Nonparametric Statistical Methods

Corresponds to Chapter 14 of Tamhane and Dunlop

Slides prepared by Elizabeth Newton (MIT)
Nonparametric Methods

• Most NP methods are based on ranks instead of original data
• Reference: Hollander & Wolfe, Nonparametric Statistical Methods
Histogram of 100 gamma(1,1) r.v.'s
Histogram of ranks of 100 r.v.'s

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# Parametric and Nonparametric Tests

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<th>Parametric</th>
<th>Nonparametric</th>
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<td>Type of test</td>
<td>Parametric</td>
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</tr>
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</tr>
</tbody>
</table>
Sign Test

- Inference on median (u) for a single sample, size n
- $H_0: u = u_0$ vs. $H_1: u \neq u_0$
- Count the number of $x_i$’s that are greater than $u_0$ and denote this $s^+$
- The number of $x_i$’s less than $u$ are $s^- = n - s^+$
- Reject $H_0$ if $s^+$ is large or if $s^-$ is small.
- Under $H_0$, $s^+$ (and $s^-$) has binomial(n,1/2) distribution
- Large sample z test

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Histogram of thermostat data

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Sign Test in S-Plus

> thermostat
[1] 202.2 203.4 200.5 202.5 206.3 198.0 203.7 200.8 201.3 199.0

> thermostat<200
[1] F F F F F T F F F T

> sum(thermostat<200)
[1] 2

> 2*pbinom(sum(thermostat<200),10,0.5)
[1] 0.109375
Wilcoxon Signed Rank Test

• Inference on median \( (u) \), single sample, size \( n \)
• Assumes population distribution is symmetric
• \( H_0: u=u_0 \) vs. \( H_1: u \neq u_0 \)
• \( d_i = x_i - u_0 \)
• Rank order \( |d_i| \)
• \( W^+ \) = sum of ranks of positive differences
• \( W^- \) = sum of ranks of negative differences
• \( W_{\text{max}} \) = maximum \( (W^+, W^-) \)
• Reject \( H_0 \) if \( W_{\text{max}} \) is large.
• Null Distribution – see text
• Large sample \( z \) test
S-Plus wilcox.test for thermostat data

> thermostat
[1] 202.2 203.4 200.5 202.5 206.3 198.0 203.7 200.8
201.3 199.0

> sum(rank(abs(thermostat-200))[-c(6,10)])
[1] 47

> wilcox.test(thermostat,mu=200)

   Exact Wilcoxon signed-rank test

data:  thermostat
signed-rank statistic V = 47, n = 10, p-value = 0.0488
alternative hypothesis: true mu is not equal to 200

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S-Plus parametric t-test for thermostat data

> t.test(thermostat, mu=200)

One-sample t-Test

data:  thermostat
t = 2.3223, df = 9, p-value = 0.0453
alternative hypothesis: true mean is not equal to 200
95 percent confidence interval:
  200.0459 203.4941
sample estimates:
  mean of x
    201.77
Location-Scale Families

• See course textbook, page 575.
2 normal pdf’s with location parameters = -1 and 1, scale parameter = 1

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Wilcoxon Rank Sum Test

• Inference on location of distribution of 2 independent random samples X and Y (e.g. from control and treatment population).
• Assume $X \sim Y + \Delta$
• $H_0: \Delta = 0$ vs. $H_1: \Delta \neq 0$
• Rank all $N = n_1 + n_2$ observations
• $W =$ sum of ranks assigned to the Y’s (or X’s, whichever has smaller sample size)
• Reject $H_0$ if $W$ is extreme
Mann-Whitney U test

• Equivalent to Wilcoxon rank sum test
• Compare each $x_i$ with each $y_i$.
• There are $n_x \times n_y$ such comparisons
• $U =$ number of pairs in which $x_i < y_i$.
• $I_{cbs} W = U + (n^*(n+1))/2$ (when no ties)
• Reject $H_0$ if $U$ is extreme.
Boxplots of times to failure for control and stressed capacitors

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S-Plus wilcox.test

> wilcox.test(cg, sg)

Exact Wilcoxon rank-sum test

data:  cg and sg
rank-sum statistic W = 95, n = 8, m = 10, p-value = 0.1011
alternative hypothesis: true mu is not equal to 0
S-Plus parametric t-test

> t.test(cg, sg)

Standard Two-Sample t-Test

data: cg and sg
t = 1.8105, df = 16, p-value = 0.089
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  -1.103506  14.018506
sample estimates:
  mean of x  mean of y
  15.53750   9.079995

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Kolmogorov-Smirnov Tests

The Kolmogorov-Smirnov test detects differences in location, scale, skewness, or whatever (any differences between two distributions), uses two empirical cumulative distribution functions (step functions).

There is also a one-sample version for testing the distance between some observed data and a specified (ideal) distribution.

Tests the maximum gap between the observed distribution and the hypothesized distribution as a function of sample size (tables or p-values).
Histograms of 100 random normal (2,1) deviates and 100 random gamma(4,2) deviates

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Kolmogorov-Smirnov Tests

> ks.gof(x,y)

Two-Sample Kolmogorov-Smirnov Test
data:  x and y
ks = 0.15, p-value = 0.2112
alternative hypothesis: cdf of x does not equal the
cdf of y for at least one sample point.

> ks.gof(y)

One sample Kolmogorov-Smirnov Test of Composite Normality
data:  y
ks = 0.0969, p-value = 0.0216
alternative hypothesis: True cdf is not the normal distn. with
estimated parameters
sample estimates:
mean of x standard deviation of x
1.865857  0.9421928

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Kruskal-Wallis Test

- Inference for several independent samples
- Assume distributions of each of the samples differ only possibly in location.
- \(X_{ij} = \theta + \tau_j + e_{ij}\).
- \(H_0: \tau_1=\tau_2=..=\tau_k\), vs. \(H_1: \tau_i \neq \tau_j\) for some \(i \neq j\)
- Rank all \(N=n_1+n_2..+n_a\) observations.
- Calculate rank sums and averages in each group
- Calculate KW test statistic=kw (see text)
- Reject \(H_0\) for large values of kw
- For large \(n_i\)’s, null dist’n of kw \(\chi^2_{a-1}\)
Test scores for four different teaching methods
(page 582)

`scm<-matrix(score,7,4)`

> `scm`

```
[1,] 14.06 14.71 23.32 26.93
[2,] 14.26 19.49 23.42 29.76
[3,] 14.59 20.20 24.92 30.43
[4,] 18.15 20.27 27.82 33.16
[5,] 20.82 22.34 28.68 33.88
[6,] 23.44 24.92 32.85 36.43
[7,] 25.43 26.84 33.90 37.04
```

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Plot.factor(f(grp),score)

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Ranks of Test Scores

> scmr <- matrix(rank(score), 7, 4)
> scmr

[1,] 1   4.0 11.0   18
[2,] 2   6.0 12.0   21
[3,] 3   7.0 14.5   22
[4,] 5   8.0 19.0   24
[5,] 9  10.0 20.0   25
[6,] 13  14.5 23.0   27
[7,] 16  17.0 26.0   28

> tmp <- apply(scmr, 2, sum)
> tmp

[1] 49.0 66.5 125.5 165.0

> (12/(28*29))*sum((tmp^2)/7)-3*29
[1] 18.13406

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Kruskal-Wallis test in S-Plus

> kruskal.test(scm, col(scm))

Kruskal-Wallis rank sum test

data:  scm and col(scm)
Kruskal-Wallis chi-square = 18.139, df = 3,
p-value = 0.0004
alternative hypothesis: two.sided
ANOVA for test scores

summary(aov(score~f(grp)))

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum of Sq</th>
<th>Mean Sq</th>
<th>F Value</th>
<th>Pr(&quot;&gt;F&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(grp)</td>
<td>3</td>
<td>830.1914</td>
<td>276.7305</td>
<td>15.93607</td>
<td>6.50918e-006</td>
</tr>
<tr>
<td>Residuals</td>
<td>24</td>
<td>416.7609</td>
<td>17.3650</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Friedman Test

- Inference for several matched samples
- $a$ treatments, $b$ blocks
- $H_0: \tau_1 = \tau_2 = \ldots = \tau_k$, vs. $H_1: \tau_i \neq \tau_j$ for some $i \neq j$
- Rank observations separately within each block
- Calculate rank sums
- Calculate the Friedman statistic, $fr$ (see text)
- Reject $H_0$ for large values of $fr$
- For $b$ large, $fr \sim \chi^2_{a-1}$
Ranks within Blocks (rows)

> scmrb<-t(apply(scm,1,rank))
> scmrb
[1,]  1   2   3   4
[2,]  1   2   3   4
[3,]  1   2   3   4
[4,]  1   2   3   4
[5,]  1   2   3   4
[6,]  1   2   3   4
[7,]  1   2   3   4

> tmp<-apply(scmrb,2,sum)
> tmp
[1]  7 14 21 28

> (12/(4*7*5))*sum(tmp^2)-3*7*5
[1] 21

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Friedman test in S-Plus

• > friedman.test(scm, col(scm), row(scm))

• Friedman rank sum test

• data: scm and col(scm) and row(scm)
• Friedman chi-square = 21, df = 3, p-value = 0.0001
• alternative hypothesis: two.sided
ANOVA test score data with blocks

> summary(aov(score~f(grp)+f(blk)))

Df Sum of Sq  Mean Sq  F Value         Pr(F)
  f(grp)  3  830.1914 276.7305 260.4768 5.220000e-015
  f(blk)  6  397.6377  66.2729  62.3804 4.558276e-011
Residuals 18   19.1232   1.0624

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Correlation Methods

• Pearson Correlation: measures only linear association.
• Spearman Correlation: correlation of the ranks
• Kendall’s Tau: based on number of concordant and discordant pairs.
Kendall’s Tau

• Assume: the n bivariate observations \((X_1, Y_1), \ldots, (X_n, Y_n)\) are a random sample from a continuous bivariate population.

• \(H_0: X_i, Y_i\) are independent

• \(H_0: F(x,y) = F(x)F(y)\)

• Measure dependence by finding the number of concordant and discordant pairs.

• Population correlation coefficient:
  \[
  \tau = 2 \cdot P\{X_2 - X_1)(Y_2 - Y_1) > 0\} - 1
  \]
Kendall’s Tau

For $1 \leq i < j \leq n$:

$$Q((X_i, Y_i), (X_j, Y_j)) = \begin{cases} 1, & \text{if } (X_i - X_j)(Y_i - Y_j) > 0 \\ 0, & \text{if } (X_i - X_j)(Y_i - Y_j) = 0 \\ -1, & \text{if } (X_i - X_j)(Y_i - Y_j) < 0 \end{cases}$$

$$K = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Q((X_i, Y_i), (X_j, Y_j))$$

$$\hat{\tau} = \frac{2K}{n(n-1)}$$
Kendall’s Tau example

> m
  1  3  2  4
1 NA  1  1 1
2 NA NA -1  1
3 NA NA NA 1
4 NA NA NA NA

> 2*sum(m,na.rm=T)/12
[1] 0.6666667

> cor.test(c(1,2,3,4),c(1,3,2,4),method="k")

  Kendall's rank correlation tau

data:  c(1, 2, 3, 4) and c(1, 3, 2, 4)
  normal-z = 1.3587, p-value = 0.1742
  alternative hypothesis: true tau is not equal to 0
  sample estimates:
    tau
    0.6666667

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\begin{align*}
x &= 1:10 \\
y &= \exp(x)
\end{align*}
Pearson Correlation

> cor.test(x,y,method="p")

Pearson's product-moment correlation
data:  x and y
t = 2.9082, df = 8, p-value = 0.0196
alternative hypothesis: true coef is not equal to 0
sample estimates:
cor
0.7168704

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Spearman Correlation

> cor.test(x,y,method="s")

Spearman's rank correlation

data:  x and y
normal-z = 2.9818, p-value = 0.0029
alternative hypothesis: true rho is not equal to 0
sample estimates:
  rho
    1

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Kendall Correlation

> cor.test(x, y, method = "k")

Kendall's rank correlation tau

data: x and y
normal-z = 4.0249, p-value = 0.0001
alternative hypothesis: true tau is not equal to 0
sample estimates:
tau
1

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Example - Environmental Data – Censored below LOD

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Resampling Methods

- Parametric methods – Inference based on assumed population distribution
- Resampling methods – No assumption about functional form of population distribution.
- Permutation Tests – 2 sample problem
- Jackknife – Delete one observation at a time
- Bootstrap – resample with replacement
Permuation Tests

• Goal: estimate difference in means (2 sample problem)
• \((x_1, x_2\ldots x_{n_1})\) and \((y_1, y_2\ldots y_{n_2})\) are independent samples drawn from \(F_1\) and \(F_2\).
• \(H_0: F_1=F_2 \Rightarrow\) all assignments of labels \(x\) and \(y\) equally likely.
• Choose SRS of size \(n_1\) from \(n_1+n_2\) observations and label as \(x\), label rest as \(y\).
• Calculate value of test statistic (e.g. difference in means) for each assignment \(\Rightarrow\) permutation distribution.
• There are \((n_1+n_2)\) choose \((n_1)\) possible distinct assignments (capacitor data set Ex14.7, \(n_1=8, n_2=10\), number of assignments=43,758)
Jackknife

- Goal: estimate distribution and standard error of statistic (e.g. median or mean)
- Draw \( n \) samples of size \( n-1 \) from original sample, by deleting one observation at a time.
- Calculate \( m_j^* \) = mean (median) from each sample

\[
JSE(m) = \sqrt{\frac{n-1}{n} \sum_{j=1}^{n} (m_j^* - \bar{m}^*)^2}
\]

- JSE is exact for mean, not necessarily very good for median
Bootstrap

• Goal: estimate distribution, standard error, confidence interval of statistic (e.g. mean, median, correlation)

• Draw B samples of size n, with replacement, from original sample

• Calculate test statistics from each sample

$$BSE(m) = \sqrt{\frac{\sum_{j=1}^{B} (m_j^* - \bar{m}^*)^2}{B - 1}}$$
Swiss Data Set in S-Plus

Fertility Data for Switzerland in 1888
SUMMARY:
The swiss.fertility and swiss.x data sets contain fertility data for Switzerland in 1888.

ARGUMENTS:

swiss.fertility
standardized fertility measure l[g] for each of 47 French-speaking provinces of
Switzerland in approximately 1888.

swiss.x
matrix with 5 columns that contain socioeconomic indicators for the provinces:
1) percent of population involved in agriculture as an occupation; 2) percent
of "draftees" receiving highest mark on army examination; 3) percent of
population whose education is beyond primary school; 4) percent of
population who are Catholic; and, 5) percent of live births who live less than
1 year (infant mortality).

SOURCE:
Unpublished data used by permission of Francine van de Walle. Population Study
Center, University of Pennsylvania, Philadelphia, PA.
Bootstrap estimates and CI for variance of education

> educ<-swiss.x[,3]
> var(educ)
[1] 92.45606

> educ.boot<-bootstrap(educ,var,trace=F)
> summary(educ.boot)
Call:
bootstrap(data = educ, statistic = var, trace = F)

Number of Replications: 1000

Summary Statistics:
   Observed    Bias  Mean    SE
t     var 92.46 -0.5972 91.86 39.14

Empirical Percentiles:
   2.5%   5%  95% 97.5%
t     var 29.98 36.26 165.3   175

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Histogram of variance estimates obtained from 1000 bootstrap samples

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QQ plot of variance estimates

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Plot of LSAT scores by GPA for a sample of 15 schools

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Bootstrap estimates and CI for correlation between LSAT and GPA

> law.boot <- bootstrap(law.data, cor(lsat, gpa), trace = F)
> summary(law.boot)

Call:
bootstrap(data = law.data, statistic = cor(lsat, gpa), trace = F)

Number of Replications: 1000

Summary Statistics:

<table>
<thead>
<tr>
<th>Param</th>
<th>Observed</th>
<th>Bias</th>
<th>Mean</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Param</td>
<td>0.7764</td>
<td>-0.00506</td>
<td>0.7713</td>
<td>0.1368</td>
</tr>
</tbody>
</table>

Empirical Percentiles:

<table>
<thead>
<tr>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Param</td>
<td>0.449</td>
<td>0.5133</td>
<td>0.947</td>
</tr>
</tbody>
</table>

BCa Confidence Limits:

<table>
<thead>
<tr>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Param</td>
<td>0.2623</td>
<td>0.4138</td>
<td>0.9232</td>
</tr>
</tbody>
</table>

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Histogram of correlation estimates obtained from 1000 bootstrap samples

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S-Plus Stack-loss data set

- **Stack-loss Data**
- **SUMMARY:**
  - The stack.loss and stack.x data sets are from the operation of a plant for the oxidation of ammonia to nitric acid, measured on 21 consecutive days.
- **ARGUMENTS:**
  - **stack.loss**
    - percent of ammonia lost (times 10).
  - **stack.x**
    - matrix with 21 rows and 3 columns representing air flow to the plant, cooling water inlet temperature, and acid concentration as a percentage (coded by subtracting 50 and then multiplying by 10).
- **SOURCE:**
Summary of stack loss regression

```r
> summary(tmp)

Call: lm(formula = stack.loss ~ Air.Flow + Water.Temp + Acid.Conc., data = stack)
Residuals:
     Min      1Q  Median       3Q      Max
-7.2380 -1.7120 -0.4551  2.3610  5.6978

Coefficients:                  Value  Std. Error  t value  Pr(>|t|)
(Intercept)          -39.9197   11.8960   -3.3557   0.0038
Air.Flow             0.7156     0.1349    5.3066   0.0001
Water.Temp           1.2953     0.3680    3.5196   0.0026
Acid.Conc.           -0.1521     0.1563   -0.9733   0.3440

Residual standard error: 3.243 on 17 degrees of freedom
Multiple R-Squared: 0.9136
F-statistic: 59.9 on 3 and 17 degrees of freedom, the p-value is 3.016e-009

Correlation of Coefficients:
                   (Intercept)  Air.Flow  Water.Temp
(Intercept)     1.0000000  0.1793334
Air.Flow        0.1793334  1.0000000
Water.Temp      -0.1488626 -0.7355857
Acid.Conc.      -0.9016442 -0.3389504
```

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Summary of stack loss bootstrap output

```r
summary(stack.boot)
Call:
bootstrap(data = stack, statistic = coef(lm(stack.loss ~ Air.Flow
 + Water.Temp + Acid.Conc., stack)), trace = F)

Number of Replications: 1000

Summary Statistics:

<table>
<thead>
<tr>
<th></th>
<th>Observed</th>
<th>Bias</th>
<th>Mean</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-39.9197</td>
<td>0.5691</td>
<td>-39.3505</td>
<td>9.3731</td>
</tr>
<tr>
<td>Air.Flow</td>
<td>0.7156</td>
<td>0.0017</td>
<td>0.7173</td>
<td>0.1777</td>
</tr>
<tr>
<td>Water.Temp</td>
<td>1.2953</td>
<td>-0.0265</td>
<td>1.2688</td>
<td>0.4798</td>
</tr>
<tr>
<td>Acid.Conc.</td>
<td>-0.1521</td>
<td>-0.0007</td>
<td>-0.1528</td>
<td>0.1261</td>
</tr>
</tbody>
</table>

Empirical Percentiles:

<table>
<thead>
<tr>
<th></th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-56.0109</td>
<td>-53.422</td>
<td>-21.9299</td>
<td>-18.7526</td>
</tr>
<tr>
<td>Air.Flow</td>
<td>0.3903</td>
<td>0.4366</td>
<td>1.0026</td>
<td>1.04605</td>
</tr>
<tr>
<td>Water.Temp</td>
<td>0.4004</td>
<td>0.5131</td>
<td>2.0738</td>
<td>2.23633</td>
</tr>
<tr>
<td>Acid.Conc.</td>
<td>-0.4285</td>
<td>-0.3740</td>
<td>0.0328</td>
<td>0.05912</td>
</tr>
</tbody>
</table>
```

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Summary of stack loss bootstrap output

summary(stack.boot)

BCa Confidence Limits:

<table>
<thead>
<tr>
<th></th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-55.6465</td>
<td>-52.6606</td>
<td>-21.451125</td>
<td>-18.55810</td>
</tr>
<tr>
<td>Air.Flow</td>
<td>0.3266</td>
<td>0.4120</td>
<td>0.992007</td>
<td>1.01855</td>
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<tr>
<td>Water.Temp</td>
<td>0.5244</td>
<td>0.6193</td>
<td>2.264165</td>
<td>2.40956</td>
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<tr>
<td>Acid.Conc.</td>
<td>-0.4629</td>
<td>-0.4101</td>
<td>-0.007724</td>
<td>0.04459</td>
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</table>

Correlation of Replicates:

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<th></th>
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</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1.00000</td>
<td>-0.17636</td>
<td>0.09902</td>
<td>-0.80236</td>
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<tr>
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<td>-0.78822</td>
<td>1.00000</td>
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<tr>
<td>Acid.Conc.</td>
<td>-0.80236</td>
<td>-0.07635</td>
<td>-0.24463</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

E Newton

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Histories of regression coefficients

(Intercept)

Air.Flow

Water.Temp

Acid.Conc.

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QQ Plots of regression coefficients

(Intercept)

Air.Flow

Water.Temp

Acid.Conc.

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