6.972: Game Theory	May 3, 2005

Lecture 22: Mechanism Design

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1 Agenda

- VCG Mechanisms and Examples
- Budget Balancedness
- d'AGV Mechanism
- Market Equilibrium - Debreu's Method

2 An Example - Shortest Path by Nisan & Ronen 99

The Problem: Given -
- A direct graph G which is biconnected
- Edges e with costs c^e which is the cost for sending a single message. This is private information.
- Two nodes X (origin) and Y (destination)

The Goal: To fing the least costly path from X to Y.

The Formulation:
- MD Framework
- Edges: Strategic Agents

Outputs: All paths from X to Y such that $p \in P$.

$$V_e(P, c^e) = -c_e \text{ if } e \in p$$
$$\emptyset \text{ otherwise}$$

Finding the LCP from X to Y $\iff$ choosing p over all P to maximize $\sum_e V_e(P, c_e)$
$\rightarrow$ Consider the VCG mechanism. Figure out the payments to the agents so that truth-telling is a dominant strategy. Recall Clarke payments:
$t_e = [\sum_{j\neq i} V_j(x^*(\theta), \theta_j)] - \sum_{j\neq i}[V_j(x^*_{-i}(\theta), \theta_j)]$

Denote $d_{G|_{c_e=k}}$ as the cost of the LCP when $c_e = K$. Thus,

$$t_e = \emptyset \text{ if } e \text{ not on the LCP}$$
$$d_{G|_{c_e=\infty}} - d_{G|_{c_e=0}} \text{ if } e \text{ is on the LCP}$$

3 Payments with examples

3.1 Feigenbaum et al. (2002)

Nodes are strategic agents

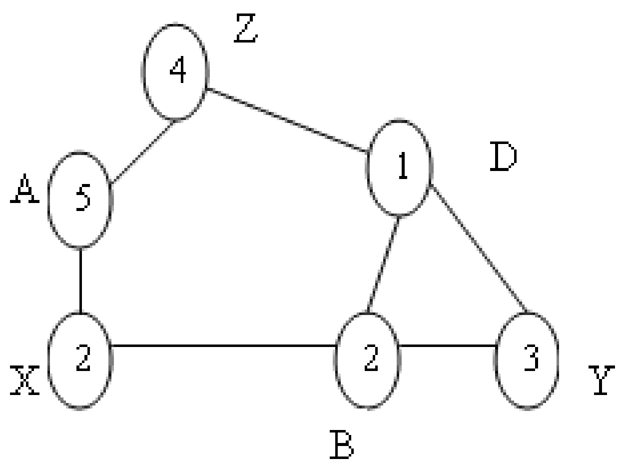

Figure 1: Finding LCP from X to Y

In Figure 1, consider the LCP between X and Y : $XBDZ$

$$t^D = 5 - 2 = 3$$
$$t^B = 5 - 1 = 4$$

Therefore the cost to induce truth telling is $3 + 4 = 7$.

Consider the situation where we have a path p with length L and a path q with length $L(1+\epsilon)$ where $\epsilon > 0$ and the number of edges on the paths are K. Both paths are from X to Y

Payments to edges on P:

Payment to edge e is $L(1+\epsilon) - L + c_e = c_e + L\epsilon$.

Total Payment: $\sum_e (c_e + L\epsilon) = L(1 + \epsilon K)$

3.2 Groves Mechanism

If $t_i(\theta) = [\sum_{j \neq i} V_j(X^*(\theta), \theta_j)] + h_i(\theta_{-i})$, then efficient allocation is truthfully implementable in dominant strategies.

Is this the only social choice function that achieves an efficient allocation in a strategy proof way?

Recall, $f(.) : (X(.), t(.))$ in a quasi-linear environment.

The answer is YES!

Proposition: [Green & Laffont] *Under some conditions (set of possible types to be sufficiently rich), a social choice function (scf) $f(.) = (X^*(.), t(.))$ with an efficient allocation is truthfully implemented in dominant strategies only if $t_i(.)$ is given by the Groves payments.*

Proof: Note that $\forall\theta$, we can write $t_i(\theta_i, \theta_{-i}) : \sum_{j\neq i} V_j(X^*(\theta_i, \theta_{-i}), \theta_j) + h_i(\theta_i, \theta_{-i})$
Show that if $f(.)$ is strategy proof $\rightarrow h_i(.)$ must be independent of θ_i. Let us suppose it is not i.e., Suppose: $\exists\theta_i, \hat{\theta}_i, \theta_{-i}$ such that, $h_i(\theta_i, \theta_{-i}) \neq h_i(\hat{\theta}_i, \theta_{-i})$

We therefore have two cases -
Case 1:
$X^*(\theta_i, \theta_{-i}) = X^*(\hat{\theta}_i, \theta_{-i})$
By IC in dominant strategies,

$$\begin{aligned} V_i(X^*(\theta_i, \theta_{-i}), \theta_i) + t_i(\theta_i, \theta{-i}) &\geq V_i(X^*(\hat{\theta}_i, \theta_{-i}), \theta_i) + t_i(\hat{\theta}_i, \theta{-i}) \\ V_i(X^*(\hat{\theta}_i, \theta_{-i}), \hat{\theta}_i) + t_i(\hat{\theta}_i, \theta{-i}) &\geq V_i(X^*(\theta_i, \theta_{-i}), \hat{\theta}_i) + t_i(\theta_i, \theta_{-i}) \end{aligned}$$

$\rightarrow t_i(\theta_i, \theta_{-i}) = t_i(\hat{\theta}_i, \theta_{-i}) = h_i(\theta_i, \theta_{-i}) = h_i(\hat{\theta}_i, \theta_{-i}) \rightarrow$ Contradiction!
Case 2:
Suppose without loss of generality that $h_i(\theta_i, \theta_{-i}) > h_i(\hat{\theta}_i, \theta_{-i})$
Define type θ_i^ϵ for some $\epsilon > 0$ as:

$$\begin{aligned} V_i(x, \theta_i^\epsilon) &= -\sum_{j\neq i} V_j(X^*(\theta_i, \theta_{-i}), \theta_j) \text{ if } X = X^*(\theta_i, \theta_{-i}) \\ &\quad -\sum_{j\neq i} V_j(X^*(\hat{\theta}_i, \theta_{-i}), \theta_j) + \epsilon \text{ if } X = X^*(\theta_i, \theta_{-i}) \\ &\quad -\infty \text{ otherwise} \end{aligned}$$

Show for sufficiently small ϵ, type θ_i^ϵ we will report θ_i.
Note that: $X^*(\theta_i^\epsilon, \theta_{-i}) = X^*(\hat{\theta}_i, \theta_{-i})$ i.e., $X^*(\hat{\theta}_i, \theta{-i})$ maximizes $V_i(X, \theta_i^\epsilon) + \sum_{j\neq i} V_j(X, \theta_j)$
Then, $V_i(X^*(\hat{\theta}_i, \theta_{-i}), \theta_i^\epsilon) + t_i(\theta_i^\epsilon, \theta_{-i}) \geq V_i(X^*(\theta_i, \theta_{-i}), \theta_i^\epsilon) + t_i(\theta_i, \theta_{-i})$
Substituting $\epsilon + h_i(\theta_i^\epsilon, \theta_{-i}) \geq h_i(\theta_i, \theta_{-i})$.
But since, $X_i^*(\theta_i^\epsilon, \theta_{-i}) = X_i^*(\hat{\theta}_i, \theta_{-i}) \rightarrow h_i(\theta_i^\epsilon, \theta_{-i}) = h_i(\hat{\theta}_i, \theta_{-i})$
The above equation combined with $\epsilon + h_i(\theta_i, \theta_{-i}) \geq h_i(\theta_i, \theta_{-i})$ yields a contradiction for sufficiently small ϵ.

4 Budget-Balancing

$u_i(y, \theta_i) = V_i(X, \theta_i) + t_i(\theta_i) \rightarrow \sum_{i=i}^{I} t_i(\theta) = 0 \forall\theta$.
Proposition: [Green, Laffont & Hurwicz] *There is no social choice unction that is IC in dominant strategies, efficient and budget-balanced.*
But this is possible in a Bayesian implementation using d'AGV mechanism. However if we add *Participation Constraints*, then even a Bayesian Nash Equilibrium does not work. There are many new papers in this area.