<u>Economic Order Quantity</u>

- **Consider a department that is purchasing white T-shirts. Demand for the product is quite constant over time. There is a fixed cost of placing an order that corresponds to the buyer's time plus some incidental costs. The store borrows money to buy its merchandise so there is a cost of capital associated with the inventory in the store. How many shirts should the buyer order?**

- **Characteristics of the problem**
  1. **Multiple period problem**
  2. **Demand is constant (can be relaxed).**
  3. **There is an ordering cost incurred each time an order is placed.**
  4. **There is an inventory holding cost for any units on-hand.**
  5. **There is no order lead-time (can be relaxed)**
  6. **Shortages are not permitted (also unnecessary since everything is deterministic).**

- **Examples**
  1. **Commodity products**
  2. **…**

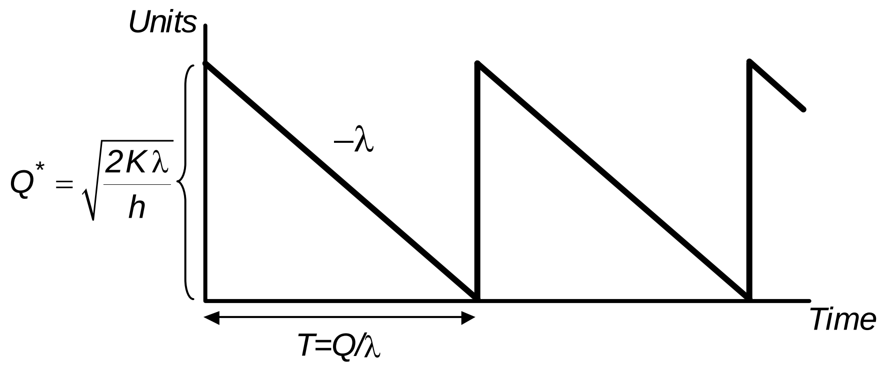
Model Basics

Inputs

- **<u>purchase price</u> – the per unit cost for the vendor to purchase the item**
  - **Let c denote the per unit cost**
- **<u>Holding cost rate</u> – the cost rate at which inventory is charged**
  - **Let I denote the holding cost rate**
- **<u>Order cost</u> – the fixed cost associated with placing an order**
  - **Let K denote the fixed order cost**
- **<u>Demand rate</u> – annual demand rate for the product.**

- **Let λ denote the demand rate**

Graphical Representation



Model Intuition

- **When will we place an order?**
  - **When inventory on hand is zero. Due to the fact that replenishment is instantaneous.**

- **Let Q denote the order quantity. Let Q* denote the optimal order quantity. Why will Q* remain unchanged over time?**
  - **If we place orders when inventory on hand equals 0, then the conditions between orders are exactly the same. Therefore, the ordering pattern should replicate.**

Decision Variables

- **<u>Order quantity</u> – the amount to order at the start of the period. Denoted by Q.**

Derivation of the Optimal Order Quantity

- **Define a cycle as the time between orders. Let T denote the length of a cycle.**

  T = Q/λ

- **Example**

  λ = 1000 units /year
  Q = 250 units

T = (250 units)/(1000 units /year) = 0.25 years

- **The purchasing and order cost per cycle, denoted C(Q), can be computed as follows:**

  C(Q) = K + cQ

  - **There are 1/T orders placed every year. Therefore, the total purchasing and order cost per year is equal to:**

$$\frac{1}{T}(K + cQ)$$

where we can substitute T = Q/λ to get

$$\frac{\lambda}{Q}(K + cQ) = \frac{K\lambda}{Q} + c\lambda$$

- **We now want to determine the inventory holding cost**
  - **The average inventory on hand equals Q/2. The maximum is Q, the minimum is 0, and the rate of consumption is constant.**
  - **Let h = Ic denote the per unit holding cost**
    - **Example**

      I = 0.20 $/$ / year
      c = 60 $/unit

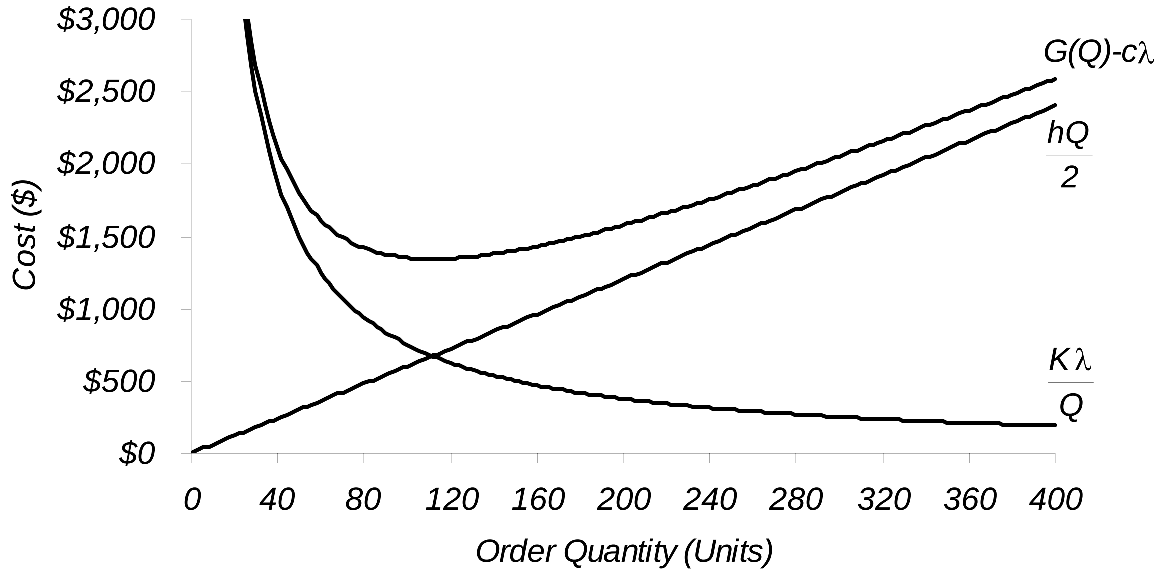
      h = (0.20 $/$ / year)*(60 $/unit) = 12.00 $/unit/year

- **We can now formulate the total annual cost for an order quantity of Q:**

$$G(Q) = \frac{K\lambda}{Q} + c\lambda + \frac{hQ}{2}$$

- **What does this function look like (K = 75 $/order)?**

Order and Inventory Holding Cost as a Function of Order Quantity



- **To find Q*, we take the derivative of G(Q) with respect to Q and set it equal to zero.**

$$Q^* = \sqrt{\frac{2K\lambda}{h}}$$

- **In the example above, Q* = 111.8. The total ordering and holding cost equals $1,341.64.**

Simple Example – Furniture Dealer

- **Sofas sell at a rate of 60 sofas/week and their purchase price is 300 $/unit. The holding cost rate is 25% and it costs $75 to place an order.**

  λ = (60 sofas/week)(52 weeks/year) = 3120 sofa/year

  h = (.25)(300 $/unit) = 75 $/unit

$$Q^* = \sqrt{\frac{(2)(75\ \$/order)(3120\ sofas/year)}{75\ \$/sofa/year}} = 79\ sofas$$

$$T^* = \frac{79\ units}{3120\ units/year} = .025\ years = 9.24\ days$$