# 6.972 Game Theory & Mechanism Design

Tuesday, February 01, 2005
1:15 PM

Goal: Deep understanding of fundamental issues in GT, MD with applications in engineering problems with applications in engineering problems while presenting open research problems

Study of multi-person decision problems

Course structure

1. **Static Games of complete information (35%)**
    - Both Matrix games and cont. games
    - Solution concepts, dominance (Nash Eq.), Rationalizability, Correl Eq.
    - Existence and Uniqueness (super-modular potential)
    - Computation of Eq.
    - Learning (Myopic, Bayesian)

2. **Dynamic Games of complete information (20%)**
    a. Sub-game perfect eq.
    b. Simple bargaining models
    c. Nash's bargaining solution
    d. Repeated games
        i. Folk Theorems
        ii. Multiplayer DP

3. **Static Games with incomplete information (15%)**
    a. Bayesian Nash Eq.
    b. Simple auctions
    c. Optimal Auctions
    d. Optimal Mechanisms

4. **Mechanism Design (15%)**
    a. Efficient Mechanisms
    b. Dominant strategy implementations
    c. Nash Implementations

5. **Network Games (15%)**
    a. Utility based resource allocation
    b. Selfish network routing
    c. Network anarchy
    d. Eq. concepts (Wardrop eq.)
    e. Pricing/Price of anarchy/stability

Main text: Game Theory by Fudenberg & Tirle MIT Press 1991

**Pigou's example (1920)**

J Users ->
L1 (gamma 1)
-> li (.) specificies delay on each link dependant on level of congestion
2/J units of flow
L2 (gamma-2)

a. Maximize total delay encountered by flows of users

Min l1 (gamma 1) gamma 1 + l2 (gamma 2) gamma 2
Xi >= 0

s.t. gamma 1= sigma []i=1 to j] Xi

Solvable using Lagrange multipliers

L1 (gamma 1) + L1' (gamma1) (gamma1) = Lambda 1
L2 (gamma 2) + (L2)' (gamma 2) (gamma 2 = Lambda 2

Lambda 1 = Lambda 2 = 2

=> Gamma 1 = 1
Gamma 2 = 1

Assumptions

- Half-up, half-down
- Centralized problem
- All users are obedient
- Everyone is symmetric

b. Selfish Users
    a. Choosing path of minimum delay

epsilon
2 units of traffic ---->
------>
(2-epsilon)

    b. => all traffic takes top link
    c. System optimum or social optimum

C1 (gamma) = 1.1. + 2.1 = 3
C2 (gamma) = 2.2 = 4
C1 (gamma)/C2 (gamma) = 3/4 ("magic #")

There are other resource allocation mechanisms where the bound is again 3/4. hence magic #

There is some form of game theory interaction

c. Large Users

L1 (Gamma) = 3/2 gamma

2 Users (1 Unit of flow each)
L2 (gamma) = 2

Represeent possible actions & payoffs (delays by a matrix)

| P2/<br>P1 | U | D |
|---|---|---|
| U | 3,3 | 3/2,2 |
| D | 2, 3/2 | 2,2 |

No incentive to deviate unilaterally

d. Induce them to choose actions that will yield centralized solution
e. => "price"

P1 = (L1)' (gamma 1) Gamma 1 = Gamma 1
P2 = (L2)' (Gamma 2) Gamma 2= 0

Choose now the link with smallest "effective cost" L1 (gamma) = P'

Tif in the eq gamma1 > 0 and gamma 2 > 0 then

L1 (gamma 1) + P1 = L2/Gamma 2 + p2
L1 (Gamma 1) = (L1)' (Gamma1) Gamma 1 = L2 (Gamma 2) + (L2)' (agmma 2) Gamma 2

Marginal congestion cost

2. Rate Control Problem

People have different service req.s

J/2 Type-1 users
L9gamma) = gamma Type 1 U(x) = 3x x belongs {0, 1/j, 3/j}

J/2 Type 2 users

a. Choose rates to maximize (total utility - total delay)
b. DVs = Xj J = 1... J/2
c. Xj' = J = 1..J/2

Max sigma (J= 1 ..J/2) U1 (Xj)+ sigma (j=1..J/2) U (X'j) - L(gamma) Gamma
s.t. gamma = sigma (J=1..J/2) Xj + sigma (j = 1..J/2) Xj'

Will people tell the truth:

1 -> will
2 -> S = I payoff 5/J - 2/J * 3/2 = 2/J

J = II
3/J - 1/J * 3/2 = 3/2/J

Exercise: Show that if you charge saying type 1 1/J units