

Exam 14.454

Fall 2004

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Question #1 -- Simple Labor Market Search Model (20 pts)

Assume that the labor market is described by the following model. Population is normalized to 1. The unemployment rate is u , v the vacancy rate, and w the wage. Let x be the output net of capital costs that is produced by a match between a worker and a job. Workers separate from jobs at the exogenous rate s ; they are hired at rate $h = h(u, v)$, where h is CRS. The following equations describe the economy.

$$\dot{u} = s(1-u) - h \quad (1)$$

$$h = h(u, v) \quad (2)$$

$$w = w\left(\frac{u}{v}\right) \quad (3)$$

$$\dot{v} = g(x - w) \quad (4)$$

Assume $g(0) = 0$.

- Give some intuition for each of the above equations. For equation (2) you should discuss the values and signs of h_u , h_v , $h(0, v)$, and $h(u, 0)$. For equation (3), you should discuss a plausible assumption about the sign of $w'(\cdot)$. For equation (4), discuss a plausible sign for $g'(\cdot)$ and the assumption that $g(0) = 0$.
- In (u, v) space, show the $\dot{u} = 0$ and $\dot{v} = 0$ curves and indicate the directions of movement around these curves.
- Suppose that there is a reduction in the production technology. Show what happens in both the short-run and the long-run. Explain in words.
- Assume $h = m\sqrt{uv}$. What happens if m increases? Show both in diagrams & words.
- Look at the below graph of unemployment and vacancy rates for Australia, 1966-1999. With the above model in mind, what kinds of shocks might explain it?



Question #2 -- Banks and Bank Runs (30 pts)

Assume there is a continuum 1 of individuals that are each endowed with one unit of currency. There are three time periods, $t = 0, 1, 2$. At $t = 0$, individuals have two options with regards to how they can invest their money. They can either stuff it in their mattress, where it gets a return equal to 1, or they can invest it in a long-term project that yields a return $R = 4$ in period two. For example, in individual that invests an amount I will receive $4I$ in period two, and have $1 - I$ stuffed under the mattress. However, individuals always have the option of withdrawing their money from the long-term project early in period one at a penalty. If they withdraw early, they only receive a return $L = 1/4$ in period 1, rather than the return $R = 4$ in period 2.

At time $t = 1$, a fraction $\pi = 1/2$ of the individuals receive a liquidity shock. These individuals are “impatient” and only value consumption in period one. The fraction $1 - \pi$ individuals that do not receive a liquidity shock are “patient” and only value consumption in period two. At time $t = 0$, each individual has an equal chance of being hit by the liquidity shock. Assume that individuals do not discount the future, so that their ex-ante expected utility is given by, $U = \pi u(c_1) + (1 - \pi)u(c_2)$, where c_1 and c_2 is the consumption period 1 and 2 respectively, and $u(c) = -1/c$.

- (a) Assume there are no markets available to individuals, so that individuals must simply invest on their own. Given that the individual has invested an amount I at time $t = 0$, what will be the optimal levels of consumption, c_1 , c_2 , if:
 - i. the individual receives a liquidity shock (i.e. is impatient)
 - ii. the individual does not receive a liquidity shock (i.e. is patient)
- (b) What is the optimal level of investment, I^* ? Given I^* , what is the ex-ante expected utility of an individual? Explain in 1-2 sentences why both patient and impatient individuals regret their initial investment decision ex-post in period 1 after their type is realized.
- (c) How would the introduction of an ex-post financial market improve the welfare of individuals in this economy? No math, just explain in 1-2 sentences.
- (d) Now suppose that when types are revealed in period 1, this information is publicly observable. Suppose there exists a social planner that individual's entrust all of their endowment to at time 0. The social planner will pay impatient individuals c_1^* in period 1 and patient individuals c_2^* in period 2.
 - i. Solving the social planner's problem, what is c_1^* and c_2^* ?
 - ii. How much does the social planner invest? (i.e. what is I^*)
 - iii. What is an individual's ex-ante expected utility now?
 - iv. Why is the social planner able to improve the individual's ex-ante utility?
- (e) Now suppose an agent's type is private information, and the social planner can only offer a contract contingent on an individual's announcement of his or her type at time 1. (i.e. she cannot condition the contract on other agents' announcements). Furthermore, at time 1, she meets each agent once with the meeting order randomly determined. If individual's report honestly, can the social planner offer the same contract as in part (d)? Is it optimal for an individual to

report honestly when everyone else does? Explain in 1-2 sentences how this planner can be interpreted as a bank.

- (f) Suppose all agents fear a bank run, and each agent reports to the bank at time 1 as being impatient. How many individuals will get paid by the bank before it runs out of money in period 1? Given this, explain why this bank run can be an equilibrium... i.e. why is it optimal for a “patient” individual to run on the bank when he/she expects a bank run?
- (g) Suppose the bank implements a policy of only paying the first π individuals that show up at time 1, and the rest will get paid at time 2. (i.e. it suspends convertibility). Will this eliminate the bank run as an equilibrium?

Question #3 -- Stock Prices, Dividends and Bubbles (20pts)

Assume you are in an economy where the stock price, p_t , is given by the standard arbitrage equation (5) and the process for dividends at time t , d_t , is given by equation (6) below:

$$p_t = \frac{1}{1+r} E_t[p_{t+1}] + \frac{1}{1+r} E_t[d_{t+1}] \quad (5)$$

$$d_t - \bar{d} = \rho(d_{t-1} - \bar{d}) + \varepsilon_t, \quad \varepsilon_t \text{ i.i.d. and } E_{t-1}[\varepsilon_t] = 0 \quad (6)$$

- (a) Use iterated expectations to solve for the price, p_t , as a function of ONLY future expected dividends. What assumption do you implicitly need to do this?
- (b) Assume that $\rho < 1/(1+r)$. Use iterated expectations to find an expression for the expectation (as of time t) for dividends at time $t+i$, $E_t[d_{t+i}]$, that is a function of only \bar{d} , ρ , and d_t .
- (c) Use your answers from (a) and (b) to find an expression for p_t , as a function of \bar{d} , ρ , and d_t . Call this solution to the arbitrage equation the fundamental price, p_t^* .
- (d) Now assume the price of the stock has a bubble component, b_t , where $b_t = (1+r)^t b_0$ and $b_0 > 0$. Prove that the price $p_t = p_t^* + b_t$ is also a solution to the arbitrage condition (5) and that our assumption from part (a) is no longer necessary.
- (e) Why are individuals willing to pay a higher price, p_t , for the stock than the fundamental price corresponding to the present value of the dividends, p_t^* ?