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SUSTAINABLE ENERGY

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RESOURCE EVALUATION AND DEPLETION ANALYSES

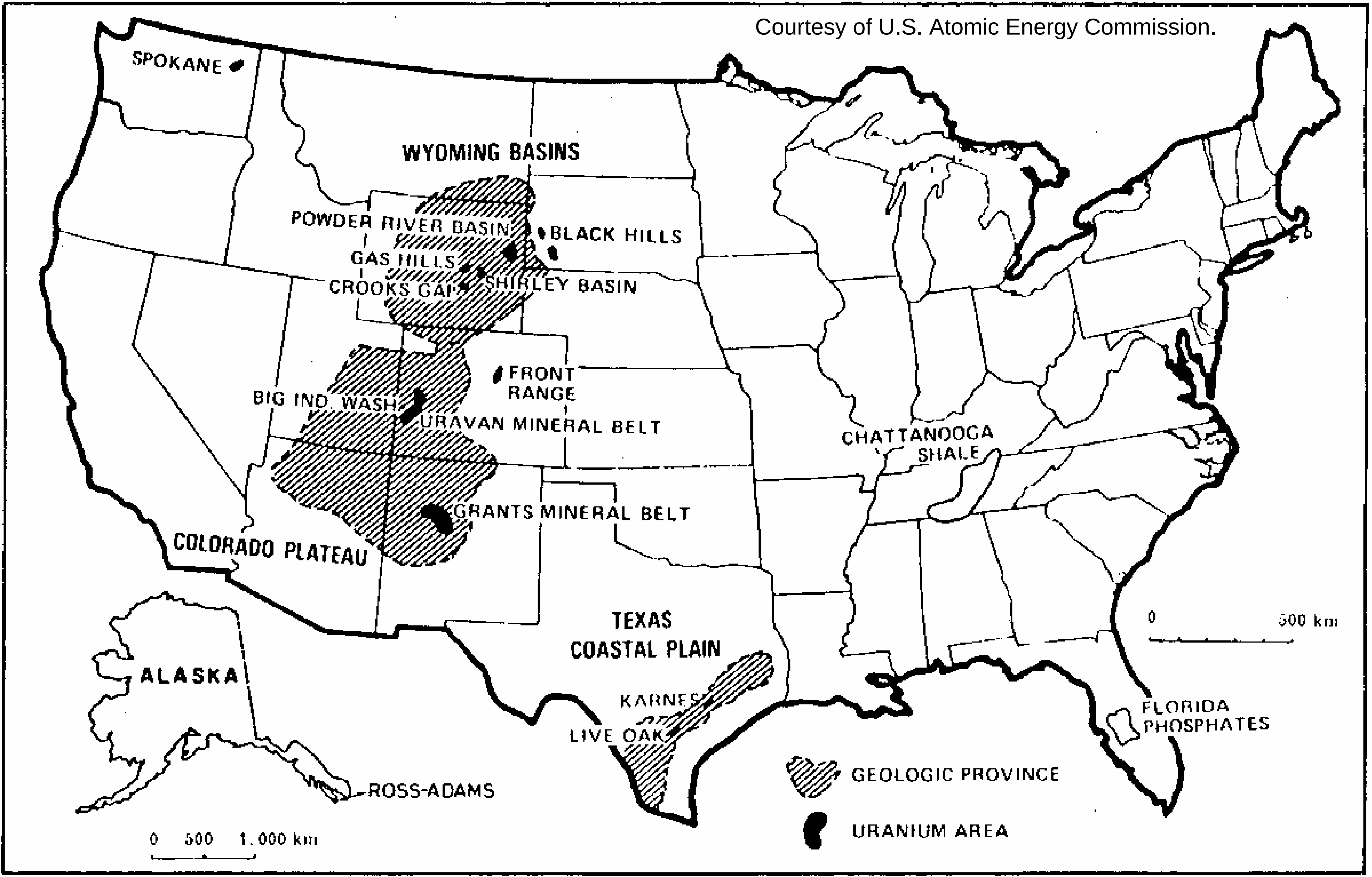
WAYS OF ESTIMATING ENERGY RESOURCES

- Monte Carlo
- “Hubbert” Method Extrapolation
- Expert Opinion (Delphi)

FACTORS AFECTING RESOURCE RECOVERY

- Nature of Deposit
- Fuel Price
- Technological Innovation
  - Deep drilling
  - Sideways drilling
  - Oil and gas field pressurization
  - Hydrofracturing
  - Large scale mechanization

URANIUM AREAS OF THE U.S.



Courtesy of U.S. Atomic Energy Commission.

MAJOR SOURCES OF URANIUM

Class 1 – Sandstone Deposits

	Share	$U^{3}O^{8}$ Concentration (Percent)	Tons $U^{3}O^{8}$
New Mexico	.49	0.25	Total
Wyoming	.36	0.20	315,000
Utah	.03	0.32	Š $10/lb
Colorado	.03	0.28	
Texas	.06	0.28	
Other	.03	0.28	
Class 2 – Vein Deposits			7,100
Class 3 – Lignite Deposits		0.01-0.05	1,200
Class 4 – Phosphate Rock		0.015	
Class 5 – Phosphate Rock Leached Zone (Fla.)		0.010	54,600
Class 6 – Chattanooga Shale		0.006	2,557,300
Class 7 – Copper Leach Solution Operations		0.0012	30,000
Class 8 – Conway Granite		0.0012-Uranium 0.0050-Thorium	1x10^{6} 4x10^{6}
Class 9 – Sea Water		0.33x10^{-6}	4x10^{9}

ESTIMATES OF URANIUM AVAILABILITY FROM GEOLOGICAL FORMATIONS AND OCEANS IN THE U.S.

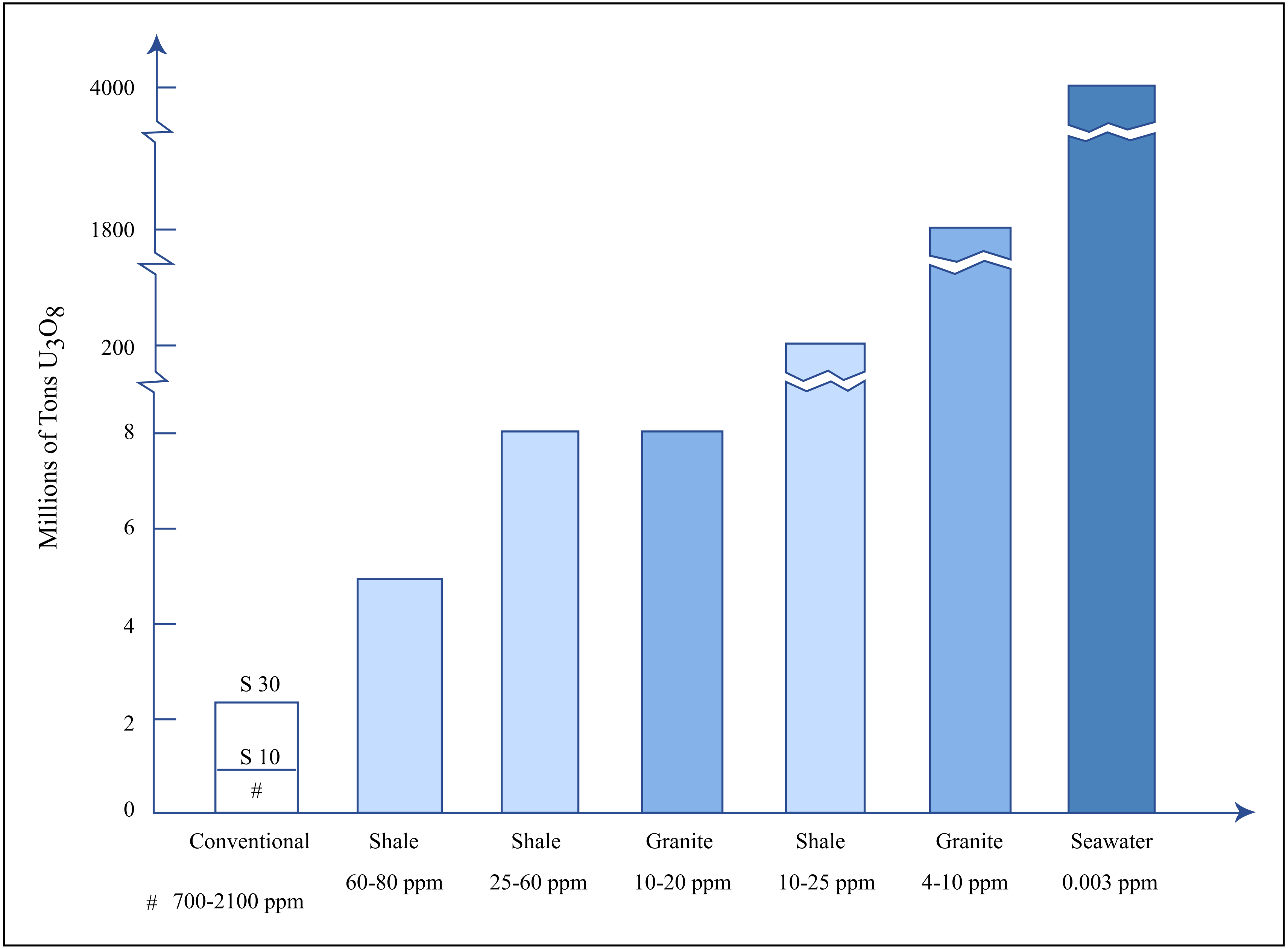


Figure by MIT OCW.

DECLINE IN GRADE OF MINED COPPER ORES SINCE 1925

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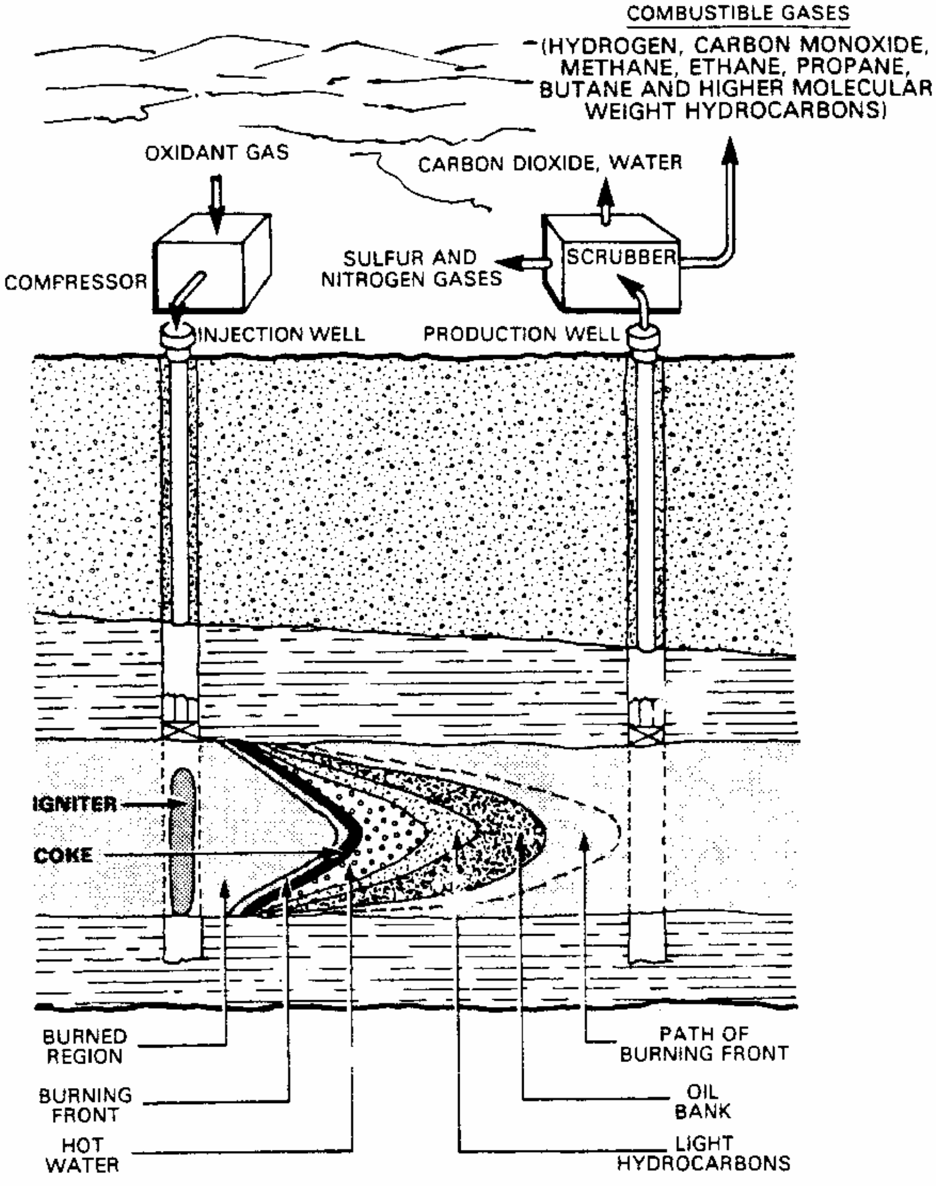
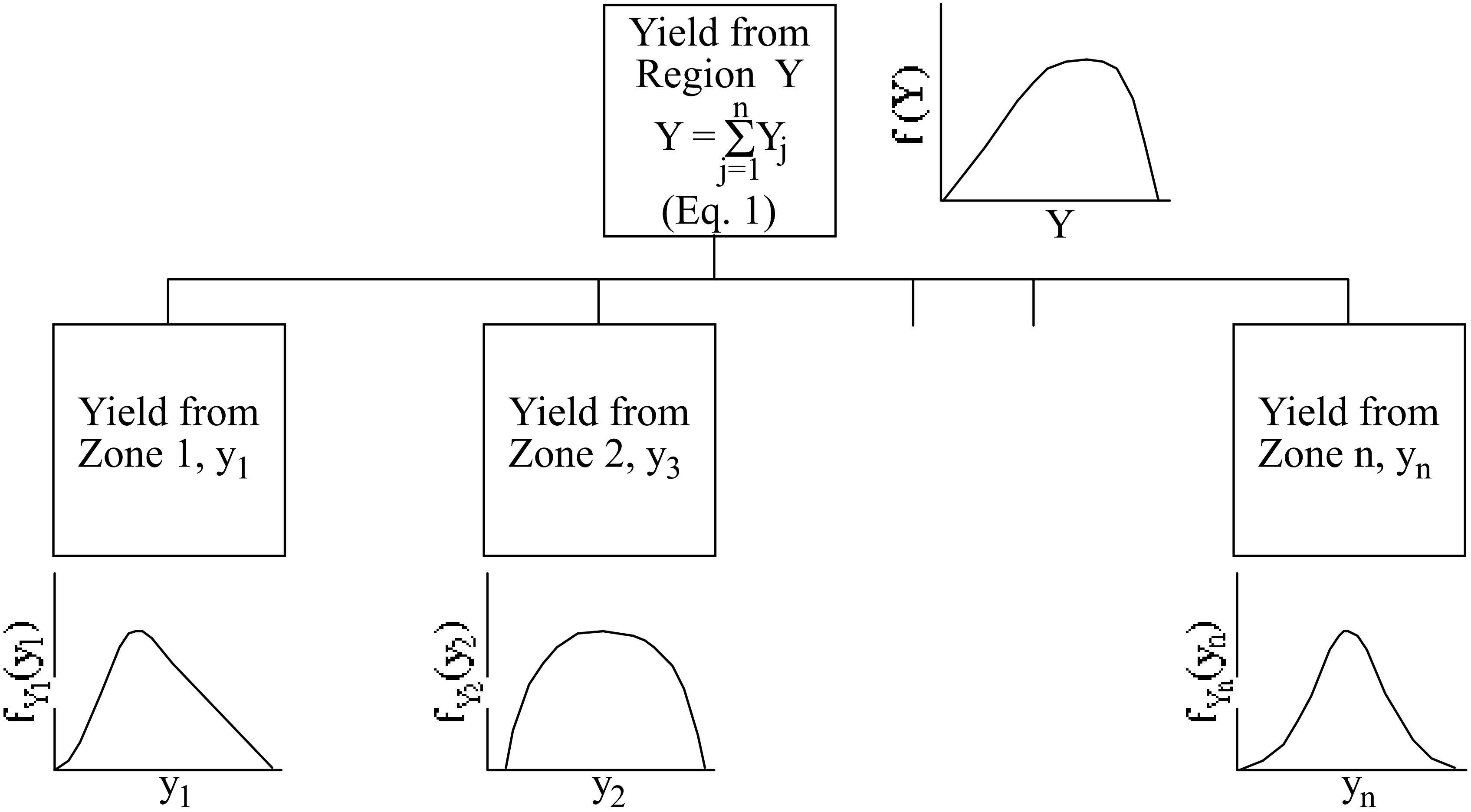
RECOVERY BY IN-SITU COMBUSTION



Source: U.S. Department of Energy, "Fossil Energy Research and Development Program of the U.S. Department of Energy, FY 1979," DOE/ET-0013(78), March 1978.

MONTE CARLO ESTIMATION



Yield from Region Y

$$Y = \sum_{j=1}^{n} Y_j$$

(Eq. 1)

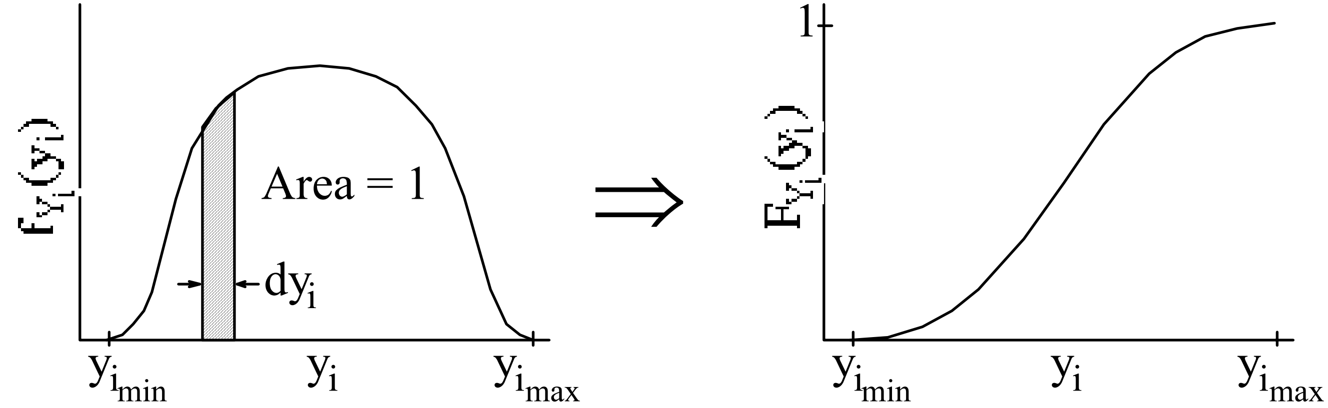
Yield from Zone 1, y_1

Yield from Zone 2, y_3

Yield from Zone n, y_n

Probability density functions are obtained subjectively, using information about deposit characteristics, fuel price, and technology used.

MONTE CARLO SAMPLING



$$\text{Prob.}\left(y_i < Y_i < y_i + \delta y_i\right) = f_{Y_i}\left(y_i\right) dy_i \quad \text{(Eq. 2)}$$

$$\text{Prob.}\left(Y_i < y_i\right) = F_{Y_i}\left(y_i\right)$$

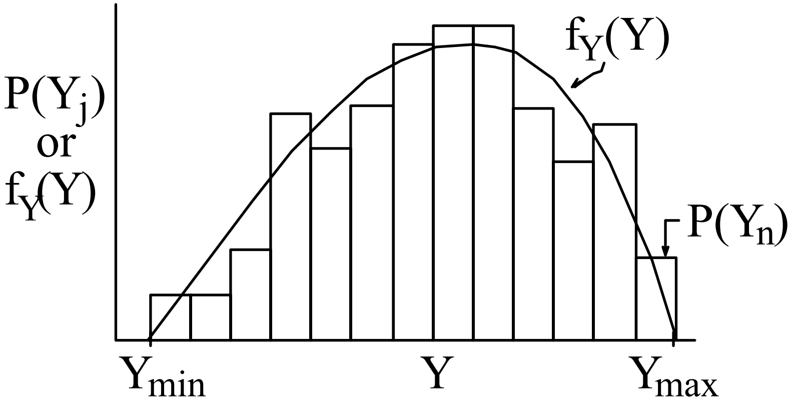
(Eq. 3)

$$= \int_{y_{i_{min}}}^{y_i} f_{Y_i}\left(y_i'\right) dy_i'$$

Consider Y_i to be a random variable within $\left[y_{i_{min}},\ y_{i_{max}}\right]$

MONTE CARLO SAMPLING, Continued

1. Utilize a random number generator to select a value of $F(y_i)$ within range [0, 1] ⇒ corresponding value of y_i (Eq. 3).
2. Repeat step 1 for all values of i and utilize selected values of $\overline{Y}_{i_1} = \left[y_{1_1}, \quad y_{2_1}, \quad \cdots, \quad y_{n_1} \right]$ to calculate a value of Y_1, (Eq. 1) (note Y is also a random variable).
3. Repeat step 2 many times and obtain a set of values of Y. Their distribution will approximate that of the variable Y as



KING HUBBERT ESTIMATION METHOD

CHARACTERISTICS OF MINERAL RESOURCE EXTRACTION

- As More Resource Is Extracted The Grade Of The Marginally Most Attractive Resources Decreases, Causing
  - Need for improved extraction technologies
  - Search for alternative deposits, minerals
  - Price increases (actually, rarely observed)

PHASES OF MINERAL RESOURCE EXTRACTION

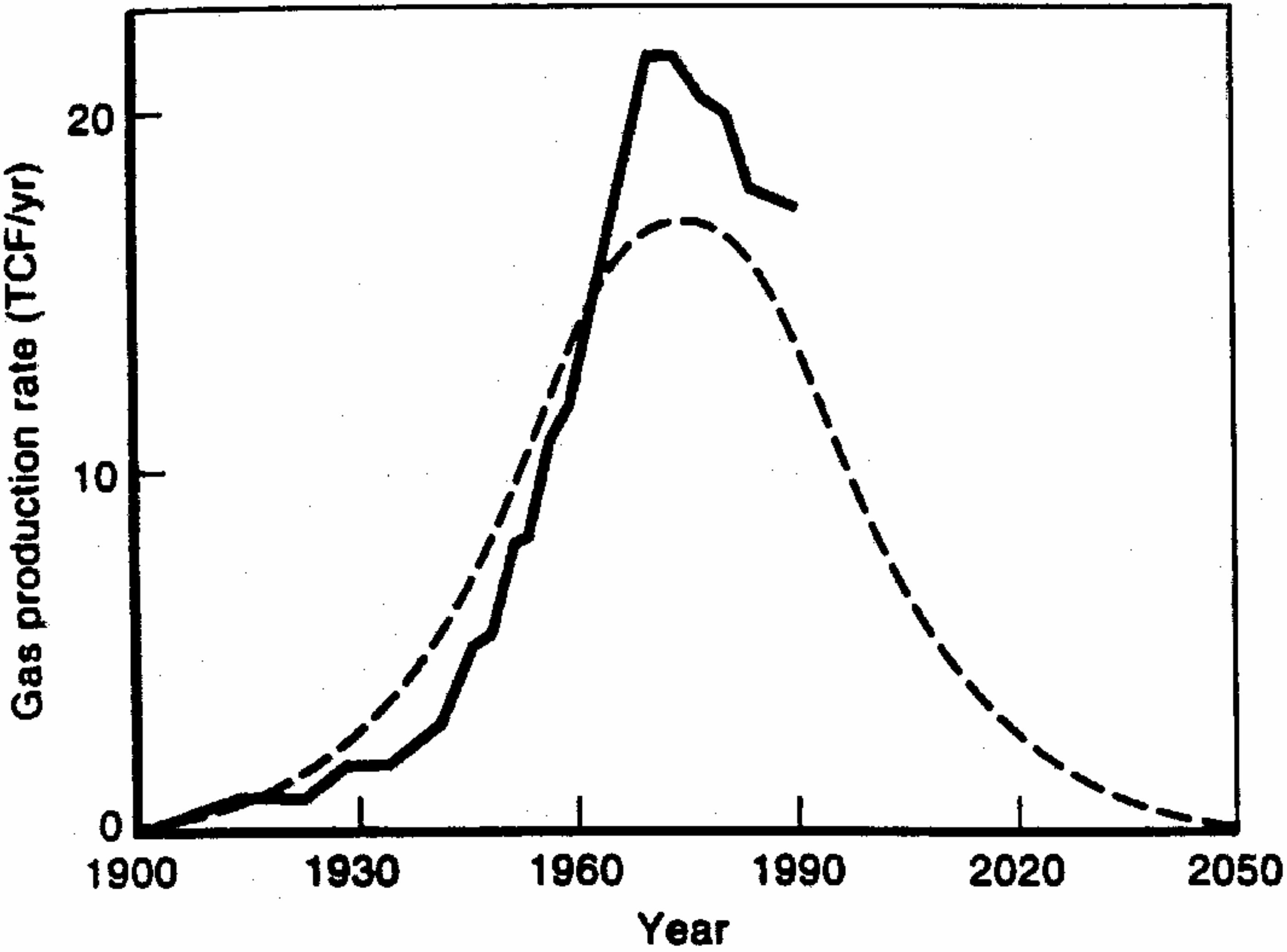
- Early: Low Demand, Low Production Costs, Low Innovation
- Growing: Increasing Demand And Discovering Rate, Production Growing With Demand, Start of Innovation
- Mature: Decreasing Demand And Discovery Rate, Production Struggling To Meet Demand, Shift To Alternatives
- Late: Low Demand, Production Difficulties, Strong Shift To Alternatives (rarely observed)

Graph removed for copyright reasons.

Natural Gas reserves, 1947-1980, from American Gas Association.

U.S. NATURAL GAS PRODUCTION

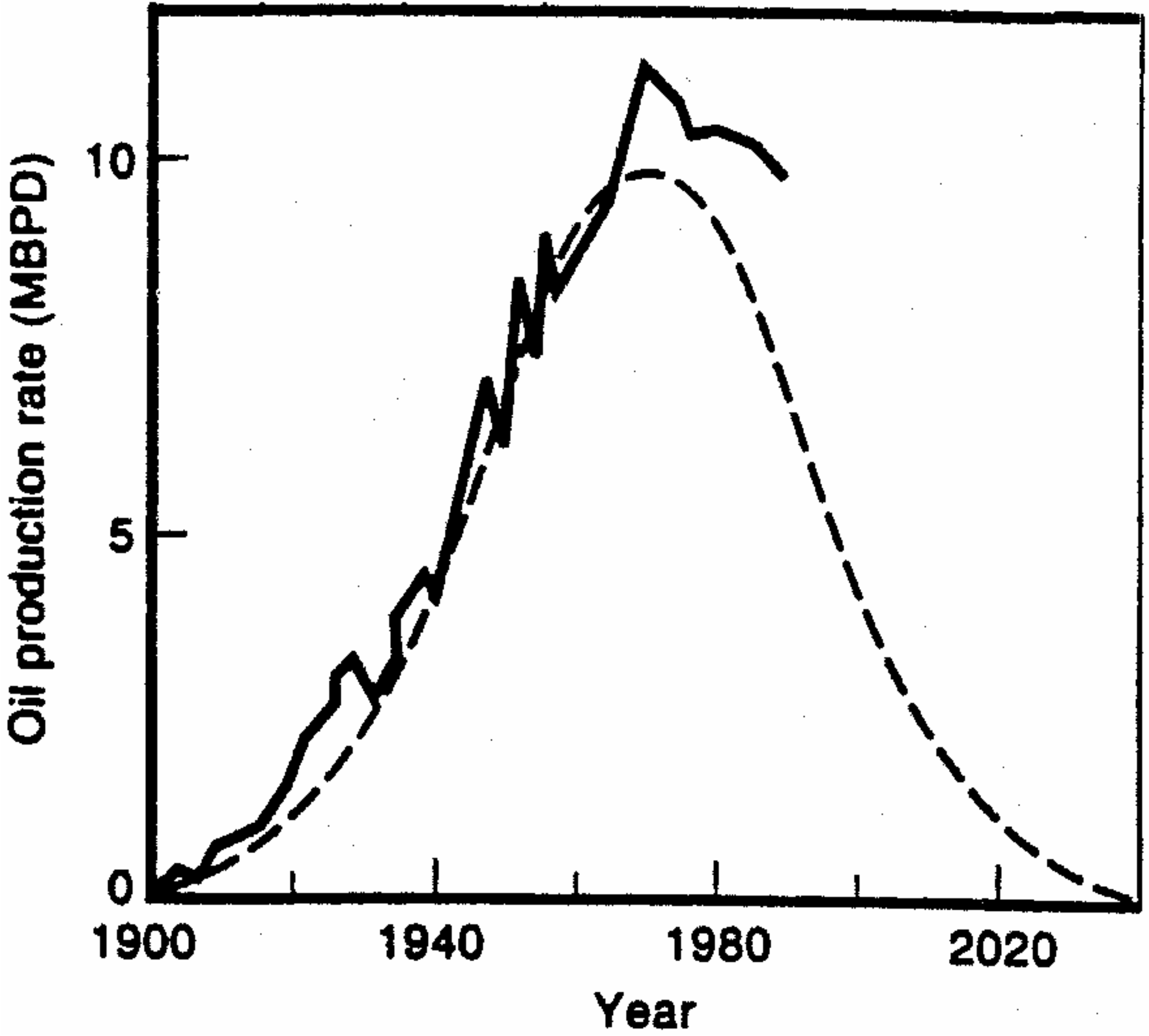
Courtesy of U.S. DOE.



Comparison of estimated (Hubbert) production curve and actual production (solid line).

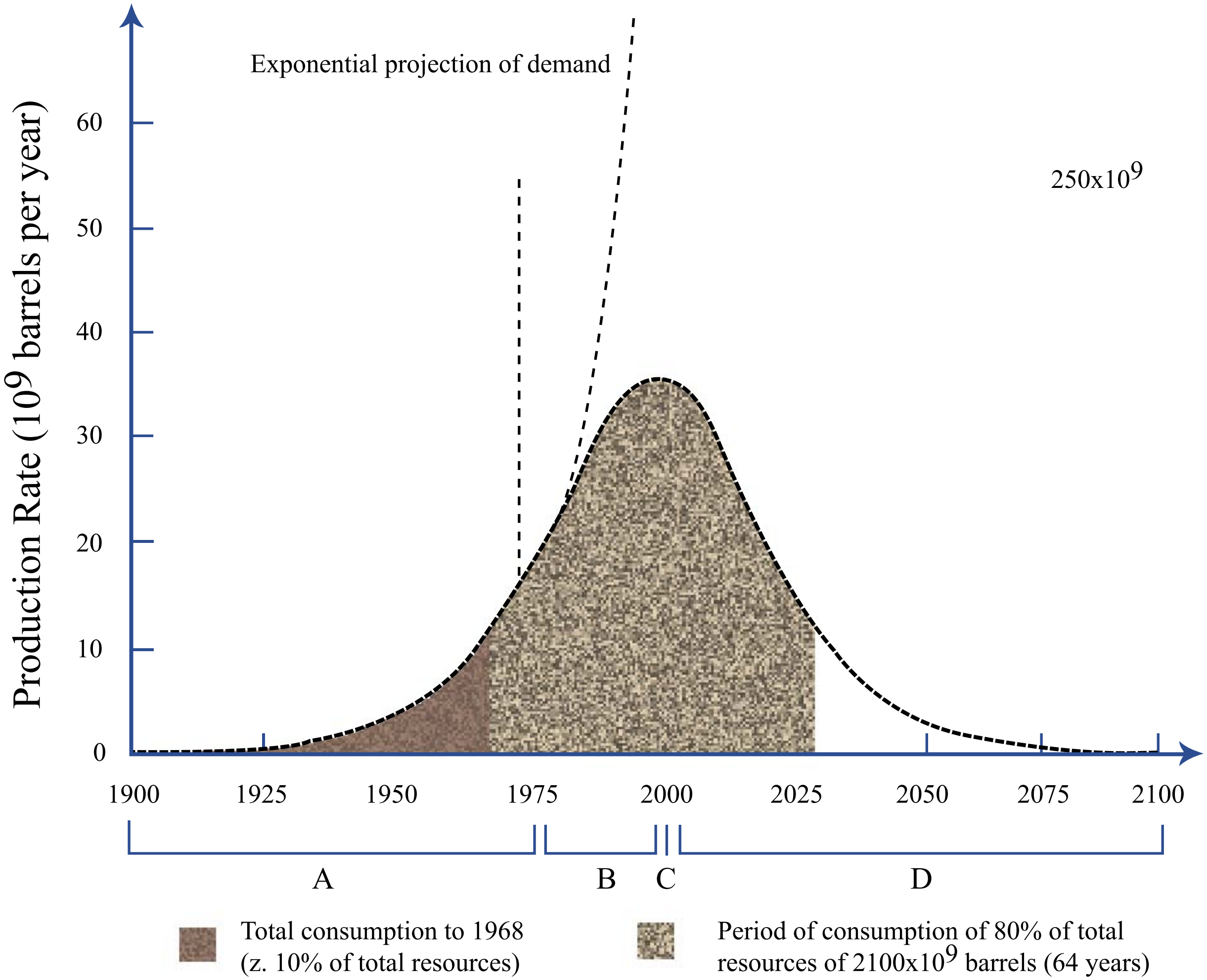
U.S. CRUDE OIL PRODUCTION

Courtesy of U.S. DOE.



Comparison of estimated (Hubbert) production curve and actual production (solid line).

COMPLETE CYCLE OF WORLD CRUDE-OIL PRODUCTION



A: Exponentially growing demands are supplied.

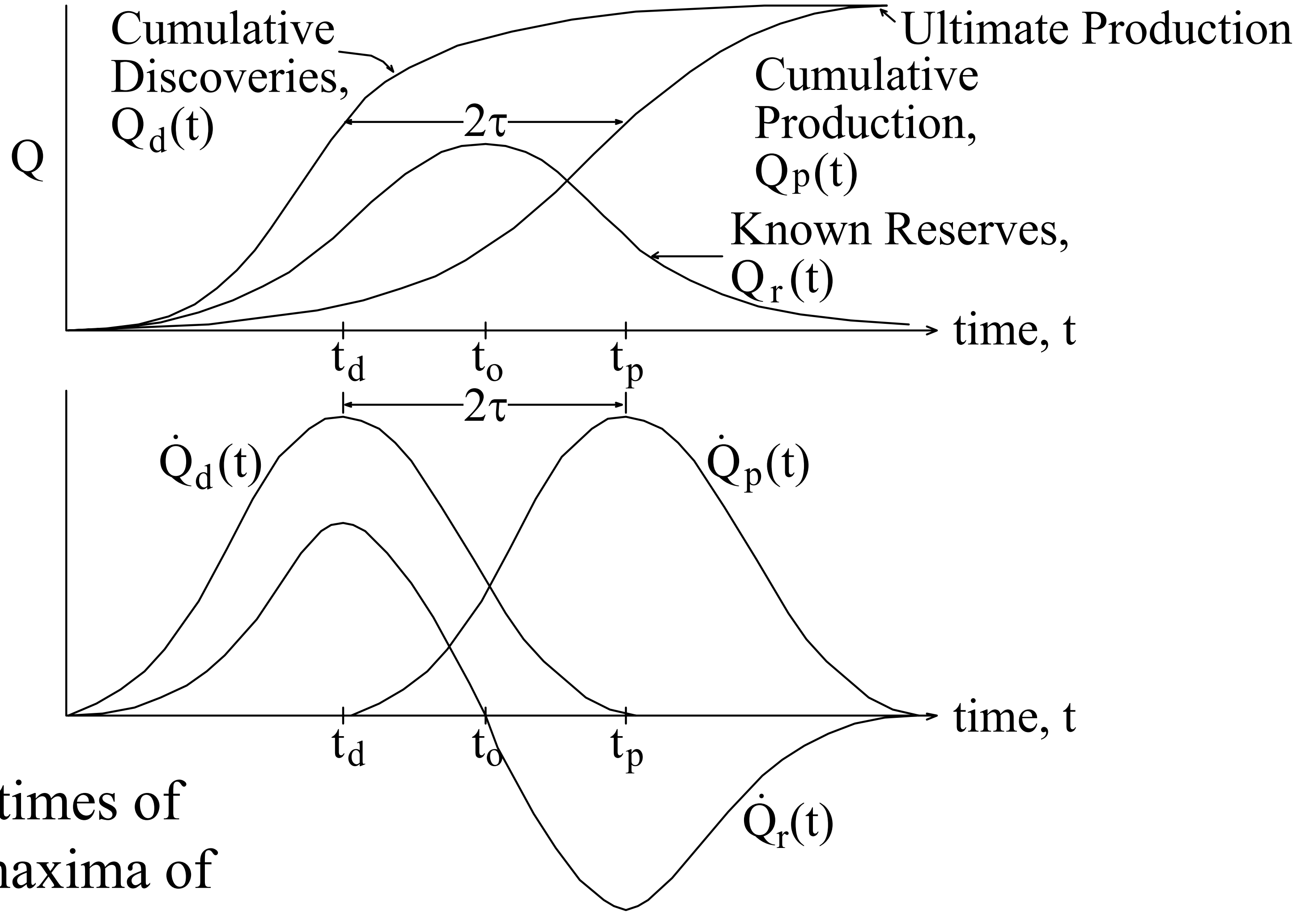
B: Discovery and extraction become increasingly difficult; production is unable to meet demands.

C: Maximum attainable production rate

D: Declining production, ultimately decaying exponentially

Figure by MIT OCW.

RESOURCE BEHAVIOR UNDER "HUBBERT" ASSUMPTIONS



Timing:

t_d, t_o, t_p are times of respective maxima of Q_d, Q_r, Q_p.

EQUATIONS

Conservation of Resource:

$$Q_d(t) = Q_r(t) + Q_p(t) \qquad \text{(Eq. 4)}$$

Rate Conservation:

$$\dot{Q}_d(t) = \dot{Q}_r(t) + \dot{Q}_p(t) \qquad \text{(Eq. 5)}$$

Approximate Results:

$$t\left(\dot{Q}_d = 0\right) - t\left(\dot{Q}_r = 0\right) = 2\tau \qquad \text{(Eq. 6)}$$

$$\tau \approx \begin{cases} \left(t_o - t_p\right) \\ \left(t_d - t_o\right) \end{cases} \qquad \text{(Eq. 7)}$$

or

$$t_o \approx \frac{1}{2}\left(t_d + t_p\right) \qquad \text{(Eq. 8)}$$

$$Q_{p_{ultimate}} \approx 2\,Q_d\left(t_d\right) \qquad \text{(Eq. 9)}$$

EQUATIONS, Continued

If we assume Gaussian distributions for $Q_r(t), \dot{Q}_d(t)$ and $\dot{Q}_p(t)$, with each having the same standard deviation, σ, obtain

$$Q_r(t) = \frac{Q_{r_o}}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{t - t_o}{\sigma}\right)^2\right] \qquad \text{(Eq. 10)}$$

$$\dot{Q}_d(t) = \frac{Q_{d_o}}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{t - t_d}{\sigma}\right)^2\right] \qquad \text{(Eq. 11)}$$

$$\dot{Q}_p(t) = \frac{Q_{p_o}}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{t - t_p}{\sigma}\right)^2\right] \qquad \text{(Eq. 12)}$$

Then, when Q_r is at a maximum $t = t_o$ and $\dot{Q}_r = 0$, or

$$\ddot{Q}_r(t_o) = \frac{Q_{r_o}}{\sigma^2} \Rightarrow \boxed{\sigma^2 = \frac{Q_r(t_o)}{\ddot{Q}_r(t_o)}} \qquad \text{(Eq. 13)}$$

EQUATIONS, Continued

When $\dot{Q}_d$ is at a maximum, $t = t_d$, and

$$\ddot{Q}_d(t_d) = 0 = \dot{Q}_r(t_d) + \dot{Q}_p(t_d)$$

$$\Rightarrow \tau \approx \sigma^2 \left(\frac{\dot{Q}_{p_o}}{Q_{r_o}} \right) e^{-(3/2)(\tau/\sigma)^2} \qquad \text{(Eq. 14)}$$

Example: US Petroleum Production

$\tau \approx 6$ years

$\sigma \approx 12$ years

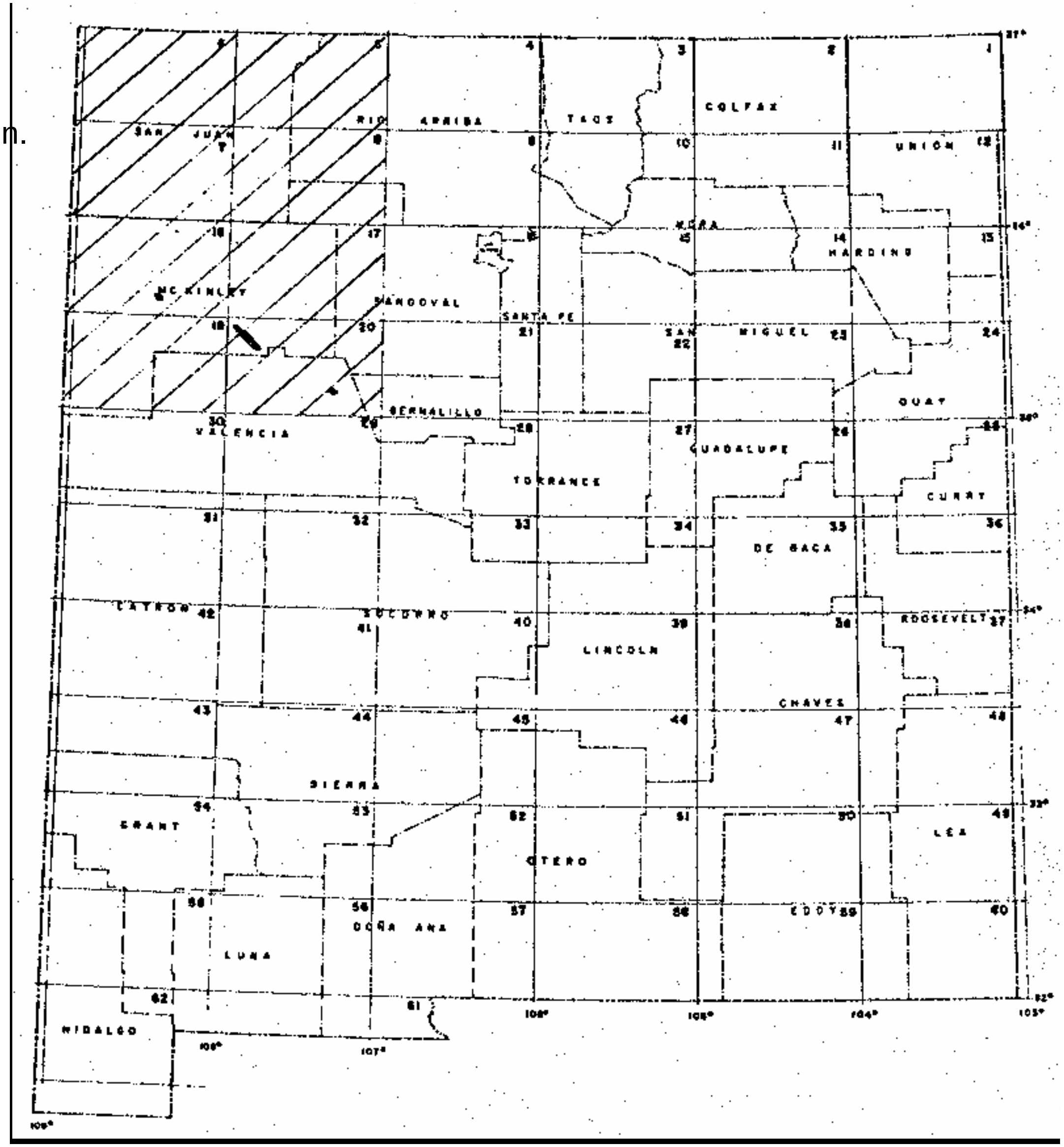
$Q_{r_o} \approx 35$ billion bbl

$\dot{Q}_{p_o} \approx 12$ million bbl/day

$t_{\text{ultimate production}} \approx 150$ years

SUBJECTIVE PROBABILITY STUDY – STATE OF NEW MEXICO

Courtesy of U.S. Atomic Energy Commission.



NEW MEXICO SUBJECTIVE PROBABILITY STUDY (AFTER DELPHI)

Courtesy of U.S. Atomic Energy Commission.