Theory uncertainties for Higgs mass and other searches using jet bins

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Bounds on the Higgs mass from the Tevatron and LHC are determined using exclusive jet bins to maximize sensitivity. Scale variation in exclusive fixed-order predictions underestimates the perturbative uncertainty for these cross sections, due to cancellations between the perturbative corrections leading to large $K$ factors and those that induce logarithmic sensitivity to the jet-bin boundary. To account for this, we propose that scale variation in the fixed-order calculations should be used to determine theory uncertainties for inclusive jet bins, whose differences yield exclusive jet cross sections. This yields a theory correlation matrix for the jet bins such that the additional uncertainty from large logarithms due to the jet boundary cancels when neighboring bins are added. This procedure is tested for $H + 0, 1$ jets, $WW + 0$ jets, and $W + 0, 1, 2$ jets, and found to be generally applicable. For a case where the higher-order resummation of the jet boundary corrections is known, we show that this procedure yields fixed-order uncertainties which are theoretically consistent with those obtained in the resummed calculation.

I. INTRODUCTION

In the search for the Higgs boson at the Tevatron and the Large Hadron Collider (LHC), the data are divided into exclusive jet bins. This is done because the background composition depends on the number of jets in the final state, and the overall sensitivity can be increased significantly by optimizing the analysis for Higgs + 0, 1, and 2 jet signals. The primary example is the $H \rightarrow WW^*$ decay channel, which dominates the current Tevatron exclusion limits around $m_H \approx 130 \text{ GeV}$ being pursued at the LHC [3,4]. The importance of the Higgs + 1 jet channel in $H \rightarrow \tau\tau$ and $H \rightarrow WW^*$ was demonstrated explicitly in Refs. [5,6]. Similarly, for $H \rightarrow \gamma\gamma$, which plays an important role for $m_H \approx 130 \text{ GeV}$, the search sensitivity can be improved by optimizing the analysis for different jet bins [7].

Since the measurements are performed in each jet bin, the perturbative uncertainties in the theoretical predictions must also be evaluated separately for each jet multiplicity [8]. Furthermore, to combine the results in the end, the correlations between the theoretical uncertainties in the different jet bins as well as in the total cross section have to be understood and taken into account.

In the winter 2011 Tevatron analyses of $gg \rightarrow H \rightarrow WW^*$ [2], the perturbative uncertainties in the signal cross section are evaluated using common scale variation for the exclusive jet bins, which yields [8]

$$\frac{\Delta \sigma_{\text{total}}}{\sigma_{\text{total}}} = 66.5\% \times \left(\frac{+5\%}{-9\%}\right) + 28.6\% \times \left(\frac{+24\%}{-22\%}\right)$$

$$+ 4.9\% \times \left(\frac{+78\%}{-41\%}\right) = \left(\frac{+14\%}{-14\%}\right).$$

(1)

The three terms are the contributions from the 0, 1, and ( $\geq 2$)-jet bins with their relative scale uncertainties in brackets. By using a common scale variation the uncertainties are effectively 100% correlated and are added linearly, such that the $\pm 14\%$ scale uncertainty in the total cross section is reproduced.

For the 0-jet bin, which is the most sensitive search channel in $H \rightarrow WW^*$, one applies a strong veto on additional jets. It is often argued that with jet veto the perturbative uncertainties improve, yielding scale uncertainties from fixed-order perturbation theory that are smaller than those in the total cross section, as seen in Eq. (1). This apparent improvement arises from cancellations between two sources, large corrections to the total cross section (large $K$ factors) and the large corrections from logarithmic dependence on the jet veto. Since the improvement arises from a cancellation between two large and predominantly independent perturbative series, it must be assessed carefully.

We propose a simple procedure to estimate more realistic perturbative uncertainties for exclusive jet bins from fixed-order perturbation theory. The method is designed for processes with large $K$ factors or large perturbative corrections in inclusive cross sections and takes into account the structure of the various perturbative series. As we will see, it can also be applied in general. The essential idea is to first independently determine the uncertainties in the inclusive $N$-jet cross sections $\sigma_{\geq N}$, and then use them to compute the uncertainty in the exclusive $N$-jet cross section $\sigma_N$ from the difference

$$\sigma_N = \sigma_{\geq N} - \sigma_{\geq N+1}. \quad (2)$$

To a first approximation the perturbative series for $\sigma_{\geq N}$ can be considered unrelated for different $N$. For instance, their series start at different orders in $\alpha_s$, and there is a priori no direct relation between the modifications to the series caused by the jet cuts that define these two inclusive
samples. Therefore, as explained in detail in Sec. II, we can work in the limit where the fixed-order perturbative uncertainties in the $\sigma_{\geq N}$'s can be taken as uncorrelated, leading to
\begin{equation}
\Delta^2_N = \Delta^2_{\geq N} + \Delta^2_{\geq N+1}.
\end{equation}

The uncertainty in the exclusive cross section is larger than that in the corresponding inclusive one, which accounts for its more complicated perturbative structure. Equation (2) also leads to an anticorrelation between the cross sections in neighboring jet bins. When neighboring bins are added the sensitivity to the boundary between them cancels and the uncertainty reduces accordingly.

For example, for the 0-jet bin in $H \to WW$ discussed above, we have $\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}$. Here, $\sigma_{\geq 1}$ contains double logarithms of the jet $p_T$ cut, whereas $\sigma_{\text{total}}$ does not involve any jet definition, so their perturbative series can be considered largely independent. Therefore, taking their perturbative uncertainties $\Delta_{\text{total}}$ and $\Delta_{\geq 1}$ as uncorrelated, the covariance matrix for $\{\sigma_0, \sigma_{\geq 1}\}$ is
\begin{equation}
\begin{pmatrix}
\Delta_{\text{total}}^2 & -\Delta_{\geq 1}^2 \\
-\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2
\end{pmatrix}.
\end{equation}

Using this matrix to compute the uncertainty in $\sigma_0 + \sigma_{\geq 1}$ reproduces $\Delta_{\text{total}}$ as it should.

We should mention that we are only discussing here the uncertainties due to unknown higher-order perturbative corrections, which are commonly estimated using scale variations. We do not discuss parametric uncertainties, such as parton distribution function (PDF) and $\alpha_s$ uncertainties, which have been extensively discussed, recently, for example, in Refs. [9–16].

In the next section we present the arguments leading to our proposal for evaluating the perturbative uncertainties for exclusive jet bins, and discuss the structure of the perturbative series. In Sec. III, we apply our method to a variety of processes. We start in Secs. III A and III B with discussion and numerical results for $gg \to H + 0$ jets and $gg \to H + 1$ jets. In Sec. III C, we consider $pp \to WW + 0$ jets, which is an important background for Higgs production. In Secs. III D, III E, and III F we consider $W + 0, 1, 2$ jets, which are important backgrounds for missing-energy searches. In Sec. IV, we consider again $gg \to H + 0$ jets and test our method for the fixed-order uncertainties against a case where the resummation of the large logarithms induced by the jet binning is known to next-to-next-to-leading logarithmic (NNLL) accuracy. We conclude in Sec. V. In the Appendix, we give expressions for the uncertainties and correlations for the case where one considers 0, 1, and ($\geq 2$)-jet bins as in Eq. (1).

II. JET-BIN UNCERTAINTIES

To examine in more detail the modification of the perturbative series that takes place for exclusive jet bins, we will consider the example of the 0-jet bin and ($\geq 1$)-jet bin. The total cross section, $\sigma_{\text{total}}$, is divided into a 0-jet exclusive cross section, $\sigma_0(p_{\text{cut}})$, and the ($\geq 1$)-jet inclusive cross section, $\sigma_{\geq 1}(p_{\text{cut}})$,
\begin{equation}
\sigma_{\text{total}} = \int_0^{p_{\text{cut}}} dp \frac{d\sigma}{dp} + \int_{p_{\text{cut}}}^{Z_{\text{cut}}} dp \frac{d\sigma}{dp} \\
= \sigma_0(p_{\text{cut}}) + \sigma_{\geq 1}(p_{\text{cut}}).
\end{equation}

Here, $p$ denotes the kinematic variable which is used to divide the cross section into jet bins. For most of our analysis we take $p = p_T^{\text{jet}}$, which for Eq. (5) is the largest $p_T$ of any jet in the event. In this case, $\sigma_0(p_{\text{cut}})$ only contains events with jets having $p_T \leq p_{\text{cut}}^{\text{jet}}$, and $\sigma_{\geq 1}(p_{\text{cut}})$ contains events with at least one jet with $p_T \geq p_{\text{cut}}^{\text{jet}}$.

In Eq. (5) both $\sigma_0$ and $\sigma_{\geq 1}$ depend on the phase space cut, $p_{\text{cut}}$, and by construction this dependence cancels in their sum. This means that the additional perturbative uncertainty induced by this cut, call it $\Delta_{\text{cut}}$, must be 100% anticorrelated between $\sigma_0(p_{\text{cut}})$ and $\sigma_{\geq 1}(p_{\text{cut}})$. That is, the contribution of $\Delta_{\text{cut}}$ to the covariance matrix for $\{\sigma_0, \sigma_{\geq 1}\}$ must be of the form
\begin{equation}
C_{\text{cut}} = \begin{pmatrix}
\Delta_{\text{cut}}^2 & -\Delta_{\text{cut}}^2 \\
-\Delta_{\text{cut}}^2 & \Delta_{\text{cut}}^2
\end{pmatrix}.
\end{equation}

The questions then are: (1) how can we estimate $\Delta_{\text{cut}}$, and (2) how is the overall perturbative uncertainty $\Delta_{\text{total}}$ of $\sigma_{\text{total}}$ related to the uncertainty for $\sigma_0$ and $\sigma_{\geq 1}$.

To answer these questions, we discuss the perturbative structure of the cross sections in more detail. By restricting the cross section to the 0-jet region, one restricts the collinear initial-state radiation from the colliding hard partons as well as the overall soft radiation in the event. This restriction on additional emissions leads to the appearance of Sudakov double logarithms of the form $L^2 = \ln^2(p_{\text{cut}}/Q)$ at each order in a perturbative expansion in the strong coupling constant $\alpha_s$, where $Q$ is the hard scale of the process. For Higgs production from gluon fusion, $Q = m_H$, and the leading double logarithms appearing at $O(\alpha_s^3)$ are
\begin{equation}
\sigma_0(p_{\text{cut}}) = \sigma_B \left( 1 - \frac{3\alpha_s}{\pi} \ln^2 \frac{p_{\text{cut}}}{m_H} + \cdots \right),
\end{equation}

where $\sigma_B$ is the Born (tree-level) cross section.

The total cross section just depends on the hard scale $Q$, which means by choosing the scale $\mu \approx Q$, the fixed-order
expansion does not contain large logarithms and has the structure

$$\sigma_{\text{total}} \approx \sigma_B [1 + \alpha_s + \alpha_s^2 + \mathcal{O}(\alpha_s^3)].$$  \hspace{1cm} (8)

The coefficients of this series can be large, corresponding to the well-known large $K$ factors. For instance, the cross section for $gg \rightarrow H$ doubles from leading order to next-to-leading order (NLO) even though $\alpha_s \sim 0.1$. As usual, varying the scale in $\alpha_s$ (and the PDFs) one obtains an estimate of the size of the missing higher-order terms in this series, corresponding to $\Delta_{\text{total}}$.

The inclusive 1-jet cross section has the perturbative structure

$$\sigma_{\geq 1}(p^{\text{cut}}) \approx \sigma_B [\alpha_s (L^2 + L + 1) + \alpha_s^2 (L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)],$$  \hspace{1cm} (9)

where the logarithms $L = \ln(p^{\text{cut}}/Q)$ arise from cutting off the IR divergences in the real emission diagrams. For $p^{\text{cut}} \ll Q$ the logarithms can get large enough to overcome the $\alpha_s$ suppression. In the limit $\alpha_s L^2 \approx 1$, the fixed-order perturbative expansion breaks down and the logarithmic terms must be resummed to all orders in $\alpha_s$ to obtain a meaningful result. For typical experimental values of $p^{\text{cut}}$ fixed-order perturbation theory can still be considered, but the logarithms cause large corrections at each order and dominate the series. This means varying the scale in $\alpha_s$ in Eq. (9) directly tracks the size of the large logarithms and therefore allows one to get some estimate of the size of missing higher-order terms caused by $p^{\text{cut}}$, that correspond to $\Delta_{\text{cut}}$. Therefore, we can approximate $\Delta_{\text{cut}} = \Delta_{\geq 1}$, where $\Delta_{\geq 1}$ is obtained from the scale variation for $\sigma_{\geq 1}$.

The exclusive 0-jet cross section is equal to the difference between Eqs. (8) and (9), and so has the schematic structure

$$\sigma_0(p^{\text{cut}}) \approx \sigma_B [1 + \alpha_s + \alpha_s^2 + \mathcal{O}(\alpha_s^3)]$$
$$- [\alpha_s (L^2 + L + 1) + \alpha_s^2 (L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)]].$$  \hspace{1cm} (10)

In this difference, the large positive corrections in $\sigma_{\text{total}}$ partly cancel against the large negative logarithmic corrections. For example, at $\mathcal{O}(\alpha_s)$ there is a value of $L$ for which the $\alpha_s$ terms in the schematic Eq. (10) cancel exactly, indicating that at this $p^{\text{cut}}$ the NLO cross section has vanishing scale dependence and is equal to the LO cross section, $\sigma_0(p^{\text{cut}}) = \sigma_B$. We will see this effect explicitly in our examples below, using the complete perturbative expressions. We will find that this occurs for values of $p^{\text{cut}}$ in the experimentally relevant region. Because of this cancellation, a standard use of scale variation in Eq. (10) does not actually probe the size of the logarithms, and thus is not suitable to estimate $\Delta_{\text{cut}}$.

Since $\Delta_{\text{cut}}$ and $\Delta_{\text{total}}$ are by definition uncorrelated, by associating $\Delta_{\text{cut}} = \Delta_{\geq 1}$ we are effectively treating the perturbative series for $\sigma_{\text{total}}$ and $\sigma_{\geq 1}$ as independent with separate (uncorrelated) perturbative uncertainties. That is, considering $\{\sigma_{\text{total}}, \sigma_{\geq 1}\}$, the covariance matrix is diagonal,

$$C = \begin{pmatrix} \Delta_{\text{total}}^2 & 0 & \Delta_{\geq 1}^2 \\ \Delta_{\text{total}}^2 & \Delta_{\geq 1}^2 & 0 \\ -\Delta_{\text{total}}^2 & -\Delta_{\text{total}}^2 & \Delta_{\geq 1}^2 \end{pmatrix}.$$  \hspace{1cm} (11)

This is consistent, since for small $p^{\text{cut}}$ the two series have very different structures. In particular, there is no reason to believe that the same cancellations in $\sigma_0$ will persist at every order in perturbation theory at a given $p^{\text{cut}}$.

From Eq. (11) it follows that the perturbative uncertainty in $\sigma_0(p^{\text{cut}})$ is given by $\Delta_{\text{total}}^2 + \Delta_{\geq 1}^2$, i.e., by summing the inclusive cross section uncertainties in quadrature. It also follows that the complete covariance matrix for the three quantities $\{\sigma_{\text{total}}, \sigma_0, \sigma_{\geq 1}\}$ is

$$C = \begin{pmatrix} \Delta_{\text{total}}^2 & \Delta_{\text{total}}^2 & 0 \\ \Delta_{\text{total}}^2 & \Delta_{\text{total}}^2 & 0 \\ -\Delta_{\text{total}}^2 & -\Delta_{\text{total}}^2 & \Delta_{\geq 1}^2 \end{pmatrix},$$  \hspace{1cm} (12)

where $\Delta_{\text{total}}$ and $\Delta_{\geq 1}$ are considered uncorrelated and are evaluated by separately varying the scales in the fixed-order predictions for $\sigma_{\text{total}}$ and $\sigma_{\geq 1}(p^{\text{cut}})$, respectively. The $\Delta_{\geq 1}$ contributions in the lower right $2 \times 2$ matrix for $\sigma_0$ and $\sigma_{\geq 1}$ are equivalent to Eq. (6) with $\Delta_{\text{cut}} = \Delta_{\geq 1}$. Note that in this $2 \times 2$ space all of $\Delta_{\text{total}}$ occurs in the uncertainty for $\sigma_0$. This is reasonable from the point of view that $\sigma_0$ starts at the same order in $\alpha_s$ as $\sigma_{\text{total}}$ and contains the same leading virtual corrections.

The limit $\Delta_{\text{cut}} = \Delta_{\geq 1}$ which Eq. (12) is based on is of course not exact but an approximation. However, the preceding arguments show that it is a more reasonable starting point than using a common scale variation for the different jet bins. The latter usually results in the cross sections being 100% correlated, as in Eq. (1), and, in particular, does not account for the additional $p^{\text{cut}}$ induced uncertainties. In our numerical examples below, we will see that our method produces more sensible uncertainty estimates for fixed-order predictions. In Sec. IV we will compare the estimates from our method with those obtained by an explicit resummation in the jet-veto variable. This provides further evidence that our method gives consistent uncertainty estimates. Resummation provides a way for improving predictions for the central value of the cross section, together with better estimates of $\Delta_{\text{cut}}$ and the structure of the theory correlation matrix, as discussed in Sec. IV.
It is straightforward to generalize the above discussion to jet bins with more jets. For the $N$-jet bin we replace 
\[ \sigma_{\text{total}} \to \sigma_{\geq N}, \sigma_0 \to \sigma_N, \] 
and \( \sigma_{\geq 1} \to \sigma_{\geq N+1}, \) and take the appropriate \( \sigma_B. \) If the perturbative series for \( \sigma_{\geq N} \) exhibits large \( \alpha_s \) corrections, then the additional large logarithms present in \( \sigma_{\geq N+1} \) will again lead to cancellations when we consider the difference \( \sigma_N = \sigma_{\geq N} - \sigma_{\geq N+1}. \) Hence, \( \Delta_{\geq N+1} \) will again give a better estimate for the \( \Delta_{\text{cut}} \) that arises from separating \( \sigma_{\geq N} \) into jet bins \( \sigma_N \) and \( \sigma_{\geq N+1}. \) Another advantage of our procedure is that it is easily generalized to more than two jet bins by iteration. The case of three jet bins is given in the Appendix.

### III. EXAMPLE PROCESSES

To elucidate the effect of \( p_T^{\text{jet}} \) vetoes on the fixed-order cross sections and demonstrate our method, we will now go through several explicit examples, considering in turn \( H + 0 \) jets, \( H + 1 \) jet, \( WW + 0 \) jets, and \( W + 0, 1, \) and 2 jets. All of our NLO \( p_T \) spectra are obtained using the MCFM code [17–20]. As our jet algorithm we use anti-\( k_T \) for the LHC results and a cone algorithm for the Tevatron results with \( R = 0.5 \) for both.

#### A. Higgs + 0 jets

In Higgs production via gluon fusion the cross section is known to next-to-next-to-leading order [21–28], and exhibits large perturbative corrections. Consider the numerical results for the Higgs production cross section for \( m_H = 165 \text{ GeV}, \) \( \mu_f = \mu_r = m_H/2, \) and MSTW2008 next-to-next-to-leading order (NNLO) PDFs [29], for which \( \alpha_s = \alpha_s(m_H/2) = 0.1189. \) Here one finds [30–33]

\[
\sigma_{\text{total}} = (3.32 \, \text{pb})[1 + 9.5 \alpha_s + 35 \alpha_s^2 + O(\alpha_s^3)], \tag{13}
\]

for the LHC at \( \sqrt{s_{\text{cm}}} = 7 \) TeV. Note that there is an \( \alpha_s^2 \) in the Born cross section, \( \sigma_B = 3.32 \) pb, but only the relative size of the corrections is important for our discussion. For the Tevatron the series is

\[
\sigma_{\text{total}} = (0.15 \, \text{pb})[1 + 9.0 \alpha_s + 34 \alpha_s^2 + O(\alpha_s^3)]. \tag{14}
\]

In both cases the large \( K \) factors are clearly visible.\(^4\) For the inclusive 1-jet cross section at the LHC one finds

\[^4\text{Using instead } \mu_f = \mu_r = m_H \text{ the coefficients of the } \alpha_s \text{ and } \alpha_s^2 \text{ terms increase to 11 and 65 for the LHC and 12 and 74 for the Tevatron, respectively. The } \alpha_s \text{ coefficients for the Tevatron, for example, arise as } 9.0 = 4.9 + 2.0 + 2.1 \text{ (} \mu = m_H/2 \text{) and } 12.0 = 4.9 + 5.7 + 1.4 \text{ (} \mu = m_H \text{), where the three contributions are, respectively, from the terms in the partonic cross section proportional to } \delta(1 - z), \text{ terms involving the plus functions } [1/(1 - z)], \text{ and } [\ln(1 - z)/(1 - z)], \text{ and the remaining terms that are nonsingular for } z \to 1. \text{ When separating these different terms we keep the overall } 1/z \text{ factor in the convolution integral with measure } dz/z.\]

The first values correspond to the ATLAS and CMS reference cuts, and the second to current ATLAS and CMS \( H \to WW \) analyses [3,4]. Similarly, for the typical cuts used in \( H \to WW \) at the Tevatron [2], one finds

\[
\sigma_{\geq 1}(p_T^{\text{jet}} \geq 20 \text{ GeV}, |\eta_{\text{jet}}| \leq 2.5) = (0.15 \, \text{pb})[4.1 \alpha_s + 27 \alpha_s^2 + O(\alpha_s^3)]. \tag{16}\]

In both Eqs. (15) and (16) one clearly sees the impact of the large logarithms on the perturbative series. Comparing to Eqs. (13) and (14) one also sees the sizable numerical cancellation between the two series at each order in \( \alpha_s. \) The extent of this cancellation depends sensitively on the value of \( p_T^{\text{cut}}. \)

The perturbative uncertainties on these inclusive cross sections can now be used to determine the exclusive cross section uncertainties. Varying the scale up and down by a factor of 2 around \( m_H/2 \) gives for the Tevatron \( \sigma_{\text{total}} = (0.386 \pm 0.040) \, \text{pb} \) and \( \sigma_{\geq 1} = (0.132 \pm 0.034) \, \text{pb} \) with the \( p_T^{\text{jet}} \) and \( \eta_{\text{jet}} \) cuts as in Eq. (16). Adding these in quadrature according to the upper-left entry in Eq. (4) gives

\[
\sigma_0 = (0.254 \pm 0.052) \, \text{pb}, \tag{17}\]

i.e., a 20% uncertainty. In contrast, when doing a scale variation directly in the fixed-order expansion for \( \sigma_0(p_T^{\text{cut}}), \) as in Eq. (1), one implicitly assumes that the perturbative uncertainties between the series for \( \sigma_{\text{total}} \) and \( \sigma_{\geq 1} \) are 100% correlated, giving \( \sigma_0 = (0.254 \pm 0.006). \) Here this leads to an underestimate for the remaining uncertainty. For the LHC, using the reference cuts, we get \( \sigma_{\text{total}} = (8.70 \pm 0.75) \, \text{pb} \) and \( \sigma_{\geq 1} = (3.08 \pm 0.59) \, \text{pb}, \) leading to

\[
\sigma_0 = (5.63 \pm 0.96) \, \text{pb}, \tag{18}\]

i.e., a 17% uncertainty. In contrast, the direct scale variation for \( \sigma_0 \) yields \( \sigma_0 = (5.63 \pm 0.15), \) which is again an underestimate.

The two procedures of evaluating uncertainties can be compared as a function of \( p_T^{\text{cut}}, \) and in the upper left panel of Fig. 1 we do so for \( \sigma_0(p_T^{\text{cut}}) \) for Higgs production. Results for \( \sigma_0 \) are obtained at NNLO for the LHC at \( \sqrt{s_{\text{cm}}} = 7 \) TeV, using MCFM to calculate the \( p_T^{\text{cut}} \) dependence, FEHIP [30,31] for the total NNLO cross section, and \( \mu = m_H/2 \) for central values. The central value is the solid blue curve, and the green dashed and dotted lines show the results of direct exclusive scale variation by a factor of 2. For small values of \( p_T^{\text{cut}} \) the cancellations that take place for \( \sigma_0(p_T^{\text{cut}}) \) cause the error bands to shrink. In particular, the direct exclusive scale uncertainty vanishes at \( p_T^{\text{cut}} \approx 25 \) GeV, where there is an almost exact cancellation between the

\[ \sigma_{\geq 1}(p_T^{\text{jet}} \geq 30 \text{ GeV}, |\eta_{\text{jet}}| \leq 3.0) = (3.32 \, \text{pb})[4.7 \alpha_s + 26 \alpha_s^2 + O(\alpha_s^3)]. \]
two series in Eq. (10), and the uncertainty curves pinch together. In contrast, the outer red solid lines show the result of our method, which combines the independent inclusive uncertainties to obtain the exclusive uncertainty, $C_1^2 C_2^1 = C_1^2 + C_1^2$. One can see that for large values of $p_{\text{cut}}$ this combined inclusive uncertainty estimate reproduces the direct exclusive scale variation, since $C_2^1(p_{\text{cut}})$ becomes small. On the other hand, for small values of $p_{\text{cut}}$, the uncertainties obtained in this way are now more realistic, because they explicitly take into account the large logarithmic corrections. The features of this plot are quite generic. In particular, the same pattern of uncertainties is observed for the Tevatron, when we take $\mu = m_H$ as our central curve with $\mu = 2m_H$ and $\mu = m_H/2$ for the range of scale variation, and whether or not we only look at jets at central rapidities. We also note that using independent variations for $\mu_f$ and $\mu_r$ does not change this picture, in particular, the $\mu_f$ variation for fixed $\mu_r$ is quite small.

Since both NLO and NNLO results for $\sigma_0(p_T^{\text{cut}})$ are available, it is also useful to consider the convergence, which we show in Fig. 2 for the Tevatron (top panels) and the LHC at 7 TeV (bottom panels). In the left panels we directly vary the scales in $\sigma_0(p_T^{\text{cut}})$ to estimate the uncertainty, while in the right panels we again propagate the uncertainties from the inclusive cross sections. As we lower $p_T^{\text{cut}}$, the direct exclusive scale variation uncertainty estimate decreases at both NLO and NNLO, and eventually becomes very small when the curves pinch and the uncertainty is clearly underestimated. In contrast, the combined inclusive scale variation gives realistic uncertainties for all values of $p_T^{\text{cut}}$. In particular, there is considerable uncertainty for small $p_T^{\text{cut}}$ where the summation of logarithms is important.

**B. Higgs + 1 jet**

As our next example we consider the 1-jet bin in Higgs production from gluon fusion. This jet bin is defined by two cuts, one which ensures that the jet with the largest $p_T$ is outside the 0-jet bin, $p_{T_1}^\text{jet} \geq p_{T_1}^{\text{cut}}$, and one which ensures
that the jet with the next largest $p_T$ is restricted, $p_{T2}^{\text{cut}} \leq p_{T1}^{\text{cut}}$, so that we do not have two or more jets. The 1-jet cross section can be computed as a difference of inclusive cross sections with these cuts,

$$\sigma_1 = \sigma_{\geq 1}(p_{T1}^{\text{cut}}) - \sigma_{\geq 2}(p_{T1}^{\text{cut}}, p_{T2}^{\text{cut}}) = \sigma_0(p_{T1}^{\text{cut}}) - \sigma_0(p_{T2}^{\text{cut}}).$$

(19)

For convenience we adopt the notation that $p_{T1}^{\text{cut}}$ is always used for the cutoff that determines the upper boundary of the jet bin under consideration, which gives the analog of the $L$ dependent terms in Eq. (10).

The inclusive cross section $\sigma_{\geq 1}$ that includes the 1-jet bin exhibits large perturbative corrections, much as $\sigma_{\text{total}}$ does for the 0-jet bin. For $\sigma_{\geq 1}$ the large corrections are caused in part by the large double logarithmic series in $\ln(p_{T1}^{\text{cut}}/m_H)$, but remains predominantly independent of the large double logarithms of $L = \ln(p_{T2}^{\text{cut}}/m_H)$ which control the series for $\sigma_{\geq 2}$. With $\mu_f = \mu_r = m_H/2, m_H = 165$ GeV, and MSTW2008 NNLO PDFs, we find

$$\sigma_{\geq 1}(p_{T1}^{\text{cut}} \geq 30 \text{ GeV}) = 2.00 \text{ pb}[1 + 5.4\alpha_s + O(\alpha_s^2)],$$

$$\sigma_{\geq 2}(p_{T1}^{\text{cut}} \geq 30 \text{ GeV}, p_{T2}^{\text{cut}} \geq 30 \text{ GeV}) = 2.00 \text{ pb}[3.6\alpha_s + O(\alpha_s^2)].$$

(20)

For $\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}$ there is a sizeable cancellation between these $\alpha_s$ terms. If we lower the cut to $p_{T2}^{\text{cut}} \geq 22$ GeV then the logarithm increases and there is an almost exact cancellation with the $5.4\alpha_s$. In the top right panel of Fig. 1 we plot $\sigma_1$ as a function of $p_{T1}^{\text{cut}}$, and we again see that this cancellation occurs in a region where there is a dramatic decrease in the direct exclusive scale dependence (green dashed and dotted curves). Using the inclusive uncertainties for $\sigma_{\geq 1}$ and $\sigma_{\geq 2}$, and adding them in quadrature, gives
the outer solid red curves, which again avoids this problem and provides a more realistic estimate for the perturbative uncertainty.

Using the result from the Appendix we can examine the full uncertainties and correlation matrix with 0, 1, and \((\geq 2)\)-jet bins in Higgs production. For the cuts in Eq. (20) varying the scale by factors of 2, we have \(\sigma_{\text{total}} = (8.70 \pm 0.75)\, \text{pb}, \sigma_{\leq 1} = (3.29 \pm 0.62)\, \text{pb},\) and \(\sigma_{\geq 2} = (8.58 \pm 0.49)\, \text{pb},\) corresponding to relative uncertainties of 8.6\%, 18.8\%, and 57\%, respectively. We let \(\delta(x)\) denote the relative percent uncertainty of the quantity \(x,\) and \(\rho(x, y)\) the correlation coefficient between \(x\) and \(y.\) The Appendix yields

\[
\delta(\sigma_0) = 18\%, \quad \delta(\sigma_1) = 32\%, \quad \rho(\sigma_0, \sigma_{\text{total}}) = 0.77, \\
\rho(\sigma_1, \sigma_{\geq 2}) = -0.62, \quad \rho(\sigma_0, \sigma_1) = -0.50,
\]

(21)

where we have only shown the nonzero correlations. Note that \(\sigma_0\) and \(\sigma_1\) as well as \(\sigma_1\) and \(\sigma_{\geq 2}\) have a substantial negative correlation because of the jet-bin boundary they share, while \(\sigma_0\) and \(\sigma_{\geq 2}\) are uncorrelated.

In contrast, the direct exclusive scale variation results in all the cross sections being 100\% correlated. Because of the cancellations between the perturbative series, this leads to much smaller (and unrealistic) uncertainties, with our choice of cuts \(\delta(\sigma_0) = 2.3\%\) and \(\delta(\sigma_1) = 5.5\%,\) which is reflected in the pinching of the dotted and dashed green lines in Fig. 1. (Note that increasing the range of scale variation or separately varying \(\mu,\) and \(\mu_f\) does not mitigate this problem.) The analog of Eq. (1) for this example would be

\[
0.62 \times 2.3\% + 0.28 \times 5.5\% + 0.10 \times 57\% = 8.6\%.
\]

(22)

When all \(\sigma_i\) are 100\% correlated, \(\sigma_0\) is forced to have a smaller relative uncertainty than \(\sigma_{\text{total}},\) as in Eq. (1), since it has to make up for the much larger uncertainties in \(\sigma_{\geq 2}\).

In addition to the cross sections in each jet bin, we can also consider the relative jet fractions \(f_0 = \sigma_0/\sigma_{\text{total}}\) and \(f_1/\sigma_{\text{total}},\) which are often used in experimental analyses. The perturbative theory uncertainties and correlations for the jet fractions follow by standard error propagation from those in Eq. (21). The general expressions are given in the Appendix, and we find

\[
\delta(f_0) = 13\%, \quad \delta(f_1) = 33\%, \quad \rho(f_0, \sigma_{\text{total}}) = 0.42, \\
\rho(f_1, \sigma_{\text{total}}) = -0.26, \quad \rho(f_0, f_1) = -0.80.
\]

(23)

Comparing to Eq. (21), the use of jet fractions with \(\sigma_{\text{total}}\) in the denominator yields a nonzero anticorrelation for \(\sigma_{\text{total}}\) with the 1-jet bin, and decreases the correlation for \(\sigma_{\text{total}}\) with the 0-jet bin.

It is also interesting to consider the case with \(p_{T1}^{\text{jet}} \geq 120\, \text{GeV},\) where the logarithms of \(p_{T1}^{\text{jet}}/m_H\) are not large. The cross section \(\sigma_{\geq 1}\) now has a smaller perturbative correction, but for a region of cuts on \(p_{T2}^{\text{jet}}\) there are still substantial cancellations in \(\sigma_1.\) For instance, for \(p_{T2}^{\text{jet}} \geq 60\, \text{GeV}\) we have

\[
\sigma_{\geq 1}(p_{T1}^{\text{jet}} \geq 120\, \text{GeV}) = (0.31\, \text{pb})[1 + 2.9\alpha_s + \mathcal{O}(\alpha_s^2)],
\]

\[
\sigma_{\geq 2}(p_{T1}^{\text{jet}} \geq 120\, \text{GeV}, p_{T2}^{\text{jet}} \geq 60\, \text{GeV}) = (0.31\, \text{pb})[3.7\alpha_s + \mathcal{O}(\alpha_s^2)],
\]

(24)

and the \(\alpha_s\) terms completely cancel around \(p_{T2}^{\text{jet}} \geq 70\, \text{GeV.}\) In the bottom right panel of Fig. 1 we plot \(\sigma_1\) as a function of \(p_{T2}^{\text{jet}}\) for this scenario. Once again the combined inclusive uncertainties (solid red curves) give a better estimate than the direct exclusive scale uncertainty determined by up/down \(\mu\) variation in \(\sigma_1\) (green dotted and dashed curves). It is interesting to notice that the curves dive and a logarithmic summation in \(p_{T2}^{\text{jet}}\) becomes important earlier now, i.e., at much larger values for \(p_{T2}^{\text{jet}}\), when the cut on \(p_{T1}^{\text{jet}}\) is raised. For \(p_{T1}^{\text{jet}} \geq 120\, \text{GeV}\) and \(p_{T2}^{\text{jet}} \leq 30\, \text{GeV}\) fixed-order perturbation theory does not yield a controlled expansion, and the resummation of the jet-veto logarithms is clearly necessary.

C. \(WW + 0\) jets

The process \(pp \to WW + 0\) jets is the dominant irreducible background for the \(H \to WW^*\) search in the 0-jet bin, and also exhibits a relatively large \(K\) factor \(\sim 1.5.\) Hence, it is interesting to contrast the scale uncertainties here with those found for \(H + 0\) jets. Including the Higgs search cuts (modulo the jet veto), the \(K\) factor for \(WW\) becomes larger than 2 [19], but we will not include those cuts in our analysis here. With \(\mu_s = \mu_f = m_W,\) NLO MSTW2008 PDFs, and \(\alpha_s = \alpha_s(m_W) = 0.1226,\) the total \(pp \to WW\) cross section is

\[
\sigma_{\text{total}} = (32.5\, \text{pb})[1 + 3.6\alpha_s + \mathcal{O}(\alpha_s^2)].
\]

(25)

while for the inclusive 1-jet cross section with logarithms of \(p_T^{\text{jet}}\) we have

\[
\sigma_{\geq 1}(p_T^{\text{jet}} \geq 30\, \text{GeV}) = (32.5\, \text{pb})[2.8\alpha_s + \mathcal{O}(\alpha_s^2)].
\]

(26)

Thus, when we consider \(\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}\) there is a sizable cancellation for the \(\alpha_s\) terms. In Fig. 1, lower left panel, we show \(\sigma_0\) for \(pp \to WW + 0\) jets as a function of \(p_T^{\text{jet}}\). Once again the dotted and dashed green curves from direct exclusive scale variation exhibit a pinching near \(p_T^{\text{jet}} \sim 30\, \text{GeV}\) due to cancellations between the two perturbative series in Eqs. (25) and (26), leading to an underestimate of the perturbative uncertainty. The combined inclusive uncertainty estimate again mitigates this problem. The pattern of uncertainties here is the same as for \(H + 0\) jets and \(H + 1\) jet, just with smaller overall uncertainties. Just like for \(H + 0\) jets using independent variations for \(\mu_f\) and \(\mu_s\) does not change the picture, the \(\mu_f\) variation for fixed \(\mu_s\) is again quite small.
D. $W+0$ jets

The exclusive process $pp \rightarrow W+N$ jets is an important benchmark process at the LHC and also an important standard model background for new physics searches looking for missing energy. In this section we consider $pp \rightarrow W+0$ jets, which provides us with a case to test our method when the perturbative corrections in the inclusive cross sections are not as large. For simplicity, we only work to NLO here. Using $\mu_f = \mu_r = m_W$ for the central value and MSTW2008 NLO PDFs, the inclusive $W$ production cross section is

$$\sigma_{\text{total}} = (80.7 \text{nb})[1 + 1.3\alpha_s + O(\alpha_s^2)],$$

(27)

where we have summed over $W^\pm$, and have not included the leptonic branching fractions. For the inclusive 1-jet cross section we have

$$\sigma_{\geq 1}(p_T^{\text{jet}} \geq 30 \text{GeV}) = (80.7 \text{nb})[0.9\alpha_s + O(\alpha_s^2)].$$

(28)

The perturbative coefficients in Eqs. (27) and (28) are much smaller than in Higgs production. The resulting predictions for $\sigma_0(p_T^{\text{cut}})$ are shown in the top left panel of Fig. 3, where the different lines have the same meaning as in Fig. 1. Since the $\alpha_s$ corrections are not very large here, the $\mu_f$ scale variation in the PDFs dominates over the $\mu_r$ variation in $\alpha_s$ and produces a 100% negative correlation between $\sigma_{\text{total}}$ and $\sigma_{\geq 1}$. (Keeping $\mu_f$ fixed at $m_W$ and only varying $\mu_r$ results in the expected pinching of the dotted and dashed green lines.) This means their scale uncertainties add linearly in $\sigma_0$, which maximizes the uncertainty in this 0-jet cross section. In this case, our method, shown by the outer solid red lines, gives an uncertainty band very similar to direct exclusive scale variation. Hence, our method of using independent inclusive uncertainties still remains consistent for this situation.

E. $W+1$ jet

For $pp \rightarrow W+1$ jet the perturbative corrections in $\sigma_{\geq 1}$ are larger than those in the $W$ total cross section, which is in part influenced by logarithms from the lower cut on $p_T^{\text{jet}}$.
the $p_T$ of the leading jet. The situation for the $W + 1$ jet bin is similar to $H + 1$ jet. Considering Eq. (19) the series for the inclusive 2-jet cross section, $\sigma_{\geq 2}$, has large double logarithms $L = \ln(p_T^{jet}/m_W)$ of the second largest jet $p_T$, which are independent of those in the perturbative series for $\sigma_{\geq 1}$. Taking $\mu = m_W$ for central values, and using MSTW2008 PDFs at NLO, the total $W^+ + W^-$ cross sections with both jet cuts at 30 GeV are

$$\sigma_{\geq 1}(p_T^{jet} \geq 30 \text{ GeV}) = (8.61 \text{ nb})[1 + 3.4\alpha_s + O(\alpha_s^2)].$$

$$\sigma_{\geq 2}(p_T^{jet} \geq 30 \text{ GeV}, p_T^{jet} \geq 30 \text{ GeV}) = (8.61 \text{ nb})[2.5\alpha_s + O(\alpha_s^2)].$$

(29)

Once again the result for $\sigma_{\geq 2}$ and the precise cancellation that occurs in $\sigma_1$ is quite sensitive to $p_T^{cut}$, the cut on $p_T^{jet}$, yielding an almost exact cancellation of the $3.4\alpha_s$ for $p_T^{jet} \geq 25 \text{ GeV}$. In the top right panel of Fig. 3 we plot $\sigma_1$ as a function of $p_T^{cut}$, with direct exclusive scale variation (green dashed and dotted curves) and those derived from independent inclusive uncertainties (solid red curves). Just like for $H + 1$ jet, the direct exclusive scale variation curves pinch, while the inclusive curves avoid this problem and remain realistic.

We can also consider what happens when we make a larger cut on $p_T^{jet}$. Here, unlike for the Higgs case, the relative size of the perturbative correction in $\sigma_{\geq 1}$ increases. For instance,

$$\sigma_{\geq 1}(p_T^{jet} \geq 80 \text{ GeV}) = (1.07 \text{ nb})[1 + 5.3\alpha_s + O(\alpha_s^2)].$$

$$\sigma_{\geq 2}(p_T^{jet} \geq 80 \text{ GeV}, p_T^{jet} \geq 60 \text{ GeV}) = (1.07 \text{ nb})[4.1\alpha_s + O(\alpha_s^2)].$$

(30)

For $p_T^{jet} \geq p_T^{cut}$ in $\sigma_{\geq 2}$ the resulting 1-jet cross section $\sigma_1$ is shown as a function of $p_T^{cut}$ in the bottom left panel of Fig. 3. The situation for the uncertainties is similar to that for the less stringent cut on $p_T^{jet}$ in the upper right panel. Much like in $H + 1$ jet the logarithms start to influence the cross section at larger values of $p_T^{cut}$ for larger $p_T^{jet}$.

**F. W + 2 jets**

As our last example we consider $W + 2$ jets, and for simplicity we only consider the case of $W^+$ production. The inclusive 2-jet and 3-jet cross sections with all jets cut at 30 GeV are

$$\sigma_{\geq 2}(p_T^{jet,2, \geq 30 \text{ GeV}}) = (1.60 \text{ nb})[1 + 1.0\alpha_s + O(\alpha_s^2)].$$

$$\sigma_{\geq 3}(p_T^{jet,2,3, \geq 30 \text{ GeV}}) = (1.60 \text{ nb})[2.3\alpha_s + O(\alpha_s^2)].$$

(31)

and the resulting exclusive 2-jet cross section as a function of the $p_T^{cut}$ on the third jet is shown in the bottom right panel in Fig. 3.

There are two different types of diagrams contributing to this process, those having two external quark lines and two gluon lines at lowest order ($qgqg$), and those having four external quark lines at lowest order ($qqqq$). The $qgqg$-type contributions have the same behavior as $W + 1$ jet, again displaying a pinching in the direct exclusive scale variation curves. On the other hand, in the $qqqq$-type contributions the PDF scale dependence dominates, similar to what we observed for $W + 0$ jets. The combination of the two leads to the behavior seen in Fig. 3 at large $p_T^{cut}$, where the scale uncertainties in the inclusive 2-jet cross section are asymmetric. Here there is some choice for how to combine the scale variation into an uncertainty estimate for $\sigma_{\geq 2}$ (green dashed and dotted curves). The choice one makes for $\sigma_{\geq 2}$ simply propagates into the equivalent choice for the exclusive 2-jet bin $\sigma_2$ (solid red curves). For simplicity in the bottom right panel of Fig. 3 we still use $\mu = m_W/2$ and $\mu = 2m_W$ to determine $\Delta_{\geq 2}$, in which case the central value should be taken as the center of the band rather than the dark solid blue line for $\mu = m_W$.

For $W + 2$ jets in Fig. 3 the pinching caused by the $qgqg$ contributions is again mitigated by combining the inclusive uncertainties. Hence, we see that our method can be applied and gives more stable uncertainty estimates even in more complicated cases where several components contribute to the cross section.

Note that we have also checked that when increasing the cuts on the two leading jets, the same effect as in $H + 1$ jets and $W + 1$ jets happens here as well. Namely, the jet-veto logarithms from restricting the third jet become more important earlier and influence the cross section at larger values of $p_T^{cut}$ for larger $p_T^{jet}$.

**IV. RESUMMATION FOR HIGGS + 0 JETS**

In Sec. III we have seen that direct exclusive scale variation often leads to an accidental underestimate of the uncertainties for exclusive jet bin cross sections for a range of experimentally relevant cuts. Instead combining independent uncertainties on inclusive cross sections yields a more uniform (and larger) uncertainty band for the exclusive jet bins. The region where direct exclusive scale variation runs into trouble borders the region where the resummation of the large logarithms of $p_T^{jet}$ becomes important. In this section, we test how realistic the fixed-order scale uncertainties are by comparing them to a case where the resummation of large logarithms induced by the jet bin are known to NNLL + NNLO accuracy.

We again consider $H + 0$ jets from gluon fusion. At NNLL order accuracy the resummation is sensitive to the precise jet algorithm used to define $p_T^{jet}$, and other complications in the required theoretical setup. To avoid these issues, we will use a slightly different variable to define the
0-jet bin, an inclusive event shape known as beam thrust [34].

\[
T_{cm} = \sum_k (E_k - |\vec{p}_k^T|). \tag{32}
\]

The sum over \( k \) runs over all particles except the Higgs decay products. Beam thrust essentially measures the thrust of an event along the \( \vec{z} \) beam axis. When \( T_{cm} \leq T_{\text{cut}} \), from Eq. (32) we see that events in \( \sigma_0(T_{cm}) \) are only allowed to contain hard radiation in the forward regions at large rapidities, and hence this cut vetoes central jets. Much like with \( p_T^\text{jet} \) the perturbative series for this \( \sigma_0 \) has double logarithms, for example, the analog of Eq. (7) is

\[
\sigma_0(T_{cm}^\text{cut}) = \sigma_B \left( 1 - \frac{3\alpha_s}{\pi} \ln \frac{T_{cm}^\text{cut}}{m_H} + \cdots \right). \tag{33}
\]

For beam thrust, the all-order resummation of perturbative corrections is known to NNLL order for both \( H + 0 \) jets and \( V + 0 \) jets [35–37]. For Higgs production the computation has been extended to fully include all NNLO corrections, and it was observed that the resummed cross section at NNLL + NNLO had larger uncertainties than the pure NNLO result for \( \sigma_0(T_{cm} \leq T_{cm}^\text{cut}) \) utilizing direct exclusive fixed-order scale variation. This led to the conclusion that the direct exclusive scale variation underestimates the fixed-order perturbative uncertainties in the 0-jet bin. In the resummed calculation, fixed-order \( \alpha_s \) expansions are carried out at three distinct scales (hard \( \mu_H \), jet/beam \( \mu_B \), and soft \( \mu_S \)) which appear in the corresponding factorization theorem. The uncertainties in the resummed cross section are obtained by varying these scales. Varying \( \mu_H \) up and down by a factor of 2 moves all three scales up and down, and hence is a scale variation that is correlated with the usual scale variation for the inclusive cross section. Varying \( \mu_B \) or \( \mu_S \) while holding

\[ \mu_H \] fixed explicitly accounts for additional higher order uncertainties induced by the presence of the large jet-veto logarithms, and hence allow us to determine \( \Delta_{\text{cut}} \).

In Fig. 4 we compare the remaining perturbative uncertainties after resummation at NNLL + NNLO, shown by the darker red bands, to the NNLO uncertainties obtained with the fixed-order method advocated here, which are shown by the lighter gray bands. The results for the NNLL + NNLO cross section are obtained from Ref. [37]. The left panel shows the results for the Tevatron and the right panel the results for the LHC at 7 TeV. The fact that the resummation reduces the perturbative uncertainties, as it should, shows that our method of using independent inclusive scale variations yields more robust fixed-order uncertainties.

In the resummed calculation, \( \sigma_{\text{total}} \) is by construction not affected by the \( \mu_S \) and \( \mu_B \) variations. We denote the combined \( \mu_S \) and \( \mu_B \) uncertainty by \( \Delta_{SB} \). It provides a direct estimate of the cut-induced uncertainty, \( \Delta_{\text{cut}} = \Delta_{SB} \), which is anticorrelated between \( \sigma_0(T_{cm}) \) and the corresponding \( \Delta_{\text{cut}}(T_{cm}^\text{cut}) = \sigma_{\text{total}} - \sigma_0(T_{cm}^\text{cut}) \). On the other hand, the \( \mu_H \) variation affects all the cross sections yielding an uncertainty component that is 100% correlated between them. In particular, it is responsible for estimating

\[ \Delta_{\text{cut}} \] uncertainties induced by the presence of the large jet-veto logarithms, and hence allow us to determine \( \Delta_{\text{cut}} \).

5We have made a small improvement to Ref. [37]. The NNLL + NNLO results of Ref. [37] fully incorporate the NNLO corrections by adding so-called nonsingular fixed-order contributions, which are terms that do not appear in an expansion of the strict NNLL result. In Ref. [37] the nonsingular contributions were obtained for the sum of \( \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) \) cross sections using FEHIP [30,31]. Here we use a much higher statistics spectrum from MCFM [17], which allows us to separately determine the nonsingular cross sections at \( \mathcal{O}(\alpha_s) \) and \( \mathcal{O}(\alpha_s^2) \). The only place this improvement is visible is for \( T_{\text{cut}}^\text{cut} \leq 3 \text{ GeV} \), where the resummed cross sections are now consistent with zero within the displayed uncertainties.
the perturbative uncertainty of $\sigma_{\text{total}}$, for which it is equivalent to the usual fixed-order scale variation, $\Delta_{\text{H0}} = \Delta_{\text{total}}$. The full covariance matrix for $\{\sigma_{\text{total}}, \sigma_0, \sigma_{\geq 1}\}$, that is the analog of Eq. (12) but for the resummed result, is then

$$C = C_{SB} + C_H, \quad C_{SB} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{SB}^2 - \Delta_{SB}^2 & -2 \Delta_{SB} \Delta_{H0} \\ 0 & -2 \Delta_{SB} \Delta_{H0} & \Delta_{H0}^2 - \Delta_{H0}^2 \end{pmatrix}, \quad C_H = \begin{pmatrix} \Delta_{H0}^2 & \Delta_{H0} \Delta_{H0} & \Delta_{H0} \Delta_{H0} \\ \Delta_{H0} \Delta_{H0} & \Delta_{H0}^2 & \Delta_{H0} \Delta_{H0} \\ \Delta_{H0} \Delta_{H0} & \Delta_{H0} \Delta_{H0} & \Delta_{H0}^2 \end{pmatrix}.$$

where $\Delta_{SB}$ is obtained from the envelope of the $\mu_S$ and $\mu_B$ variations, and $C_{SB}$ is equivalent to $C_{\text{cut}}$ in Eq. (6). The $\Delta_{H0}$ are obtained from the $\mu_H$ variation and satisfy $\Delta_{\text{H0}} = \Delta_{H0} + \Delta_{H_{\geq 1}}$. The full uncertainty in the 0-jet bin shown by the darker red bands in Fig. 4 is then given by $\Delta_{SB}^2 + \Delta_{H0}^2$, which is the 0-bin entry on the diagonal of $C$.

Compared to Eq. (34), using a direct exclusive scale variation at fixed order would correspond to taking $\Delta_{SB} = 0$ and obtaining the analog of the $\Delta_{H0}$ by scale variation without resummation ($\mu_H = \mu_B = \mu_S$). On the other hand, our proposed fixed-order method would correspond to taking $\Delta_{SB} = \Delta_{\geq 1}$ and $\Delta_{H_{\geq 1}} = 0$, such that $\Delta_{H0} = \Delta_{\text{H0}} \rightarrow \Delta_{\text{total}}$. Hence, the resummation of the jet veto logarithms allows one to capture both types of uncertainties appearing in the two different fixed-order methods. Note that the numerical dominance of $\Delta_{SB}^2$ over $\Delta_{H0} \Delta_{H_{\geq 1}}$ in the 0-jet region is another way to justify the preference for using the combined inclusive scale variation over the direct exclusive scale variation when given a choice between these two methods.

As an example, consider $T_{\text{cut}} = 20$ GeV. At fixed NNLO, the inclusive cross sections are $\sigma_{\text{total}} = (8.70 \pm 0.75)$ pb and $\sigma_{\geq 1} = (2.25 \pm 0.62)$ pb. Using Eq. (12), this gives

$$\delta(\sigma_0) = 15\%, \quad \delta(\sigma_{\geq 1}) = 28\%, \quad \rho(\sigma_0, \sigma_{\text{total}}) = 0.77, \quad \rho(\sigma_{\geq 1}, \sigma_{\text{total}}) = 0, \quad \rho(\sigma_0, \sigma_{\geq 1}) = -0.64.$$

For $\sigma_0$ this corresponds to the lighter gray bands in Fig. 4, and the structure here is very similar to what we saw in Eq. (21).

From our resummed result using Eq. (34) we obtain

$$\delta(\sigma_0) = 11.8\%, \quad \delta(\sigma_{\geq 1}) = 19.7\%, \quad \rho(\sigma_0, \sigma_{\text{total}}) = 0.04, \quad \rho(\sigma_{\geq 1}, \sigma_{\text{total}}) = 0.33, \quad \rho(\sigma_0, \sigma_{\geq 1}) = -0.82,$$

which for $\sigma_0$ corresponds to the darker red bands in Fig. 4. After resummation neither of $\sigma_0$ and $\sigma_{\geq 1}$ is strongly correlated with $\sigma_{\text{total}}$, anymore, which at first sight is perhaps a bit surprising. However, for small $T_{\text{cut}}$ this is not unexpected and is simply due to the fact that the central values and remaining perturbative uncertainties are dominated by the resummed logarithmic series (i.e., $\Delta_{SB}$ dominates numerically over $\Delta_{H0}$ and $\Delta_{H_{\geq 1}}$). In fact, this supports our arguments in Sec. II that the uncertainties from higher-order terms in the logarithmic series for $\sigma_{\geq 1}$ and the fixed-order series for $\sigma_{\text{total}}$ can and should be considered independent, which lead to Eq. (11).

Comparing Eqs. (35) and (36), we see that the uncertainties obtained from our fixed-order method follow a similar pattern for the relative uncertainties for $\sigma_0$ and $\sigma_{\geq 1}$ as observed in the resummed result, with a strong negative correlation between them. Since resummation provides an improved treatment of the cut-induced effects, we take this as further evidence that the method of using inclusive fixed-order cross section uncertainties provides a consistent way to obtain reliable estimates of perturbative uncertainties in exclusive jet bins. In particular it provides a suitable starting point for an uncertainty estimate, that can be further refined when an appropriate resummed result becomes available.

V. CONCLUSIONS

We have proposed a method to estimate perturbative uncertainties in fixed-order predictions of exclusive jet cross sections that accounts for the presence of large logarithms at higher orders caused by the jet binning. The method uses the fixed-order calculations of inclusive cross sections, $\sigma_{\geq N}$ and $\sigma_{\geq N+1}$, for which the standard scale variation provides reasonable uncertainty estimates, and combines these inclusive uncertainties into an estimate for the corresponding exclusive $N$-jet cross section $\sigma_N = \sigma_{\geq N} - \sigma_{\geq N+1}$, treating the inclusive cross sections as uncorrelated.

We have illustrated this procedure for a variety of processes, including analysis of $H + 0, 1$ jets, $WW + 0$ jets, and $W + 0, 1, 2$ jets with MCFM, and showed that it yields more robust estimates of theory uncertainties than direct exclusive scale variation. We have also shown for a specific case with $H + 0$ jets that it leads to fixed-order uncertainties that are theoretically consistent with the corresponding resummed predictions. In jet bins used for new physics searches, we anticipate that it should yield realistic uncertainty estimates for standard model backgrounds. We also

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expect that it provides a suitable fixed-order starting point for the central values, uncertainties, and jet bin correlations, which can be improved by higher-order logarithmic resummation.

Our treatment of the fixed-order exclusive and inclusive cross sections has followed the standard approach of always using cross section results at the same order in $\alpha_s$. It would be interesting to study whether this can be relaxed when using differences of inclusive cross sections to compute the central values for the jet bins. For example, for $gg \rightarrow H$ one could independently compute $\sigma_{\text{total}}$ at NNLO, and $\sigma_{\geq 1}$ and $\sigma_{\geq 2}$ each at NLO, and then use these to compute the jet bins as $\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}$ and $\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}$. Since we argued that the inclusive series can be treated independently, it may be consistent to include them to different orders to compute the central value and uncertainties of $\sigma_1$. This would have the advantage of allowing one to utilize the NLO result for $\sigma_{\geq 2}$ without destroying the consistent perturbative expansion for $\sigma_{\geq 1}$ and $\sigma_{\text{total}}$ when the jet bins are added together. Since in this case the perturbative order of the jet boundary between $\sigma_1$ and $\sigma_{\geq 2}$ does not match up, this deserves a dedicated study before being used in practice.

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APPENDIX: CASE OF THREE JET BINS

In this appendix we generalize Eq. (12) to the case of 0, 1, and ( $\geq 2$)-jet bins that is actually used in current Higgs searches. Since only neighboring jet bins will be correlated, the generalization to more than three jet bins is not any more complicated.

We start from the inclusive cross sections $\sigma_{\text{total}}$, $\sigma_{\geq 1}$, $\sigma_{\geq 2}$, and denote their absolute uncertainties by $\Delta_{\text{total}}$, $\Delta_{\geq 1}$, $\Delta_{\geq 2}$ and their relative uncertainties by $\delta_i = \Delta_i/\sigma_i$. We define the exclusive cross sections and event fractions

$$
\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}, \quad f_0 = \frac{\sigma_0}{\sigma_{\text{total}}},
$$

$$
\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}, \quad f_1 = \frac{\sigma_1}{\sigma_{\text{total}}}. \quad (A1)
$$

The covariance matrix for the four quantities $\{\sigma_{\text{total}}, \sigma_0, \sigma_1, \sigma_{\geq 2}\}$ is given by

$$
C = \begin{pmatrix}
\Delta_{\text{total}}^2 & \frac{\Delta_{\text{total}}^2}{2} & 0 & 0 \\
\frac{\Delta_{\text{total}}^2}{2} & \Delta_{\geq 1}^2 + \frac{\Delta_{\geq 2}^2}{2} & -\Delta_{\geq 1}^2 & 0 \\
0 & -\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2 + \frac{\Delta_{\geq 2}^2}{2} & -\Delta_{\geq 2}^2 \\
0 & 0 & -\Delta_{\geq 2}^2 & \Delta_{\geq 2}^2
\end{pmatrix}. \quad (A2)
$$

Of course, only three of these four quantities are independent. For example, $\sigma_{\text{total}} = \sigma_0 + \sigma_1 + \sigma_{\geq 2}$, and it is easy to check that $\Delta(\sigma_0 + \sigma_1 + \sigma_{\geq 2}) = \Delta_{\text{total}}$, which is given by the sum of all entries in the lower $3 \times 3$ matrix. The relative uncertainties of $\sigma_{\geq i}$ following from Eq. (A2), written in terms of relative quantities, are

$$
\delta(\sigma_0)^2 = \frac{1}{f_0^2} \delta_{\text{total}}^2 + \left(\frac{1}{f_0} - 1\right)^2 \delta_{\geq 1}^2, \quad (A3)
$$

$$
\delta(\sigma_1)^2 = \left(\frac{1 - f_0}{f_1}\right)^2 \delta_{\geq 1}^2 + \left(\frac{1 - f_0}{f_1} - 1\right)^2 \delta_{\geq 2}^2. \quad (A4)
$$

Similarly, the correlation coefficients for $\sigma_0$ and $\sigma_1$ following from Eq. (A2) are

$$
\rho(\sigma_0, \sigma_{\text{total}}) = \left[1 + \frac{\delta_{\text{total}}^2}{\delta_0^2} \left(1 - f_0\right)^2\right]^{-1/2},
$$

$$
\rho(\sigma_0, \sigma_1) = \left[1 + \frac{\delta_{\text{total}}^2}{\delta_0^2} \left(1 - f_0\right)^2\right]^{-1/2} \times \left[1 + \frac{\delta_{\geq 2}^2}{\delta_1^2} \left(1 - \frac{f_1}{f_0} - 1\right)^2\right]^{-1/2},
$$

$$
\rho(\sigma_0, \sigma_{\geq 2}) = 0,
$$

$$
\rho(\sigma_1, \sigma_{\text{total}}) = 0,
$$

$$
\rho(\sigma_1, \sigma_{\geq 2}) = \left[1 + \frac{\delta_{\geq 1}^2}{\delta_{\geq 2}^2} \left(1 - \frac{f_1}{f_0} - 1\right)^2\right]^{-1/2}. \quad (A5)
$$

The relative uncertainties for $f_0$ and $f_1$ are

$$
\delta(f_0)^2 = \left(\frac{1}{f_0} - 1\right)^2 \left(\delta_{\text{total}}^2 + \delta_{\geq 1}^2\right),
$$

$$
\delta(f_1)^2 = \delta_{\text{total}}^2 + \left(\frac{1 - f_0}{f_1}\right)^2 \delta_{\geq 1}^2 + \left(\frac{1 - f_0}{f_1} - 1\right)^2 \delta_{\geq 2}^2, \quad (A5)
$$

and their correlations are

$$
\rho(f_0, \sigma_{\text{total}}) = \left[1 + \frac{\delta_{\text{total}}^2}{\delta_0^2} \left(1 - f_0\right)^2\right]^{-1/2},
$$

$$
\rho(f_0, f_1) = \left(1 + \frac{1 - f_0}{f_1} \delta_{\geq 1}^2\right) \left(\frac{1}{f_0} - 1\right) \delta(f_0) \delta(f_1),
$$

$$
\rho(f_1, \sigma_{\text{total}}) = -\delta(\sigma_1)^2 \delta_0 f_0 \delta(f_1), \quad (A6)
$$

$$
\rho(f_1, \sigma_{\geq 2}) = -\delta(\sigma_{\geq 2})^2 \delta_0 f_0 \delta(f_1) \delta_1. \quad (A7)
$$