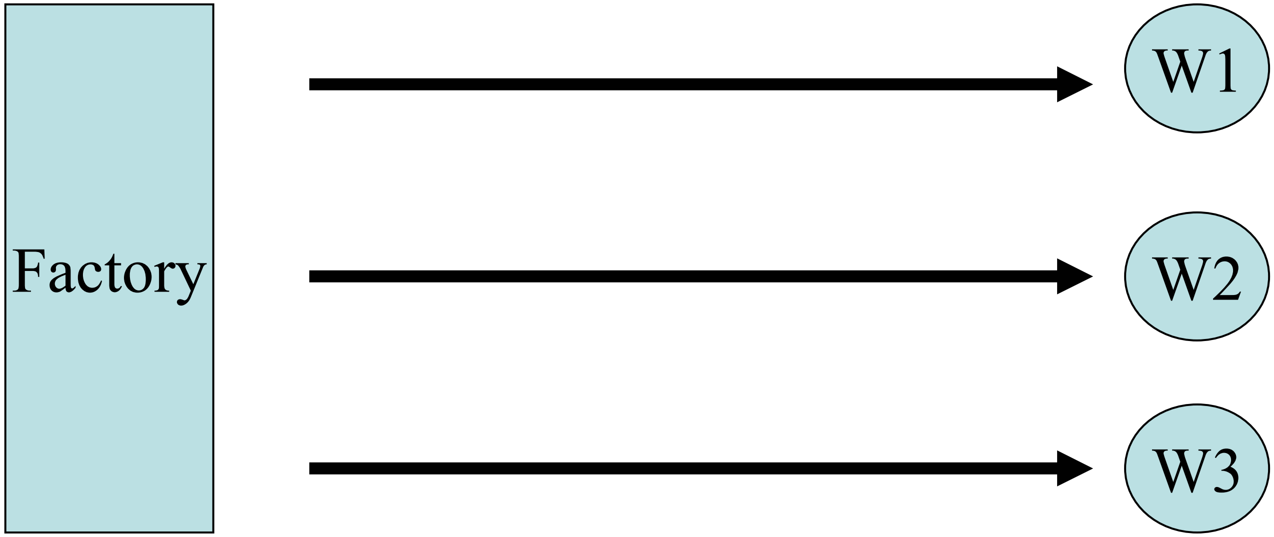
- *Better service from regional locations*
- *Transportation economies*
- *Mixing functions*
- *Risk pooling over the manufacturing or procurement lead time*
- *Differentiated stocking and service policies*

Design and Planning Issues

- *Number of echelons*
- *Number and location of distribution centers*
- *Stock location: what items to stock at each DC*
- *Replenishment policies – inventory & transportation; who serves whom*
- *Information systems*

Risk Pooling: Centralized Control

- *Assume periodic review, base-stock policies*
- *Centralized control, with global information*
- *Cross-dock operation – no inventory held at a "central distribution center"*

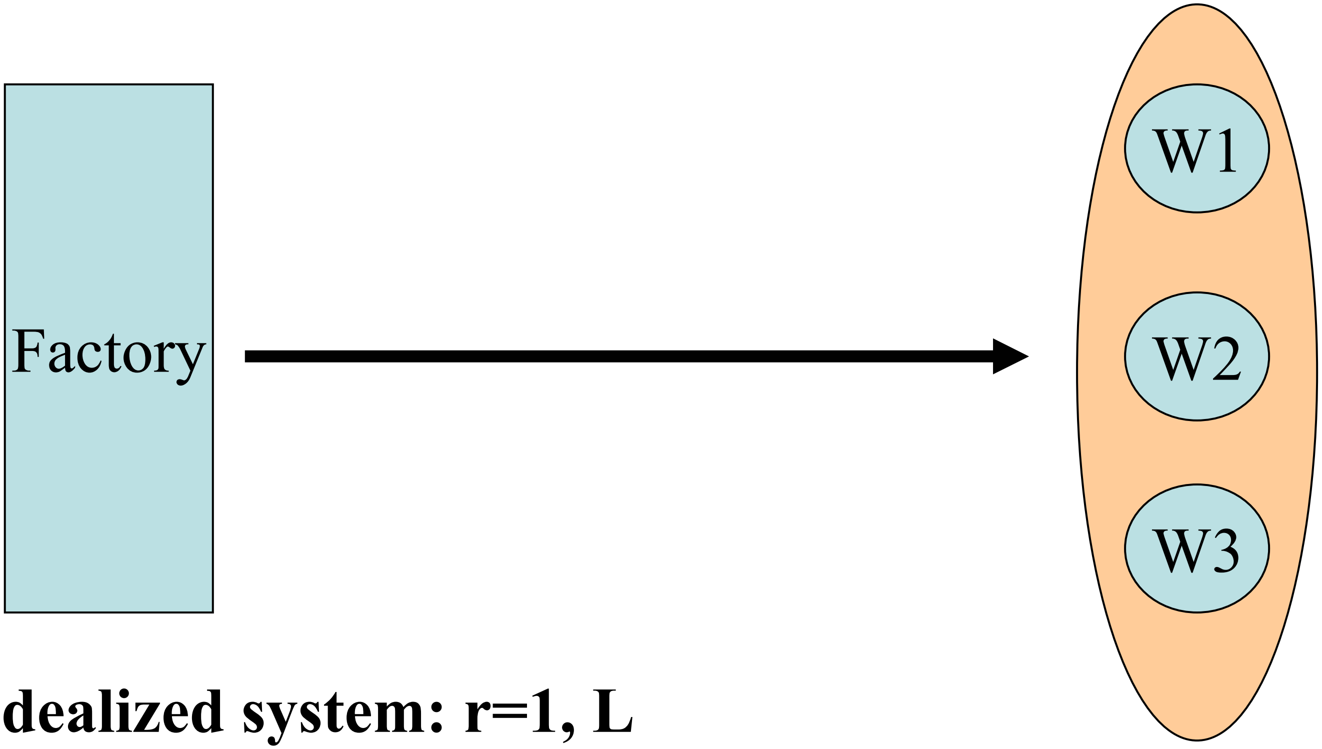


Decentralized system: r = 1, L

Safety stock at each warehouse proportional to $\sigma_i \sqrt{L+1}$

Total safety stock $\approx z \sum_{i=1}^{3} \sigma_i \sqrt{L+1}$

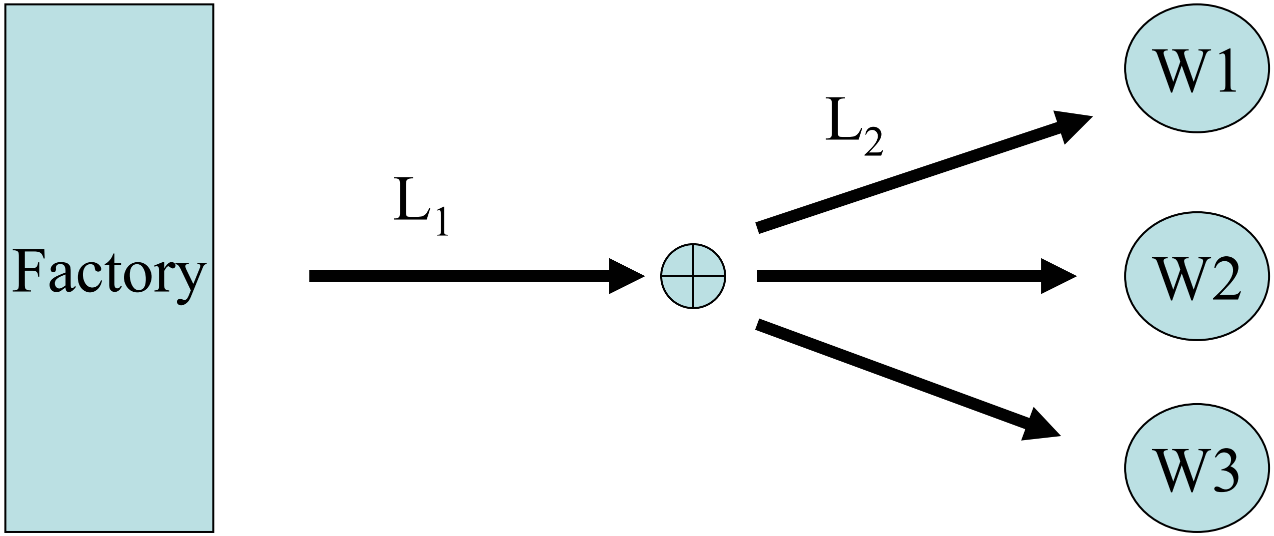
Cycle stock? Pipeline stock?



Idealized system: r=1, L

Total safety stock $$\approx z\sqrt{\sum_{i=1}^{3}\sigma_i^2}\,\sqrt{L+1}$$

Safety stock at each warehouse $$\approx z\,\frac{\sigma_i}{\sum_{i=1}^{3}\sigma_i}\sqrt{\sum_{i=1}^{3}\sigma_i^2}\,\sqrt{L+1}$$



Cross-dock system: $r = 1$, $L = L_1 + L_2$

Each period:

- **System replenishment order = $d_1 + d_2 + d_3$**
- **Allocate stock receipts to balance inventories**
- **Transship allocations**

Cross-dock System

Safety stock at each warehouse:

$$\approx z \sqrt{\left(\frac{\sigma_j}{\sum_{i=1}^{3} \sigma_i}\right)^2 L_1 \sum_{i=1}^{3} \sigma_i^2 + \left(L_2 + 1\right)\sigma_j^2}$$

Total safety stock:

$$\approx z \sum_{j=1}^{3} \sqrt{\left(\frac{\sigma_j}{\sum_{i=1}^{3} \sigma_i}\right)^2 L_1 \sum_{i=1}^{3} \sigma_i^2 + \left(L_2 + 1\right)\sigma_j^2}$$

	Total safety stock
Decentralized	$\approx \sigma n \sqrt{L+1}$
Idealized	$\approx \sigma \sqrt{n} \sqrt{L+1}$
Cross-dock	$\approx \sigma n \sqrt{\frac{L_1}{n} + L_2 + 1}$

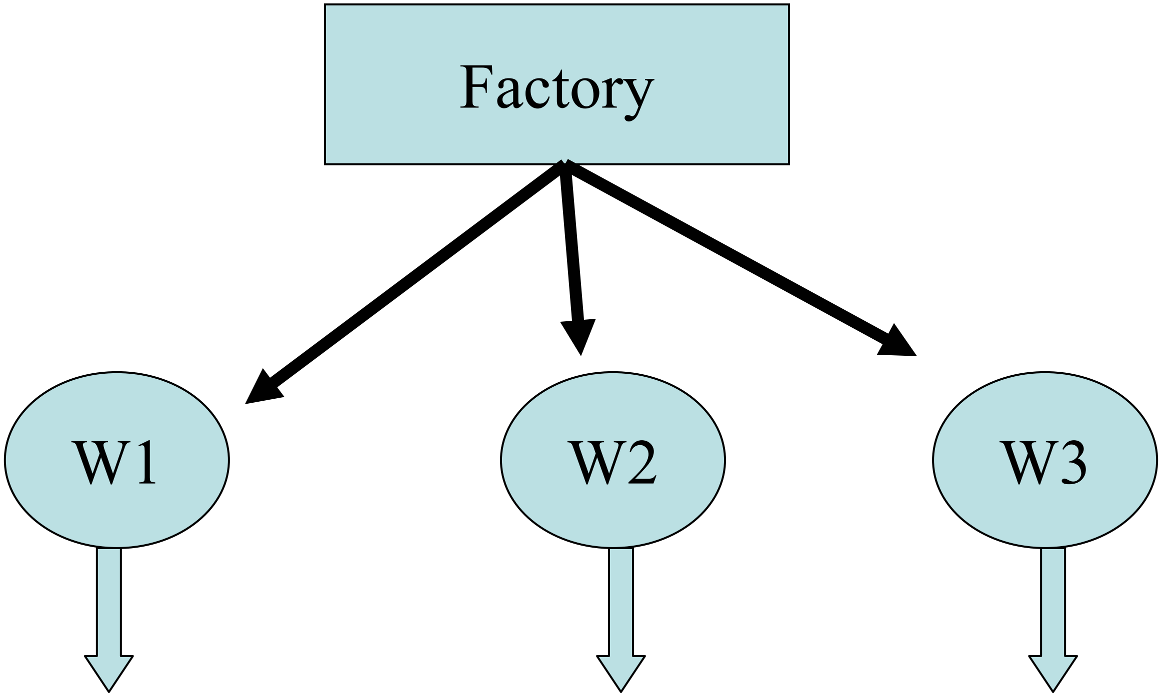
n "identical" warehouses,
each with standard deviation of demand = σ

	L_1=0	L_1=2	L_1=5	L_1=8	L_1=10	Ideal
n=1	166	166	166	166	166	166
n=2	332	316	292	265	245	235
n=3	497	466	415	357	312	287
n=5	829	766	661	536	433	371
n=10	1658	1517	1275	975	707	524

Safety stock
L = 10, σ = 50, n warehouses with same σ

Risk Pooling: Decentralized Control

- Assume periodic review, base-stock policies
- Decentralized control at warehouses, ordering on a central distribution center
- Central distribution center holds inventory, and operates without global information
- Central distribution center operates with high service level



Factory

W1

W2

W3

Independent warehouses

r = 1 week

L = 7 (production) + 1 (transportation) = 8 weeks

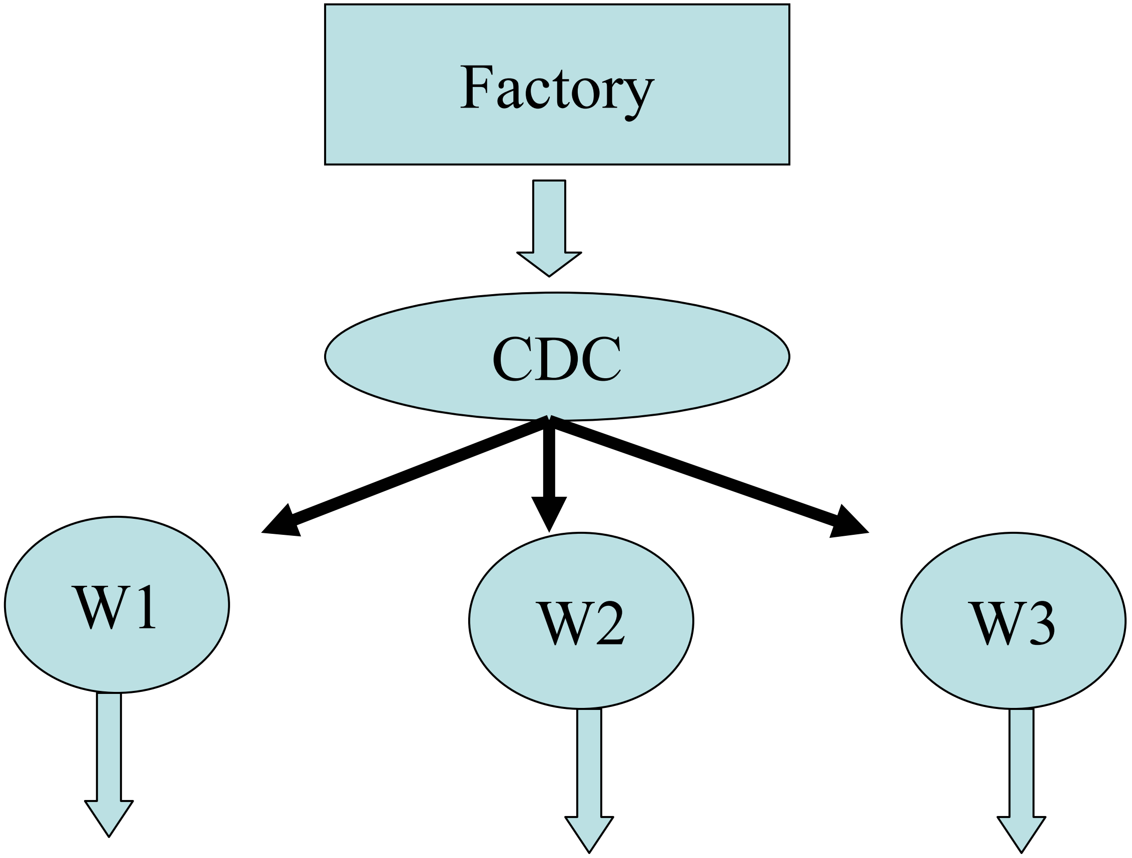
E[D] = 500 / week **σ[D] = 50 / week**

No CDC: How much safety stock?

At each W: $\approx \sigma\sqrt{r+L} = (50)\sqrt{1+8} = 150$

For n independent warehouses: $\approx n\sigma\sqrt{r+L} = 150n$

$= 450$ for $n = 3$



W: r = 1 week **L = 1 week (transportation)**

CDC: r = 1 week **L = 7 weeks (production)**

W: E[D] = 500 / week **σ[D] = 50 / week**

CDC: How much safety stock?

At each W: $\approx \sigma\sqrt{r+L} = (50)\sqrt{1+1} = 71$

At CDC: $\approx \sigma\sqrt{n}\sqrt{r+L} = (50)\sqrt{3}\sqrt{1+7} = 245$

For system: $\approx n\sigma\sqrt{r+L} + \sigma\sqrt{n}\sqrt{r+L}$

$= (3)(50)\sqrt{2} + (50)\sqrt{3}\sqrt{1+7}$

$= 212 + 245 = 457$

# of Ws	*One-echelon*	*Two-echelon*
n=1	*150*	*212*
n=2	*300*	*342*
n=3	*450*	*457*
n=4	*600*	*566*
n=5	*750*	*671*

Findings for Two-echelon Systems

- *For high demand low cost items – CDC should not buffer: e. g. 50% fill rate target at CDC; 99% fill rate target at Ws*
- *For low demand high cost items – CDC should carry some safety stock, but usually less than normally assumed. Actual amount is very dependent on numbers.*
- *For some low demand items – stock only at CDC and use premium transport, or provide slower service*