Interpretation of the I-Regime and transport associated with relevant heavy particle modes

Interpretation of the I-Regime and Transport Associated with Relevant Heavy Particle Modes

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The excitation of a novel kind of heavy particle [1, 2] mode at the edge of the plasma column is considered as the signature of the I-confinement Regime [3–7]. The outward transport of impurities produced by this mode is in fact consistent with the observed expulsion of them from the main body of the plasma column (a high degree of plasma purity is a necessary feature for fusion burning plasmas capable of approaching ignition). Moreover, the theoretically predicted mode phase velocity, in the direction of the electron diamagnetic velocity, has been confirmed by relevant experimental analyses [8] of the excited fluctuations (around 200 kHz). The plasma "spontaneous rotation" in the direction of the ion diamagnetic velocity is also consistent, according to the Accretion Theory [9] of this phenomenon, with the direction of the mode phase velocity. Another feature of the mode that predicted by the theory is that the I-Regime exhibits a knee of the ion temperature at the edge of the plasma column but not one of the particle density as the mode excitation factor is the relative main ion temperature gradient exceeding the local relative density gradient. The net plasma current density appearing in the saturation stage of the relevant instability, where the induced particle and energy fluxes are drastically reduced, is associated with the significant amplitudes of the poloidal magnetic field fluctuations [6, 7] observed to accompany the density fluctuations. The theoretical implications of the significant electron temperature fluctuations [10] observed are discussed.

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1. INTRODUCTION

The collective modes that are associated with the presence of heavy particle populations in a hydrogenic plasma have been found originally [1] in the context of predicting the transport of impurities that could be observed by the experiments. Recent investigations of the so called I-Regime by the Alcator C-Mod experimental machine have brought to light features of this regime that can be explained by the excitation of a new kind of heavy particle mode at the edge of the plasma column [2]. This mode involves both density and magnetic field fluctuations. The transport of impurities that is produced by it is outward, that is in the direction of their assumed density gradient, while the main plasma ions are transported inward. These features are consistent with a mode having a frequency about 200 kHz that has been found experimentally in the I-Regime and with the observation that, in this regime, impurities are expelled toward the edge of the plasma column [3–7]. These modes have a phase velocity in the direction of the electron diamagnetic velocity, a feature that was found first theoretically [2] and that has been later observed experimentally [8]. The mode is dissipative in that its growth rate depends inversely on the effective finite longitudinal thermal conductivity of the main ion population for which the relative temperature gradient exceeds the relative density gradient. Under this condition, the mode driving factor is a combination of finite impurity temperature and of the main ion temperature gradient. The observed spontaneous rotation is in the direction of the main ion diamagnetic velocity and is assumed, according to the Accretion Theory [9], to originate from a recoil process at the edge of the plasma column associated with ejection of angular momentum in the opposite direction by the considered mode. The electromagnetic fluctuations [6, 7] that are observed experimentally together with the density fluctuations are explained as related to the vanishing of the fluxes of the main ion and impurity populations in the saturated stage of the relevant instability. The assumption made that the effects of the longitudinal (to the magnetic field) thermal conductivity for the heavy particle population are relatively small are crucial for the validity of the presented theory. The surprising observation of significant electron temperature fluctuations [10] is associated with a process [11] that can decrease the longitudinal electron thermal conductivity relative to its collisional value.

This paper is organized as follows. In Section 2 we derive a model dispersion relation referring to a plane geometry simulating relevant confinement configurations. The longitudinal thermal conductivity of the main ion population as well as that of the electrons is considered to be relatively large. In Section 3 a discussion of the model dispersion relation is given relating it to that previously known [1] for impurity driven modes. In Section 4 the newly found mode with frequency relatively close to the impurity acoustic frequency is shown to become unstable as a result of the combined effects of the radial temperature gradient and of the longitudinal thermal conductivity of the main ion population. In Section 5 the considered mode is shown to produce an inward transport of the

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main ion population and an outward transport of the impurity population [12]. The thermal energy of the main ion population is transported outward as well. A relative density gradient of the impurity population toward which this is expected to evolve and for which the mode growth rate vanishes is identified. In Section 6 the geometrical characteristics [13] of the mode that can be excited in a simplified toroidal confinement configuration are described. Then the theory for the density fluctuations of the main ion population is carried out for the relevant collisionless regime. In Section 7 we extend the derivation of the dispersive dispersion relation obtained in Section 2 by deriving the normal mode equation appropriate for the toroidal mode representation [12–14] discussed in Section 6. In Section 8 we evaluate the effects of the finite longitudinal main ion thermal conductivity for the toroidal modes considered earlier and the corresponding effects of the relevant mode particle resonances. In Section 9 we propose an explanation, consistently with the experimental observations, for the magnetic field fluctuations accompanying the density perturbations that have been observed. Thus the limitations of the electrostatic approximation adopted in the previous sections are made evident. In Section 10 we discuss the experimental observations of electron temperature fluctuations in the context of the considered mode. In Section 11 we discuss the processes that can be involved in the onset of the observed spontaneous rotation. In Section 12 the main conclusions are given based on the theory presented in the earlier sections.

2. PLANE GEOMETRY FORMULATION

In order to identify the main characteristics of the modes of interest we consider a plane plasma configuration where the equilibrium magnetic field $\mathbf{B} \simeq B\,\mathbf{e_z}$ and all quantities depend on x in the unperturbed state. The plasma components are the electrons with density $n_e(x)$, the main ion population with density $n_i(x)$ and $Z = 1$ and the impurity population with density $n_I(x)$ and mass number $A_I \gg 1$. Clearly $n_e = n_i + Zn_I$. We use standard symbols and limit our analysis to electrostatic modes represented by $\hat{\mathbf{E}} \simeq -\nabla\hat{\phi}$.

The simplest model of the modes that we shall consider for a toroidal configuration is of the standing type along the magnetic field and propagating across it,

$$\hat{\phi} = \tilde{\phi}(x)\sin(k_\shortparallel z)\exp(-i\omega t + ik_y y)\,. \qquad (2.1)$$

Since the expected dispersion relation depends on $k_\shortparallel^2$, we may consider, instead of standing modes represented by Eq. (2.1), propagating modes along the magnetic field, for which

$$\hat{\phi} = \tilde{\phi}(x)\exp(-i\omega t + ik_\shortparallel z + ik_y y)\,, \qquad (2.2)$$

where, for the sake of simplicity, we consider $k_y^2 \gg \left|\frac{1}{\tilde{\phi}}\frac{d^2\tilde{\phi}}{dx^2}\right|$. The relevant longitudinal phase velocities are in the range

$$v_{thI}^2 < |\omega/k_\shortparallel|^2 < v_{thi}^2 < v_{the}^2 \qquad (2.3)$$

where $v_{thj}^2 \equiv 2T_j/m_j$, $j = i$, e and I. Under these conditions the key equations that describe the modes of interest are

$$Z\hat{n}_I \simeq \hat{n}_e - \hat{n}_i \qquad (2.4)$$

$$0 \simeq -ik_\shortparallel\hat{n}_e T_e + ik_\shortparallel e n_e\hat{\phi} \qquad (2.5)$$

$$0 \simeq -ik_\shortparallel\hat{n}_i T_i - ik_\shortparallel e n_i\hat{\phi} \qquad (2.6)$$

where $\hat{n}_I, \hat{n}_i$ and $\hat{n}_e$ indicate the density perturbations of the involved particle populations. Moreover,

$$-i\omega\hat{n}_I + \hat{v}_{Ex}\frac{dn_I}{dx} + ik_\shortparallel n_I\hat{u}_{I\shortparallel} \simeq 0, \qquad (2.7)$$

$$\hat{v}_{Ex} = -ik_y c\,\hat{\phi}\Big/B \qquad (2.8)$$

and

$$-i\omega n_I m_I\hat{u}_{I\shortparallel} \simeq -ik_\shortparallel\left(\hat{n}_I T_I + n_I\hat{T}_I\right) - ik_\shortparallel eZn_I\hat{\phi}. \qquad (2.9)$$

The thermal energy balance for the impurity population in the limit of negligible thermal conductivity is

$$\frac{3}{2}n_I\left(-i\omega\hat{T}_I + \hat{v}_{Ex}\frac{dT_I}{dx}\right) + ik_\shortparallel n_I T_I\hat{u}_{I\shortparallel} \simeq 0 \qquad (2.10)$$

where

$$\hat{u}_{I\shortparallel} \simeq \frac{\omega}{k_\shortparallel}\frac{\hat{n}_I}{n_I} + i\frac{\hat{v}_{Ex}}{k_\shortparallel}\frac{1}{n_I}\frac{dn_I}{dx}.$$

Here we assume that $T_e \simeq T_i \simeq T_I$. Moreover, if we consider the impurity density profile to be relatively peaked, that is $\left|\frac{1}{T_I}\frac{dT_I}{dx}\right| \ll \left|\frac{1}{n_I}\frac{dn_I}{dx}\right|$, Eq. (2.10) reduces simply to

$$\frac{\hat{T}_I}{T_I} \simeq \frac{2}{3}\left(\frac{\hat{n}_I}{n_I} + i\frac{\hat{v}_{Ex}}{\omega}\frac{1}{n_I}\frac{dn_I}{dx}\right), \qquad (2.11)$$

or

$$\hat{u}_{I\shortparallel} \simeq \frac{3}{2}\frac{\omega}{k_\shortparallel}\frac{\hat{T}_I}{T_I}; \qquad (2.12)$$

and Eq. (2.9) to

$$\frac{\hat{T}_I}{T_I} \simeq \frac{2}{5}\frac{\omega_{IA}^2}{\omega^2 - 2\omega_{IA}^2/5}\left(\frac{\hat{n}_I}{n_I} + \frac{Ze\hat{\phi}}{T_I}\right) \qquad (2.13)$$

where $\omega_{IA}^2 \equiv 5k_\shortparallel^2 T_I/(3m_I)$. Then Eq. (2.11) and (2.13) give rise to

$$\omega\left(\omega^2 - \omega_{IA}^2\right)\frac{\hat{n}_I}{n_I} + \left[\omega_{**}^I\left(\omega^2 - \frac{2}{5}\omega_{IA}^2\right) - \frac{3}{5}\omega\omega_{IA}^2\right]\frac{Ze\hat{\phi}}{T_I} = 0 \qquad (2.14)$$

where $\omega_{**}^I \equiv (\, ck_y T_I/ZeB\,)\,(\, dn_I/dx\;/n_I\,)$.

By combining Eq. (2.4), (2.5) and (2.6) we have

$$Z\hat{n}_I \simeq e\hat{\phi}\,\bar{\bar{n}}\Big/\bar{\bar{T}} \tag{2.15}$$

where $\bar{\bar{n}}\Big/\bar{\bar{T}} \equiv n_i/T_i + n_e/T_e\,$.

Finally, Eqs. (2.14) and (2.15) lead to the dispersion relation

$$\left(\omega^2 - \omega_{IA}^2\right)(\omega + \omega_{*I}) = \omega_{SI}^2\left(\omega - \omega_{**}^I\right), \tag{2.16}$$

where

$$\omega_{SI}^2 \equiv \frac{3}{5}\omega_{IA}^2\Delta, \quad \Delta \equiv Z^2\frac{n_I}{\bar{\bar{n}}}\frac{\bar{\bar{T}}}{T_I} \quad \text{and} \quad \omega_{*I} \equiv \omega_{**}^I\Delta.$$

3. DISCUSSION OF THE MODEL DISPERSION RELATION

We observe that, for the derivation of dispersion relation (2.16), the thermal conductivity of the main ion population has been considered to be large while that of the impurity population has been considered to be negligibly small.

Now we note that the dispersion relation for the known "impurity" [1] driven modes

$$\omega\,(\omega + \omega_{*I}) - \omega_{SI}^2 = 0 \tag{3.1}$$

does not involve the frequencies ω_{IA}^2 or ω_{**}^I as it is derived in the "hypersonic" limit where $\omega^2 > 2T_I\,k_{\shortparallel}^2/m_I$ and $\omega \sim \omega_{*I} \sim \omega_{SI}$. In this limit we have an "impurity drift" mode

$$\omega \simeq -\omega_{*I} \quad \text{for} \quad \omega_{SI}^2 < \omega_{*I}^2/4\,, \tag{3.2}$$

that has a phase velocity in the direction of the ion diamagnetic velocity v_{di} and an "impurity sound" mode

$$\omega \simeq \omega_{SI} - \omega_{*I}/2 \quad \text{for} \quad \omega_{SI}^2 > \omega_{*I}^2/4\,, \tag{3.3}$$

that has a phase velocity in the opposite (electron diamagnetic) direction. Clearly, the hypersonic condition applied to the root (3.3) requires that $\Delta > 2$.

We observe that an evident solution of Eq. (2.16) is

$$\omega = \omega_{IA} = \omega_{**}^I. \tag{3.4}$$

More generally, if we consider the case where $\Delta < 1$ and a mode with a phase velocity in the direction of the electron diamagnetic velocity ($\omega > 0$ for $k_y > 0$) the approximate solution of Eq. (2.16) is

$$\omega \simeq \omega_{IA} + \frac{3}{10}\Delta\left(\omega_{IA} - \omega_{**}^I\right). \tag{3.5}$$

As will be shown this mode can be driven unstable by the local temperature gradient of the main ion population for $\omega_{**}^I < \omega_{IA}$. In particular, we can argue that (see Section 5) the transport of the impurity population toward the edge of the plasma column raises the value of $dn_I/dx\,/n_I$ until the marginal stability condition (3.4) is reached.

Finally, in order to analyze the cubic equation (2.16) in more detail we define $\bar{\omega} \equiv \omega/\omega_{SI}$ and $U \equiv \omega_{*I}/\omega_{SI}$ and rewrite it as

$$f\,(\bar{\omega}, U) \equiv \left[\bar{\omega}^2 - 5/(3\Delta)\right](\bar{\omega} + U) - (\bar{\omega} - U/\Delta\,) = 0. \tag{3.6}$$

Obviously, this dispersion function reduces to that in Eq. (3.1) when $\Delta \gg 1$. We observe that there are three roots of Eq. (2.16) separated by one local maximum and one minimum of $f\,(\bar{\omega}, U)$. Two of the roots are negative, and the other is positive as shown in Fig. 1. The positive root corresponds to the mode in the direction of the electron diamagnetic velocity represented by Eq. (3.5) in the relevant asymptotic limit. Fig. 2 shows how the new root $\bar{\omega}_2$ emerges and how the two roots $\bar{\omega}_1$ and $\bar{\omega}_3$ continue when Δ varies from above 1 to below it.

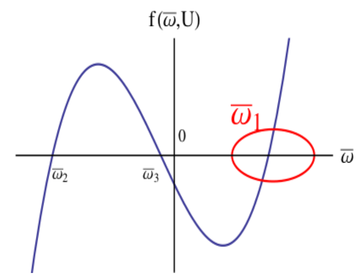

FIG. 1: Graphic representation of the solution of Eq. (3.6) for $\Delta = 0.3$ and $U = 1$.

4. "DISSIPATIVE" EFFECTS

The analysis of the data concerning the I-Regime of which a sample is given in Appendix indicates that the main ion population has to be treated as collisionless. Considering a plane geometry derivation for by the relevant mode-particle resonance ($\omega = k_{\shortparallel}v_{\shortparallel}$) can be evaluated easily. On the other hand, a good insight on the

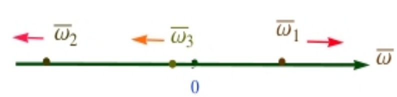

FIG. 2: The arrows indicate how roots move when Δ varies from $\Delta \gg 1$ to $\Delta < 1$.

effect of this can be given by considering the effect of finite longitudinal thermal conductivity in the limit where $D^{th}_{\shortparallel i} k^2_{\shortparallel} > |\omega|$, $D^{th}_{\shortparallel i}$ being the main ion longitudinal thermal diffusion coefficient. Then, considering for simplicity the case where the thermal energy balance equation for the main ion population reduces to

$$\hat{v}_{Ex}\frac{dT_i}{dx}\left(1-\frac{2}{3\eta_i}\right)\simeq -k^2_{\shortparallel} D^{th}_{\shortparallel i}\hat{T}_i, \tag{4.1}$$

where $\eta_i = (d\ln T_i/dx)/(d\ln n_i/dx)$, and

$$\frac{\hat{n}_i}{n_i}\simeq -\frac{e\hat{\phi}}{T_i}-\frac{\hat{T}_i}{T_i}\simeq -\frac{e\hat{\phi}}{T_i}\left(1+i\varepsilon_i'\right) \tag{4.2}$$

where $\varepsilon_i' \equiv \left[\omega^T_{*i}/\left(k^2_{\shortparallel} D^{th}_{\shortparallel i}\right)\right]\left[1-2/(3\eta_i)\right]$ and $\omega^T_{*i} \equiv (ck_y/eB)(dT_i/dx)$. Then the quasi-neutrality condition becomes

$$Z\hat{n}_I \simeq \frac{e\hat{\phi}}{\bar{\bar{T}}}\bar{\bar{n}}\left(1+i\varepsilon_i\right) \tag{4.3}$$

where $\varepsilon_i \equiv \left(n_i\bar{\bar{T}}\big/\bar{\bar{n}}T_i\right)\varepsilon_i'$. Consequently, the dispersion relation becomes

$$\left(\omega^2-\omega^2_{IA}\right)\left[\omega\left(1+i\varepsilon_i\right)+\omega_{*I}\right]=\frac{3}{5}\left(\omega-\omega^I_{**}\right)\omega^2_{IA}\Delta. \tag{4.4}$$

We consider, for instance, $\Delta < 1$ and take $\omega = \omega_{IA} + \delta\omega$. Then we have

$$\delta\omega\left[\omega_{IA}\left(1+i\varepsilon_i\right)+\omega_{*I}\right]\simeq\frac{3\Delta}{10}\left(\omega_{IA}-\omega^I_{**}\right)\omega_{IA} \tag{4.5}$$

that is

$$\mathrm{Im}\delta\omega \simeq -\frac{3}{5}\frac{\varepsilon_i\Delta}{\left(1+\frac{\omega_{*I}}{\omega_{IA}}\right)^2+\varepsilon_i^2}\left(\omega_{IA}-\omega^I_{**}\right). \tag{4.6}$$

Therefore, for $dT_i/dx < 0$, $\eta_i > \frac{2}{3}$ and $k_y > 0$ (phase velocity in the electron diamagnetic velocity direction) the dispersion equation gives a positive growth rate when $\omega_{IA} > \omega^I_{**}$.

The longitudinal thermal conductivity of the impurity population, when included in the theory, introduces a damping for the considered mode. In particular, if consider the limit where $\omega \simeq \omega_{IA} + \delta\omega$ and $D^{th}_{\shortparallel I}k^2_{\shortparallel}/\omega_{IA} < 1$, $D^{th}_{\shortparallel I}$ being the longitudinal thermal diffusion coefficient for the impurity population, we obtain the damping

$$\mathrm{Im}\delta\omega_D \simeq -\frac{2}{15}k^2_{\shortparallel}D^{th}_{\shortparallel I}. \tag{4.7}$$

Finally, we note that considering collisionless regimes instead of Eq. (4.2) we obtain, for the same plane one dimensional model,

$$\frac{\hat{n}_i}{n_i}\simeq -\frac{e\hat{\phi}}{T_i}\left[1+i\sqrt{\pi}\frac{\omega^T_{*i}}{|k_{\shortparallel}|\,v_{thi}}\left(\frac{1}{2}-\frac{1}{\eta_i}\right)\right]. \tag{4.8}$$

We observe that in this case the one-dimensional nature of the mode-particle resonance leads to increase the critical value of η_i by a factor 3 relative to that found with the finite thermal conductivity model as shown by Eq. (4.2).

5. QUASI-LINEAR TRANSPORT

The directions of the particle flows produced by the considered mode can be estimated by the quasi-linear approximation that, for the sake of simplicity, we apply to the plane geometry model analyzed in the previous sections. In particular, we indicate conventional averages over y and z by $\langle\ \rangle$ and note that, since $\hat{n}_e$ and $\hat{\phi}$ are in phase,

$$\Gamma_e = \langle\hat{n}_e\hat{v}_{Ex}\rangle = 0 \tag{5.1}$$

that is, there is no net electron flow. Therefore, charge conservation requires that

$$Z\left\langle\hat{n}_I\hat{v}_{Ex}\right\rangle = -\left\langle\hat{n}_i\hat{v}_{Ex}\right\rangle \tag{5.2}$$

and if the transport of the main ion population is inward that of the impurity is outward. In particular, since $\langle\hat{n}_i\hat{v}_{Ex}\rangle = -n_i\left\langle\hat{T}_i\hat{v}_{Ex}\right\rangle\big/T_i$ we obtain

$$\langle\hat{n}_i\hat{v}_{Ex}\rangle \simeq \frac{1}{T_i}\frac{dT_i}{dx}\frac{n_i}{k^2_{\shortparallel}D^{th}_{\shortparallel i}}\left\langle|\hat{v}_{Ex}|^2\right\rangle\left(1-\frac{2}{3\eta_i}\right), \tag{5.3}$$

after using Eq. (4.2). This shows that the main ion transport is inward for $\eta_i > 2/3$, while the impurity transport is outward, a feature consistent with the fact that the impurities are observed to be confined at the edge of the plasma column in the I-Regime. As indicated earlier, we expect that the mode will increase $(dn_I/dx)/n_I$ until the condition (3.5) will be reached.

Clearly, the effective transverse diffusion coefficient, $D^{eff}_{\perp i}$, for the main ion thermal energy that can be inferred from Eq. (5.3) for reasonable estimates (see, for instance, Ref. [15]) of $|\hat{v}_{Ex}|$ will tend to increase as the mode transverse wavelengths increase. Therefore it is important to obtain information on the dependence of the mode wavelengths on the main plasma parameters from the experiments and correlate them with the inferred scaling for the corresponding energy confinement times. Now in spite of the advances made in the investigation of the I-Regime this information is still missing.

Therefore we may consider the condition (3.4) that gives the perpendicular wavenumber $k_\perp$,

$$\frac{k_{\shortparallel}}{k_\perp}\left(\frac{5T_I}{3m_I}\right)^{1/2}\simeq\frac{cT_I}{ZeB}\frac{1}{n_I}\frac{dn_I}{dx}, \tag{5.4}$$

and, since $k_{\shortparallel} \simeq l_0/(qR_0)$ (see Section 6), we argue that the perpendicular wavelength $\lambda_\perp = 2\pi/k_\perp$ has the most significant and favorable dependence on the magnetic field. Then we may suggest that $D^{eff}_{\perp i}$ should depend on the other plasma parameters mostly through their relationship to the mode mode growth rate $\mathrm{Im}\delta\omega$.

6. TOROIDAL MODES

We refer to the simplest model of a toroidal configuration having magnetic surfaces with circular cross sections and large aspect ratios ($\varepsilon_0 \equiv r/R_0 \ll 1$) represented by

$$\mathbf{B} \simeq \frac{1}{1+\varepsilon_0 \cos\theta}\left[B_\zeta(r)\,\mathbf{e}_\zeta + B_\theta(r)\,\mathbf{e}_\theta\right]. \qquad (6.1)$$

The electrostatic modes that are radially localized around a given magnetic surface $r = r_0$ and can be excited in this configuration, where magnetic shear is present, are described by the potential

$$\hat{\phi} \simeq \tilde{\phi}(r_0,\theta)\exp\left\{-i\omega t + in^0\left[\zeta - q(r)\theta\right] + in^0\left[q(r) - q_0^0\right]F(\theta)\right\}. \qquad (6.2)$$

Here $q(r) = rB_\zeta/(RB_\theta)$ is the unwinding parameter, n^0 is the toroidal mode number and $\tilde{\phi}(r_0,\theta)$ is a periodic function of θ with

$$\left|\frac{1}{\tilde{\phi}}\frac{d\tilde{\phi}}{d\theta}\right| \ll n^0 q_0^0,$$

$q_0^0 = m^0/n^0 = q(r = r_0)$, $m^0 \gg 1$ is an integer and $r = r_0$ is a rational surface. Moreover, $F(\theta)$ is an odd function of θ that is vanishing for $-\pi < \theta < \pi$, with $F(\theta = \pm\pi) = \pm 1$ and ensures that $\hat{\phi}$ is a periodic function of θ while $q(r) \neq q_0^0$. In particular, the conditions $\tilde{\phi} = 0$ and $d\tilde{\phi}\big/d\theta = 0$ are chosen for $|\theta| = \pi$. In previous literatures this has been referred to as the "disconnected mode approximation" [12–14] and adopted to describe ballooning modes for which $\tilde{\phi}(r_0,\theta)$ is an even function of θ and it has maximum at $\theta = 0$. For the radially localized modes we consider $q(r) \simeq q_0^0\left[1 + \hat{s}(r/r_0 - 1)\right]$ where $\hat{s} \equiv d\ln q/d\ln r$ is the magnetic shear parameter. We note that for $-\pi < \theta < \pi$

$$\mathbf{B}\cdot\nabla\hat{\phi} \simeq \frac{B_\theta}{r_0}\frac{d\tilde{\phi}}{d\theta}\frac{\hat{\phi}}{\tilde{\phi}}. \qquad (6.3)$$

The considered equilibrium distribution for the main ion population is

$$f_i \simeq f_{Mi}\left(1 + \Delta_i\right) \qquad (6.4)$$

where

$$f_{Mi} = \frac{n_i(r)}{\left[2\pi\frac{T_i(r)}{m_i}\right]^{3/2}}\exp\left[-\frac{\mathcal{E}}{T_i(r)}\right], \qquad (6.5)$$

$$\Delta_i \simeq -\frac{v_\zeta}{\Omega_\theta^i}\left[\frac{1}{n_i}\frac{dn_i}{dr} - \frac{1}{T_i}\frac{dT_i}{dr}\left(\frac{3}{2} - \frac{\mathcal{E}}{T_i}\right)\right], \qquad (6.6)$$

$\mathcal{E} = (m_i/2)\left(v_\shortparallel^2 + v_\perp^2\right)$ and $\Omega_\theta^i = eB_\theta/(m_i c)$. Then, following a standard procedure,

$$\hat{f}_i = -\frac{e}{T_i}f_{Mi}\left[\hat{\phi} + i\left(\omega - \omega_{*i}^T\right)\int_\infty^t dt'\hat{\phi}(t')\right] \qquad (6.7)$$

where the integration is taken along unperturbed particle orbits,

$$\omega_{*i}^T(\mathcal{E}) \equiv \omega_*^i\left[1 - \eta_i\left(\frac{3}{2} - \frac{\mathcal{E}}{T_i}\right)\right], \qquad (6.8)$$

$$\omega_*^i \equiv -\frac{n^0}{R_0}\frac{cT_i}{eB_\theta}\frac{1}{n_i}\frac{dn_i}{dr} \qquad (6.9)$$

and

$$\eta_i \equiv \frac{d\ln T_i}{dr}\bigg/\frac{d\ln n_i}{dr}. \qquad (6.10)$$

Modes for which $\tilde{\phi}(r_0,\theta)$ is even or odd in θ have different characteristics. In particular, the average potential

$$\tilde{\phi}^{(0)}(\mathcal{E},\mu) = \oint\frac{dl}{|v_\shortparallel|}\tilde{\phi}(l)\bigg/\oint\frac{dl}{|v_\shortparallel|}, \qquad (6.11)$$

where $dl = qR_0 d\theta$, vanishes when $\tilde{\phi}(r_0,\theta)$ is an odd function of θ. This is the case we consider. Therefore, the relevant modes will not "see" the locally (in θ) unfavorable curvature "seen" by the trapped (main) ions. In particular, we take

$$\tilde{\phi}(r_0,\theta) = \tilde{\phi}_0(r_0)\sin(l_0\theta)\left\{1 - \exp\left[-\frac{(\pi - |\theta|)^2}{\delta_0^2}\right]\right\} \qquad (6.12)$$

where $\delta_0 \ll 1$ and l_0 is an integer, the modes we consider have frequency

$$\omega^2 < l_0^2\bar{\omega}_{ti}^2 \qquad (6.13)$$

where $\bar{\omega}_{ti} = v_{thi}/(qR_0)$ is the average transit frequency for circulating particles. This has the property that $d\tilde{\phi}\big/d\theta = 0$ for $|\theta| = \pi$ where $F(\theta) = \pm 1$.

We observe that an even mode that vanishes at $\theta = \pm\pi$ would be represented, for instance, by

$$\tilde{\phi}(r_0,\theta) = \tilde{\phi}_0(r_0)\left[1 + \cos(l_0\theta)\right] \qquad (6.14)$$

where l_0 is an odd integer. As we can verify [see Eq. (8.4)] this function does not give the expression for $\hat{T}_i$ that is relevant to the theory presented here.

Then, referring to the circulating particle population and adopting the form (6.2) for the considered modes we employ the decomposition

$$\tilde{\phi}(r_0,\theta) = \sum_{|p^0|\geq 1}\tilde{\phi}^{(p^0)}(\mathcal{E},\Lambda,r_0)\exp\left[ip^0\sigma\omega_t t(\theta)\right], \qquad (6.15)$$

where $\Lambda \equiv \mu B_0/\mathcal{E}$, $\omega_t(\mathcal{E},\Lambda) \equiv 2\pi/\tau_t$, $\tau_t = \int_{-\pi}^{\pi} d\theta\Big/\left|\dot{\theta}\right|$ is the transit period for the main ion population, $t(\theta) = \int_0^\theta d\theta\Big/\left|\dot{\theta}\right|$, $\dot{\theta} = v_\shortparallel/(qR_0)$, $v_\shortparallel =$

$\sigma\left[2\mathcal{E}/m_i\right]^{1/2}\left(1-\Lambda B/B_0\right)^{1/2}$ and $\sigma=\mathrm{sgn}\left(v_{\shortparallel}\right)$. Clearly,

$$\tilde{\phi}^{(p^0)}\left(\mathcal{E},\mu\right)=\frac{1}{\tau_t}\int_{-\pi}^{\pi}\frac{d\theta}{\left|\dot{\theta}\right|}\tilde{\phi}\left(r_0,\theta\right)\exp\left[-ip^0\sigma\omega_t t\left(\theta\right)\right]. \tag{6.16}$$

We neglect the relevant magnetic field curvature and gradient drifts and obtain

$$\begin{aligned}\int_{-\infty}^{t}dt'\hat{\phi}\left(t'\right) \;=\; & i\sum_{|p^0|\geq 1}\frac{\tilde{\phi}^{(p^0)}\left(\mathcal{E},\Lambda,r_0\right)}{\omega-p^0\sigma\omega_t}\exp\left[i\left(p^0\sigma\omega_t\right)t\left(\theta\right)\right]\\ & \cdot\exp\left[-i\omega t+in^0\left(\zeta-q\theta\right)\right].\end{aligned}$$

Then, referring to Eq. (6.7),

$$\begin{aligned}\hat{n}_i \;=\; & -\frac{e\hat{\phi}}{T_i}n_i-\frac{e}{T_i}\int d^3\mathbf{v}f_{Mi}\left[\omega-\omega_{*i}\left(1-\frac{3}{2}\eta_i+\frac{\mathcal{E}}{T_i}\eta_i\right)\right]\\ & \cdot\sum_{|p^0|\geq 1}\frac{\tilde{\phi}^{(p^0)}\left(\mathcal{E},\Lambda,r_0\right)}{\omega-p^0\sigma\omega_t}\exp\left[ip^0\sigma\omega_t t\left(\theta\right)\right]\\ & \cdot\exp\left[-i\omega t+in^0\left(\zeta-q\theta\right)\right].\end{aligned} \tag{6.17}$$

Referring to p^0 harmonics such that $\omega<\left|p^0\omega_t\right|$,

$$\frac{1}{\omega-p^0\sigma\omega_t}\simeq-\left[\frac{1}{p^0\sigma\omega_t}+\frac{\omega}{\left(p^0\sigma\omega_t\right)^2}+i\pi\delta\left(\omega-p^0\sigma\omega_t\right)\right]. \tag{6.18}$$

Thus,

$$\begin{aligned}\hat{n}_i \;\simeq\; & -\frac{e}{T_i}\Bigg\{n_i\hat{\phi}+i\pi\omega_*^i\Bigg[\int d^3\mathbf{v}f_{Mi}\left(1-\frac{3}{2}\eta_i+\frac{\mathcal{E}}{T_i}\eta_i\right)\\ & \cdot\sum_{|p^0|\geq 1}\tilde{\phi}^{(p^0)}\left(\mathcal{E},\Lambda,r_0\right)\exp\left[i\left(p^0\sigma\omega_t\right)t\left(\theta\right)\right]\\ & \cdot\delta\left(\omega-p^0\sigma\omega_t\right)\exp\left[-i\omega t+in^0\left(\zeta-q\theta\right)\right]\Bigg]\Bigg\}.\end{aligned} \tag{6.19}$$

For the analysis that follows (Section 8) it is convenient to evaluate the quadratic form

$$\begin{aligned}\int_{-\pi}^{\pi}d\theta\hat{\phi}^*\hat{n}_i \;\simeq\; & -\frac{en_i}{T_i}\Bigg\{I_0+i\sqrt{\pi}^3\omega_*^i\sum_{\sigma}\int_0^{1-\varepsilon_0}d\Lambda L\left(\Lambda,\varepsilon_0\right)\\ & \cdot\int_0^{\infty}\frac{d\mathcal{E}}{T_i}\sqrt{\frac{\mathcal{E}}{T_i}}\exp\left(-\frac{\mathcal{E}}{T_i}\right)\left[1-\left(\frac{3}{2}-\frac{\mathcal{E}}{T_i}\right)\eta_i\right]\\ & \cdot\sum_{|p^0|\geq 1}\left|\tilde{\phi}^{(p^0)}\left(\mathcal{E},\Lambda,r_0\right)\right|^2\delta\left(\omega-p^0\sigma\omega_t\right)\Bigg\}\end{aligned} \tag{6.20}$$

where $L\left(\Lambda,\varepsilon_0\right)\equiv\frac{1}{2\pi}\int_{-\pi}^{\pi}d\theta\left[1-\Lambda/(1+\varepsilon_0\cos\theta)\right]^{-1/2}$ and $I_0\equiv\int_{-\pi}^{\pi}d\theta\left|\tilde{\phi}\left(r_0,\theta\right)\right|^2=\pi\left|\tilde{\phi}_0\left(r_0\right)\right|^2$ for the eigenfunction given in Eq. (6.12). Finally, we observe that given the sign chosen in Eq. (6.2), the wave number used for the plane one-dimensional model k_y corresponds to $-m^0/r$. Therefore, the phase velocity of a mode is $\omega R_0/n^0$ in the toroidal direction and $-\omega r_0/m^0$ in the poloidal direction.

7. SIMPLEST TOROIDAL DISPERSION RELATION

The simplest theoretical formulation of the modes of interest for a toroidal geometry can be based on the following equations

$$\hat{n}_e\simeq\frac{e\hat{\phi}}{T_e}n_e, \tag{7.1}$$

$$\hat{n}_i\simeq-\frac{e\hat{\phi}}{T_i}n_i, \tag{7.2}$$

$$Z\hat{n}_I=\hat{n}_e-\hat{n}_i, \tag{7.3}$$

$$\hat{n}_I\simeq-\frac{\hat{V}_E^r}{\omega}\frac{dn_I}{dr}+\frac{n_I}{i\omega}\nabla_{\shortparallel}\hat{u}_{I\shortparallel}, \tag{7.4}$$

where

$$\hat{V}_E^r\equiv-\frac{m^0}{r_0}\frac{c}{B}\hat{\phi}, \tag{7.5}$$

$$\nabla_{\shortparallel}\hat{u}_{I\shortparallel}\simeq\left(\frac{1}{qR_0}\frac{1}{\tilde{u}_{I\shortparallel}}\frac{d\tilde{u}_{I\shortparallel}}{d\theta}\right)\hat{u}_{I\shortparallel}. \tag{7.6}$$

Moreover,

$$n_I\hat{u}_{I\shortparallel}\simeq\frac{1}{i\omega m_I}\nabla_{\shortparallel}\left[\hat{p}_I+Zen_I\hat{\phi}\right], \tag{7.7}$$

$$\hat{p}_I=\hat{n}_IT_I+n_I\hat{T}_I\simeq\frac{5}{3}T_I\hat{n}_I+\frac{2}{3}T_I\frac{\hat{V}_E^r}{\omega}\frac{dn_I}{dr}, \tag{7.8}$$

where we have considered

$$\left|\frac{1}{n_I}\frac{dn_I}{dr}\right|\gg\left|\frac{1}{T_I}\frac{dT_I}{dr}\right|, \tag{7.9}$$

$T_I\sim T_i\sim T_e$, $\omega\gg D_{\shortparallel I}^{th}\nabla_{\shortparallel}^2$ that corresponds to neglecting the longitudinal ion thermal conductivity for the impurity population and

$$\nabla_{\shortparallel}^2\simeq\frac{1}{\left(qR_0\right)^2}\frac{1}{\tilde{\phi}}\frac{d^2\tilde{\phi}}{d\theta^2}. \tag{7.10}$$

In particular, we retrace the steps that lead to the dispersion relation (2.16) and we obtain

$$\left(\omega+\omega_{*I}\right)\left(\omega^2+\bar{\omega}_{IA}^2\frac{d^2}{d\theta^2}\right)\tilde{\phi}+\bar{\omega}_{SI}^2\left(\omega-\omega_{**}^I\right)\frac{d^2\tilde{\phi}}{d\theta^2}=0, \tag{7.11}$$

where $\bar{\omega}_{IA}^2\equiv(5/3)\,T_I\big/\left(m_Iq^2{R_0}^2\right)$ and $\bar{\omega}_{SI}^2\equiv\left(Z^2n_I/\bar{\bar{n}}\right)\bar{\bar{T}}\Big/\left(m_Iq^2{R_0}^2\right)$. Therefore, we may consider the appropriate solution to be of the form represented in

Eq. (6.12), where we take $l_0 \geq 9$ to ensure that $\omega^2 < l_0^2 \bar{\omega}_{ti}^2$ referring to the parameters quoted in Appendix and to the experiments carried out by the Alcator C-Mod machine. In particular, for $R_0 \simeq 68$ cm, $T_i \simeq 600\,\mathrm{eV}$ and $q \simeq 2.5$ we have $\bar{\omega}_{ti} \simeq 1.4 \times 10^5\,\mathrm{Hz} \simeq 8.9 \times 10^5\,\mathrm{rad \cdot sec^{-1}}$. Therefore we may take $l_0 \geq 9$. The relevant dispersion relation is

$$\left(\omega^2 - \bar{\bar{\omega}}_{IA}^2\right)(\omega + \omega_{*I}) = \left(\omega - \omega_{**}^I\right)\bar{\bar{\omega}}_{SI}^2, \qquad (7.12)$$

where $\bar{\bar{\omega}}_{IA}^2 \equiv \bar{\omega}_{IA}^2 l_0^2$, and $\bar{\bar{\omega}}_{SI}^2 \equiv \bar{\omega}_{SI}^2 l_0^2$, and we have returned to consider a dispersion equation that is very close to that obtained for the simplest plane geometry model.

8. MODE-PARTICLE RESONANCE EFFECTS

The considered mode can acquire a growth rate by a resonant interaction with the main ion population represented by

$$\omega = p^0 \sigma \omega_t (\mathcal{E}, \mu) = p^0 \sigma \, \frac{2\pi}{qR_0} \Bigg/ \int_{-\pi}^{\pi} \frac{d\theta}{|v_{\shortparallel}|} \qquad (8.1)$$

with $p^0 \sigma > 0$. We note that the effect of the mode-particle resonance (8.1) can be simulated by a finite longitudinal thermal conductivity as was done for the plane geometry model if odd parity modes are considered. In particular, for $|\omega| < l_0 \bar{\omega}_{ti}$

$$\frac{\hat{n}_i}{n_i} \simeq -\frac{e\hat{\phi}}{T_i} - \frac{\hat{T}_i}{T_i} \qquad (8.2)$$

where

$$\left\{ \begin{array}{c} \hat{n}_i \\ \hat{\phi} \\ \hat{T}_i \end{array} \right\} = \left\{ \begin{array}{c} \tilde{n}_i(\theta) \\ \tilde{\phi}(\theta) \\ \tilde{T}_i(\theta) \end{array} \right\} \exp\left\{-i\omega t + in^0\left[\zeta - q(r)\theta\right] + in^0\left[q(r) - q_0^0\right]F(\theta)\right\}, \qquad (8.3)$$

$$-i\tilde{V}_E^r \left(1 - \frac{2}{3\eta_i}\right)\frac{dT_i}{dr} \simeq \frac{D_{\shortparallel i}^{th}}{(qR_0)^2}\frac{d^2}{d\theta^2}\tilde{T}_i(\theta) \qquad (8.4)$$

and

$$\tilde{V}_E^r \equiv -\frac{c}{B}\frac{m^0}{r_0}\tilde{\phi}(\theta),$$

where $\tilde{\phi}(\theta)$ is given in Eq. (6.12).

Therefore, by directly integrating Eq. (8.4) we obtain

$$\tilde{T}_i \simeq ie\omega_{*i}^T \frac{(qR_0)^2}{l_0^2 D_{\shortparallel i}^{th}}\left(1 - \frac{2}{3\eta_i}\right)\tilde{\phi}_0(r_0)\sin(l_0\theta) \qquad (8.5)$$

where

$$\omega_{*i}^T \equiv -\frac{m^0}{r_0}\frac{c}{eB}\frac{dT_i}{dr}.$$

It follows from Eqs. (8.2), (8.5) and (6.12) that

$$\frac{1}{n_i}\int_{-\pi}^{\pi} d\theta \tilde{\phi}(\theta)\tilde{n}_i(\theta) \simeq -\frac{eI_0}{T_i}\left[1 + i\omega_{*i}^T \frac{(qR_0)^2}{l_0^2 D_{\shortparallel i}^{th}}\left(1 - \frac{2}{3\eta_i}\right)\right]. \qquad (8.6)$$

Then, by virtue of the quasi-neutrality [Eq. (7.3)]

$$Z\int_{-\pi}^{\pi} d\theta \tilde{n}_I(\theta)\tilde{\phi}(\theta) = \frac{e\bar{\bar{n}}}{\bar{\bar{T}}} I_0 (1 + i\bar{\varepsilon}_i), \qquad (8.7)$$

where

$$\bar{\varepsilon}_i \equiv \frac{n_i}{\bar{\bar{n}}}\frac{\bar{\bar{T}}}{T_i}\omega_{*i}^T \frac{(qR_0)^2}{l_0^2 D_{\shortparallel i}^{th}}\left(1 - \frac{2}{3\eta_i}\right), \qquad (8.8)$$

and instead of Eq. (7.12) we are led to consider

$$\left(\omega^2 - \bar{\bar{\omega}}^2{}_{IA}\right)\left[\omega(1 + i\bar{\varepsilon}_i) + \omega_{*I}\right] = \left(\omega - \omega_{**}^I\right)\bar{\bar{\omega}}_{SI}^2. \qquad (8.9)$$

Clearly, the conclusions on the mode stability reached in Section 4 can be extended to this case.

If we consider the collisionless limit where the mode-particle resonances discussed in Section 6 become important, then the dispersion relation (8.9) can be used with $\bar{\varepsilon}_i$ being replaced by

$$\bar{\bar{\varepsilon}}_i \equiv \frac{\sqrt{\pi}^3 \omega_*^i}{I_0}\sum_\sigma \left(\int_0^{\Lambda_0} d\Lambda + \int_{\Lambda_0}^{1-\varepsilon_0} d\Lambda\right) L(\Lambda, \varepsilon_0) \cdot \int_0^\infty \frac{d\mathcal{E}}{T_i}\sqrt{\frac{\mathcal{E}}{T_i}}\exp\left(-\frac{\mathcal{E}}{T_i}\right)\left[1 - \left(\frac{3}{2} - \frac{\mathcal{E}}{T_i}\right)\eta_i\right] \cdot \sum_{|p^0|\geq 1}\left|\tilde{\phi}^{(p^0)}(\mathcal{E}, \Lambda, r_0)\right|^2 \delta\left(\omega - p^0\sigma\omega_t\right), \qquad (8.10)$$

which can be read off in Eq. (6.20). Here the interval $0 \leq \Lambda < \Lambda_0 < 1 - \varepsilon_0$ refers to well circulating particles for which $\bar{\omega}_{ti} \simeq v_{\shortparallel}/(qR_0)$, $p^0 \simeq \pm l_0$ and $\tilde{\phi}^{(p^0)}(\mathcal{E}, \mu) \simeq \tilde{\phi}^{(\pm l_0)}(\mathcal{E}, \mu)$. Therefore,

$$\bar{\bar{\varepsilon}}_i \simeq \frac{\sqrt{\pi}\omega_*^i}{l_0 \bar{\omega}_{ti}}\left[(\eta_i - 2)\Lambda_0 + (\eta_i - \eta_i^c)(1 - \varepsilon_0 - \Lambda_0)\right] \qquad (8.11)$$

where η_i^c is the threshold for η_i associated with weakly circulating particles. In this context we observe that in the case where even modes are considered that involve resonances with trapped particles [14] we have a contribution represented by

$$\bar{\bar{\varepsilon}}_i^b \propto \frac{\omega_{*i}^T}{\bar{\omega}_{bi}}\frac{\omega^2}{\bar{\omega}_{bi}^2}\left(1 - \frac{2}{3\eta_i}\right), \qquad (8.12)$$

where $\bar{\omega}_{bi}$ is the average bounce frequency of trapped main ions. Clearly, in this case the value of threshold parameter η_c is reduced by a factor 3 relative to that found for $\varepsilon_0 = 0$ and is equal to that found when the effects of finite longitudinal thermal conductivity prevail as shown by Eq. (8.5).

In the rest of this section, we evaluate $\bar{\bar{\varepsilon}}_i$ defined in Eq. (8.10). We note that

$$\delta\left(\omega - p^0\sigma\omega_t\right) = \frac{L\left(\Lambda,\varepsilon_0\right)}{|p^0\sigma|\bar{\omega}_{ti}}\delta\left(\bar{\bar{v}} - \bar{\bar{v}}_{\rm res}\right) \tag{8.13}$$

where $\bar{\bar{v}} \equiv \sqrt{\mathcal{E}/T_i}$ and

$$\bar{\bar{v}}_{\rm res} \equiv \frac{\omega}{p^0\sigma\bar{\omega}_{ti}} L\left(\Lambda,\varepsilon_0\right), \tag{8.14}$$

Eq. (8.10) can be rewritten as

$$\begin{aligned}\bar{\bar{\varepsilon}}_i &= \frac{2\sqrt{\pi}^3}{I_0}\frac{\omega_*^i}{\bar{\omega}_{ti}}\sum_\sigma\int_0^{1-\varepsilon_0} d\Lambda L^2\left(\Lambda,\varepsilon_0\right) \\ &\cdot\int_0^\infty d\bar{\bar{v}}\bar{\bar{v}}^2\exp\left(-\bar{\bar{v}}^2\right)\left[1-\left(\frac{3}{2}-\bar{\bar{v}}^2\right)\eta_i\right] \\ &\cdot\sum_{|p^0|\geq 1}\frac{1}{|p^0|}\left|\tilde{\phi}^{(p^0)}\left(\bar{\bar{v}}^2,\Lambda,r_0\right)\right|^2\delta\left(\bar{\bar{v}}-\bar{\bar{v}}_{\rm res}\right).\end{aligned} \tag{8.15}$$

Upon performing the trivial $\bar{\bar{v}}$-integral, Eq. (8.15) reduces to

$$\begin{aligned}\bar{\bar{\varepsilon}}_i &= \frac{2\sqrt{\pi}^3}{I_0}\frac{\omega_*^i}{\bar{\omega}_{ti}}\sum_\sigma\sum_{|p^0|>1}\frac{1}{|p^0|}\int_0^{1-\varepsilon_0} d\Lambda \\ &\cdot L^2\left(\Lambda,\varepsilon_0\right)\bar{\bar{v}}_{\rm res}^2\exp\left(-\bar{\bar{v}}_{\rm res}^2\right) \\ &\cdot\left(1-\frac{3}{2}\eta_i+\bar{\bar{v}}_{\rm res}^2\eta_i\right)\left|\tilde{\phi}^{(p^0)}\left(\bar{\bar{v}}_{\rm res}^2,\Lambda,r_0\right)\right|^2.\end{aligned} \tag{8.16}$$

Denoting the two types of Λ-integrals in Eq. (8.16) by

$$\begin{aligned}\Pi_1 &\equiv 2\pi\sum_\sigma\sum_{|p^0|>1}\frac{1}{|p^0|}\int_0^{1-\varepsilon_0} d\Lambda L^2\left(\Lambda,\varepsilon_0\right) \\ &\cdot\bar{\bar{v}}_{\rm res}^2\exp\left(-\bar{\bar{v}}_{\rm res}^2\right)\left|\tilde{\phi}^{(p^0)}\left(\bar{\bar{v}}_{\rm res}^2,\Lambda,r_0\right)\right|^2\end{aligned}$$

and

$$\begin{aligned}\Pi_2 &\equiv 2\pi\sum_\sigma\sum_{|p^0|>1}\frac{1}{|p^0|}\int_0^{1-\varepsilon_0} d\Lambda L^2\left(\Lambda,\varepsilon_0\right) \\ &\cdot\bar{\bar{v}}_{\rm res}^4\exp\left(-\bar{\bar{v}}_{\rm res}^2\right)\left|\tilde{\phi}^{(p^0)}\left(\bar{\bar{v}}_{\rm res}^2,\Lambda,r_0\right)\right|^2,\end{aligned}$$

Eq. (8.16) can be cast into the form analogous to Eq. (8.8), i.e.,

$$\bar{\bar{\varepsilon}}_i = -\alpha_i^c\frac{\omega_*^i\eta_i}{\bar{\omega}_{ti}}\left(1-\frac{\eta_i^c}{\eta_i}\right), \tag{8.17}$$

Where

$$\alpha_i^c \equiv \sqrt{\pi}\frac{\Pi_1}{I_0\eta_i^c} \tag{8.18}$$

and

$$\eta_i^c \equiv \left(\frac{3}{2}-\frac{\Pi_2}{\Pi_1}\right)^{-1}. \tag{8.19}$$

The Λ-integrals in $\Pi_{1,2}$ can be evaluated analytically in the large aspect ratio limit in which $\delta \equiv \Lambda\,\varepsilon_0/(1-\Lambda) < 1$. We notice that

$$\dot{\theta} = \sigma\bar{\omega}_{ti}\bar{\bar{v}}\sqrt{1-\frac{\Lambda}{1+\varepsilon_0\cos\theta}} \simeq \sigma\bar{\omega}_{ti}\bar{\bar{v}}\sqrt{1-\Lambda}\sqrt{1+\delta\cos\theta},$$

then

$$\frac{1}{\left|\dot{\theta}\right|} \simeq \frac{1}{\bar{\omega}_{ti}\bar{\bar{v}}\sqrt{1-\Lambda}}\left[1+\frac{3\delta^2}{16}-\frac{\delta}{2}\cos\theta+\frac{3\delta^2}{16}\cos\left(2\theta\right)\right],$$

$$\begin{aligned}t\left(\theta\right) = \int_0^\theta\frac{d\theta}{\left|\dot{\theta}\right|} &\simeq \frac{1}{\bar{\omega}_{ti}\bar{\bar{v}}\sqrt{1-\Lambda}}\left[\left(1+\frac{3\delta^2}{16}\right)\theta\right. \\ &\left.-\frac{\delta}{2}\sin\theta+\frac{3\delta^2}{32}\sin\left(2\theta\right)\right],\end{aligned}$$

$$\omega_t \simeq \bar{\omega}_{ti}\bar{\bar{v}}\sqrt{1-\Lambda}\left(1-\frac{3\delta^2}{16}\right),$$

and

$$\omega_t t\left(\theta\right) \simeq \theta - \frac{\delta}{2}\sin\theta + \frac{3}{32}\delta^2\sin\left(2\theta\right).$$

Thus,

$$\begin{aligned}\exp\left[-ip^0\omega_t t\left(\theta\right)\right] &\simeq \exp\left(-ip^0\theta\right)\cdot\left[1-i\frac{3p^0\delta^2}{32}\sin\left(2\theta\right)\right] \\ &\cdot\left\{1-\left(\frac{p^0\delta}{4}\right)^2\left[1-\cos\left(2\theta\right)\right]+i\frac{p^0\delta}{2}\sin\theta\right\}\end{aligned}$$

for $\left|p^0\right|\delta < 1$. Furthermore, we note that to the order of $(l_0\varepsilon_0)^2$ in the evaluation of $\bar{\bar{\varepsilon}}_i$ the only relevant $\tilde{\phi}^{(p^0)}$ in $\Pi_{1,2}$ are those with $p^0 = \pm\left(l_0, l_0\pm 1\right)$. For the trial function $\tilde{\phi}\left(\theta\right) = \tilde{\phi}_0\sin\left(l_0\theta\right)$, Eq. (6.16) gives

$$\tilde{\phi}^{(l_0)}\left(\bar{v},\Lambda\right) \simeq \sigma\frac{\tilde{\phi}_0}{2i}\left[1-\left(\frac{l_0\delta}{4}\right)^2\right] \tag{8.20}$$

and

$$\tilde{\phi}^{(l_0\pm 1)}\left(\bar{v},\Lambda\right) \simeq \pm\sigma\frac{\tilde{\phi}_0}{2i}\frac{l_0\delta}{4}. \tag{8.21}$$

Other factors in $\Pi_{1,2}$ can also be expanded in orders of δ. In particular,

$$L\left(\Lambda,\varepsilon_0\right) \simeq \frac{1}{\sqrt{1-\Lambda}}\left(1+\frac{3\delta^2}{16}\right), \tag{8.22}$$

$$\bar{\bar{v}}_{\rm res}^2 \simeq \bar{\bar{v}}_{\rm res0}^2\left(1+\frac{3\delta^2}{8}\right), \tag{8.23}$$

and

$$\exp\left(-\bar{\bar{v}}_{\rm res}^2\right) \simeq \exp\left(-\bar{\bar{v}}_{\rm res0}^2\right)\left(1-\frac{3\delta^2}{8}\bar{\bar{v}}_{\rm res0}^2\right) \quad (8.24)$$

where

$$\bar{\bar{v}}_{\rm res0}^2 \equiv \left(\frac{\omega}{p^0\bar{\omega}_{ti}}\right)^2\frac{1}{1-\Lambda}. \quad (8.25)$$

We denote each p^0-th component of $\Pi_{1,2}$ by $\Pi_{1,2}^{(p^0)}$, respectively. All Λ-integrals in $\Pi_{1,2}^{(p^0)}$ can be transformed into $\lambda \equiv \bar{\bar{v}}_{\rm res0}^2$-integrals of which the integrands are products of certain polynomials in λ and $\exp(-\lambda)$. For instance,

$$\begin{aligned}\Pi_1^{(l_0)} \simeq\ & \frac{\pi|\tilde{\phi}_0|^2}{l_0}\int_0^{1-\varepsilon_0} d\Lambda\left(\frac{\omega}{l_0\bar{\omega}_{ti}}\right)^2\frac{1}{(1-\Lambda)^2} \\ & \cdot\left[1-\frac{\delta^2}{8}\left(2l_0^2-6+3\bar{\bar{v}}_{\rm res0}^2\right)\right]\exp\left(-\bar{\bar{v}}_{\rm res0}^2\right) \\ \simeq\ & \frac{\pi|\tilde{\phi}_0|^2}{l_0}\int_b^{\frac{b}{\varepsilon_0}} d\lambda e^{-\lambda}\left[1-\left(l_0^2-6+3\lambda\right)(b-\lambda)^2\frac{\varepsilon_0^2}{8b^2}\right]\end{aligned}$$

where $b \equiv \omega^2/l_0^2\bar{\omega}_{ti}^2$. Likewise,

$$\Pi_2^{(l_0)} \simeq \frac{\pi|\tilde{\phi}_0|^2}{l_0}\int_b^{\frac{b}{\varepsilon_0}} d\lambda\lambda e^{-\lambda}\left[1-\left(l_0^2-9+3\lambda\right)(b-\lambda)^2\frac{\varepsilon_0^2}{8b^2}\right],$$

$$\Pi_1^{(l_0+1)} \simeq \frac{\pi|\tilde{\phi}_0|^2}{l_0+1}\frac{l_0^2\varepsilon_0^2}{16}\int_{b_+}^{\frac{b_+}{\varepsilon_0}} d\lambda\left(1-\frac{\lambda}{b_+}\right)^2 e^{-\lambda}$$

where $b_+ \equiv \omega^2\Big/\left[(l_0+1)^2\bar{\omega}_{ti}^2\right]$,

$$\Pi_2^{(l_0+1)} \simeq \frac{\pi|\tilde{\phi}_0|^2}{l_0+1}\frac{l_0^2\varepsilon_0^2}{16}\int_{b_+}^{\frac{b_+}{\varepsilon_0}} d\lambda\lambda\left(1-\frac{\lambda}{b_+}\right)^2 e^{-\lambda},$$

$$\Pi_1^{(l_0-1)} \simeq \frac{\pi|\tilde{\phi}_0|^2}{l_0-1}\frac{l_0^2\varepsilon_0^2}{16}\int_{b_-}^{\frac{b_-}{\varepsilon_0}} d\lambda\left(1-\frac{\lambda}{b_-}\right)^2 e^{-\lambda}$$

where $b_- \equiv \omega^2\Big/\left[(l_0-1)^2\bar{\omega}_{ti}^2\right]$, and

$$\Pi_2^{(l_0-1)} \simeq \frac{\pi|\tilde{\phi}_0|^2}{l_0-1}\frac{l_0^2\varepsilon_0^2}{16}\int_{b_-}^{\frac{b_-}{\varepsilon_0}} d\lambda\lambda\left(1-\frac{\lambda}{b_-}\right)^2 e^{-\lambda}.$$

Upon performing all the λ-integrals, to the leading orders of b and $(l_0\varepsilon_0/b)^2$ we obtain

$$\begin{aligned}\Pi_1 &= \Pi_1^{(l_0)}+\Pi_1^{(l_0-1)}+\Pi_1^{(l_0+1)} \\ &\simeq \frac{I_0}{l_0}\exp(-b)\left(1-\frac{3}{4}\frac{\varepsilon_0^2}{b^2}\right)\end{aligned} \quad (8.26)$$

and

$$\begin{aligned}\Pi_2 &= \Pi_2^{(l_0)}+\Pi_2^{(l_0-1)}+\Pi_2^{(l_0+1)} \\ &\simeq \frac{I_0}{l_0}\exp(-b)\left[1-\frac{9}{4}\frac{\varepsilon_0^2}{b^2}+b\right]\end{aligned} \quad (8.27)$$

where we have assumed that $l_0\varepsilon_0 < b < 1$.

Thus,

$$\alpha_i^c \simeq \frac{\sqrt{\pi}}{2l_0}\left(1+\frac{9}{4}\frac{\varepsilon_0^2}{b^2}-3b\right) \quad (8.28)$$

and

$$\eta_i^c \simeq 2\left(1-\frac{3\varepsilon_0^2}{b^2}+2b\right). \quad (8.29)$$

We have recovered the critical value for η_i for the plane geometry as the second term in Eq. (8.29) is neglected. The last term in Eq. (8.29) corresponds to the correction proportional to $\omega^2\Big/(k_\shortparallel v_{thi})^2$, which can be included in Eq. (4.8). For the values of ε_0 such that $3\varepsilon_0^2 > 2b^3$ but still compatible with $\varepsilon_0 l_0 < b$, we have $\eta_i^c < 2$.

9. POLOIDAL MAGNETIC FIELD FLUCTUATIONS

In order to justify the observed electromagnetic fluctuations, we note that $\hat{E}_\shortparallel = -ik_\shortparallel\hat{\phi} + i\omega\,\hat{A}_\shortparallel\Big/c$ and up to this point we have neglected $i\omega\,\hat{A}_\shortparallel\Big/c$. The estimated density fluctuations are [16] $\tilde{n}_e/n_e \simeq 10^{-2}\times\alpha_n$, where $\alpha_n \sim 1$, while the estimated poloidal magnetic field fluctuations are [17] $\tilde{B}_\theta \simeq (3-8)\times 10^{-4}\,\mathrm{T}$. Then we may compare

$$\tilde{n}_e/n_e \quad \text{to} \quad |\omega/k_\shortparallel|\,[e/(cT_e)]\int\tilde{B}_\theta dr. \quad (9.1)$$

If we consider $|\omega/k_\shortparallel| \sim v_{thi}$ and $T_e \simeq T_i \simeq 600\,\mathrm{eV}$, we may take $|\omega/k_\shortparallel| \simeq 2\times10^7\,\mathrm{cm/sec}$ in Eq. (9.1). Moreover, for

$$\tilde{A}_\shortparallel \simeq \int\tilde{B}_\theta dr \simeq \bar{\tilde{B}}_\theta\Delta r \quad (9.2)$$

where $\Delta r \sim 1\,\mathrm{cm}$ [16] the comparison (9.1) becomes

$$\frac{\tilde{n}_e}{n_e} \quad \text{to} \quad 1.7\times10^{-3}\left(\frac{|\omega/k_\shortparallel|}{2\times10^7\,\mathrm{cm/sec}}\right)\left(\frac{\bar{\tilde{B}}_\theta}{5\,\mathrm{G}}\right)\left(\frac{\Delta r}{1\,\mathrm{cm}}\right)\left(\frac{600\,\mathrm{eV}}{T_e}\right).$$

Thus the electrostatic approximation used in the previous sections is acceptable but the estimated values of $\tilde{B}_\theta$ have to be justified. Clearly,

$$\hat{A}_\shortparallel \simeq \frac{4\pi\hat{\boldsymbol{J}}_\shortparallel}{ck^2} \simeq -i4\pi\frac{\nabla\cdot\hat{\mathbf{J}}_\perp}{ck^2k_\shortparallel}. \quad (9.3)$$

In particular, $\nabla\cdot\hat{\boldsymbol{J}}_\perp = \nabla\cdot[(n_i + Zn_I - n_e)\,\hat{\boldsymbol{v}}_E] = 0$ at the stage where the linear description of the mode is valid.

We argue that in the saturated state $\hat{n}_e/n_e$ remains $\simeq e\hat{\phi}\Big/T_e$ while the electron radial velocity is $\hat{v}_{Ex}$. Thus $\Gamma_{ex} \equiv \langle \hat{n}_e\hat{v}_{Ex}\rangle = 0$. Consistently with the experimental observation that injected impurities are promptly expelled, the impurity radial flux is considered to become greatly reduced relative to that estimated by quasi-linear theory that is valid at the start of the mode evolution. Consequently, $|\Gamma_{ix}| \equiv \left|\langle \hat{n}_i\hat{v}^i_x\rangle\right| \ll |\langle \hat{n}_i\hat{v}_{Ex}\rangle|$, where $\hat{\boldsymbol{v}}^i$ is the main ion radial velocity in the saturated state. Assuming that $\hat{n}_i$ remains of the form

$$\hat{n}_i \simeq -\frac{e\hat{\phi}}{T_i} n_i\left(1 + i\varepsilon^s_i\right),$$

in the saturated state, starting from the initial stage where $\hat{n}_i \simeq -e\hat{\phi}n_i\left(1 + i\bar{\bar{\varepsilon}}_i\right)$ and $\bar{\bar{\varepsilon}}_i$ is given by Eq. (8.17), we argue that

$$\hat{v}^i_x \simeq \hat{v}_{Ex} + \Delta\hat{v}^i_x$$

where $\Delta\hat{v}^i_x \simeq -i\varepsilon^s_i\hat{v}_{Ex}$. Correspondingly,

$$\hat{v}^I_x \simeq \hat{v}_{Ex} + \Delta\hat{v}^I_x$$

and

$$\Delta\hat{v}^I_x \simeq -i\varepsilon^s_i\frac{n_i\bar{\bar{T}}}{\bar{\bar{n}}T_i}\hat{v}_{Ex}$$

since $Z\hat{n}_I \simeq \hat{n}_e - \hat{n}_i$. Then a current density

$$\begin{aligned}\hat{J}_x &\simeq e\left(n_i\Delta\hat{v}^i_x + Zn_I\Delta\hat{v}^I_x\right)\\ &= -i\varepsilon^s_i e n_i\hat{v}_{Ex}\left(1 + \frac{Zn_I\bar{\bar{T}}}{\bar{\bar{n}}T_i}\right)\\ &\simeq -i\varepsilon^s_i e n_i\hat{v}_{Ex}\end{aligned}$$

has to be considered. We shall take $\nabla\cdot\hat{\boldsymbol{J}}_\perp \simeq \alpha_J\,\partial\hat{J}_x\Big/\partial x \simeq \alpha_J\,\hat{J}_x\Big/\Delta r$ and in view of Eqs (9.2) and (9.3) we obtain, for $d_i^2 \equiv c^2/\omega^2_{pi}$,

$$\begin{aligned}\bar{\bar{B}}_\theta &\simeq i\frac{4\pi\varepsilon^s_i e n_i\tilde{\phi}}{k\Delta r k_\shortparallel\Delta r B}\\ &\simeq i\frac{1}{k\Delta r k_\shortparallel\Delta r}\frac{cT_i}{eB}\frac{B}{d_i^2\Omega_{ci}}\times\varepsilon^s_i\times\alpha_n\times\alpha_J\times 10^{-2}\end{aligned}\tag{9.4}$$

For $k_\shortparallel \simeq l_0/(qR_0) \simeq 2/35\ \mathrm{cm}^{-1}\,(l_0/10)\,(2.5/q)$ $\cdot\,(70\ \mathrm{cm}/R_0)$, $\Delta r \sim 1\,\mathrm{cm}$, $k \sim 2\,\mathrm{cm}^{-1}$, $T_i \simeq 600\,\mathrm{eV}$, $n_i \simeq 1.25\times 10^{14}\,\mathrm{cm}^{-3}$ and $B \simeq 5\times 10^4\,\mathrm{G}$, referring to the relevant experiments, Eq. (9.4) reduces to

$$\begin{aligned}\left|\frac{\bar{\bar{B}}_\theta}{B}\right| &\simeq \frac{1}{2}\times\varepsilon^s_i\times\alpha_n\times\alpha_J\times 10^{-4}\left[\frac{2}{k\Delta r}\right]\left[\frac{2/35}{k_\shortparallel\Delta r}\right]\\ &\cdot\left[\frac{cT_i}{eB}\Big/\left(1.2\times 10^6\ \mathrm{cm}^2/\mathrm{sec}\right)\right]\left[\frac{3\ \mathrm{cm}}{d_i}\right]^2\\ &\cdot\left[\frac{2.4\times 10^8\ \mathrm{rad/sec}}{\Omega_{ci}}\right]\end{aligned}$$

and we can see that the resulting values of $\bar{\bar{B}}_\theta$ are in the range that has been estimated from the experimental observations.

10. ELECTRON TEMPERATURE FLUCTUATIONS

If we include the electron temperature fluctuations in the plasma quasi-neutrality condition, this becomes

$$Z\hat{n}_I = \frac{e\hat{\phi}}{\bar{\bar{T}}}\bar{\bar{n}} - n_e\frac{\hat{T}_{e\shortparallel}}{T_e} + n_i\frac{\hat{T}_{i\shortparallel}}{T_i}.\tag{10.1}$$

It is clear that when considering the ratio of the relevant collisional longitudinal thermal conductivities or of the relevant linearized mode-particle resonances $\hat{T}_{e\shortparallel}/T_e$ can be neglected. On the other hand it is possible that, since other microscopic modes may be present, the longitudinal electron thermal conductivity may be depressed and that electron temperature fluctuations become significant. In fact, recently A. White concluded that electron temperature fluctuations of $1\sim 2$ % were observed in the Alcator C-Mod experiments on the I-Regime [10].

Indicating the mean free paths of ions and electrons by λ_i and λ_e, we note that for the regimes we consider $k_\shortparallel\lambda_i \simeq k_\shortparallel\lambda_e \simeq 4.0\times 10^2\left[k_\shortparallel\big/\left(2/35\ \mathrm{cm}^{-1}\right)\right]\cdot\left[\lambda_i \simeq 7.0\times 10^3\ \mathrm{cm}\,(T_i/600\ \mathrm{eV})^2\right]\cdot\left(10^{14}\mathrm{cm}^{-3}/n_i\right)$ referring to a series of relevant Alcator experiments. Taking this into account we write an effective longitudinal thermal energy balance equation as

$$k^2_\shortparallel D^e_{\shortparallel\mathrm{eff}}\hat{T}_{e\shortparallel} + \hat{v}_{Ex}\frac{dT_e}{dx} + \eta^c_e T_e\nabla_\shortparallel\hat{u}_{e\shortparallel} = 0;\tag{10.2}$$

implying that $D^e_{\shortparallel\mathrm{eff}}$ is due to the effect of microscopic modes that should be present. Then we have

$$\frac{\hat{T}_{e\shortparallel}}{T_e}n_e \simeq \frac{\hat{v}_{Ex}}{k^2_\shortparallel}\frac{n_e}{D^e_{\shortparallel\mathrm{eff}}}\frac{1}{T_e}\frac{dT_e}{dx}\left(1 - \frac{\eta^c_e}{\eta_e}\right)\tag{10.3}$$

where $\eta_e = (d\ln T_e/dx)/(d\ln n_e/dx)$ and η^c_e has to be evaluated on the basis of the processes that determine $D^e_{\shortparallel\mathrm{eff}}$. Consequently, the condition for the instability of the mode we consider will depend on the difference $\left(\hat{T}_{i\shortparallel}\Big/T_i\right)n_i - \left(\hat{T}_{e\shortparallel}\Big/T_e\right)n_e$.

Finally we note that if we refer to regimes for which $k_\shortparallel\lambda_i < 1$, another class of collisional modes can be found with longitudinal phase velocities

$$(k_\shortparallel\lambda_i)\,v_{thi} < \frac{\omega}{k_\shortparallel} < v_{thi}.$$

11. SPONTANEOUS ROTATION

The "spontaneous rotation" of the plasma column observed in the I-Regime is in the same direction as that of the main ion diamagnetic velocity (co-current). The range of this velocity is [6] 20 – 90 km/sec. According to the Accretion Theory [9] of this phenomenon the presence of two processes is required:

i). a process to scatter angular momentum to the wall generating a plasma recoil in the opposite direction, that of the main ion diamagnetic velocity;

ii). an "inflow" process to transport the generated angular momentum from the edge of the plasma column toward the center.

The heavy particle mode that we introduced and that is excited at the edge of the plasma column can provide the scattering of angular momentum in the same direction as that of electron diamagnetic velocity to the wall. For this the formation and launching of plasma blobs may be envisioned.

As for the inward transport process we envision it to be represented by a composite transport equation involving a "diffusion" term and an "inflow" term as described in Ref. 9. During the transient phase, when the rotation velocity profile is peaked at the edge of the plasma column the diffusion term produce inward transport. When the velocity profile is settled and is peaked at the center of the plasma column the diffusion term is counteracted by the inflow term. According to the theory given in Ref. 9 the main inflow is associated with the ion temperature gradient and it is the result of the excitation of modes with $|\omega|^2 \Big/ k_{\shortparallel}^2 \gtrsim v_{thi}^2$.

12. CONCLUSIONS

We have identified a new mode that can be excited in a multi-component plasma with a massive particle (impurity) population, embedded in a relatively strong magnetic field. Among the salient characteristics of the new mode there are: i) its phase velocity that is in the direction of the electron diamagnetic velocity, ii) the outward transport of the heavy particle population connected with the inward transport of the main (light) ion population that it can produce, iii) the driving factors for the mode being the combined effects of the light ion population temperature gradient and of the relevant effective thermal conductivity along the magnetic field and iv) having a frequency that exceeds slightly the acoustic frequency of the heavy particle population. These characteristics and the fact that the mode is shown to produce measurable electromagnetic fluctuations make it a suitable candidate to explain the features of the 200 kHz density fluctuations found experimentally in the so-called I-confinement Regime. In particular, in this regime where the main ions and the electrons are well confined, impurities have been observed to be localized around the edge of the plasma column.

One of the strongest limitations of the theory that has been presented is its formulation for a toroidal confinement configuration having magnetic surfaces of a circular cross section. In fact, a closer comparison with the experiments that would require a more accurate description of the mode poloidal profiles, with the characteristic odd parity that we have identified, will require an extension of the analysis to geometries like that of characterizing the Alcator C-Mod machine that have an X-point.

Moreover, given the promising confinement characteristics of the I-Regime in view of future experiments on fusion burning plasmas, it is worth considering the possibility to influence the onset and the evolution of the "heavy" particle mode that we have investigated. In fact, an experimental effort in this direction is being undertaken by T. Golfinopoulos. The presence of the electron temperature fluctuations associated with the observed density fluctuations will require further investigations on the process by which the effects of finite electron thermal conductivity are substantially stronger than those expected from existing theory.

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Appendix

In this section we give the approximate numerical estimates for a set of parameters that are involved in the theory of the Heavy Particle Mode discussed earlier. These estimates are based on the relevant experimental observations made by the Alcator C-Mod machine.

- Frequency Range

$$f \gtrsim 200 \text{ kHz}, \qquad \omega \gtrsim 1.25 \times 10^6 \ \text{ rad} \cdot \text{sec}^{-1}.$$

- Spontaneous Rotation Velocity [6]

$$u_\phi \simeq 10 - 120 \text{ km} \cdot \text{sec}^{-1},$$

which corresponds to the direction of the main ion diamagnetic velocity.

- Toroidal Mode Number, estimated from the experiments [17]

$$n^0 \simeq 25 \pm 5$$

- Major Radius of the Plasma Column

$$R_0 \simeq 68 \text{ cm}$$

- Mode Localization Radius

$$R_L = R_0 + \Delta R \simeq 88.5 \text{ cm}$$

- Unwinding Parameter at $R \simeq R_L$

$$q(\psi_L) \simeq 2.5$$

- Electron Temperature at $R \simeq R_L$

$$T_e \simeq 600 \text{ eV}$$

- Electron Density at $R \simeq R_L$

$$n_e \simeq 10^{20} \text{ m}^{-3}$$

- Dominant Impurity

$$\text{O}^{+6}$$

- Impurity Parameter

$$\Delta \equiv \frac{Z^2 n_I}{T_I}\left(\frac{n_e}{T_e}+\frac{n_i}{T_i}\right)^{-1} \simeq 0.3$$

for $T_e \simeq T_i \simeq T_I$ and considering $Z_{eff} \simeq 1.5$ at $R \simeq R_L$.

- Poloidal Wavelengths

$$\lambda_\theta = \frac{2\pi}{\langle k_\theta \rangle_\psi} \simeq 3-6 \text{ cm}$$

if the flux surface averaged wavenumber $\langle k_\theta \rangle_\psi \simeq 1-2 \text{ cm}^{-1}$ as indicated by the experiments [8].

- Toroidal Wavelengths

$$\lambda_\phi = \frac{2\pi R_0}{n^0} \simeq (14-21) \text{ cm} \left[\frac{R_0}{68 \text{ cm}}\right]\left[\frac{20-30}{n^0}\right]$$

- Inferred Main Poloidal Mode Number

$$q(\psi_L)\, n^0 \simeq 60 \left[\frac{5q(\psi_L)}{12}\right]\left[\frac{n_0}{25}\right]$$

We note that the flux surface averaged poloidal mode number, $\langle m \rangle_\psi$ can be connected to $\langle k_\theta \rangle_\psi$ by $\langle m \rangle_\psi = \langle k_\theta \rangle_\psi C_\theta/(2\pi)$ where C_θ is the poloidal circumference of the magnetic surface whose outermost radius is R_L. If we take $C_\theta/(2\pi) \simeq 30$ cm, we obtain $\langle m \rangle_\psi \simeq 30-60$ for $\langle k_\theta \rangle_\psi \simeq 1-2 \text{ cm}^{-1}$. The largest value would be consistent with the inferred main poloidal mode number $q(\psi_L)\, n^0$.

- Thermal Velocities

$$v_{thi} \simeq 2.4 \times 10^7 \left[\frac{T_i}{600\,\text{eV}}\right]^{\frac{1}{2}} \text{cm}\cdot\text{sec}^{-1} \qquad \text{deuteron}$$

$$v_{thI} \simeq 8.5 \times 10^6 \left[\frac{T_I}{600\,\text{eV}}\right]^{\frac{1}{2}} \left[\frac{16\,m_p}{m_I}\right] \text{cm}\cdot\text{sec}^{-1} \text{impurity}$$

$$v_{the} \simeq 1.5 \times 10^9 \left[\frac{T_e}{600\,\text{eV}}\right]^{\frac{1}{2}} \text{cm}\cdot\text{sec}^{-1} \qquad \text{electron}$$

- Collisional Frequencies

deuterons

$$\nu_{ii} \simeq 3.5 \times 10^3 \left[\frac{n_i}{10^{14}\,\text{cm}^{-3}}\right]\left[\frac{\ln\Lambda}{15}\right]\left[\frac{600\,\text{eV}}{T_i}\right]^{\frac{3}{2}} \text{sec}^{-1}$$

impurity - deuteron

$$\nu_{Ii} \simeq 1.2 \times 10^4 \left[\frac{\nu_{ii}}{3.5 \times 10^3 \text{ sec}^{-1}}\right]\left[\frac{Z}{6}\right]^2\left[\frac{m_i}{2m_p}\right] \cdot \left[\frac{16\,m_p}{m_I}\right] \text{sec}^{-1}$$

impurity - impurity

$$\nu_{II} \simeq 4.4 \times 10^4 \left[\frac{Z^2 n_I}{10^{14}\,\text{cm}^{-3}}\right]\left[\frac{\ln\Lambda}{15}\right]\left[\frac{600\,\text{eV}}{T_I}\right]^{\frac{3}{2}} \cdot \left[\frac{Z}{6}\right]^2\left[\frac{16\,m_p}{m_I}\right]^{\frac{1}{2}} \text{sec}^{-1}$$

- Mean Free Paths

$$\lambda_{ii} \simeq 6.9 \times 10^3 \left[\frac{v_{thi}}{2.4 \times 10^7\,\text{cm}\cdot\text{sec}^{-1}}\right] \cdot \left[\frac{3.5 \times 10^3\,\text{sec}^{-1}}{\nu_{ii}}\right] \text{cm}$$

$$\lambda_{Ii} \simeq 7.1 \times 10^2 \left[\frac{v_{thI}}{8.5 \times 10^6\,\text{cm}\cdot\text{sec}^{-1}}\right] \cdot \left[\frac{1.2 \times 10^4\,\text{sec}^{-1}}{\nu_{Ii}}\right] \text{cm}$$

$$\lambda_{II} \simeq 1.9 \times 10^2 \left[\frac{v_{thI}}{8.5 \times 10^6\,\text{cm}\cdot\text{sec}^{-1}}\right] \cdot \left[\frac{4.4 \times 10^4\,\text{sec}^{-1}}{\nu_{II}}\right] \text{cm}$$

- Main Ion Collisionality Parameter

$$\frac{qR_0}{l_0\lambda_{ii}} \simeq \frac{1}{410}\left[\frac{10}{l_0}\right]\left[\frac{6.9 \times 10^3\,\text{cm}}{\lambda_{ii}}\right]\left[\frac{q}{2.5}\right]\left[\frac{R_0}{68\,\text{cm}}\right]$$

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