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Abstract

The conditions under which seismic interferometry (SI) leads to the exact Green's function (GF) are rarely met in practice, resulting in errors in the recovered GF. To alleviate this problem, we employ additional information than what is typically used in SI. This information comes from the collection of crosscorrelated traces, one for each source for a pair of receivers, which we shall refer to as the crosscorrelogram. It is by stacking the crosscorrelogram in the source dimension that we obtain an interferometric GF. In general, this crosscorrelogram has both stationary energy that contributes to the estimated GF and non-stationary energy that does not. Stationary energy in the crosscorrelogram is characterized by linearity, coherency, low wavenumber, and thus nearly in-phase events along the source dimension. Non-stationary energy by contrast is characterized by non-linearity, incoherency, high wavenumber, and out-of-phase events along the source dimension. We exploit these differences to separate the two parts of the energy in the crosscorrelogram to obtain more accurate GF estimates for non-ideal cases.

In order to perform this separation and extract more information from the crosscorrelograms we use the singular value decomposition (SVD). We find that SVD is able to enhance physical arrivals that are not properly recovered using standard stacking in SI and in many cases to recover arrivals that would otherwise be obscured by noise. Here, we filter the crosscorrelograms by using a lower-rank approximation, computed with SVD by keeping only the largest singular values, to enhance events that are coherent across multiple sources, thus isolating this stationary energy that gives the primary contribution to the GF. We illustrate this method with synthetic results for both homogeneous and scattering media simulating a possible application in microseismic monitoring with downhole receivers.

INTRODUCTION

The conditions under which seismic interferometry (SI) leads to the exact Green's function (GF) are rarely met in practice. As a result, we generally recover only estimates of the true GF. This raises the questions: How good an approximation to the GF can SI give? Can we improve this estimated GF?

To recover the full GF between two receivers using SI requires that these two receivers be surrounded by a surface of sources, with both monopole and dipole sources required for accurate amplitude estimates. Since dipole sources are rarely available in practice, here we focus primarily on recovering traveltimes. Accurate estimation of these traveltimes still requires full (monopole) source coverage, however, an assumption that is rarely met in practice. This results in a degradation of the quality of the recovered GF, which then needs to be carefully interpreted. In the ideal case, Snieder (2004) showed that the sources that give the main contribution to the causal and anti-causal GFs are the ones located along the ray path between the two receivers, and those in the Fresnel zone around these sources. Snieder came to this conclusion using the method of stationary phase;

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the sources along the raypath are the sources at which the phase is stationary. Energy emanated by sources outside the Fresnel zone should cancel out, again assuming full source coverage, as they are outside the zone in which the phase is stationary.

When the coverage is not ideal, or when the source/receiver locations and raypaths are not well known this non-stationary energy will not cancel resulting in errors in the recovered GF. To alleviate this problem, we employ additional information than is typically used. This information comes from the collection of crosscorrelated traces, one for each source for a pair of receivers, which we shall refer to as the crosscorrelogram. It is by stacking the crosscorrelogram in the source dimension that we obtain an interferometric GF. In general, this crosscorrelogram has both stationary energy, that should contribute to the estimated GF and non-stationary energy that should not. Stationary energy in the crosscorrelogram is characterized by linearity, coherency, low wavenumber, and thus nearly in-phase events along the source dimension. Non-stationary energy by contrast is characterized by non-linearity, incoherency, high wavenumber, and out-of-phase events along the source dimension. We exploit these differences to separate the two parts of the energy in the crosscorrelogram to obtain more accurate GF estimations for non-ideal cases.

In order to perform this separation and extract more information from the crosscorrelograms to ultimately improve the GF, we follow Melo et al. (2010) in which the singular value decomposition (SVD) (see e.g. Golub and van Loan (1996)) is used to do this separation. SVD is a numerical technique commonly used in seismic data processing (see e.g. Ulrych et al. (1988); Sacchi et al. (1998)), to increase the signal to noise ratio and filter linear events. Melo et al. (2010) showed how SVD is able to enhance physical arrivals that are not properly recovered using standard stacking in SI and generally recover arrivals that would otherwise be obscured by noise. Here we further investigate the relationship between SVD and SI in the microseismic context discussed below.

To understand why SVD is able to separate stationary and non-stationary energy we must first understand the relationship between frequency and singular values. This relationship is discussed by Hansen et al. (2006) where they explain the relationship between singular values and frequency (or source-wavenumber in our case) - large singular values correspond to low frequencies and small singular values correspond to high frequencies. As they correspond to low-frequencies, large singular values are associated with events that are in phase in the crosscorrelogram: stationary sources whose energy contribute to the GF. In the context of waveguides, Philippe et al. (2008) exploit the connection between singular values and frequency for characterization of targets. They show that the first singular value associated with a given target is proportional to the backscattering form function of the target, and that the second singular value is proportional to the second derivative of the angular form function. Then, they use SVD to extract the backscattered frequency signature of a target in a waveguide. Here, we filter the crosscorrelograms by using a lower-rank approximation, computed with SVD using the largest singular values, to enhance events that are coherent across multiple sources, thus isolating this stationary energy. In this way, we exploit the fact that stationary signal is at lower wavenumbers than non-stationary signal to separate it from non-stationary signal. We illustrate this method with synthetic results for both homogeneous and scattering media simulating a possible application with downhole receivers.

These examples are meant to illustrate the particular application we have in mind for this technique, which is the estimation of the GF between two sources in a geothermal reservoir. While most applications of SI estimate the GF between two receivers surrounded by sources, Curtis et al. (2009) show, using reciprocity, that it is also possible to use SI to estimate the GF between a pair of sources. In the microseismic context this would greatly increase the available data set as there

are generally few receivers and many sources. In addition, as is well known, noise is a major issue with microseismic data. Noise contaminated data lead to poor event locations, which creates uncertainty as to which receivers are in the Fresnel zone for a given pair of sources. Simply summing the responses from all receivers will not solve this problem because the receiver array is generally sparse. Here we show how using SVD to decompose the crosscorrelogram before stacking helps to alleviate this problem. In addition to these properties, we find that SVD also allows some level of separation of the GF into different components - main arrivals (direct, singly reflected, and refracted waves), multiple scattering, and noise. To separate signal from noise directly in the GF is difficult, especially for coda waves, because the noise may have comparable amplitude and temporal frequency content to the coda. Doing this separation is important because coda waves contain information about the inhomogeneities in the medium, while noise does not. We show preliminary results illustrating that it may be possible to extract information about these different components in the crosscorrelated traces before stacking them to form the GF.

METHOD

We now consider the crosscorrelogram as a matrix, C, where each row is the crosscorrelation of the signals at the two receivers from one source. Thus, the vertical dimension of C is source and the horizontal is time, as shown in figure 1.
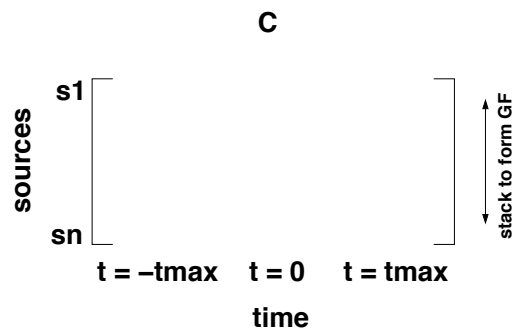

Figure 1: Crosscorrelogram matrix C. Stacking over sources gives interferometric GF.

Next, we decompose the crosscorrelogram using SVD (see e.g. Golub and van Loan (1996) for a description of SVD). The SVD decomposition of the crosscorrelogram matrix is, $C = U \Sigma V^t$, where U and V are the left and right singular vector matrices, and Σ is the diagonal matrix whose elements are the singular values of C. Figure 2 shows how we obtain a lower-rank approximation $\hat{C}$ of the crosscorrelogram by selecting only the largest singular values of the SVD decomposition of C. Stacking the rows of C gives the standard interferometric GF, G, and stacking the rows of the approximation $\hat{C}$ gives the modified interferometric GF, G_j, where j is the rank of $\hat{C}$ (the number of singular values retained). In the examples that follow, we compare these two GFs.

We now illustrate this procedure with a simple example. The model for this example is a constant velocity and density model with no reflectors, so the GF consists of the direct wave only. We examine how well we can approximate the true GF in three cases: (i) the case where there are stationary sources only, (ii) non-stationary sources only, and (iii) both stationary and non-stationary. In all three cases there are gaps in the source coverage and all the GF are normalized as we do not have dipole sources.

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Figure 2:   Crosscorrelogram matrix C and its lower-rank approximation C obtained through SVD.

First, consider a case where there are sources only in the stationary-phase zone, figure 3(a). The energy from these sources contributes constructively to the GF. Comparing the standard, figure 3(b), and rank-2, figure 3(c), crosscorrelograms and the respective estimated GFs, G and G_2, figure 3(d)-(e), we see that the standard and the lower-rank crosscorrelograms and GFs are quite similar. We use a rank-2 approximation of the crosscorrelogram because there are two stationary-phase zones in the crosscorrelogram and thus two signals we wish to reconstruct. In this simple example, it is obvious what the rank of the crosscorrelogram approximation should be, which is not the case in general. This is a case where standard interferometry works well and the SVD technique is not necessary, although it is also not detrimental.

In case (ii) there are only non-stationary sources, figure 4(a). Ideally, all of this non-stationary energy should cancel but if there are gaps in the source coverage residual energy will remain because of edge effects. As is clear in figure 4(d), G is not a good estimate of the correct GF, but it appears as if it contains a physical arrival. While G, figure 4(d), contains two non-physical arrivals due to edge effects, G_2, figure 4(e), does not. The rank-2 crosscorrelogram, figure 4(c), in this case does not enhance any linearity and does not even resemble the original crosscorrelogram, figure 4(b). The rank-2 crosscorrelogram and G_2 thus act as a diagnostic of non-physical arrivals.

Case (iii) mixes the two previous cases. Figure 5(a) shows sources uniformly distributed in each stationary zone and each non-stationary zone, but with gaps in between. The crosscorrelogram, figure 5(b)-(c), thus has energy contributing to the GF and energy that should cancel out completely but, because of the gaps, does not. The rank-2 approximation filters the pseudo-noise caused by the imperfect cancellation of non-stationary energy, and G_2 is more accurate then G as seen in figures 5(d)-(e).

EXAMPLES

This example approximately mimics an idealized source/receiver geometry of a downhole monitoring of microseismic activity in a geothermal reservoir. We use a single borehole with 35 receivers and estimate the GF between two micro-quakes as shown in both figure 6(a) and figure 7(a). The reference and interferometic GFs shown here are all normalized. The idea is to obtain the interfer-
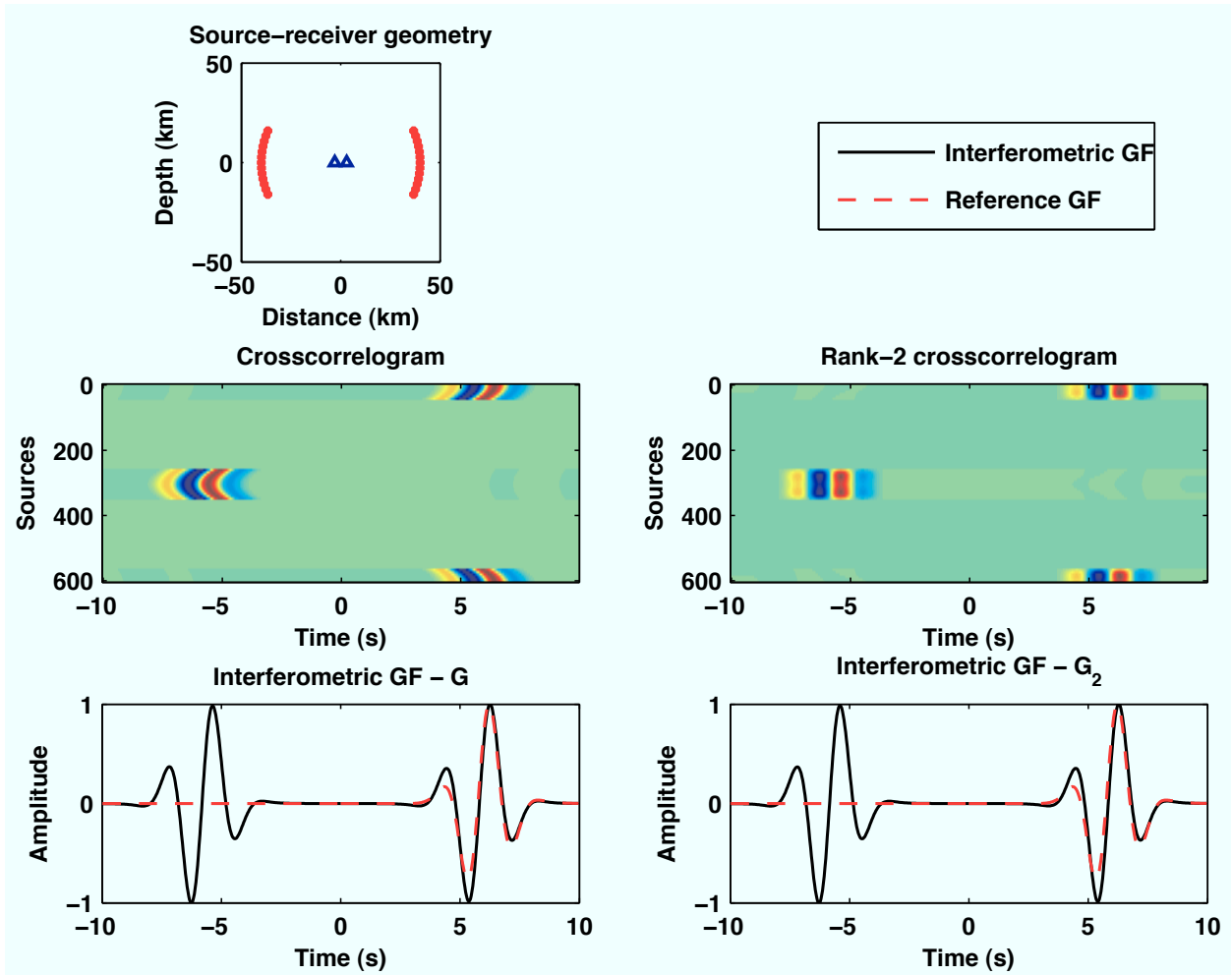
Figure 3: (a) source-receiver geometry with 13 evenly distributed sources (red stars) in each of the stationary zones of the receivers (blue triangles); (b) original crosscorrelogram; (c) rank-2 cross-correlogram; (d) standard interferometric GF, G; (e) rank-2 GF, G_2. The black line corresponds to the interferometric GFs and the red line to the true GF. The GF in (d) and (e) are similar.
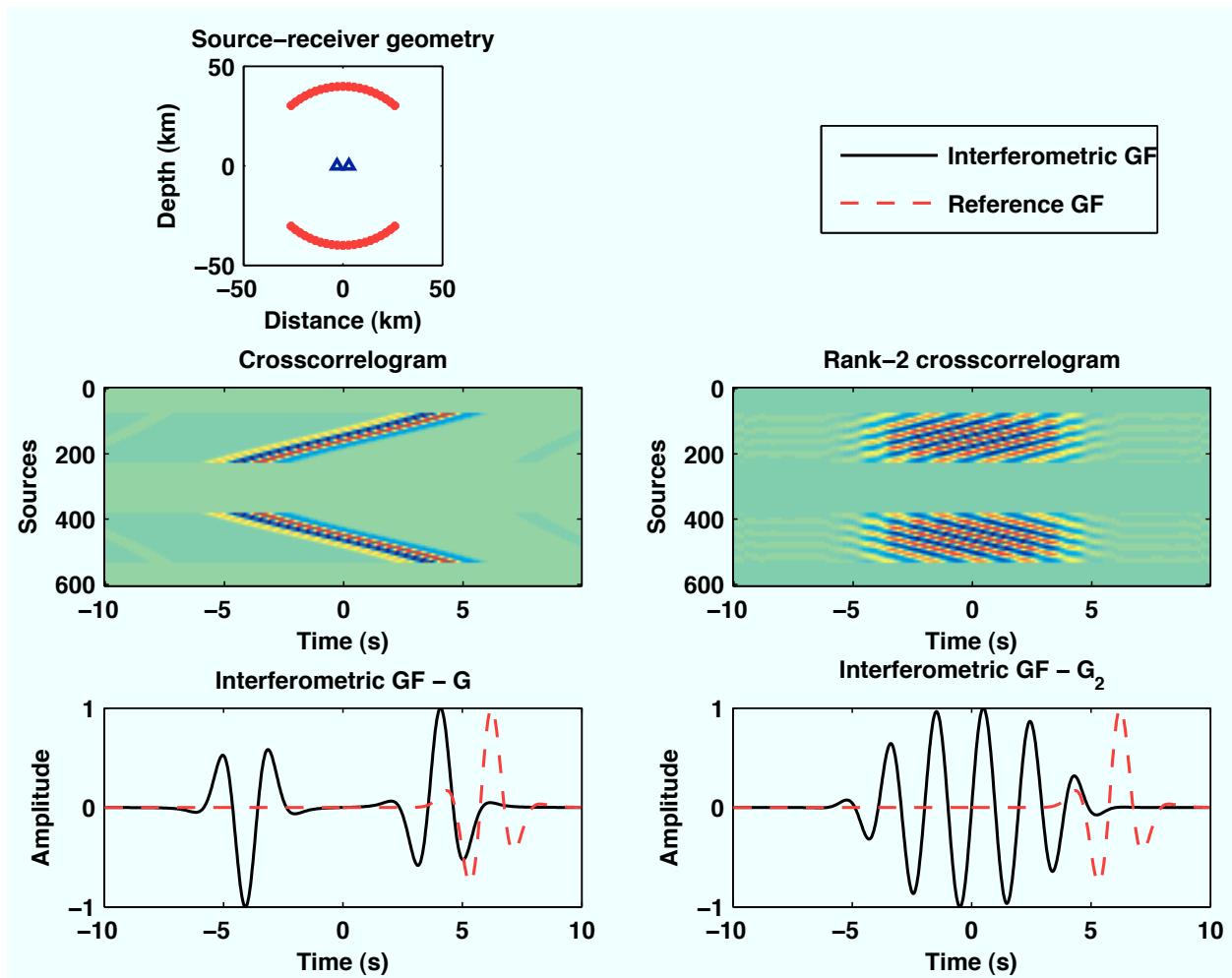
Figure 4: (a) source-receiver geometry with 21 evenly distributed sources (red stars) in each of the non-stationary zones of the receivers (blue triangles); (b) original crosscorrelogram; (c) rank-2 crosscorrelogram; (d) standard interferometric GF, G; (e) rank-2 GF, G_2. The black line corresponds to the interferometric GFs and the red line to the true GF. In (e) the GF does not contain the edge effect present in (d).
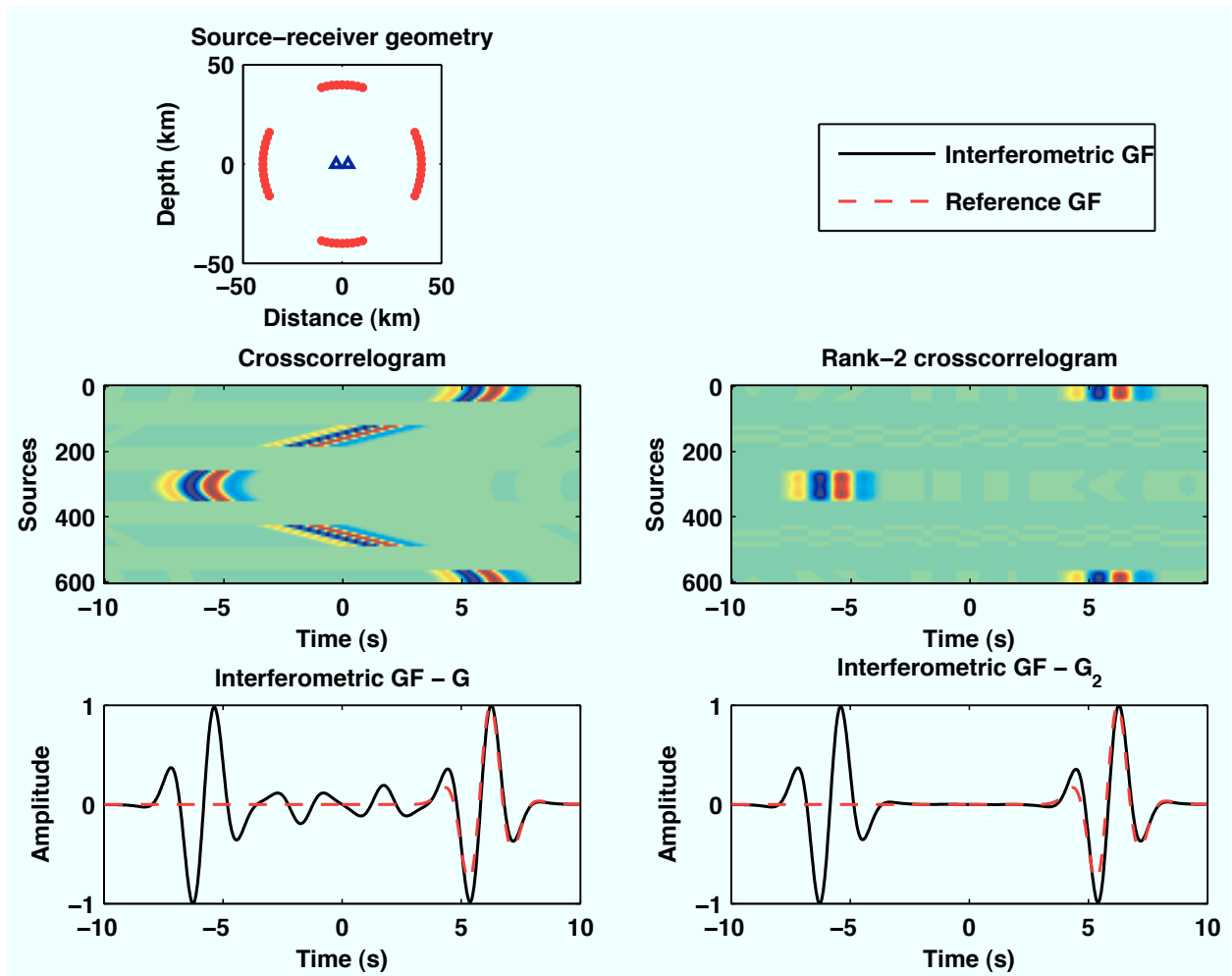
Figure 5: (a) source-receiver geometry with 13 and 9 evenly distributed sources (red stars) in each of the stationary and non-stationary zones of the receivers (blue triangles), respectively. (b) original crosscorrelogram; (c) rank-2 crosscorrelogram; (d) standard interferometric GF, G; (e) rank-2 GF, G_2. The black line corresponds to the interferometric GFs and the red line to the true GF. In (e) the fluctuations are reduced and the GF is clearer than in (d).

ometric GF between a pair of sources (micro-quakes) instead of a pair of receivers, similar to what is done in Curtis et al. (2009) for a larger scale problem. The medium is weakly scattering with a constant background velocity and density. We study two cases: first, we do the crosscorrelations in a clean dataset, figure 6, and second in a noisy dataset, figure 7. The additive noise and random scattering we use here are realizations of a Gaussian random field with prescribed correlation lengths along given directions. We focus our observations on two things: the phase of the direct wave and the energy in the coda. We added enough noise to completely obscure the direct wave and distort the waveform of the coda wave in the interferometric GF and show how SVD improves both of these measures.

In the first example, figure 6, even though the data are noise free, there is enough non-stationary energy in the crosscorrelogram from receivers outside the Fresnel zone, figure 6(b), to create high-amplitude fluctuations that hide the direct wave. Here we chose the rank-1 approximation, figure 6(c), because the GF consists only of one direct wave and thus there is only one zone of stationary-phase energy in the crosscorrelogram. In figure 6(e) the fluctuations were damped in G_1 as compared with G in figure 6(d), leading to a much clearer GF. Crosscorrelation of both GFs, G and G_1, with the reference GF peaks at 0.2835 and 0.0020 s (time sampling interval is 0.0005 s), respectively, showing a significantly more accurate phase estimate with G_1 than with G. We use the L_2-norm (square-root of sum of squares) of the coda waves as a measure of the energy in the coda. Comparing the reference and the interferometric coda waves, we find relative errors of 98% for the coda wave in G and only 12% for G_1, demonstrating that G_1 is a better approximating of the true GF than G, in this norm.

To make this example more realistic, we add weakly correlated noise to the data in figure 6. In figure 7(d) G appears strongly contaminated by noise and neither the direct arrival nor the coda wave are visible. In figure 7(e) fluctuations and random noise are strongly attenuated in G_1, revealing not only the direct arrival but also reducing the noise close to the coda wave level. The phase differences between G and G_1 and the reference GF are 0.2850 and 0.0015 s, respectively. The relative errors in the L_2-norm of the coda are 148% for G and 11% for G_1. We see that SVD eliminates most of the noise in the coda wave, as well as the fluctuations before the direct wave, demonstrating its stability with respect to noise.

For the noisy case, we performed tests for a variety of receiver apertures, noise levels, spacing between receivers. We find that the absolute improvement obtained through SVD varies from case to case but the phase of direct wave and the coda energy, in general, are closer to correct with SVD than without. This noise attenuation is particularly important in microseismic studies as the data are typically quite noisy. Stability with respect to aperture is also important because errors in the location of micro-quakes can be significant.

Discussions, conclusions, and future work

The accurate estimation of the GF with non-ideal source coverage remains a significant problem in SI. We have shown how using SVD to approximate crosscorrelograms before stacking is a promising approach to alleviate this problem. In general, for the SVD technique to work there must be more stationary energy than non-stationary energy in the crosscorrelogram, although this requirement can be relaxed somewhat through normalization of the traces in the crosscorrelogram. How much more energy is necessary and how much noise can be accommodated are subjects of ongoing research.
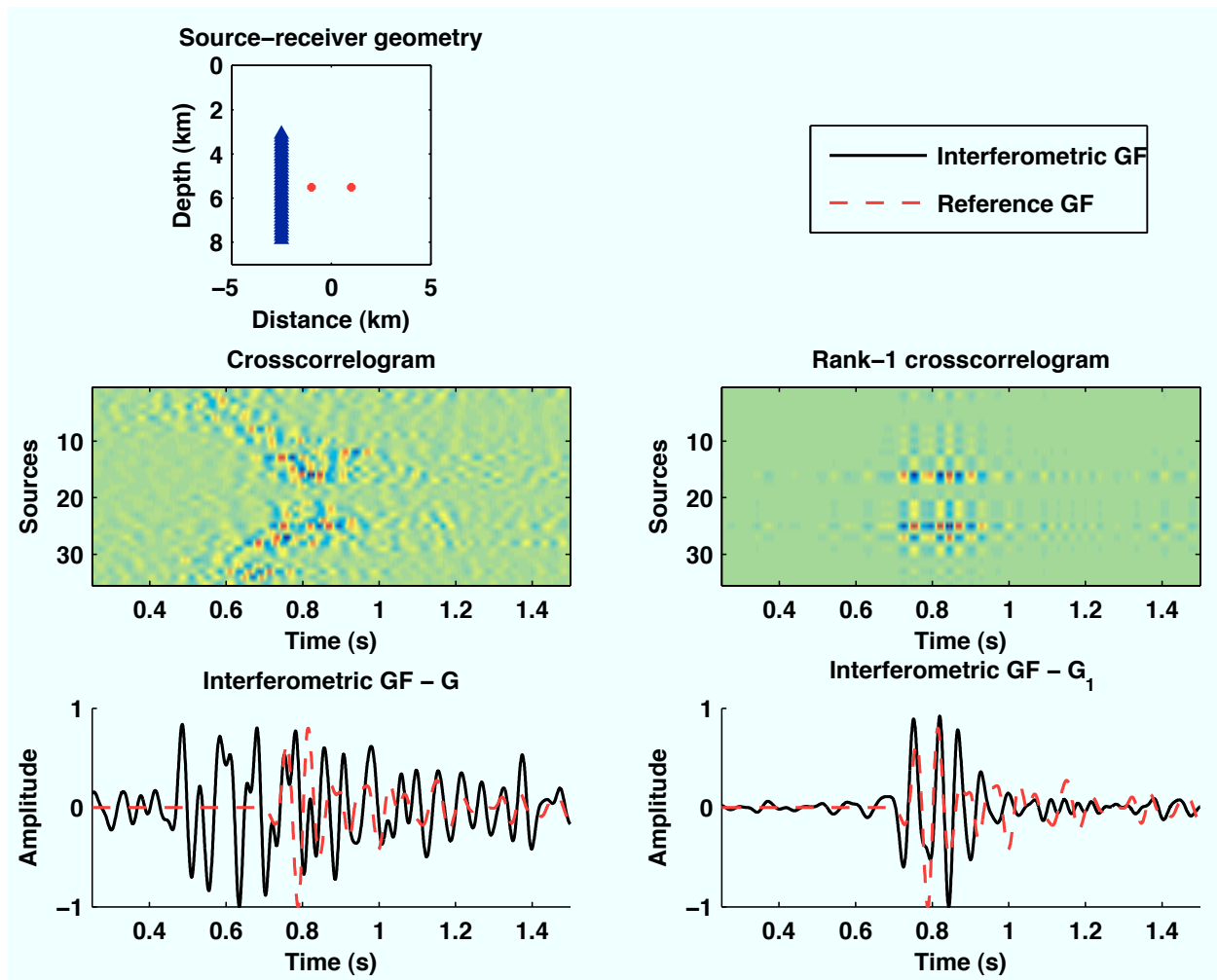
Figure 6: (a) source-receiver geometry: one borehole with 35 receivers (blue triangles) and two micro-quakes (red stars); (b) original crosscorrelogram; (c) rank-1 crosscorrelogram; (d) standard interferometric GF, G; (e) rank-1 GF, G_1. The black line corresponds to the interferometric GFs and the red line to the true GF. In (e) the fluctuations are reduced and the GF is clearer than in (d).
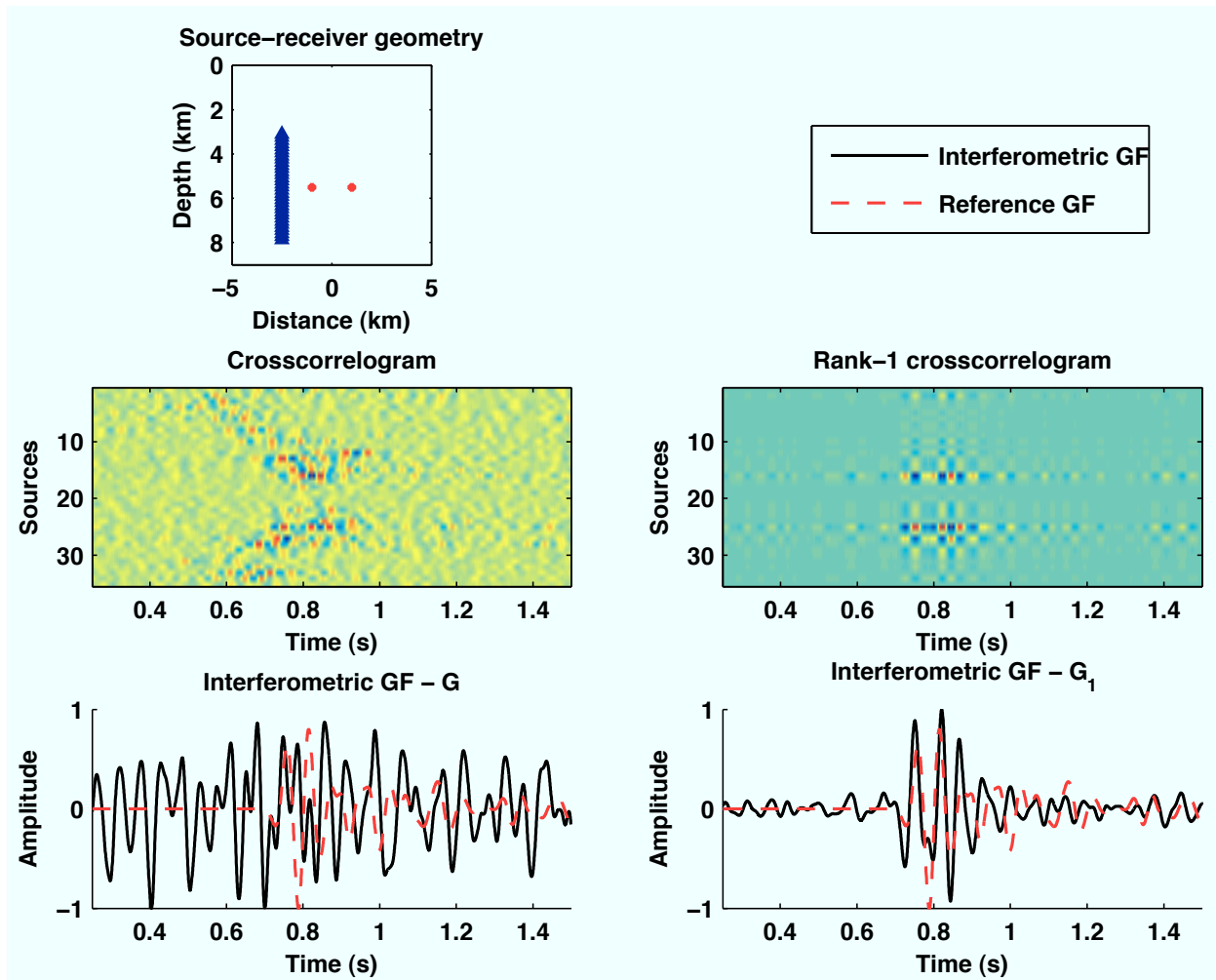
Figure 7: (a) source-receiver geometry: one borehole with 35 receivers (blue triangles) and two micro-quakes (red stars); (b) original crosscorrelogram; (c) rank-1 crosscorrelogram; (d) standard interferometric GF, G; (e) rank-1 GF, G_1. The black line corresponds to the interferometric GFs and the red line to the true GF. In (e) the random noise is reduced to the level of the coda and the GF is clearer than in (d).

We are also continuing to investigate which properties of coda waves can be accurately inferred from the GF obtained through SVD; the preliminary results shown here indicate that such properties are better recovered with SVD than stacking alone. Separating real signal from noise would lead to a coda that truly reflects the scattering characteristics of the medium thus allowing for the use of coda waves to retrieve information about the scattering strength and through this about fracture characteristics in a reservoir.

Acknowledgments

REFERENCES

Curtis, A., H. Nicolson, D. Halliday, J. Trampert, and B. Baptie, 2009, Virtual seismometers in the subsurface of the earth from seismic interferometry: Nature Geoscience, **2**, 700 – 704.

Golub, G., and C. van Loan, 1996, Matrix computations.

Hansen, P. C., M. E. Kilmer, and R. Kjeldsen, 2006, Exploiting residual information in the parameter choice for discrete ill-posed problems.: BIT, **46**, 41–59.

Melo, G., A. Malcolm, D. Mikessel, and K. van Wijk, 2010, Using svd for improved interferometric green's functions: SEG Technical Program Expanded Abstracts, **29**, 3986–3990.

Philippe, F. D., C. Prada, J. de Rosny, D. Clorennec, J.-G. Minonzio, and M. Fink, 2008, Characterization of an elastic target in a shallow water waveguide by decomposition of the time-reversal operator: The Journal of the Acoustical Society of America, **124**, 779–787.

Sacchi, M. D., T. J. Ulrych, and Magnuson, 1998, Eigen-image analysis of common offset sections: Signal-to-noise enhancement and pre-stack data compression.

Snieder, R., 2004, Extracting the green's function from correlation of coda waves: A derivation based on stationary phase.: Phys. Rev. E, **69**, 046610.

Ulrych, T. J., S. Freire, and P. Siston, 1988, Eigenimage processing of seismic sections: SEG Technical Program Expanded Abstracts, **7**, 1261–1265.