Inference in a Nutshell

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Corresponds to Chapters 6-9 of
Tamhane and Dunlop

Outline

Chapter 6: Basic Concepts of Inference

- Mean Square Error
- Confidence Interval
- Hypothesis Test

Chapter 7: Inference for Single Samples

- Mean - Large Sample - z
- Mean - Small Sample – t
- Variance – Chi-square
- Prediction and Tolerance Intervals

Outline (continued)

Chapter 8 – Inference for Two Samples

- Comparing Means, Independent, Large Sample –z
- Comparing Means, Independent, Small Sample
  - Variances equal – t
  - Variances not equal – t with df from SEM
- Matched Pairs – test differences – t
- Comparing Variances – F

Outline (continued)

Chapter 9 - Inferences for Proportions and Count Data

- Proportion, Large sample – z
- Proportion, Small sample – binomial
- Comparing 2 Proportions, large – z or Chi-square
- Comparing 2 Proportions, small – Fisher's Exact
- Matched Pairs – McNemar's Test
- One way Count – Chi square
- Two-way Count – Chi square
- Goodness of Fit – Chi square
- Odds ratio - z

Confidence Interval on the Mean

$\hat{u} \pm cd$ is a two-sided CI for mean u

where:

$\hat{u}$ = estimator of u = sample mean

d=standard deviation of $\hat{u}$.

c=critical constant, for instance, $z_{\alpha/2}$ or $t_{n-1,a/2}$.

$z_{\alpha/2}$ is such that $P(Z > z_{\alpha/2})=\alpha/2$.

$z_{\alpha/2}=\Phi^{-1}(1-\alpha/2)$ = qnorm(1-α/2) = -qnorm(α/2)

If a=0.05 then $z_{\alpha/2}$= 1.96.

If draw many samples and construct 95% CI's from them, 95% would contain true value of u.

Confidence Intervals

(See Figure 6.2 on page 205 of the course textbook.)

Hypothesis Tests

- H_0: null hypothesis, no change, no effect, for instance $u=u_0$
- H_1: alternative hypothesis, $u \neq u_0$
- α = P(Type I error = P(reject H_0 | H_0 true)
- β = P(Type II error = P(accept H_0 | H_0 false)
- Power = function of u = P(reject H_0 | u)
- A two-sided hypothesis test rejects H_0 when $|\hat{u}-u_0|/d > c \leftrightarrow |\hat{u}-u_0| > cd \leftrightarrow$ $\hat{u}<u0-cd$ or $\hat{u}>u0+cd$

Level α Tests

(See Table 7.1 on page 240 of the course textbook.)

P-Values

- P-Value is the probability of obtaining the observed result or one more extreme
- Two-sided P-Value
  - $= P(|Z|>|(\hat{u}-u_0)|/d$
  - $= 2[1-\Phi[|(\hat{u}-u_0)|/d]$
  - $= 2*(1-pnorm(abs(\hat{u}-u_0)/d))$ in S-Plus

P-Values

(See Table 7.2 on page 241 of the course textbook.)

Power Function

Power is the probability of rejecting H_0 for a given value of u.

$$\pi(u) = P(\hat{u} < u_0 - cd \mid u) + P(\hat{u} > u_0 + cd \mid u)$$

$$= \Phi[-c + (u_0 - u)/d] + \Phi[-c + (u - u_0)/d]$$

Power

(See Figure 7.3 on page 245 of the course textbook.)

Reject H_0

(1) If u_0 falls outside interval $\hat{u} \pm cd$.

(2) if $\hat{u}$ falls outside interval $u_0 \pm cd$.

(3) if p-value is small.