Note on Inventory Service Levels

For inventory models we often refer to two types of service: Type I and Type II.

Type I

The service level is defined as the **probability of stocking out when there is an order event**. (equivalently we can define it as the probability of **not** stocking out when there is an order event).

For a Q R model, the Type I service level can be expressed as:

$$\text{Probability of a stockout when you reorder} = \int_{x=R}^{x=\infty} f_L(x)\, dx$$

where $f_L(x)$ is the probability density function for demand over the lead time L. Suppose demand over the lead time has a normal distribution with mean μ and standard deviation σ, and suppose we $R = \mu + z\sigma$, where we call z the safety factor. Then

$$\text{Probability of a stockout} = \int_{x=R}^{x=\infty} f_L(x)\, dx = 1-\Phi(z)$$

where $\Phi(z)$ is the cumulative probability function for a standard normal distribution. Thus, when $z = 1.64$, the Type I service level is 0.05: we expect that the lead time demand will exceed the reorder point in 5% of the order events.

Type II

The service level is defined as the percentage of demand that is not met from inventory, or equivalently, the percentage of demand that is met from inventory (also known as the fill rate).

For a Q R model, the Type II service level can be expressed as:

$$\text{Fraction of demand not met from inventory} = \frac{\int_{x=R}^{x=\infty} (x-R) f_L(x)\, dx}{Q}$$

The numerator is the expected backorders for each order event -- that is , the expected amount of demand that is not met from inventory during an order cycle; the denominator is the expected demand between order events. Thus the ratio is the expected fraction of demand not met from stock.

The fill rate is the fraction of demand that is filled immediately from inventory, and is usually calculated as follows:

$$\text{fill rate} = 1 - \frac{\text{E[backorders per order]}}{Q} = 1 - \frac{\int_{x=R}^{x=\infty} (x-R) f_L(x)\, dx}{Q}$$

The integral for computing the expected backorders is known as the "partial loss function."

Suppose demand over the lead time has a normal distribution with mean μ and standard deviation σ, and suppose we $R = \mu + z\sigma$, where we call z the safety factor. Then, with some modest effort, one can express the partial loss function as:

$$\int_{x=R}^{x=\infty} (x-R) f_L(x)\, dx = \sigma \int_{y=z}^{y=\infty} (y-z)\phi(y)\, dy$$

where $\phi(y)$ is the probability density function for a standard normal variable. The integral

$$\int_{y=z}^{y=\infty} (y-z)\phi(y)\, dy$$

is tabled for the normal distribution; for instance in the Nahmias book, this is called the partial expectation function, denoted by L(z).

For instance, when the safety factor z = 1.64, L(z) = .0211. To compute the Type II service level, we need values for Q as well as for the demand standard deviation. As an example, suppose Q = 100 and σ = 50, then

The fraction of demand not met from inventory = 50 * .0211/100 = .01,
and the fill rate = 0.99.