

6.972 Game Theory & Mechanism Design

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1:15 PM

Goal: Deep understanding of fundamental issues in GT, MD with applications in engineering problems with applications in engineering problems while presenting open research problems

Study of multi-person decision problems

Course structure

1. Static Games of complete information (35%)

- Both Matrix games and cont. games
- Solution concepts, dominance (Nash Eq.), Rationalizability, Correl Eq.
- Existence and Uniqueness (super-modular potential)
- Computation of Eq.
- Learning (Myopic, Bayesian)

2. Dynamic Games of complete information (20%)

- a. Sub-game perfect eq.
- b. Simple bargaining models
- c. Nash's bargaining solution
- d. Repeated games
 - i. Folk Theorems
 - ii. Multiplayer DP

3. Static Games with incomplete information (15%)

- a. Bayesian Nash Eq.
- b. Simple auctions
- c. Optimal Auctions
- d. Optimal Mechanisms

4. Mechanism Design (15%)

- a. Efficient Mechanisms
- b. Dominant strategy implementations
- c. Nash Implementations

5. Network Games (15%)

- a. Utility based resource allocation
- b. Selfish network routing
- c. Network anarchy
- d. Eq. concepts (Wardrop eq.)
- e. Pricing/Price of anarchy/stability

Main text: Game Theory by Fudenberg & Tirle MIT Press 1991

Pigou's example (1920)

J Users ->

L1 (gamma 1)

-> $l_i(\cdot)$ specifies delay on each link dependant on level of congestion

2/J units of flow

L2 (gamma-2)

- a. Maximize total delay encountered by flows of users

Min $l_1(\gamma_1) \gamma_1 + l_2(\gamma_2) \gamma_2$

$x_i \geq 0$

s.t. $\gamma_1 = \sum [x_i]$

Solvable using Lagrange multipliers

$$L_1(\gamma_1) + L_1'(\gamma_1)(\gamma_1) = \lambda_1$$

$$L_2(\gamma_2) + (L_2)'(\gamma_2)(\gamma_2) = \lambda_2$$

$$\lambda_1 = \lambda_2 = 2$$

$$\Rightarrow \gamma_1 = 1$$

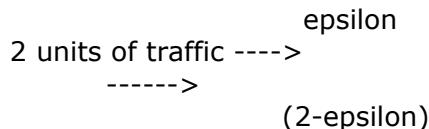
$$\gamma_2 = 1$$

Assumptions

- o Half-up, half-down
- o Centralized problem
- o All users are obedient
- o Everyone is symmetric

- b. Selfish Users

- a. Choosing path of minimum delay



- b. => all traffic takes top link

- c. System optimum or social optimum

$$C_1(\gamma) = 1.1 + 2.1 = 3$$

$$C_2(\gamma) = 2.2 = 4$$

$$C_1(\gamma)/C_2(\gamma) = 3/4 \text{ ("magic #")}$$

There are other resource allocation mechanisms where the bound is again 3/4. hence magic #

There is some form of game theory interaction

c. Large Users

$$L_1(\Gamma) = \frac{3}{2}\gamma$$

2 Users (1 Unit of flow each)
 $L_2(\gamma) = 2$

Represent possible actions & payoffs (delays by a matrix)

P2/	
	U
P1	D
U	
3,3	3/2,2
D	
2, 3/2	2,2

No incentive to deviate unilaterally

- d. Induce them to choose actions that will yield centralized solution
- e. => "price"

$$P_1 = (L_1)'(\Gamma_1) \Gamma_1 = \Gamma_1$$

$$P_2 = (L_2)'(\Gamma_2) \Gamma_2 = 0$$

Choose now the link with smallest "effective cost" $L_1(\gamma) = P'$

If in the eq $\gamma_1 > 0$ and $\gamma_2 > 0$ then

$$L_1(\gamma_1) + P_1 = L_2/\Gamma_2 + P_2$$

$$L_1(\gamma_1) = (L_1)'(\Gamma_1) \Gamma_1 = L_2(\Gamma_2) + (L_2)'(\Gamma_2) \Gamma_2$$

Marginal congestion cost

2. Rate Control Problem

People have different service req.s

J/2 Type-1 users

$$L_9(\gamma) = \gamma \text{ Type 1 } U(x) = 3x \quad x \in \{0, 1/j, 3/j\}$$

J/2 Type 2 users

- a. Choose rates to maximize (total utility - total delay)
- b. DVs = X_j $j = 1 \dots J/2$
- c. $X'_j = j = 1 \dots J/2$

Max $\sigma (j=1 \dots J/2) U_1 (X_j) + \sigma (j=1 \dots J/2) U (X'_j) - L(\gamma) \Gamma$
s.t. $\gamma = \sigma (j=1 \dots J/2) X_j + \sigma (j=1 \dots J/2) X'_j$

Will people tell the truth:

1 -> will

2 -> $S = I$ payoff $5/J - 2/J * 3/2 = 2/J$

$J = II$

$3/J - 1/J * 3/2 = 3/2/J$

Exercise: Show that if you charge saying type 1 $1/J$ units

