Problem Set #1

Course 14.454 – Macro IV

Distributed: October 26, 2004

Due: Tuesday, November 9, 2004 [in class]

1. **Imperfect Monitoring in Labor Markets**

Consider an economy similar a la Shapiro and Stiglitz with a continuum of measure 1 of infinitely lived workers with linear preferences. Time is discrete. The workers' discount factor is $\beta = 1/(1+r)$. Denote the wage by w_t. Each period a worker can decide whether to work or shirk. In the first case, he receives the wage w_t, spends effort e, and is fired if the job is exogenously terminated, which happens with probability b at the end of period t. In the second case, he receives the wage w_t but with probability q he is caught shirking at the end of the period t and is fired. When unemployed, the worker gets a zero utility flow, and he is hired with probability a_{t+1} at the beginning of next period. This probability is determined endogenously. There is a continuum of measure 1 of plots of land on which a firm can open a "shop". Each shop is identical and produces $y > e$ units of consumption good using one unit of labor and one unit of land.

(a) Derive expressions for the expected present value at time t of being unemployed (V_t^U), employed shirking (V_t^S), and employed not-shirking (V_t^{NS}) in terms of the parameters and the expected present values at time $t+1$.

(b) Introduce the no shirking condition as an equality $V_t^{NS} = V_t^S$ and use it to solve for the employment premium $V_t^{NS} - V_t^U$ as a function of e, r, and q.

(c) Combine the expressions for V_t^U and V_t^{NS} so as to find an expression for the wage w_t as a function only of the parameters and of the time varying probability a_{t+1}. How does an increase in a_{t+1} affect w_t? Explain briefly.

(d) Write the flow equation that links L_t and L_{t+1}, and use it to find a_{t+1}. *[Hint: Use the following assumption about timing: L_t is the number of shops active at time t. At the end of the period t, a proportion b of the existing shops are closed, the workers that were in these shops stay in the unemployed pool for one period, and can be hired at the beginning of period $t+2$. The plots of land freed upon termination are rented by new firms opening shops at the beginning of period $t+1$, which hire only workers who were in the pool of unemployed during time t.]*

(e) Derive the labor demand assuming that the market for land plots is competitive. Use your answers in (c) and (d) to derive the steady state level of employment. Explain how each of the parameters q, b, e, and y affects steady state employment.

(f) Prove that the steady state level of employment is less than full employment. Would unemployed workers in this economy be willing to work for less than the going wage? Why doesn't the wage fall to accommodate the unemployed?

(g) Suppose the government implements a welfare system such that the unemployed receive a payment of f, and the government finances this with a tax, τ, on output such that a firm's output is now only $(1-\tau)y$. How will this policy affect the steady state level of employment? *[Do not resolve the problem… just think through the math and give an intuitive answer].*

2. **Labor Markets and Appropriation Problems**

Consider an economy as in Caballero and Hammour's "Fundamental Transformation" paper. Suppose that there is a continuum of measure 1 supply of labor and capital. Labor in autarky sector U (unemployment/home production) has a production function: $Y_H = 2U - U^2$

Alternatively, there is the joint-production sector where one unit of labor and one unit of capital will produce $y=3$ units of the consumption good. Denote L as the amount of labor in the joint-production sector. Therefore, market clearing in the labor market requires: $U + L = 1$

There are pre-existing production units that employ $L_0 = 1/2$ of the labor force (capital used in these units can't be recycled, so just assume that it is not counted in the current unit supply). The productivity of these units is denoted by x and is distributed uniformly between 1 and 3. Denote L_1 as the labor in newly formed joint-production units and L_2 as labor in pre-existing joint-production units that are not scrapped. Thus, $L_1 + L_2 = L$.

The timing is as follows: First, existing units decide whether to separate or not, D units are destroyed and $L_0 - D$ workers remain in old joint-production units.[1] Then, separated workers (D) and already unemployed workers $(1-L_0)$ look for jobs in the L_1 new units of joint-production. Finally, the workers that do not find a job in the new joint-production units go into home production and capital in the autarky sector (think of this as investment abroad) has a constant return of $w_k = 1$.

(a) Derive the efficient allocation of labor and capital in each sector, efficient destruction margin, $\hat{x}$, for existing joint-production units, and total production of the economy. *[Hint: Setup an optimization problem to maximize the sum of total production in the old joint-production units, new joint-production units, home production, and the return on investment abroad. Then, derive the optimal destruction margin* $\hat{x}$*, optimal creation* L_1*, and optimal unemployment* U *].*

(b) Consider now the economy with appropriability problems as described in Caballero & Hammour. Write the ex-ante participation constraint for capital. Derive the wage in the autarky sector and the wage in the joint-production sector. Derive the probability of finding a job in the joint-production sector as a function of the cutoff productivity for pre-existing units $\hat{x}$. Derive the cutoff productivity for pre-existing units, equilibrium L_1, unemployment U and total production.

(c) Compare your answers from (a) and (b) in terms of the levels of production, creation and destruction. Explain your results.

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[1] Hint: You can easily show that there will be a cutoff rule $\hat{x}$ such that $D = L_0 \int_1^{\hat{x}} \frac{1}{3-1} dx$

(d) Suppose the government wishes to correct the appropriability problem and return to the first best outcome of (a). To do this, it tries to increase creation in the economy by providing each new joint production unit a lump-sum subsidy, $\tau = 1$. Determine the new equilibrium using the same approach as in (b). Has creation returned to its first-best level? What about the level of destruction and output?

(e) Now suppose the government decides to also decrease destruction in the economy by taxing workers who willingly leave old joint-production units by one unit of consumption. What is the new equilibrium of the economy? Why would the government want to increase creation while simultaneously reducing destruction?

3. **A Simple Labor Market Search Model**

Assume that the labor market is described by the following model. Population is normalized to 1. The unemployment rate is u, v the vacancy rate, and w the wage. Let x be the output net of capital costs that is produced by a match between a worker and a job. Workers separate from jobs at the exogenous rate s; they are hired at rate $h = h(u,v)$, where h is CRS. The following equations describe the economy.

$$\overset{\circ}{u} = s(1-u) - h \tag{1.1}$$

$$h = h(u,v) \tag{1.2}$$

$$w = w\left(\frac{u}{v}\right) \tag{1.3}$$

$$\overset{\circ}{v} = g(x-w) \tag{1.4}$$

Assume $g(0) = 0$.

(a) Give some intuition for each of the above equations. For equation (1.1) you should explain where each term comes from, and why it differs from Blanchard-Diamond's equation (6). For equation (1.2) you should discuss the values and signs of $h_u,\ h_v,\ h(0,v),$ and $h(u,0)$. For equation (1.3), you should discuss a plausible assumption about the sign of $w'(\ )$. For equation (1.4), discuss a plausible sign for $g'(\ )$ and the assumption that $g(0) = 0$.

(b) In (u,v) space, show the $\overset{\circ}{u} = 0$ and $\overset{\circ}{v} = 0$ curves and indicate the directions of movement around these curves.

(c) Suppose that there is a reduction in the production technology. Show what happens in both the short-run and the long-run. Explain in words.

(d) Assume $h = m\sqrt{uv}$. What happens if m increases? Show both in diagrams & words.

(e) Look at the below graph[2] of unemployment and vacancy rates for Australia, 1966-1999. With the above model in mind, what kinds of shocks might explain it?

[2] From Groenewold, Nicholaas. "Long-Run Shifts of the Beveridge Curve and the Frictional Unemployment Rate in Australia" http://www.econs.ecel.uwa.edu.au/economics/Research/2001/DW%2001.09.pdf


Figure 1: Unemployment and Vacancy Rates, Australia, 1966:Q3-1999:Q1
Vacancy Rate
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1.6
3.2
4.8
6.4
8.0
9.6
11.2
Unemploment Rate